ECE 302: Lecture 2.4 Conditional Probability

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Outline

- 2.1 Set theory
- 2.2 Probability space
- 2.3 Axioms of probability
- 2.4 Conditional probability
  - Definition
  - Examples
  - Axioms
- 2.5 Independence
- 2.6 Bayes theorem

Conditional Probability

Definition

Assume $\mathbb{P}[B] \neq 0$. The **conditional probability** of A given B is

$$\mathbb{P}[A \,|\, B] \stackrel{\text{def}}{=} \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}. \tag{1}$$

The difference between $\mathbb{P}[A \,|\, B]$ and $\mathbb{P}[A \cap B]$ is the denominator they carry:

$$\mathbb{P}[A \,|\, B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]} \quad \text{and} \quad \mathbb{P}[A \cap B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[\Omega]}. \tag{2}$$

Conditional Probability

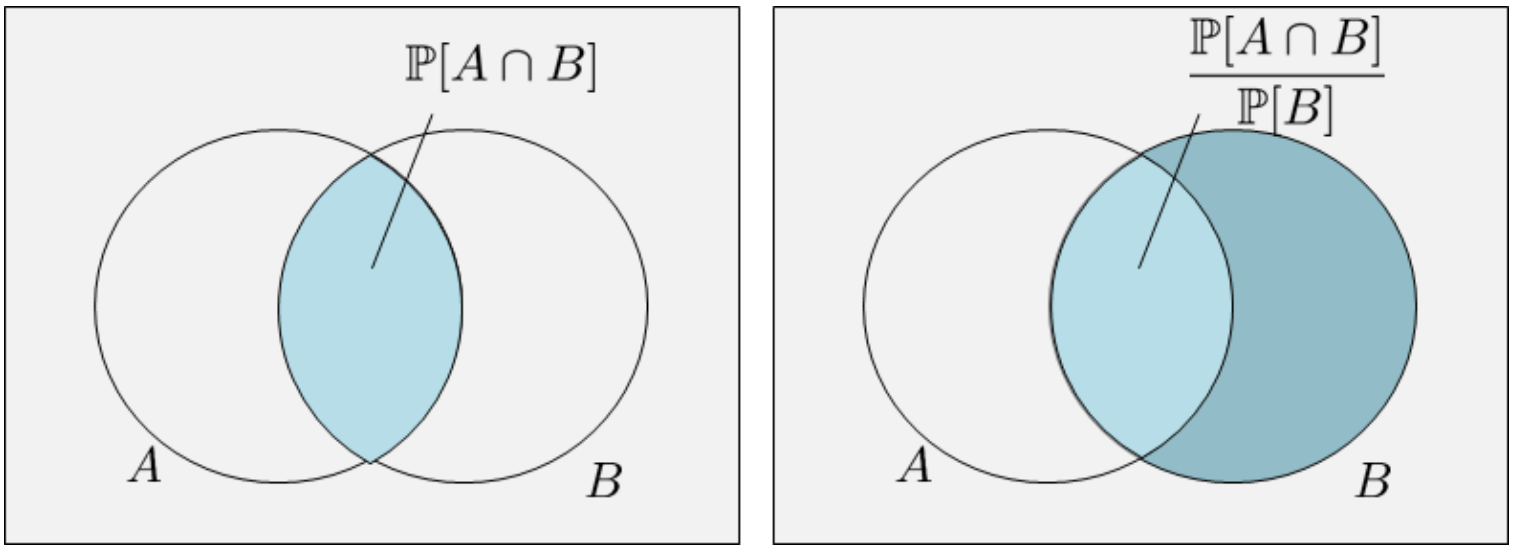

Figure: Illustration of conditional probability and its comparison with $\mathbb{P}[A \cap B]$.

