Outline

3.1 Random variables
3.2 Probability mass functions (PMF)
3.3 Cumulative distribution functions (discrete case)
3.4 Expectation
   3.4.1 Understanding expectation
   3.4.2 Properties of expectation
3.5 Moments and variance
3.6 Bernoulli random variables
3.7 Binomial random variables
3.8 Geometric random variables
3.9 Poisson random variables
Properties of $\mathbb{E}[X]$

**Property (1. Function of $X$)**

*For any function $g$,*

$$
\mathbb{E}[g(X)] = \sum_x g(x)p_X(x).
$$
Properties of $\mathbb{E}[X]$

Property (2. Linearity)

For any function $g$ and $h$,

$$\mathbb{E}[g(X) + h(X)] = \mathbb{E}[g(X)] + \mathbb{E}[h(X)].$$
Properties of $\mathbb{E}[X]$

Property (3. Scale)

For any constant $c$,

$$\mathbb{E}[cX] = c\mathbb{E}[X].$$
Property (4. DC Shift)

For any constant $c$, 

$$\mathbb{E}[X + c] = \mathbb{E}[X] + c.$$
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Moment

Definition

The \textit{k-th moment} of a random variable $X$ is

$$
\mathbb{E}[X^k] = \sum_x x^k p_X(x).
$$

Example. Flip a coin 3 times. Let $X$ be the number of heads. Then,

$$
p_X(0) = \frac{1}{8}, \quad p_X(1) = \frac{3}{8}, \quad p_X(2) = \frac{3}{8}, \quad p_X(3) = \frac{1}{8}.
$$

The second moment $\mathbb{E}[X^2]$ is
Variance

Definition

The **variance** of a random variable $X$ is

$$\text{Var}[X] = \mathbb{E}[(X - \mu_X)^2],$$

where $\mu_X = \mathbb{E}[X]$ is the expectation of $X$. $\sqrt{\text{Var}[X]}$ is called the **standard deviation**.

**Example.** $X =$ coin flip with probability $p$. Find variance of $X$. 
Properties of Variance

Property

The variance of a random variable $X$ has the following properties

(a) **Moment.**

$$\text{Var}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2.$$
(b) **Scale.** For any constant $c$,

$$\text{Var}[cX] = c^2 \text{Var}[X].$$
Properties of Variance

(c) **DC Shift.** For any constant $c$, 

$$\text{Var}[X + c] = \text{Var}[X].$$
Coming back to this problem ...

Add 10 points to everyone. Then,

- Will the mean change? Yes, \( \mathbb{E}[X] + 10 \).
- Will the standard deviation change? No. Remains \( \sqrt{\text{Var}[X]} \).
- If the letter grades are curved, will this change the grades? No.
Questions?