ECE 302: Lecture 1.3 Integration

Prof Stanley Chan

School of Electrical and Computer Engineering
Purdue University
Outline

1.1 Infinite Series
   - 1.1.1. Geometric Series
   - 1.1.2. Binomial Series

1.2 Approximations
   - 1.2.1. Taylor Approximation
   - 1.2.2. Exponential Series
   - 1.2.3. Logarithmic Approximation

1.3 Integration
   - 1.3.1. Odd and Even Functions
   - 1.3.2. Fundamental Theorem of Calculus

1.4 Linear Algebra (Optional)
   - 1.4.1. Inner Products (Optional)
   - 1.4.2. Matrix Calculus (Optional)
   - 1.4.3. Matrix Inversion (Optional)

1.5 Combinatorics
   - 1.5.1. Permutation
   - 1.5.2. Combination
Integration

How many ways to do integration?

Most of you know these two tricks:

- Substitution
- Integration by part

There are two more tricks:

- Odd and even functions.
- Integrating a probability density function $= 1$. (We will talk about this later)
Even and Odd Functions

Definition
A function \( f : \mathbb{R} \rightarrow \mathbb{R} \) is **even** if for any \( x \in \mathbb{R} \),

\[
f(x) = f(-x),
\]

and \( f \) is **odd** if

\[
f(x) = -f(-x),
\]

- **Even function:**
  \[
  \int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx.
  \]
- **Odd function:**
  \[
  \int_{-a}^{a} f(x) \, dx = 0.
  \]
Example

(a) Even function

(b) Odd function

Figure: An even function and an odd function.
Example 1

Find

\[
\int_{-1}^{1} x^2 - 0.4x^4 \, dx
\]
Example 2

Find

$$\int_{-1}^{1} xe^{-\frac{x^2}{2}} \, dx$$
Outline

1.1 Infinite Series
   - 1.1.1. Geometric Series
   - 1.1.2. Binomial Series

1.2 Approximations
   - 1.2.1. Taylor Approximation
   - 1.2.2. Exponential Series
   - 1.2.3. Logarithmic Approximation

1.3 Integration
   - 1.3.1. Odd and Even Functions
   - 1.3.2. Fundamental Theorem of Calculus

1.4 Linear Algebra (Optional)
   - 1.4.1. Inner Products (Optional)
   - 1.4.2. Matrix Calculus (Optional)
   - 1.4.3. Matrix Inversion (Optional)

1.5 Combinatorics
   - 1.5.1. Permutation
   - 1.5.2. Combination
Fundamental theorem of calculus

Theorem (Fundamental Theorem of Calculus)

Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function defined on a closed interval $[a, b]$. Then,

$$f(x) = \frac{d}{dx} \int_{a}^{x} f(t) dt,$$

(3)

for any $x \in (a, b)$.

Proof. See Lecture note.
Corollary

Let $f : [a, b] \to \mathbb{R}$ be a continuous function defined on a closed interval $[a, b]$. Let $g : \mathbb{R} \to [a, b]$ be a continuously differentiable function. Then,

$$\frac{d}{dx} \int_a^{g(x)} f(t) dt = g'(x) \cdot f(g(x)), \quad (4)$$

for any $x \in (a, b)$.

Proof.
Example

Evaluate the integral

$$\frac{d}{dx} \int_0^{x-\mu} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{t^2}{2\sigma^2} \right\} \, dt.$$  

This result will be useful when discussing Gaussian random variables.
Questions?