ECE 302: Lecture 1.3 Integration

Prof Stanley Chan

School of Electrical and Computer Engineering
Purdue University

PURDUE
UNIVERSITY

Outline

Integration

How many ways to do integration?

Most of you know these two tricks:

- Substitution
- Integration by part

There are two more tricks:

- Odd and even functions.
- Integrating a probability density function $= 1$. (We will talk about this later)

Even and Odd Functions

Definition

A function $f : \mathbb{R} \to \mathbb{R}$ is **even** if for any $x \in \mathbb{R}$,

$$f(x) = f(-x), \tag{1}$$

and f is **odd** if

$$f(x) = -f(-x), \tag{2}$$

- Even function:
$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx.$$
- Odd function:
$$\int_{-a}^{a} f(x)dx = 0.$$

Example

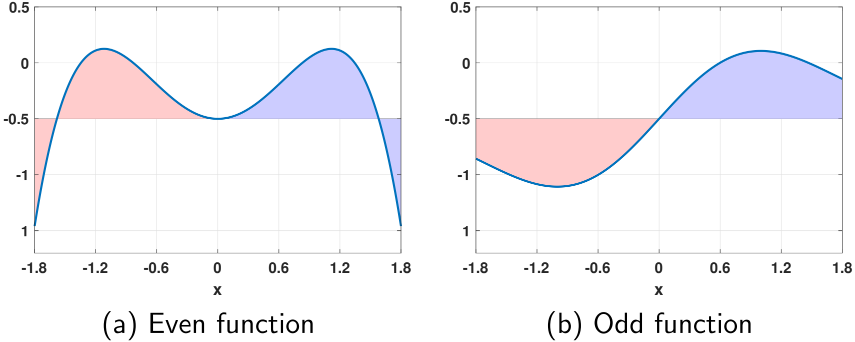

Figure: An even function and an odd function.

Example 1

Find

$$\int_{-1}^{1} x^2 - 0.4x^4 dx$$

Example 2

Find

$$\int_{-1}^{1} xe^{-\frac{x^2}{2}}\,dx$$

Outline

Fundamental theorem of calculus

Theorem (Fundamental Theorem of Calculus)

Let $f : [a, b] \to \mathbb{R}$ be a continuous function defined on a closed interval $[a, b]$. Then,

$$f(x) = \frac{d}{dx} \int_a^x f(t)dt, \tag{3}$$

for any $x \in (a, b)$.

Proof. See Lecture note.

Corollary

Corollary

Let $f : [a, b] \to \mathbb{R}$ *be a continuous function defined on a closed interval* $[a, b]$*. Let* $g : \mathbb{R} \to [a, b]$ *be a continuously differentiable function. Then,*

$$\frac{d}{dx}\int_a^{g(x)} f(t)dt = g'(x) \cdot f(g(x)), \tag{4}$$

for any $x \in (a, b)$*.*

Proof.

Example

Evaluate the integral

$$\frac{d}{dx}\int_0^{x-\mu} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{t^2}{2\sigma^2}\right\} dt.$$

This result will be useful when discussing **Gaussian random variables**.

Questions?