# ECE 302: Lecture 2.3 Axioms of Probability 

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## Outline

- 2.1 Set theory
- 2.2 Probability space
- 2.3 Axioms of probability
- 2.3.1 The three axioms
- 2.3.2 Corollaries derived from the axioms
- 2.3.3 Examples
- 2.4 Conditional probability
- 2.5 Independence
- 2.6 Bayes theorem


## Probability Law

## Definition

A probability law is a function $\mathbb{P}: \mathcal{F} \rightarrow[0,1]$ that maps an event $A$ to a real number in $[0,1]$. The function must satisfy three axioms known as Probability Axioms.
I. Non-negativity:
II. Normalization:

## Probability Law

III. Additivity:

For any disjoint subsets $\left\{A_{1}, A_{2}, \ldots\right\}$, it holds that

$$
\mathbb{P}\left[\bigcup_{n=1}^{\infty} A_{n}\right]=\sum_{n=1}^{\infty} \mathbb{P}\left[A_{n}\right] .
$$

## Understanding additivity

If $A$ and $B$ are disjoint, then

$$
\begin{equation*}
\mathbb{P}[A \cup B]=\mathbb{P}[A]+\mathbb{P}[B] \tag{1}
\end{equation*}
$$



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## Properties of Probability

(1) $\mathbb{P}\left[A^{c}\right]=1-\mathbb{P}[A]$.
(3) For any $A \subseteq \Omega, \mathbb{P}[A] \leq 1$.
(- $\mathbb{P}[\emptyset]=0$.

## Properties of Probability

(9) For any $A$ and $B$,

$$
\mathbb{P}[A \cup B]=\mathbb{P}[A]+\mathbb{P}[B]-\mathbb{P}[A \cap B]
$$



## Properties of Probability

## Proof.

## Properties of Probability

(5) (Union Bound) For any $A$ and $B$,

$$
\mathbb{P}[A \cup B] \leq \mathbb{P}[A]+\mathbb{P}[B]
$$

## Properties of Probability

(6) If $A \subseteq B$, then $\mathbb{P}[A] \leq \mathbb{P}[B]$

Example. $A=\{t \leq 5\}$, and $B=\{t \leq 10\}$, then $\mathbb{P}[A] \leq \mathbb{P}[B]$.

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## Example

Let the events $A$ and $B$ have $\mathbb{P}[A]=x, \mathbb{P}[B]=y$ and $\mathbb{P}[A \cup B]=z$. Find the following probabilities.
(a) $\mathbb{P}[A \cap B]$
(b) $\mathbb{P}\left[A^{c} \cap B^{c}\right]$

## Example

(c) $\mathbb{P}\left[A^{c} \cup B^{c}\right]$
(d) $\mathbb{P}\left[A \cap B^{c}\right]$

## Questions?