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Examples

Example 1. Let

$$A = \{\text{Eat 2 burgers}\} \quad \text{and} \quad B = \{\text{Finish a football game}\}.$$

In this example,

$$\begin{aligned}
\mathbb{P}[A] &= \text{Probability that you eat 2 burgers} \\
\mathbb{P}[B] &= \text{Probability that you just finish a football game} \\
\mathbb{P}[A \cap B] &= \text{Prob. that you just finish a game and you eat 2 burgers} \\
\mathbb{P}[A \,|\, B] &= \text{Prob. that you eat 2 burgers given that you just finish a game.}
\end{aligned}$$

Examples

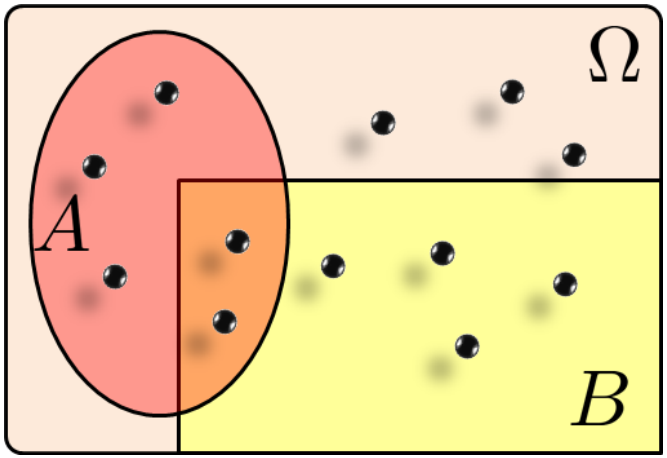
Example 2. Throw a dice. Let

$$A = \{\text{Get 3}\} \quad \text{and} \quad B = \{\text{odd numbers}\}.$$

Find $\mathbb{P}[A \,|\, B]$ and $\mathbb{P}[B \,|\, A]$.

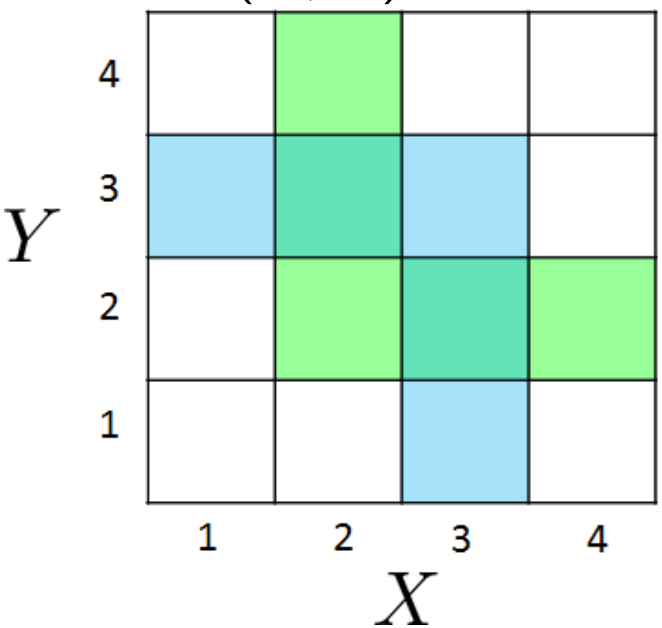
Examples

Example 3. Consider the situation below. There are 12 points with equal probability of happening. Find the probabilities $\mathbb{P}[A|B]$ and $\mathbb{P}[B|A]$.



Examples

Example 4. Consider a 4 sided dice. Let X be the first roll and Y be the second roll. Let B be the event that $\min(X, Y) = 2$ and M be the event that $\max(X, Y) = 3$. Find $\mathbb{P}[M|B]$.



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Axioms

Proposition

Let $\mathbb{P}[B] > 0$. The conditional probability $\mathbb{P}[A \mid B]$ satisfies Axiom I to Axiom III.

Axiom 1:

Axiom 2:

Axioms

Axiom 3:

Questions?