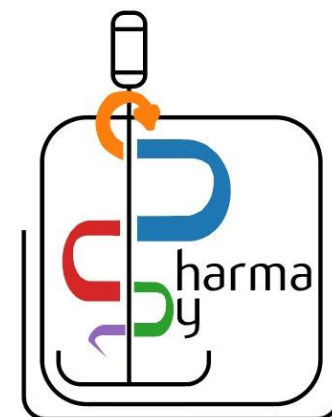
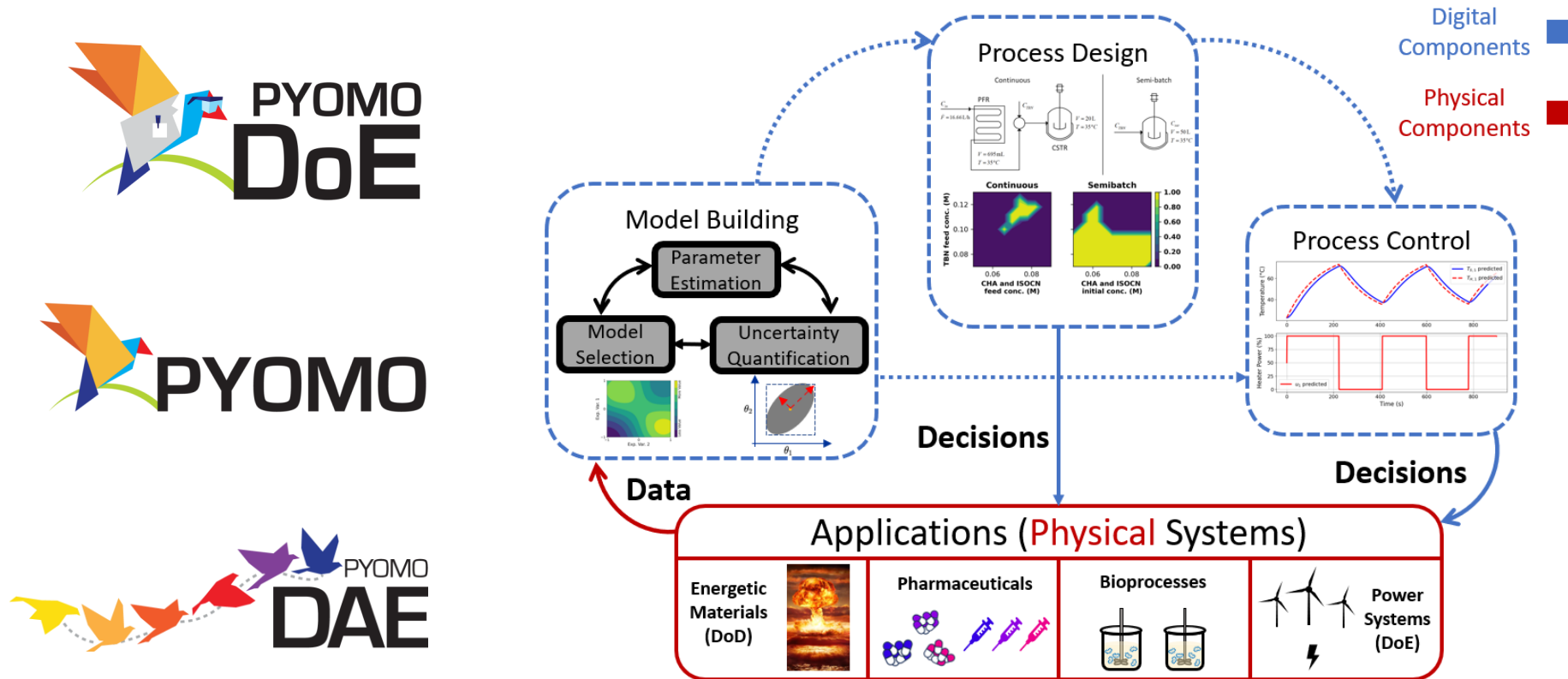


Digitalization in Chemical Engineering: Building Blocks of Fit-for-Purpose Digital Twins

Fit-for-Purpose Digital Twins



Daniel Laky, Ph.D.

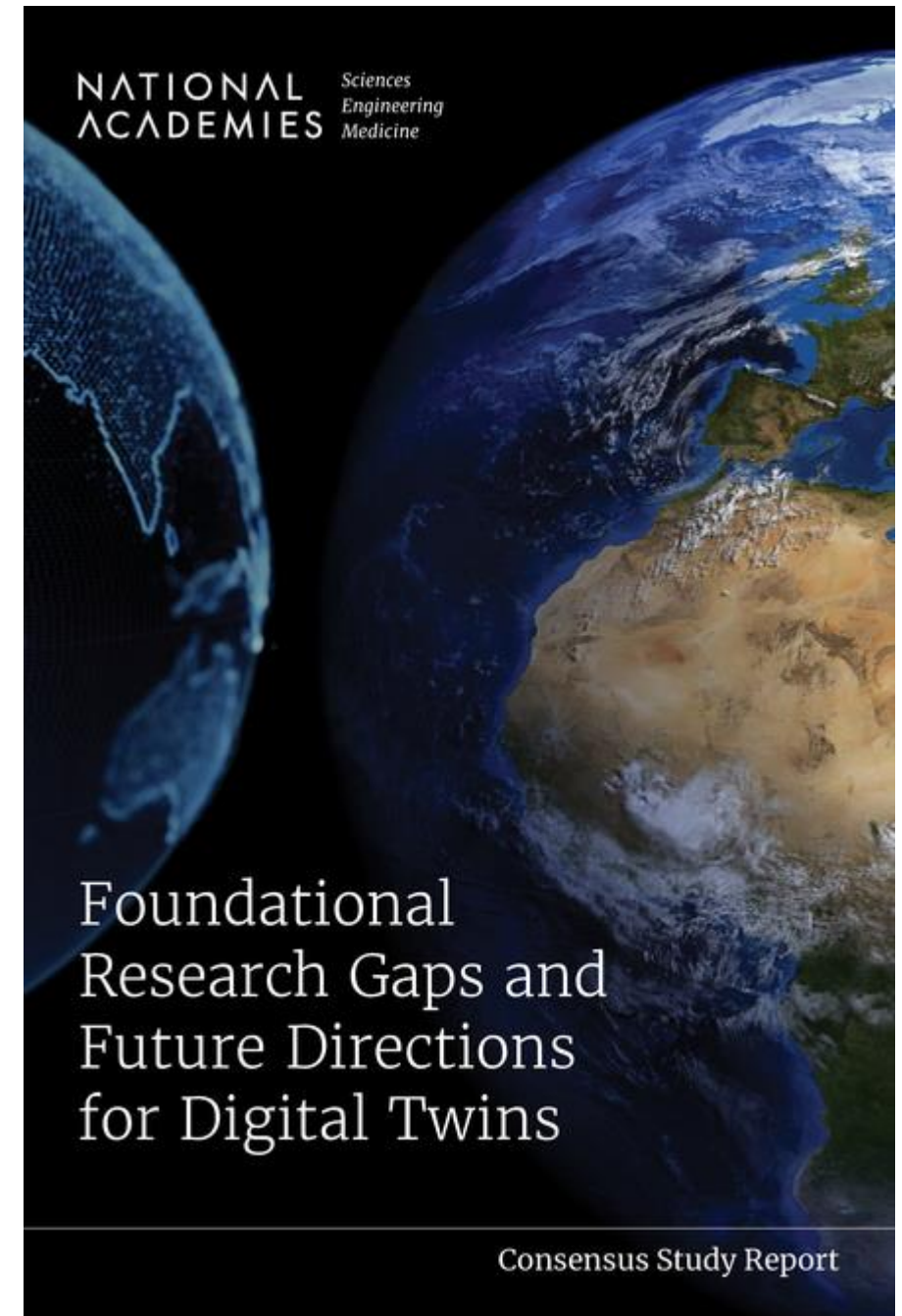
Department of Chemical and Biomolecular Engineering
Computational Molecular Science & Engineering Laboratory
University of Notre Dame, Notre Dame, IN

September 6th, 2024



Definition of “Digital Twin”

*A **digital twin** is a set of **virtual information constructs** that mimics the structure, context, and behavior of a **natural, engineered, or social system** (or **system-of-systems**), is **dynamically updated** with data from its physical twin, has a **predictive capability**, and **informs decisions that realize value**. The **bidirectional interaction** between the virtual and the physical is central to the digital twin.*



Digital Twins Provide Opportunities to Accelerate Scientific Discovery and Improve Manufacturing

- High-throughput applications
 - Self-driving laboratories
 - Automated and adaptive experimental campaigns

