ECE 302: Lecture A.2 Mean Function

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What is a random process?

A random process is a function indexed by a random key.

Mean function

The **mean function** $\mu_X(t)$ of a random process $X(t)$ is

$$\mu_X(t) = \mathbb{E}\left[X(t)\right].$$

More explicitly, the mean function is

$$\mu_X(t) = \mathbb{E}[X(t)] = \int_\Omega X(t,\xi)p(\xi)d\xi$$

Example 1

Example 1: Let $A \sim$Uniform$[0,1]$, and let $X(t) = A\cos(2\pi t)$, find $\mu_X(0)$, and $\mu_X(t)$.

$$\mu_X(0) = \mathbb{E}[X(0)] = \mathbb{E}[A\cos(0)] = \mathbb{E}[A] = \frac{1}{2}$$

$$\mu_X(t) = \mathbb{E}[X(t)] = \mathbb{E}[A\cos(2\pi t)] = \cos(2\pi t)\mathbb{E}[A] = \frac{1}{2}\cos(2\pi t).$$

Example 1

More formally, we can rewrite $X(t)$ as

$$X(t, \xi) = A(\xi) \cos(2\pi t).$$

Then, when we take the expectation, we are actually taking the expectation on A:

$$\begin{aligned}
\mu_X(t) &= \int_\Omega X(t, a) p_A(a) da \\
&= \int_0^1 a \cos(2\pi t) \cdot 1 \; da \\
&= \cos(2\pi t) \left[\frac{a^2}{2} \right]_0^1 = \frac{1}{2} \cos(2\pi t).
\end{aligned}$$

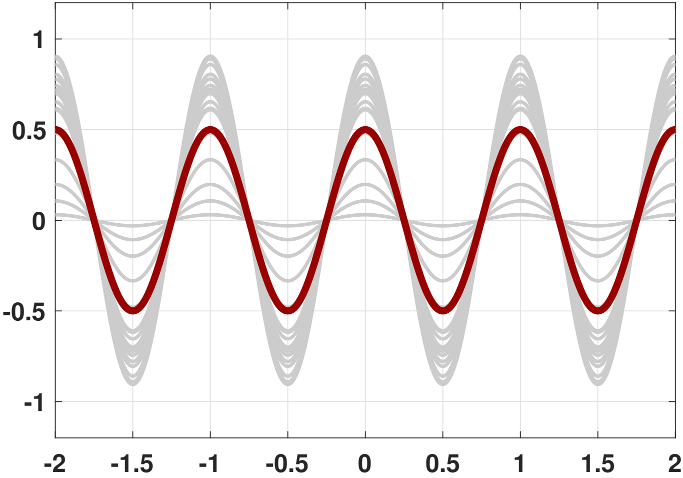

Figure: The mean function of $X(t) = A\cos(2\pi t)$.

Example 2

Example 2: Let $\Theta\sim$Uniform$[-\pi,\pi]$, and let $X(t)=\cos(\omega t+\Theta)$. Find $\mu_X(t)$.

$$\mu_X(t)=\mathbb{E}\left[\cos(\omega t+\Theta)\right]=\int_{-\pi}^{\pi}\cos(\omega t+\theta)\frac{1}{2\pi}d\theta=0$$

Example 2

More formally, write $X(t)$ as

$$X(t,\xi) = \cos(\omega t + \Theta(\xi)).$$

Then, the expectation is

$$\begin{aligned}\mu_X(t) &= \int_\Omega \cos(\omega t + \theta) p_\Theta(\theta) d\theta \\ &= \int_{-\pi}^{\pi} \cos(\omega t + \theta) \cdot \frac{1}{2\pi}\, d\theta \\ &= 0.\end{aligned}$$

Example 2

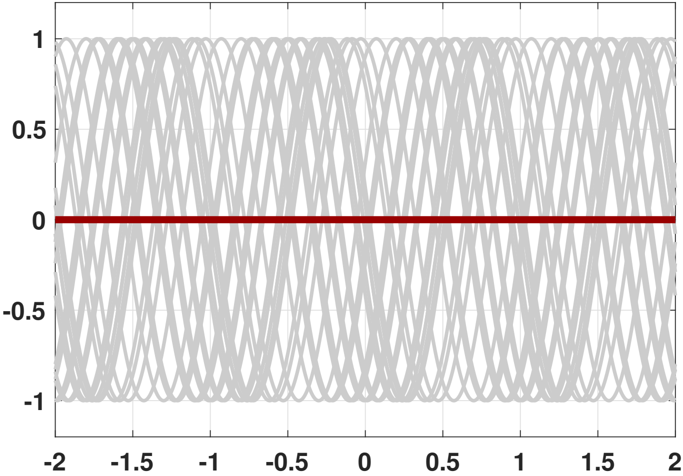

Figure: The mean function of $X(t) = \cos(\omega t + \Theta)$.

Example 3

Example 3: We can work on discrete-time random processes. Let $X[n] = S^n$, where $S \sim$Uniform$[0,1]$. Find $\mu_X[n]$.

$$\mu_X[n] = \mathbb{E}[s^n] = \int_0^1 s^n ds = \frac{1}{n+1}.$$

In this example, the randomness goes with the constant s. Thus, if we write $X[n]$ as

$$X[n,\xi] = [S(\xi)]^n,$$

then the expectation is

$$\mathbb{E}[X[n]] = \int_\Omega s^n p_S(s) ds = \int_0^1 s^n \cdot 1 ds = \frac{1}{n+1}.$$

Example 3

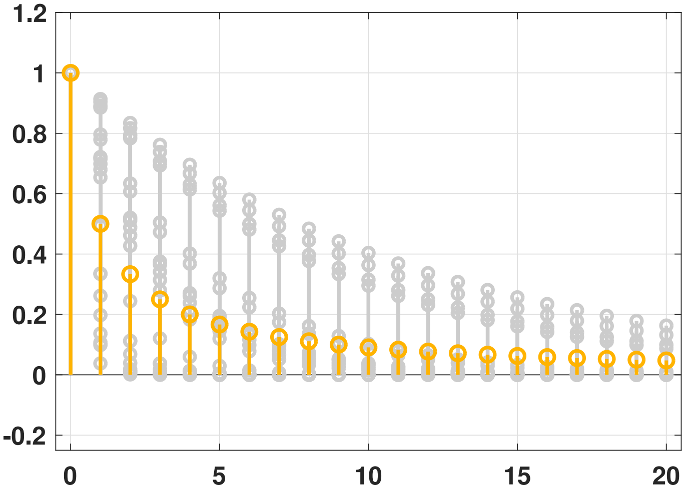

Figure: The mean function of $X[n] = S^n$, where $S \sim \text{Uniform}[0,1]$.

Questions?