ECE 302: Lecture 2.5 Independence

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Outline

The game of throw dices — easy case

Throw a dice twice. Let

$$A = \{\text{1st dice is 3}\} \quad \text{and} \quad B = \{\text{2nd dice is 4}\}.$$

Are A and B *independent*?

- What is independence?
- One event does not affect the other event!
- Are A and B independent then?

The game of throw dices — hard case

Throw a dice twice. Let

$$A = \{\text{1st dice is } 1\} \quad \text{and} \quad B = \{\text{sum is } 7\}.$$

Are A and B independent?

- Not as trivial ...
- If you know the sum is 7, then the pair has to be (1,6), (2,5), (3,4), (4,3), (5,2), (6,1).
- The chance of getting first dice $= 1$ is still $1/6$. It has been not been changed by B.

Mathematical definition

Definition

Two events A and B are statistically **independent** if

Disjoint VS Independent.

Independence Via Conditional Probability

- Recall that $\mathbb{P}[A \mid B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$.
- If A and B are independent, then $\mathbb{P}[A \cap B] = \mathbb{P}[A]\,\mathbb{P}[B]$

Therefore,

$$\mathbb{P}[A \mid B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]} = \frac{\mathbb{P}[A]\,\mathbb{P}[B]}{\mathbb{P}[B]} = \mathbb{P}[A].$$

Interpretation.

Pictorial Illustration. Conditional probability

$$\mathbb{P}[A|B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]} = \text{ratio of A in B} = \mathbb{P}[A] = \text{ratio of A in } \Omega$$

Outline

Example 1

Example 1. Throw a dice twice. Let

$$A = \{\text{1st dice is 3}\} \quad \text{and} \quad B = \{\text{2nd dice is 4}\}.$$

Are A and B independent?

Example 2

Example 2. Throw a dice twice. Let

$$A = \{\text{1st dice is 1}\} \quad \text{and} \quad B = \{\text{sum is 7}\}.$$

Are A and B independent?

Example 2(b)

How about we change the problem in this way?

$$A = \{\text{1st dice is 1}\} \quad \text{and} \quad B = \{\text{sum is 8}\}.$$

Are A and B independent?

Example 3

Example 3. Throw a dice twice. Let

$$A = \{\text{1st dice is 2}\} \quad \text{and} \quad B = \{\text{sum is 8}\}.$$

Are A and B independent?

Example 3 (continue)

Interpreting the answer for Example 3:

$$A = \{\text{1st dice is 2}\} \quad \text{and} \quad B = \{\text{sum is 8}\}.$$

- Think about $\mathbb{P}[A|B]$.
- If you know the sum is 8, then the pair has to be (2,6), (3,5), (4,4), (5,3), (6,2).
- The chance of getting first dice $= 2$ is no longer $1/6$. It has been changed by B.
- So dependent.

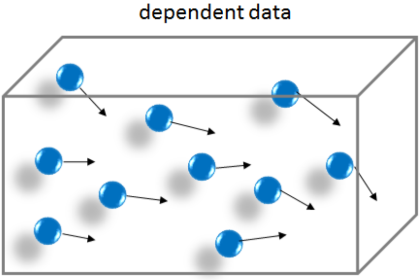
Example 4

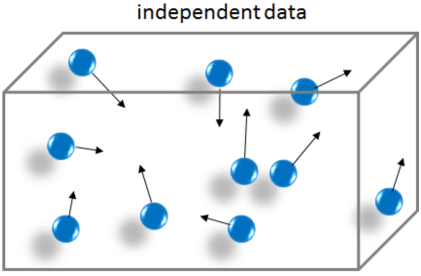
Example 4. Throw a dice twice. Let

$$A = \{\text{max is } 2\} \quad \text{and} \quad B = \{\text{min is } 2\}.$$

Are A and B independent?

Why border independence?





Questions?