ECE 302: Lecture 3.5 Moment and Variance

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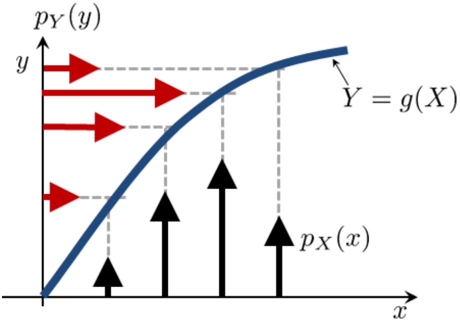
Outline

Properties of $\mathbb{E}[X]$

Property (1. Function of X)

For any function g,

$$\mathbb{E}[g(X)] = \sum_x g(x)p_X(x).$$



Properties of $\mathbb{E}[X]$

Property (2. Linearity)

For any function g and h,

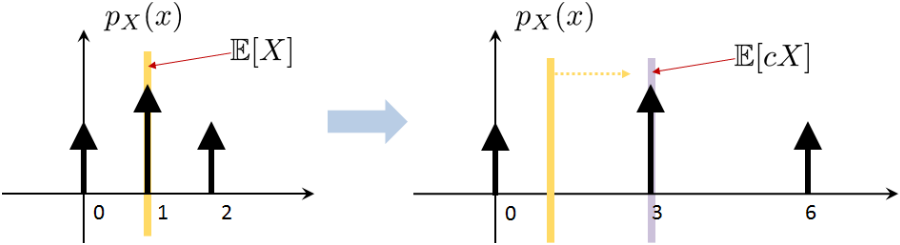
$$\mathbb{E}[g(X) + h(X)] = \mathbb{E}[g(X)] + \mathbb{E}[h(X)].$$

Properties of $\mathbb{E}[X]$

Property (3. Scale)

For any constant c,

$$\mathbb{E}[cX] = c\mathbb{E}[X].$$

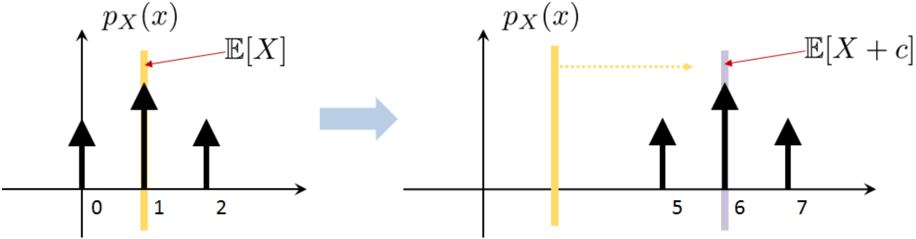


Properties of $\mathbb{E}[X]$

Property (4. DC Shift)

For any constant c,

$$\mathbb{E}[X + c] = \mathbb{E}[X] + c.$$



Outline

Moment

Definition

The k-**th moment** of a random variable X is

$$\mathbb{E}[X^k] = \sum_x x^k \, p_X(x).$$

Example. Flip a coin 3 times. Let X be the number of heads. Then,

$$p_X(0) = \frac{1}{8}, \;\; p_X(1) = \frac{3}{8}, \;\; p_X(2) = \frac{3}{8}, \;\; p_X(3) = \frac{1}{8}.$$

The second moment $\mathbb{E}[X^2]$ is

Variance

Definition

The **variance** of a random variable X is

$$\mathrm{Var}[X] = \mathbb{E}[(X - \mu_X)^2],$$

where $\mu_X = \mathbb{E}[X]$ is the expectation of X. $\sqrt{\mathrm{Var}[X]}$ is called the **standard deviation**.

Example. $X =$ coin flip with probability p. Find variance of X.

Properties of Variance

Property

The variance of a random variable X has the following properties

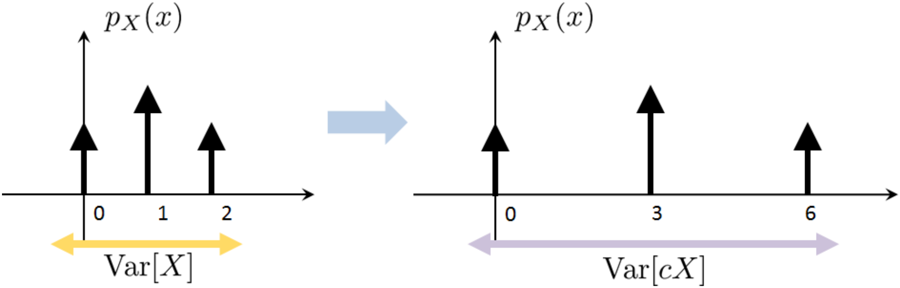
(a) **Moment**.

$$\mathrm{Var}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2.$$

Properties of Variance

(b) **Scale**. For any constant c,

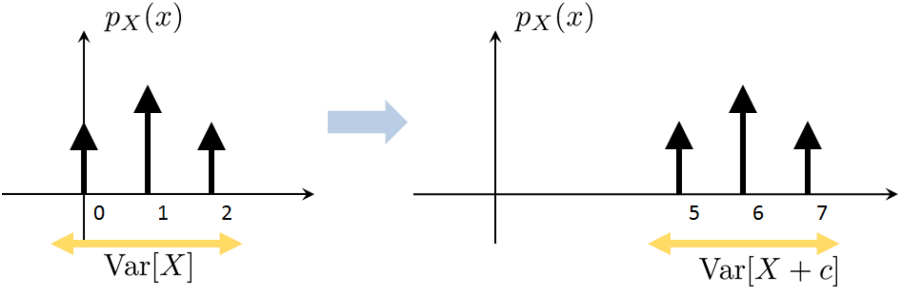
$$\mathrm{Var}[cX] = c^2\mathrm{Var}[X].$$



Properties of Variance

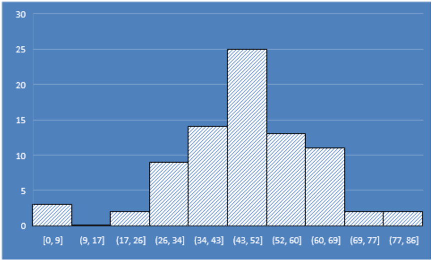
(c) **DC Shift**. For any constant c,

$$\mathrm{Var}[X + c] = \mathrm{Var}[X].$$



Coming back to this problem ...

Student 1	86.00
Student 2	76.20
Student 3	29.10
Student 4	26.38
Student 5	60.86
...	...
Student 71	48.04
Student 72	30.20
Student 73	55.44
Student 74	49.92
Student 75	17.60



Add 10 points to everyone. Then,

- Will the mean change? Yes, $\mathbb{E}[X] + 10$.
- Will the standard deviation change? No. Remains $\sqrt{Var[X]}$
- If the letter grades are curved, will this change the grades? No.

Questions?