ECE 302: Lecture 2.5 Independence

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2.1 Set theory
2.2 Probability space
2.3 Axioms of probability
2.4 Conditional probability
2.5 Independence
  2.5.1 What is independent?
  2.5.2 Examples
2.6 Bayes theorem
The game of throw dices — easy case

Throw a dice twice. Let

\[ A = \{1\text{st dice is 3}\} \quad \text{and} \quad B = \{2\text{nd dice is 4}\}. \]

Are \( A \) and \( B \) independent?

- What is independence?
- One event does not affect the other event!
- Are \( A \) and \( B \) independent then?
The game of throw dices — hard case

Throw a dice twice. Let

\[ A = \{ \text{1st dice is 1} \} \quad \text{and} \quad B = \{ \text{sum is 7} \}. \]

Are \( A \) and \( B \) independent?

- Not as trivial ...
- If you know the sum is 7, then the pair has to be \((1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\).
- The chance of getting first dice = 1 is still 1/6. It has been not been changed by \( B \).
Definition

Two events $A$ and $B$ are statistically **independent** if

Disjoint VS Independent.
Independence Via Conditional Probability

- Recall that $\mathbb{P}[A \mid B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$.
- If $A$ and $B$ are independent, then $\mathbb{P}[A \cap B] = \mathbb{P}[A] \mathbb{P}[B]$

Therefore,

$$\mathbb{P}[A \mid B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]} = \frac{\mathbb{P}[A] \mathbb{P}[B]}{\mathbb{P}[B]} = \mathbb{P}[A].$$

**Interpretation.**

**Pictorial Illustration.** Conditional probability

$$\mathbb{P}[A \mid B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]} = \frac{\mathbb{P}[A]}{\Omega} = \mathbb{P}[A] = \text{ratio of } A \text{ in } \Omega$$
Outline

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- 2.2 Probability space
- 2.3 Axioms of probability
- 2.4 Conditional probability
- 2.5 Independence
  - 2.5.1 What is independent?
  - 2.5.2 Examples
- 2.6 Bayes theorem
Example 1

Example 1. Throw a dice twice. Let

\[ A = \{ \text{1st dice is 3} \} \quad \text{and} \quad B = \{ \text{2nd dice is 4} \}. \]

Are \( A \) and \( B \) independent?
Example 2. Throw a dice twice. Let

\[ A = \{ \text{1st dice is 1} \} \quad \text{and} \quad B = \{ \text{sum is 7} \}. \]

Are \( A \) and \( B \) independent?
Example 2(b)

How about we change the problem in this way?

\[ A = \{1st \text{ dice is 1}\} \quad \text{and} \quad B = \{\text{sum is 8}\}. \]

Are \( A \) and \( B \) independent?
Example 3. Throw a dice twice. Let
\[ A = \{\text{1st dice is 2}\} \quad \text{and} \quad B = \{\text{sum is 8}\}. \]
Are \( A \) and \( B \) independent?
Example 3 (continue)

Interpreting the answer for Example 3:

\[ A = \{ \text{1st dice is 2} \} \quad \text{and} \quad B = \{ \text{sum is 8} \}. \]

- Think about \( P[A|B] \).
- If you know the sum is 8, then the pair has to be (2,6), (3,5), (4,4), (5,3), (6,2).
- The chance of getting first dice = 2 is no longer 1/6. It has been changed by \( B \).
- So dependent.
Example 4. Throw a dice twice. Let

\[ A = \{ \text{max is 2} \} \quad \text{and} \quad B = \{ \text{min is 2} \}. \]

Are \( A \) and \( B \) independent?
Why border independence?

dependent data

independent data
Questions?