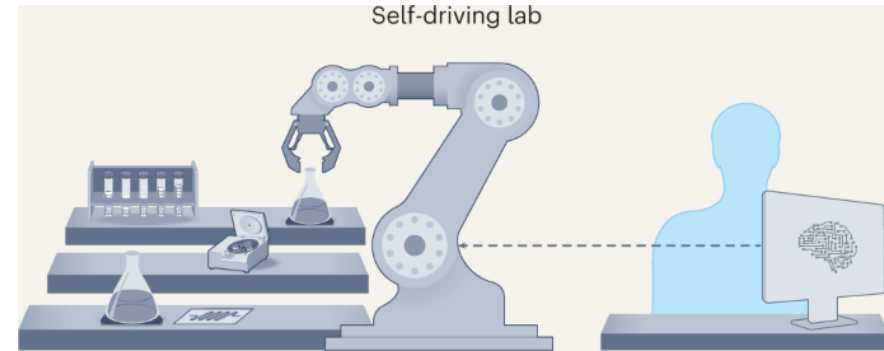


Figure from Abolhasani & Kumacheva (2023), *Nature Syn.*

Digital Twins Provide Opportunities to Accelerate Scientific Discovery and Improve Manufacturing

- High-throughput applications
 - Self-driving laboratories
 - Automated and adaptive experimental campaigns
- Robust digital twins enable optimal manufacturing
 - Optimal design
 - Traditional process design
 - Data flow optimization
 - Optimal control
 - Real-time decision making
 - Applications: pharmaceuticals and energy systems, among others^{1,2,3,4,5}

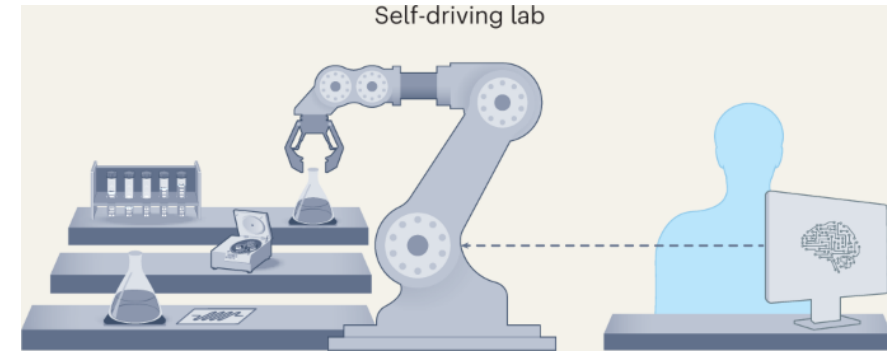
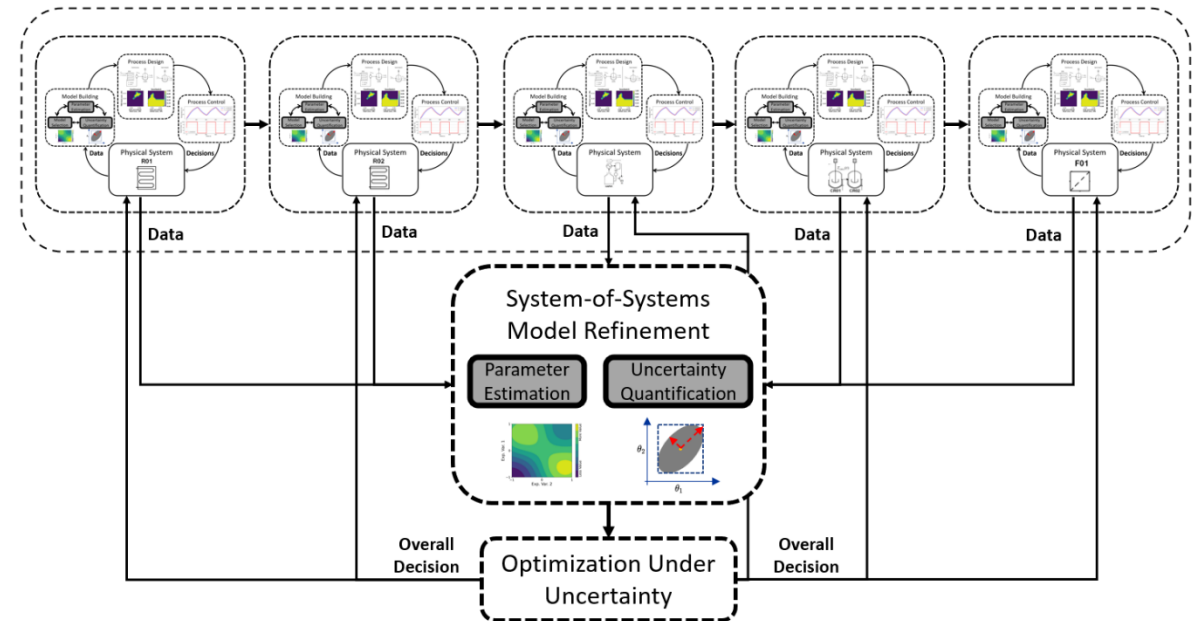



Figure from Abolhasani & Kumacheva (2023), *Nature Syn.*

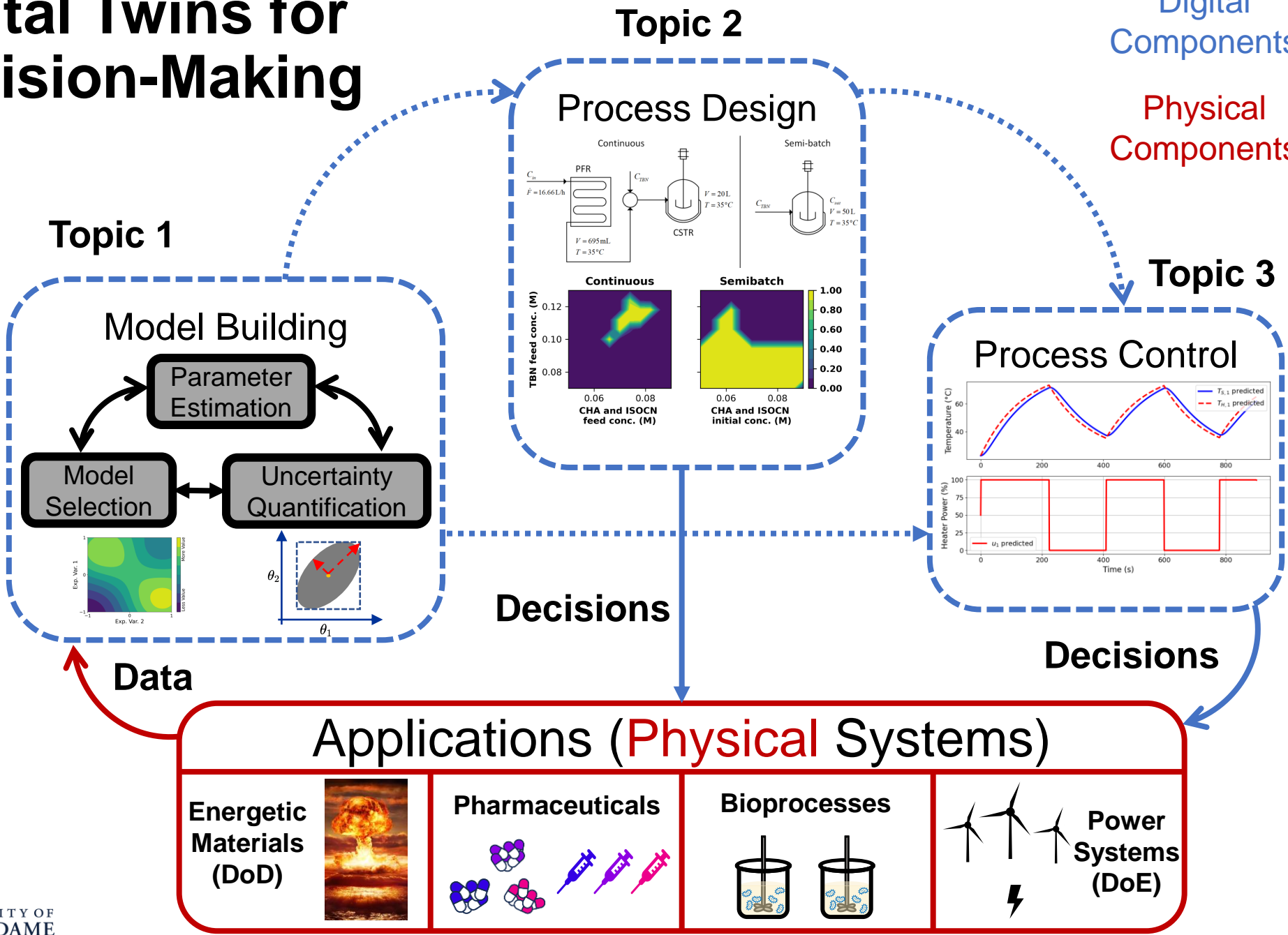
System-of-Systems Digital Twins



Digital Twins for Decision-Making

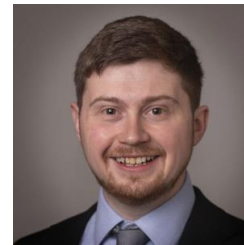
Digital Components 

Physical Components 

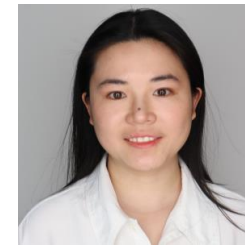


Science-Based Design of Experiments (SBDoe)

