ECE 302: Lecture 2.3 Axioms of Probability

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Outline

Probability Law

Definition

A **probability law** is a function $\mathbb{P} : \mathcal{F} \rightarrow [0, 1]$ that maps an event A to a real number in [0, 1]. The function must satisfy three axioms known as **Probability Axioms**.

I. Non-negativity:

II. Normalization:

Probability Law

III. Additivity:

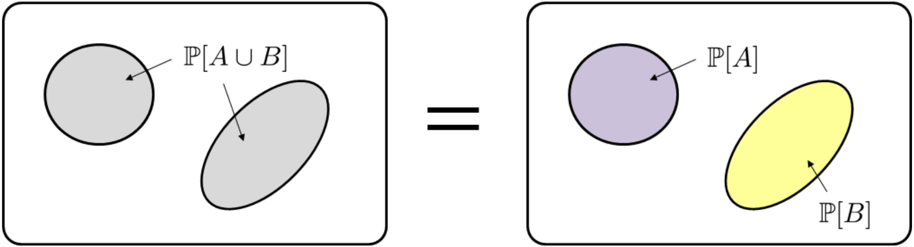
For any **disjoint** subsets $\{A_1, A_2, \ldots\}$, it holds that

$$\mathbb{P}\left[\bigcup_{n=1}^{\infty} A_n\right] = \sum_{n=1}^{\infty} \mathbb{P}[A_n].$$

Understanding additivity

If A and B are disjoint, then

$$\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B], \tag{1}$$



Outline

Properties of Probability

1. $\mathbb{P}[A^c] = 1 - \mathbb{P}[A]$.

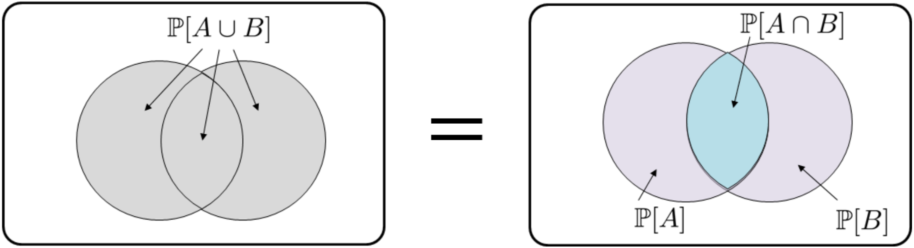
2. For any $A \subseteq \Omega$, $\mathbb{P}[A] \leq 1$.

3. $\mathbb{P}[\emptyset] = 0$.

Properties of Probability

4. For any A and B,

$$\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B] - \mathbb{P}[A \cap B].$$



Properties of Probability

Proof.

Properties of Probability

5. (Union Bound) For any A and B,

$$\mathbb{P}[A \cup B] \leq \mathbb{P}[A] + \mathbb{P}[B].$$

Properties of Probability

6. If $A \subseteq B$, then $\mathbb{P}[A] \leq \mathbb{P}[B]$

Example. $A = \{t \leq 5\}$, and $B = \{t \leq 10\}$, then $\mathbb{P}[A] \leq \mathbb{P}[B]$.

Outline

Example

Let the events A and B have $\mathbb{P}[A] = x$, $\mathbb{P}[B] = y$ and $\mathbb{P}[A \cup B] = z$. Find the following probabilities.

(a) $\mathbb{P}[A \cap B]$

(b) $\mathbb{P}[A^c \cap B^c]$

Example

(c) $\mathbb{P}[A^c \cup B^c]$

(d) $\mathbb{P}[A \cap B^c]$

Questions?