ECE 302: Lecture 2.3 Axioms of Probability

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Outline

- 2.1 Set theory
- 2.2 Probability space
- 2.3 Axioms of probability
  - 2.3.1 The three axioms
  - 2.3.2 Corollaries derived from the axioms
  - 2.3.3 Examples
- 2.4 Conditional probability
- 2.5 Independence
- 2.6 Bayes theorem
Probability Law

Definition

A **probability law** is a function $\mathbb{P} : \mathcal{F} \rightarrow [0, 1]$ that maps an event $A$ to a real number in $[0, 1]$. The function must satisfy three axioms known as **Probability Axioms**.

I. **Non-negativity:**

II. **Normalization:**
III. Additivity:

For any disjoint subsets \( \{A_1, A_2, \ldots \} \), it holds that

\[
\mathbb{P} \left[ \bigcup_{n=1}^{\infty} A_n \right] = \sum_{n=1}^{\infty} \mathbb{P}[A_n].
\]
If $A$ and $B$ are disjoint, then

$$\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B],$$

(1)
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Properties of Probability

1. $\mathbb{P}[A^c] = 1 - \mathbb{P}[A]$.

2. For any $A \subseteq \Omega$, $\mathbb{P}[A] \leq 1$.

3. $\mathbb{P}[\emptyset] = 0$. 
Properties of Probability

4. For any $A$ and $B$,

$$P[A \cup B] = P[A] + P[B] - P[A \cap B].$$
Properties of Probability

Proof.
(Union Bound) For any $A$ and $B$,

$$P[A \cup B] \leq P[A] + P[B].$$
Properties of Probability

If \( A \subseteq B \), then \( \mathbb{P}[A] \leq \mathbb{P}[B] \)

Example. \( A = \{ t \leq 5 \} \), and \( B = \{ t \leq 10 \} \), then \( \mathbb{P}[A] \leq \mathbb{P}[B] \).
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Example

Let the events $A$ and $B$ have $P[A] = x$, $P[B] = y$ and $P[A \cup B] = z$. Find the following probabilities.

(a) $P[A \cap B]$

(b) $P[A^c \cap B^c]$
Example

(c) $\mathbb{P}[A^c \cup B^c]$

(d) $\mathbb{P}[A \cap B^c]$
Questions?