University of Notre Dame
Sandia National Lab



Prof. Alexander
Dowling



Dr. Jialu Wang



Hailey Lynch



Dr. Bethany
Nicholson



Dr. John
Siirola

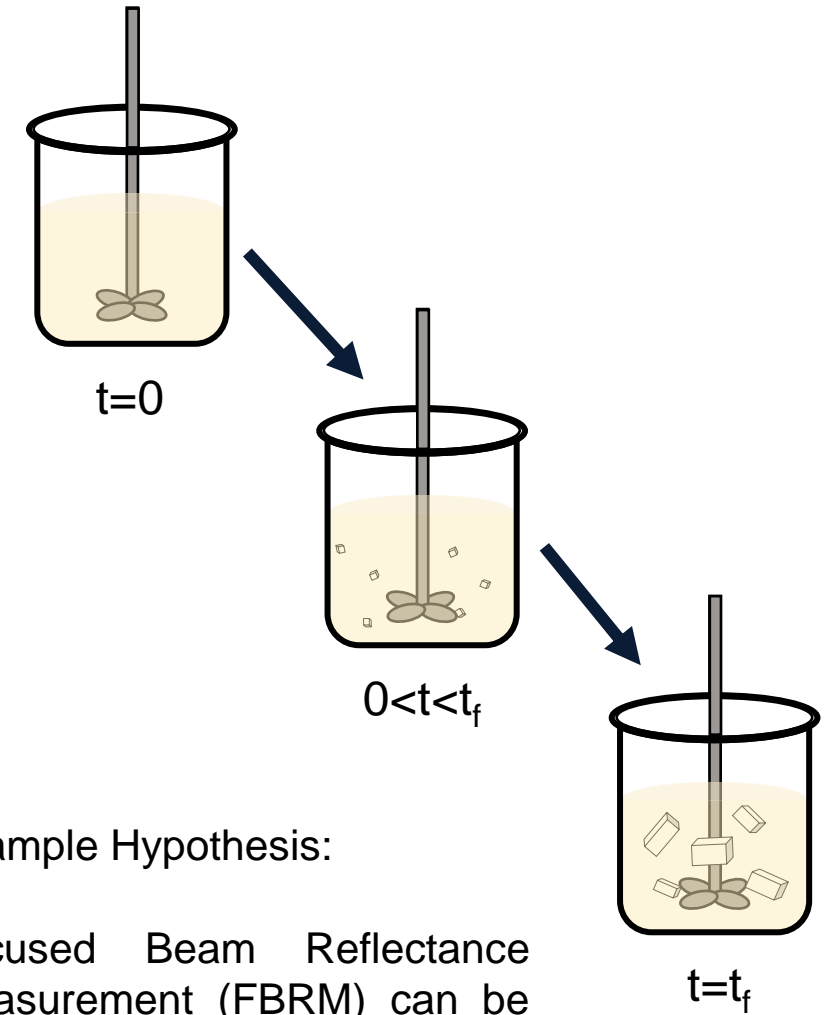
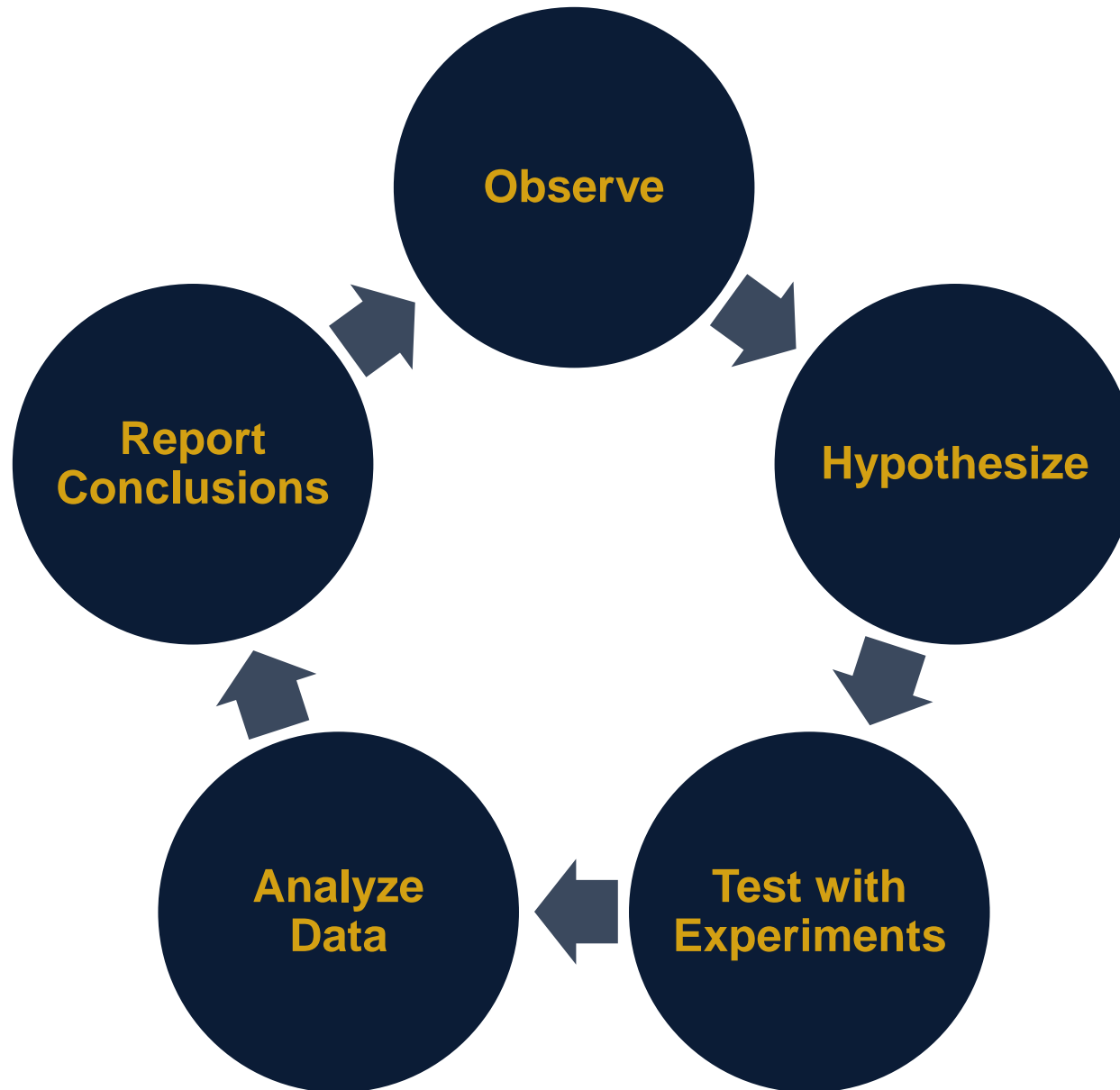


Dr. Shawn
Martin



Katherine
Klise

Scientific Method



Example Hypothesis:

Focused Beam Reflectance Measurement (FBRM) can be used to control particle size in crystallization

(Direct Nucleation Control [DNC])

How easy is it to “Test with Experiments”?



