ECE 302: Lecture 1.1 Infinite Series

Prof Stanley Chan

School of Electrical and Computer Engineering
Purdue University

Outline

Sum of **Finite** Geometric Series

Theorem

The sum of a **finite geometric series** *of power n is*

$$\sum_{k=0}^{n} r^k = 1 + r + r^2 + \ldots + r^n = \frac{1 - r^{n+1}}{1 - r}. \tag{1}$$

Sum of **Infinite** Geometric Series

Corollary

Let $0 < r < 1$*. The sum of an* **infinite geometric series** *is*

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + \ldots = \frac{1}{1-r}. \tag{2}$$

Can $r > 1$?

So here is the infinite series for $0 < r < 1$.

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + \ldots = \frac{1}{1-r}. \tag{3}$$

What happens if $r > 1$?

What happens if $r = 0$?

Example 1

Compute the infinite series $\sum_{k=2}^{\infty} \frac{1}{2^k}$.

Derivative

Corollary

Let $0 < r < 1$. *It holds that*

$$\sum_{k=1}^{\infty} k r^{k-1} = 1 + 2r + 3r^2 + \ldots = \frac{1}{(1-r)^2}. \tag{4}$$

Example 2

Compute the infinite sum $\sum_{k=1}^{\infty} k \cdot \frac{1}{3^k}$.

Outline

Combination: n choose k

Definition

The symbol $\binom{n}{k}$ denotes n choose k, and is defined as

$$\binom{n}{k} \overset{\text{def}}{=} \frac{n!}{k!(n-k)!}. \tag{5}$$

Example. Compute $\binom{5}{3}$ and $\binom{6}{2}$.

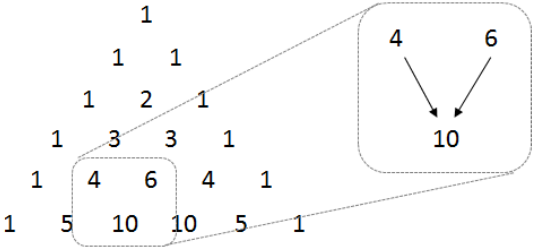
Pascal Identity

Theorem (Pascal Identity)

Let n and k be positive integers such that $k \leq n$. Then,

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}. \tag{6}$$

Proof: See note.



Binomial Series

Theorem

For any real numbers a and b, the binomial series of power n is

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k, \tag{7}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Proof: See note.

Example. $(1+x)^3 =$

Example

Let $0 < p < 1$. Find $\sum_{k=0}^{n} \binom{n}{k} p^{n-k}(1-p)^k$.

Questions?