ECE 302: Lecture 2.4 Conditional Probability

Prof Stanley Chan

School of Electrical and Computer Engineering
Purdue University
Outline

- 2.1 Set theory
- 2.2 Probability space
- 2.3 Axioms of probability
- 2.4 Conditional probability
  - Definition
  - Examples
  - Axioms
- 2.5 Independence
- 2.6 Bayes theorem
Conditional Probability

**Definition**
Assume $P[B] \neq 0$. The *conditional probability* of $A$ given $B$ is

$$P[A \mid B] \overset{\text{def}}{=} \frac{P[A \cap B]}{P[B]}.$$  \hspace{1cm} (1)

The difference between $P[A \mid B]$ and $P[A \cap B]$ is the denominator they carry:

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} \quad \text{and} \quad P[A \cap B] = \frac{P[A \cap B]}{P[\Omega]}.$$  \hspace{1cm} (2)
Figure: Illustration of conditional probability and its comparison with $\mathbb{P}[A \cap B]$. 
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Examples

Example 1. Let

\[ A = \{\text{Eat 2 burgers}\} \quad \text{and} \quad B = \{\text{Finish a football game}\}. \]

In this example,

\[ \mathbb{P}[A] = \text{Probability that you eat 2 burgers} \]
\[ \mathbb{P}[B] = \text{Probability that you just finish a football game} \]
\[ \mathbb{P}[A \cap B] = \text{Prob. that you just finish a game and you eat 2 burgers} \]
\[ \mathbb{P}[A | B] = \text{Prob. that you eat 2 burgers given that you just finish a game}. \]
Examples

Example 2. Throw a dice. Let

\[ A = \{ \text{Get 3} \} \quad \text{and} \quad B = \{ \text{odd numbers} \}. \]

Find \( \mathbb{P}[A \mid B] \) and \( \mathbb{P}[B \mid A] \).
Example 3. Consider the situation below. There are 12 points with equal probability of happening. Find the probabilities $P[A|B]$ and $P[B|A]$. 
Example 4. Consider a 4 sided dice. Let $X$ be the first roll and $Y$ be the second roll. Let $B$ be the event that $\min(X, Y) = 2$ and $M$ be the event that $\max(X, Y) = 3$. Find $\mathbb{P}[M|B]$. 

\[ \begin{array}{cccc}
   & 1 & 2 & 3 & 4 \\
 1 & & & & \\
 2 & & & & \\
 3 & & & & \\
 4 & & & & \\
\end{array} \]
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Axioms

Proposition

Let $P[B] > 0$. The conditional probability $P[A \mid B]$ satisfies Axiom I to Axiom III.

Axiom 1:

Axiom 2:
Axioms

Axiom 3:
Questions?