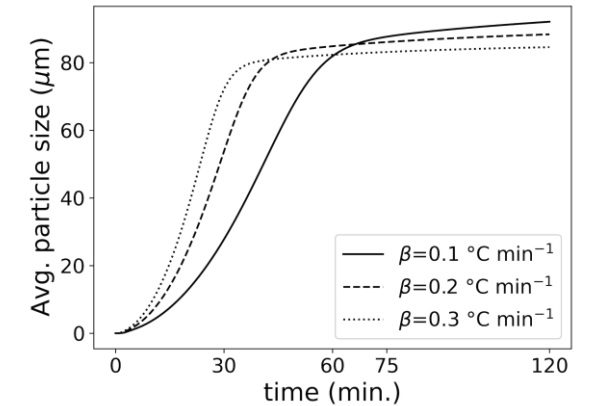
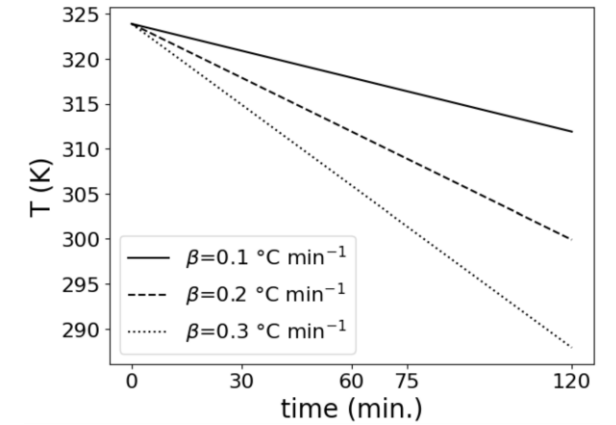
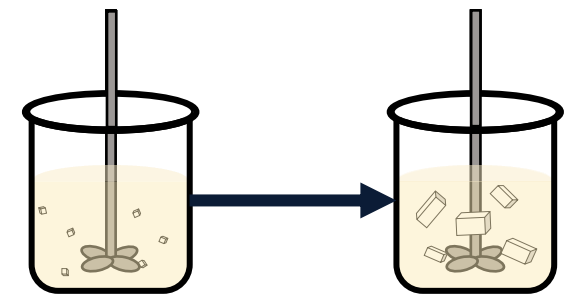
Test with Experiments

- How many experiments should I perform?
- Which experiment(s) will inform our problem best?
- How expensive are the experiments?
 - Material cost
 - Monetary cost
 - Time cost
- Are we developing a model for use in other engineering tasks?

How easy is it to “Test with Experiments”?

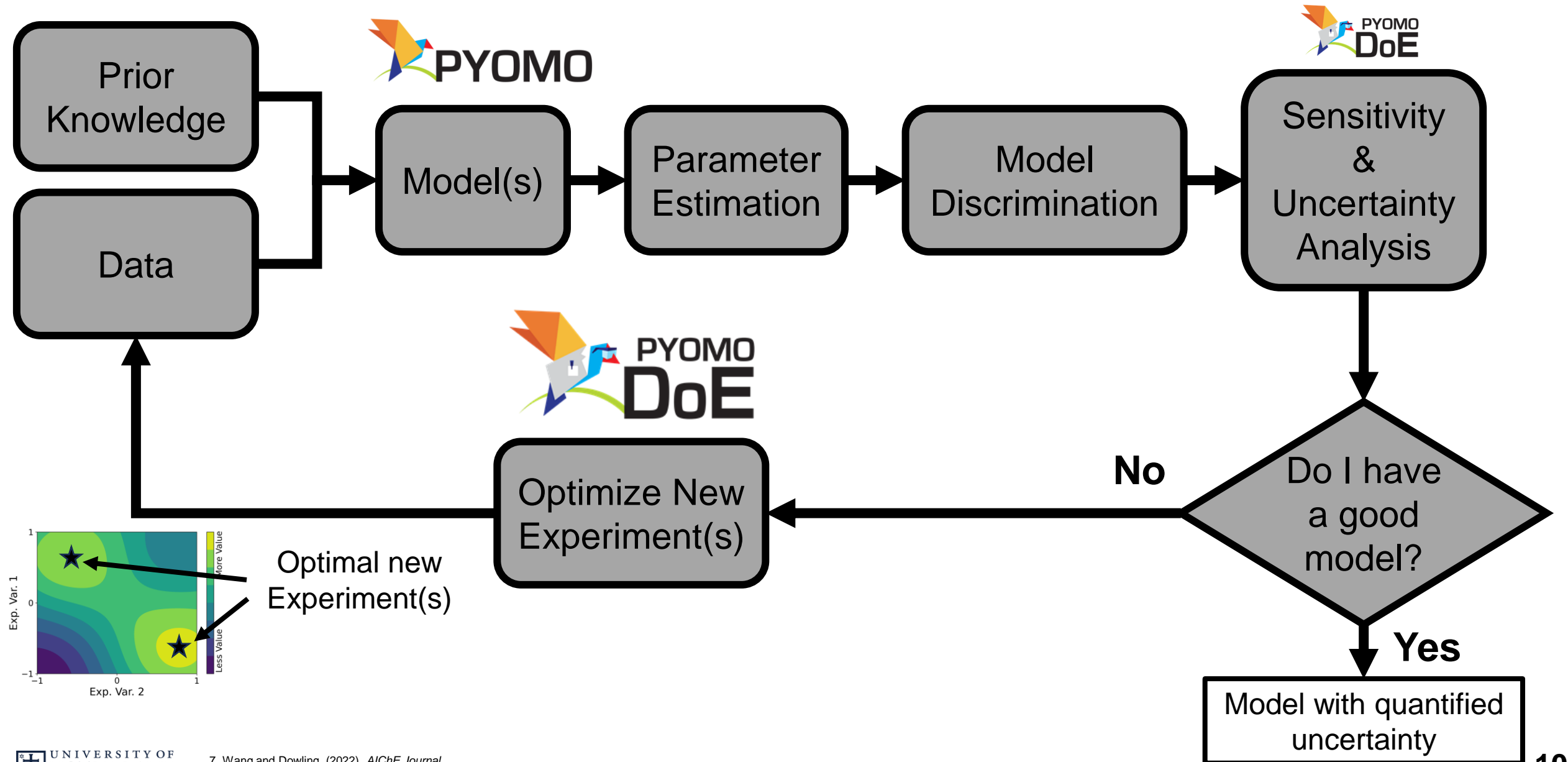
Test with Experiments

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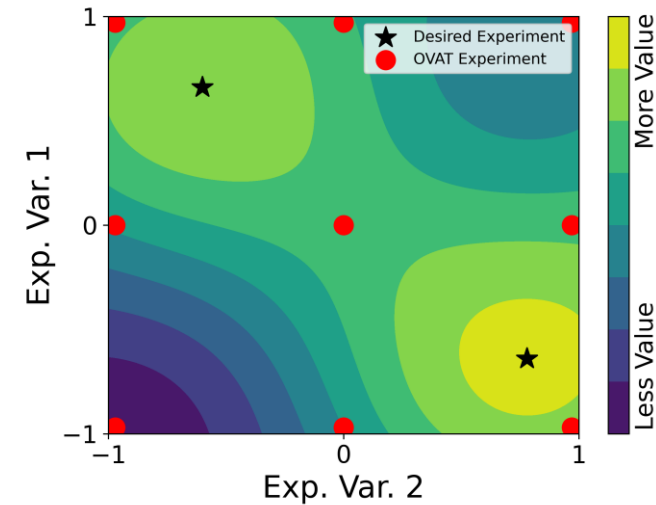
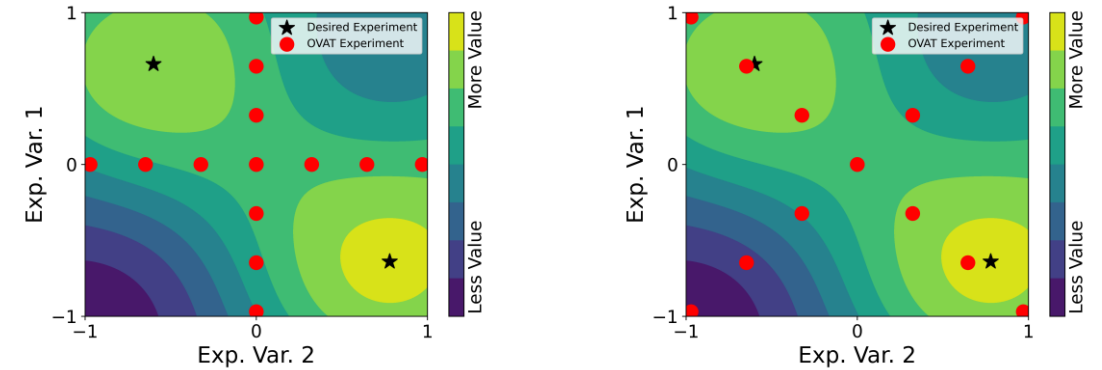
Figures from [6]

