

# ECE 302: Lecture 1.1 Infinite Series

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# Outline

- 1.1 Infinite Series
  - 1.1.1. Geometric Series
  - 1.1.2. Binomial Series
- 1.2 Approximations
  - 1.2.1. Taylor Approximation
  - 1.2.2. Exponential Series
  - 1.2.3. Logarithmic Approximation
- 1.3 Integration
  - 1.3.1. Odd and Even Functions
  - 1.3.2. Fundamental Theorem of Calculus
- 1.4 Linear Algebra (Optional)
  - 1.4.1. Inner Products (Optional)
  - 1.4.2. Matrix Calculus (Optional)
  - 1.4.3. Matrix Inversion (Optional)
- 1.5 Combinatorics
  - 1.5.1. Permutation
  - 1.5.2. Combination

## Sum of **Finite** Geometric Series

### Theorem

The sum of a **finite geometric series** of power  $n$  is

$$\sum_{k=0}^n r^k = 1 + r + r^2 + \dots + r^n = \frac{1 - r^{n+1}}{1 - r}. \quad (1)$$

## Sum of **Infinite** Geometric Series

### Corollary

Let  $0 < r < 1$ . The sum of an **infinite geometric series** is

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + \dots = \frac{1}{1-r}. \quad (2)$$

## Can $r > 1$ ?

So here is the infinite series for  $0 < r < 1$ .

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + \dots = \frac{1}{1-r}. \quad (3)$$

What happens if  $r > 1$ ?

What happens if  $r = 0$ ?

## Example 1

Compute the infinite series  $\sum_{k=2}^{\infty} \frac{1}{2^k}$ .

# Derivative

## Corollary

Let  $0 < r < 1$ . It holds that

$$\sum_{k=1}^{\infty} kr^{k-1} = 1 + 2r + 3r^2 + \dots = \frac{1}{(1-r)^2}. \quad (4)$$

## Example 2

Compute the infinite sum  $\sum_{k=1}^{\infty} k \cdot \frac{1}{3^k}$ .



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## Combination: $n$ choose $k$

### Definition

The symbol  $\binom{n}{k}$  denotes  $n$  choose  $k$ , and is defined as

$$\binom{n}{k} \stackrel{\text{def}}{=} \frac{n!}{k!(n-k)!}. \quad (5)$$

**Example.** Compute  $\binom{5}{3}$  and  $\binom{6}{2}$ .

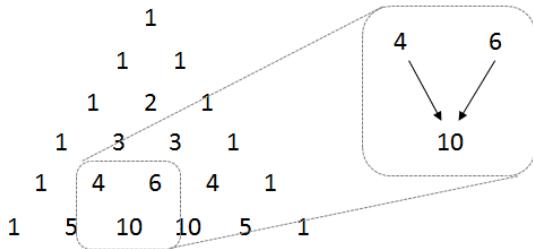
# Pascal Identity

## Theorem (Pascal Identity)

Let  $n$  and  $k$  be positive integers such that  $k \leq n$ . Then,

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}. \quad (6)$$

Proof: See note.



# Binomial Series

## Theorem

For any real numbers  $a$  and  $b$ , the binomial series of power  $n$  is

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k, \quad (7)$$

where  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ .

Proof: See note.

**Example.**  $(1 + x)^3 =$

## Example

Let  $0 < p < 1$ . Find  $\sum_{k=0}^n \binom{n}{k} p^{n-k} (1-p)^k$ .

**Questions?**