ECE 302: Lecture 3.8 Geometric Random Variables

Prof Stanley Chan

School of Electrical and Computer Engineering
Purdue University

Outline

- 3.1 Random variables
- 3.2 Probability mass functions (PMF)
- 3.3 Cumulative distribution functions (discrete case)
- 3.4 Expectation
- 3.5 Moments and variance
- 3.6 Bernoulli random variables
- 3.7 Binomial random variables
- 3.8 Geometric random variables
- 3.9 Poisson random variables

Flipping a coin

Figure: Suppose you flip a coin until you see a head. This requires you to have $N-1$ tails, and then followed by a head in the N-th coin flip. The probability of this sequence of events are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ..., which forms an infinite series.

Geometric Random Variable

Definition

Let X be a **Geometric** random variable. Then, the PMF of X is

$$p_X(k) = (1-p)^{k-1} p, \qquad k = 1, 2, \ldots,$$

where $0 < p < 1$ is the Geometric parameter. We write

$$X \sim \text{Geometric}(p)$$

to say that X is drawn from a Geometric distribution with a parameter p.

Interpretation: Flip a coin until you get a head.

$$p_X(k) = \underbrace{(1-p)^{k-1}}_{k-1 \text{ failures}} \underbrace{p}_{\text{final success}}.$$

Probability mass function

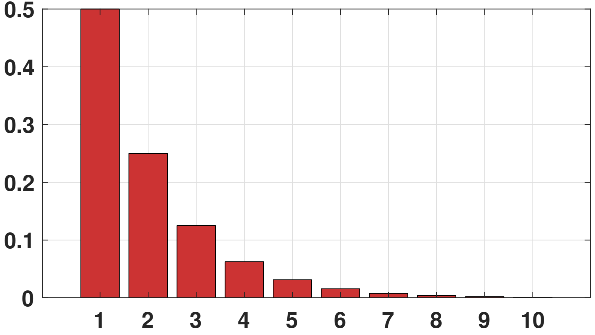


Figure: The histogram of flipping a coin until we see a Head. The x-axis denotes the number of coins we need to flip, and the y-axis denotes the probability.

Probability of success

If $p = \frac{1}{2}$, then

$$
\begin{aligned}
\mathbb{P}[\text{success after 1 attempt}] &= \frac{1}{2} = 0.5 \\
\mathbb{P}[\text{success after 2 attempts}] &= \frac{1}{2} + \frac{1}{4} = 0.75 \\
\mathbb{P}[\text{success after 3 attempts}] &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 0.875 \\
\mathbb{P}[\text{success after 4 attempts}] &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 0.9375.
\end{aligned}
$$

Shape of a Geometric Random Variable

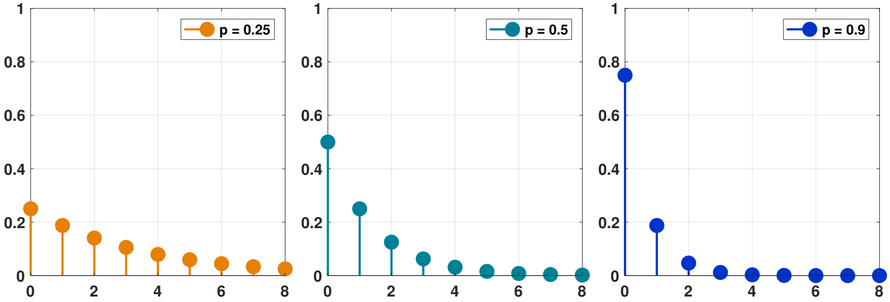


Table: PMFs of a geometric random variable $X \sim \text{Geometric}(p)$.

CDF of a Geometric Random Variable

Theorem

The CDF of a geometric random variable is

$$F_X(k) = 1 - (1-p)^k. \tag{1}$$

Moments of Geometric Random Variables

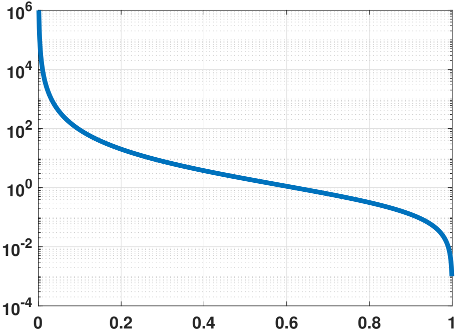
Property

If $X \sim \text{Geometric}(p)$, *then*

$$\begin{aligned}
\mathbb{E}[X] &= \frac{1}{p}, \\
\mathbb{E}[X^2] &= \frac{2}{p^2} - \frac{1}{p}, \\
\text{Var}[X] &= \frac{1-p}{p^2}.
\end{aligned}$$

Largest variance happens when ...

Since $\mathrm{Var}[X] = \frac{1-p}{p^2}$, when will you have the largest variance?



Questions?