Sequential Optimal Experiment Design (Model-Building)⁷



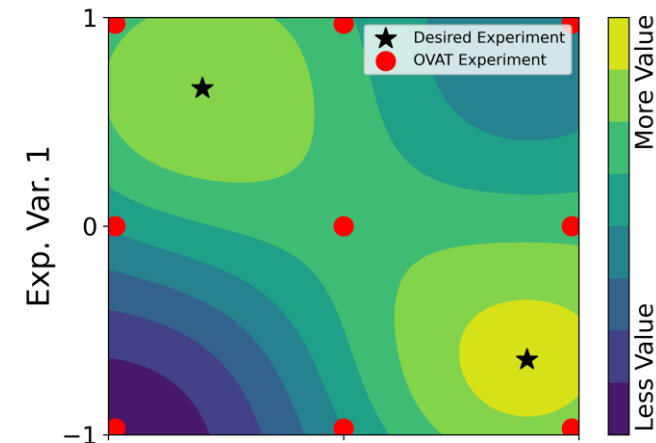
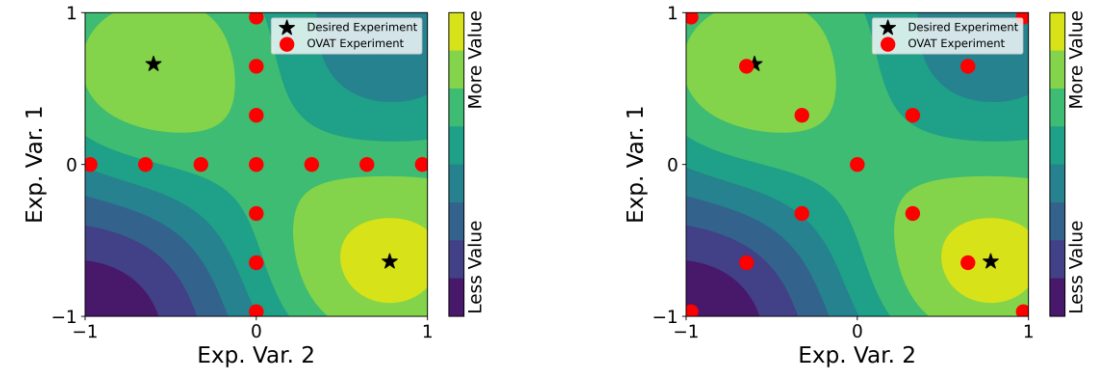
Traditional Experimental Design Strategies Miss Model Information

- One variable at a time⁷
 - Fix variable 1, adjust variable 2
 - Requires some knowledge of the surface to explore critical regions
- Factorial Design⁸
 - Set of values for each design variable
 - Number of experiments grows exponentially with number of design variables



Traditional Experimental Design Strategies Miss Model Information

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 - Number of experiments grows exponentially with number of design variables



- 1. Many experiments required to explore design space**
- 2. Often miss critical regions of the design space**

Brief Crystallization Overview (Case Study)

$$\frac{d\mu_0}{dt} = B_p + B_s$$

$$\frac{d\mu_1}{dt} = G\mu_0$$

$$\frac{d\mu_2}{dt} = 2G\mu_1$$

$$\frac{d\mu_3}{dt} = 3G\mu_2$$

$$\frac{dT}{dt} = \alpha(t)$$

$$\frac{dC_{\text{crys}}}{dt} = -\rho_{\text{crys}} k_v \frac{d\mu_3}{dt}$$

$$B_p = k_{b_p} S^p$$

$$B_s = k_{b_s} S^{s_1} (k_v \mu_3)^{s_2}$$

$$G = k_g S^{g_1}$$

$$S = \frac{C - C_{\text{sat}}}{C_{\text{sat}}}$$

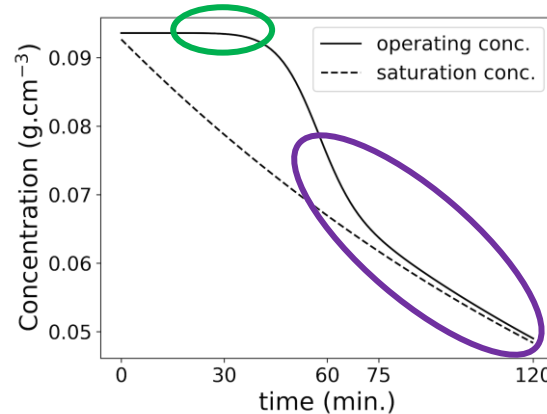
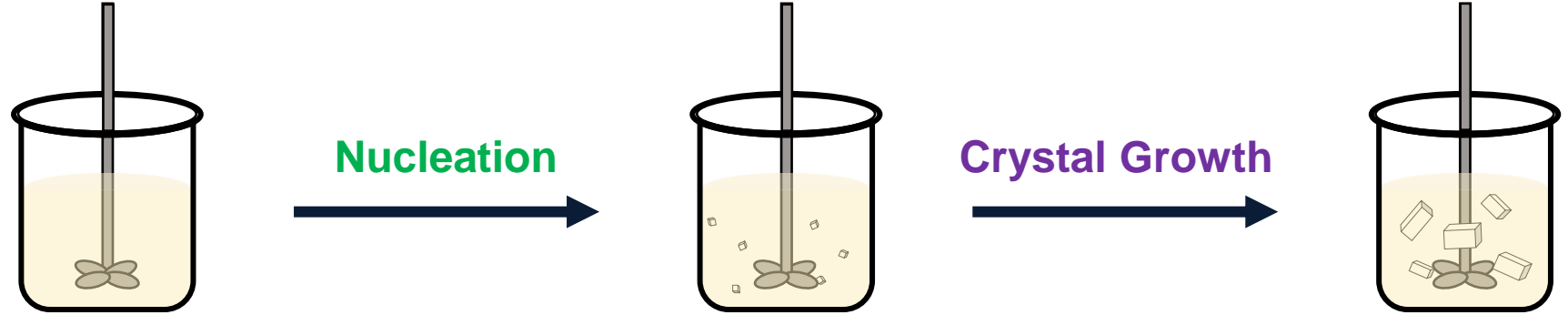
$$C_{\text{sat}} = A + BT + CT^2$$

$$\mu_i(0) = \mu_{i_{\text{seed}}}$$

$$T(0) = T_0$$

$$C_{\text{crys}}(0) = C_0$$

$$i \in (0, 1, 2, 3)$$



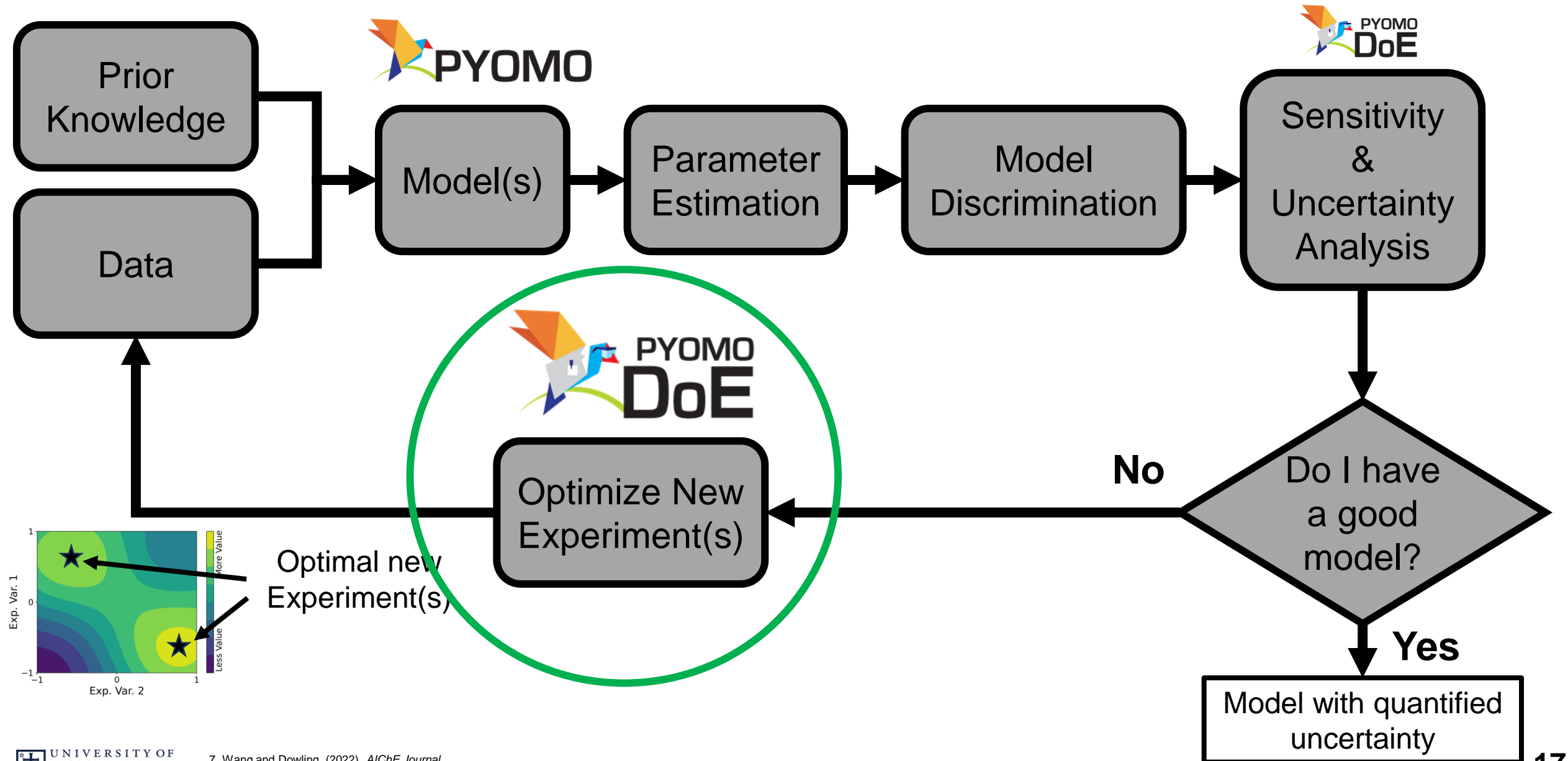
- Cooling crystallization: Solubility reduced by lowering temperature → causes supersaturation

$$S = \frac{C - C_{\text{sat}}}{C_{\text{sat}}}$$

$$C_{\text{sat}} = A + BT + CT^2$$

- At some point, **nucleation** occurs, causing seed crystals to form
- **Growth** becomes the dominant mechanism as supersaturation depletes

