

ECE 302: Lecture 1.3 Integration

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Outline

- 1.1 Infinite Series
 - 1.1.1. Geometric Series
 - 1.1.2. Binomial Series
- 1.2 Approximations
 - 1.2.1. Taylor Approximation
 - 1.2.2. Exponential Series
 - 1.2.3. Logarithmic Approximation
- 1.3 Integration
 - 1.3.1. Odd and Even Functions
 - 1.3.2. Fundamental Theorem of Calculus
- 1.4 Linear Algebra (Optional)
 - 1.4.1. Inner Products (Optional)
 - 1.4.2. Matrix Calculus (Optional)
 - 1.4.3. Matrix Inversion (Optional)
- 1.5 Combinatorics
 - 1.5.1. Permutation
 - 1.5.2. Combination

Integration

How many ways to do integration?

Most of you know these two tricks:

- Substitution
- Integration by part

There are two more tricks:

- Odd and even functions.
- Integrating a probability density function = 1. (We will talk about this later)

Even and Odd Functions

Definition

A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is **even** if for any $x \in \mathbb{R}$,

$$f(x) = f(-x), \quad (1)$$

and f is **odd** if

$$f(x) = -f(-x), \quad (2)$$

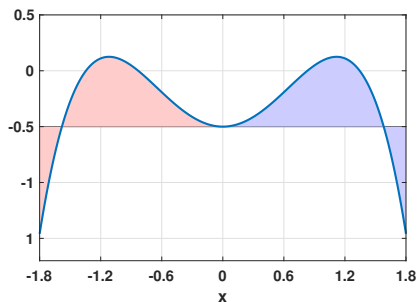
- Even function:

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx.$$

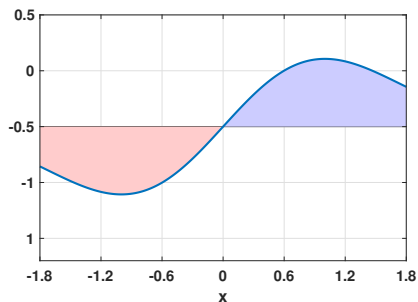
- Odd function:

$$\int_{-a}^a f(x) dx = 0.$$

Example



(a) Even function



(b) Odd function

Figure: An even function and an odd function.

Example 1

Find

$$\int_{-1}^1 x^2 - 0.4x^4 dx$$

Example 2

Find

$$\int_{-1}^1 x e^{-\frac{x^2}{2}} dx$$

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Fundamental theorem of calculus

Theorem (Fundamental Theorem of Calculus)

Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function defined on a closed interval $[a, b]$. Then,

$$f(x) = \frac{d}{dx} \int_a^x f(t) dt, \quad (3)$$

for any $x \in (a, b)$.

Proof. See Lecture note.

Corollary

Corollary

Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function defined on a closed interval $[a, b]$. Let $g : \mathbb{R} \rightarrow [a, b]$ be a continuously differentiable function. Then,

$$\frac{d}{dx} \int_a^{g(x)} f(t) dt = g'(x) \cdot f(g(x)), \quad (4)$$

for any $x \in (a, b)$.

Proof.

Example

Evaluate the integral

$$\frac{d}{dx} \int_0^{x-\mu} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{t^2}{2\sigma^2}\right\} dt.$$

This result will be useful when discussing **Gaussian random variables**.

Questions?