Sequential Optimal Experiment Design (Model-Building)⁷



Crystallization Model In General Format

$$\frac{d\mu_0}{dt} = B_p + B_s$$

$$\frac{d\mu_1}{dt} = G\mu_0$$

$$\frac{d\mu_2}{dt} = 2G\mu_1$$

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$$C_{\text{sat}} = A + BT + CT^2$$

$$\mu_i(0) = \mu_{i_{\text{seed}}} \quad i \in (0, 1, 2, 3)$$

$$T(0) = T_0$$

$$C_{\text{crys}}(0) = C_0$$

Measurement
Estimates

Unknown
Parameters

Experimental
Design

f : differential equations

g : algebraic equations

h : measurement functions

\hat{y} : measurement response variable prediction

y : measurement response variable

\dot{x} : derivate of x wrt time

x : dynamic state variable

z : algebraic state variables

u : time-varying control variables

\bar{w} : constant control variables

t : control timepoints

θ : unknown model parameters

ϕ : experimental design vector

Sensitivity Matrix Used to Explain Information Content of Experimental Data

$$\mathbf{Q}_r = \begin{bmatrix} \left. \frac{\partial \hat{y}_r}{\partial \theta_1} \right|_{t_1} & \cdots & \left. \frac{\partial \hat{y}_r}{\partial \theta_p} \right|_{t_1} \\ \vdots & \ddots & \vdots \\ \left. \frac{\partial \hat{y}_r}{\partial \theta_1} \right|_{t_{n_r}} & \cdots & \left. \frac{\partial \hat{y}_r}{\partial \theta_p} \right|_{t_{n_r}} \end{bmatrix}$$

Sensitivity matrix

$$\mathbf{V}(\hat{\boldsymbol{\theta}}, \boldsymbol{\phi}) \approx \left[\sum_r \sum_{r'} \tilde{\sigma}_{(r,r')} \mathbf{Q}_r^T \mathbf{Q}_{r'} + \overbrace{\mathbf{V}_0(\hat{\boldsymbol{\theta}})^{-1}}^{\text{Prior}} \right]^{-1}$$

Parameter covariance matrix

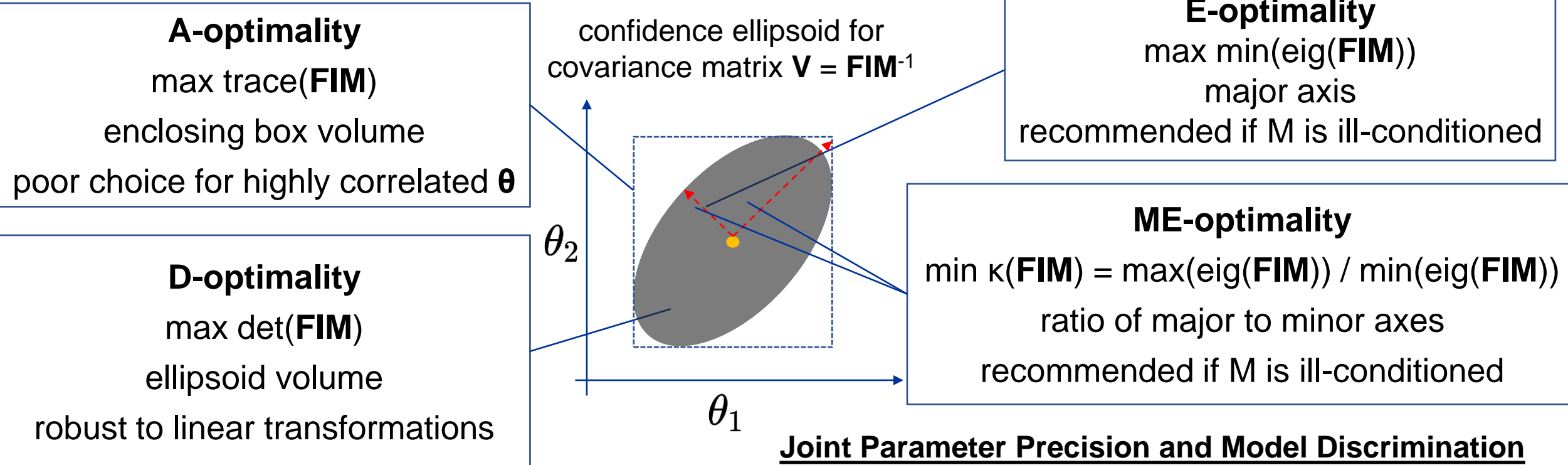
$$\mathbf{FIM} \approx \left[\mathbf{V}(\hat{\boldsymbol{\theta}}, \boldsymbol{\phi}) \right]^{-1}$$

Fisher information matrix (FIM)

- Fisher Information Matrix⁹: Used to understand how an experiment or set of experiments contribute to “information” explained

Alphabetic Design Criteria Measure Information Content

Figure adapted from: Franceschini, G., & Macchietto, S. (2008). *Chem. Eng. Sci.*, 63(19), 4846-4872.



Model Discrimination

Hunter, W.G. and Reiner, A.M., 1965. Designs for discriminating between two rival models. *Technometrics*, 7(3), pp.307-323.

Buzzi-Ferraris, G. and Forzatti, P., 1983. A new sequential experimental design procedure for discriminating among rival models. *Chemical engineering science*, 38(2), pp.225-232.

Ferraris, G.B., Forzatti, P., Emig, G. and Hofmann, H., 1984. Sequential experimental design for model discrimination in the case of multiple responses. *Chemical engineering science*, 39(1), pp.81-85.

Joint Parameter Precision and Model Discrimination

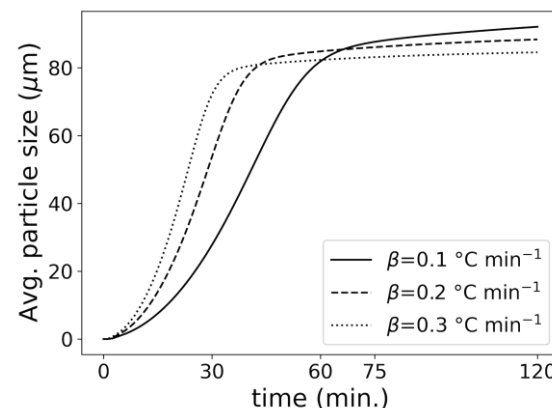
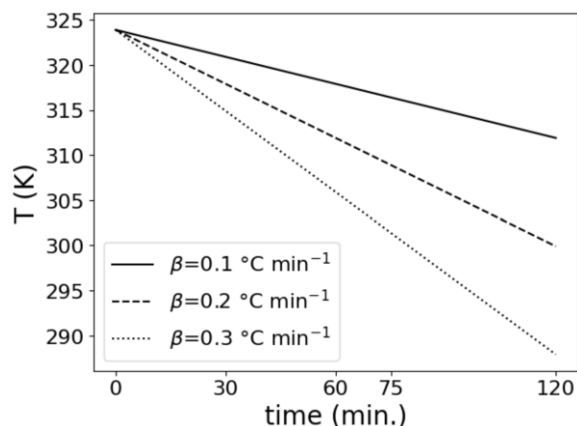
Alberton, A.L., Schwaab, M., Lobão, M.W.N. and Pinto, J.C., 2011. Experimental design for the joint model discrimination and precise parameter estimation through information measures. *Chemical Engineering Science*, 66(9), pp.1940-1952.

Galvanin, F., Cao, E., Al-Rifai, N., Gavrilidis, A. and Dua, V., 2016. A joint model-based experimental design approach for the identification of kinetic models in continuous flow laboratory reactors. *Computers & Chemical Engineering*, 95, pp.202-215.

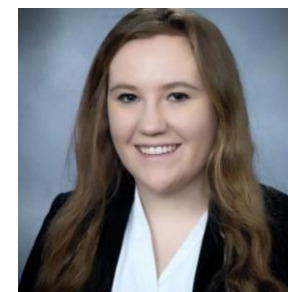
Galvanin, F., Cao, E., Al-Rifai, N., Dua, V. and Gavrilidis, A., 2015. Optimal design of experiments for the identification of kinetic models of methanol oxidation over silver catalyst. *Chimica Oggi-Chemistry Today*, 33(3), pp.51-56.

Pankajakshan, A., Waldron, C., Quaglio, M., Gavrilidis, A. and Galvanin, F., 2019. A Multi-Objective Optimal Experimental Design Framework for Enhancing the Efficiency of Online Model Identification Platforms. *Engineering*, 5(6), pp.1049-1059.

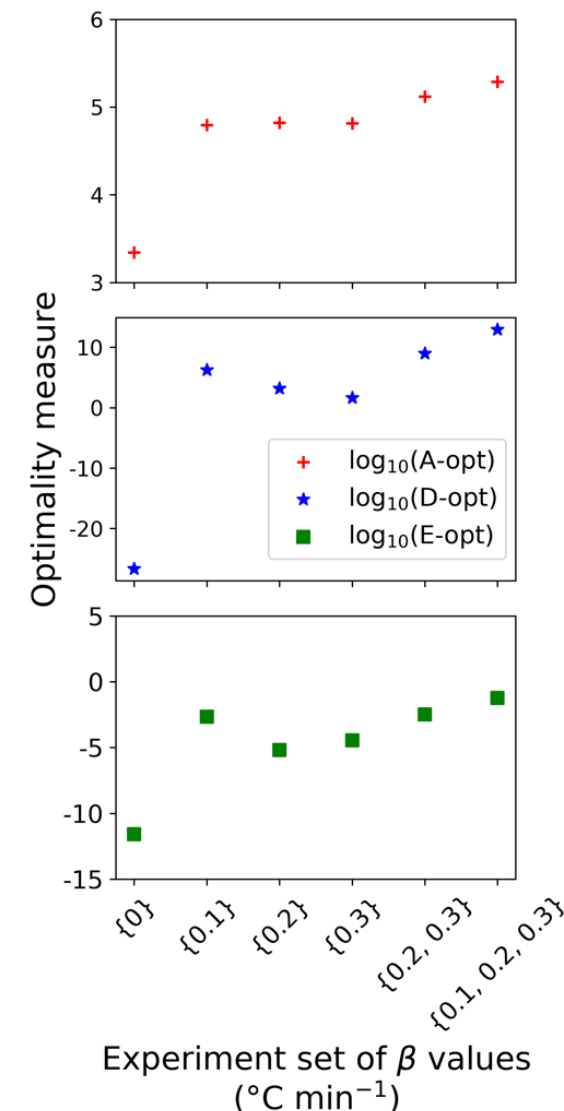
Most Experimental Information at Lowest Cooling Rate



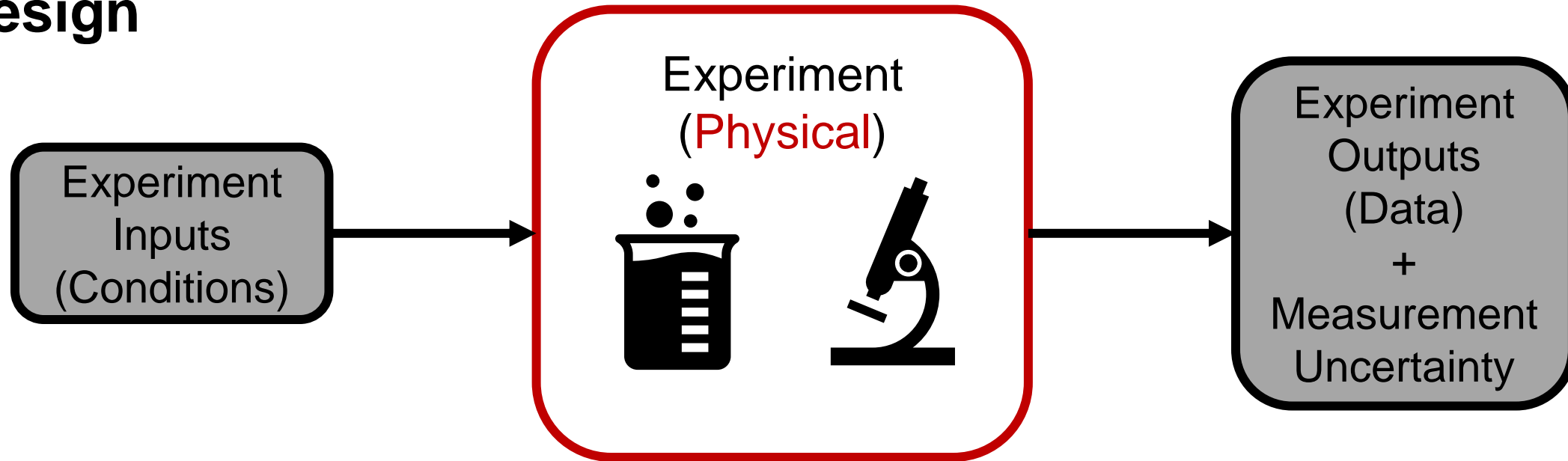
- Explore information content of different cooling rates ($0.1, 0.2$, and $0.3 \text{ } ^\circ\text{C}\cdot\text{min}^{-1}$)
 - Traditional “factorial” DoE target slower cooling rates for growth kinetics
- Low cooling rate is most information-rich experiment (aligns with expert intuition)
- Multiple experiments lead to higher E-optimality, A-optimality, and especially D-optimality
 - D-Optimality \rightarrow Smaller Confidence Region



Hailey Lynch



“Experiment” Abstraction Streamlines Closed-Loop Experiment Design



Dr. Bethany Nicholson



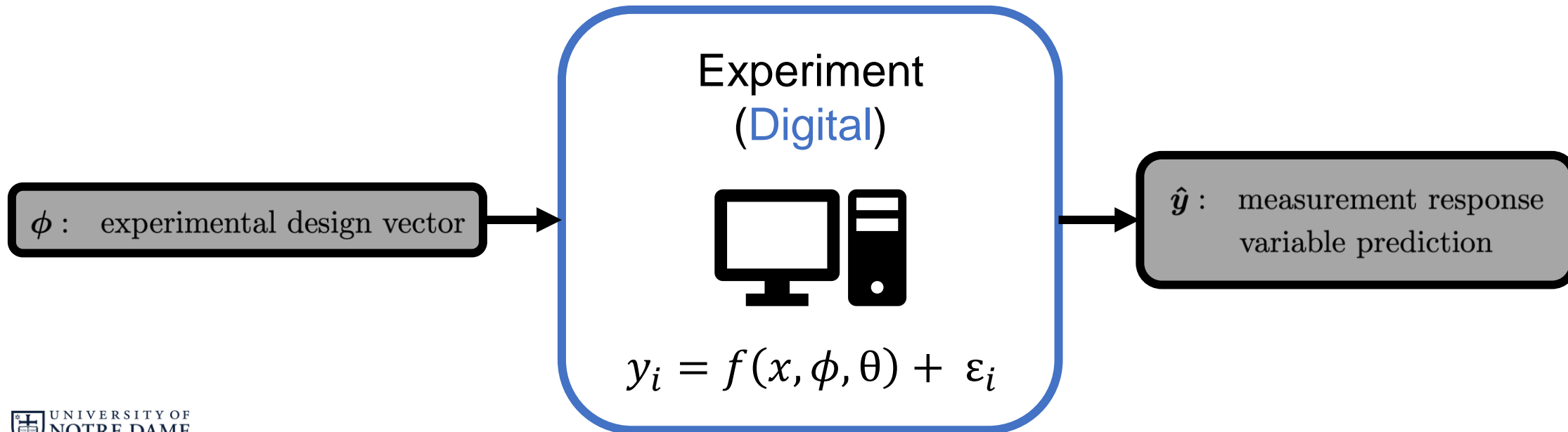
Dr. John Siirola



Dr. Shawn Martin



Katherine Klise



“Experiment” Abstraction Streamlines Closed-Loop Experiment Design



Known Variables

ϕ : experimental design vector
 y : measurement response variable

Experiment (Digital)



$$y_i = f(x, \phi, \theta) + \varepsilon_i$$

Unknown Variables

θ : unknown model parameters

$$\min_{\theta} \sum_i (y_i - \hat{y}_i)^2$$



Dr. Bethany
Nicholson



Dr. John
Siirola



Experiment (Digital)



$$y_i = f(x, \phi, \theta) + \varepsilon_i$$

θ : unknown model parameters

ϕ : experimental design vector

$$\max_{\phi_l \leq \phi \leq \phi_u} \Psi (\text{FIM} (\theta, \phi))$$



Dr. Shawn
Martin



Katherine
Klise

Temperature Control Lab (TC-Lab) – Closed-Loop Experimental Design with New Experiment Abstraction

Parameter Estimation

$$C_p^H \frac{dT_{H,1}}{dt} = U_a(T_{amb} - T_{H,1}) + U_b(T_{S,1} - T_{H,1}) + \alpha P_1 u_1$$

$$C_p^S \frac{dT_{S,1}}{dt} = U_b(T_{H,1} - T_{S,1})$$

$$\theta \equiv \{C_P^H, C_P^S, U_a, U_b\}$$

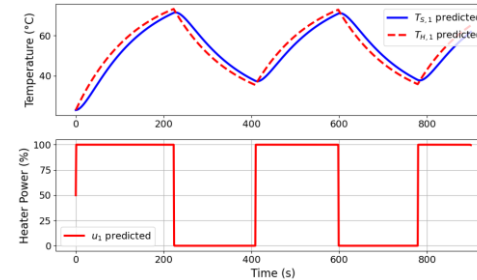
$$y \equiv \{T_{S,1}\}$$

$$\phi \equiv \{u_1\}$$

 **PYOMO** - parmes



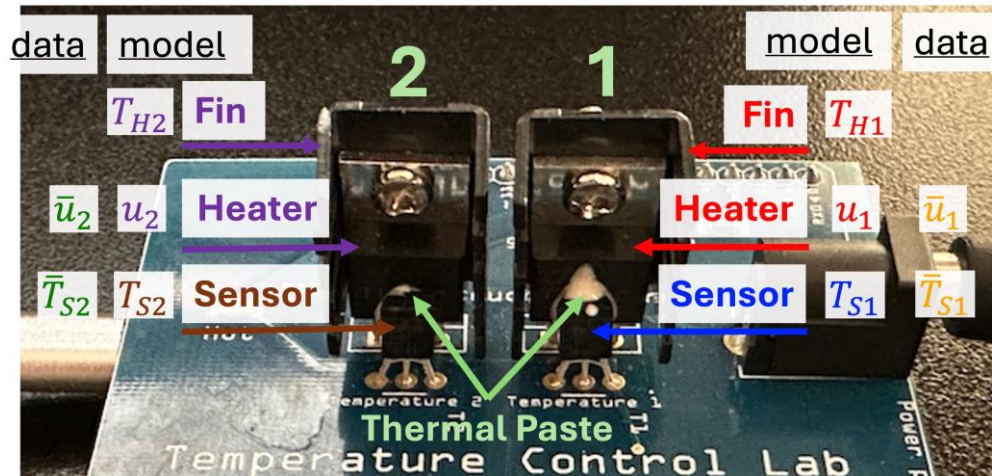
Optimal Experiment Design/Optimal Control



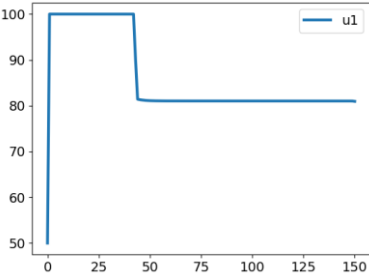
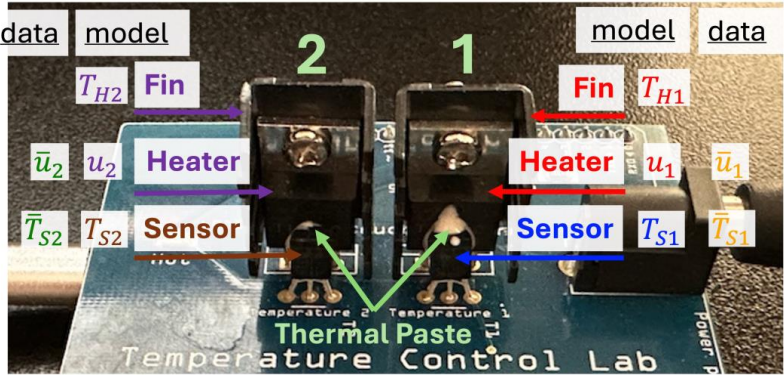
Digital Components 

Physical Components 

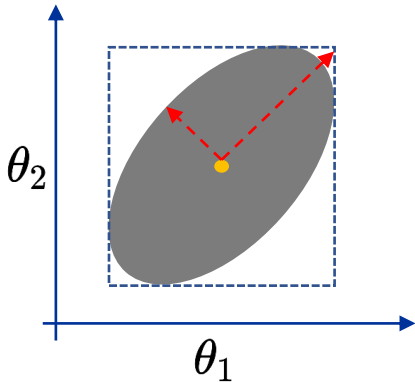
Physical System



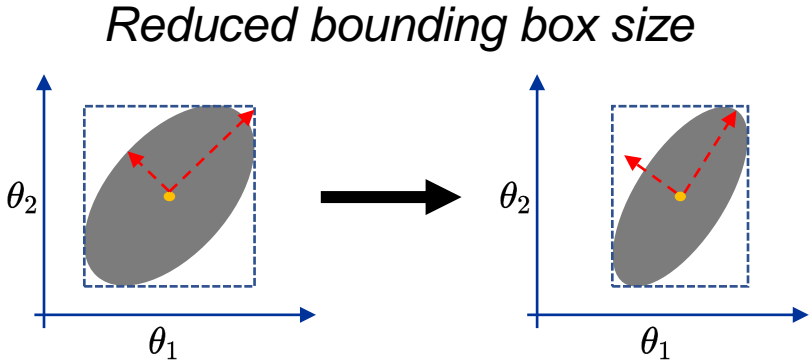
Optimal Experiments Reduce Uncertainty Depending on FIM Criteria



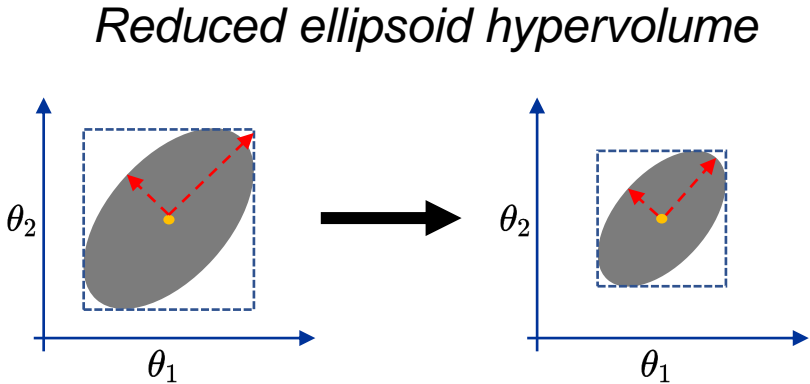
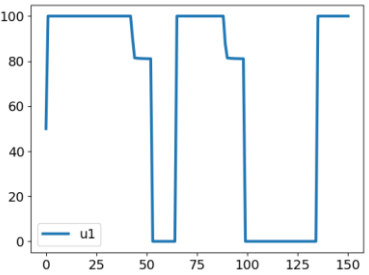
Initial Uncertainty



A-optimal

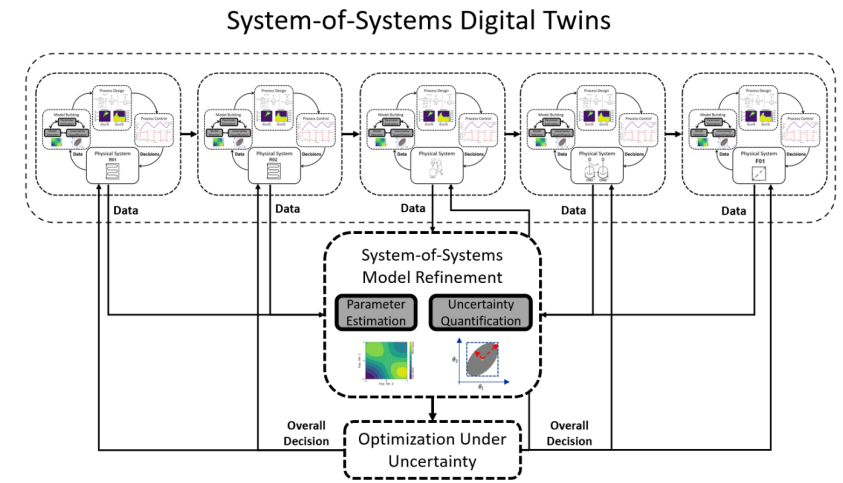
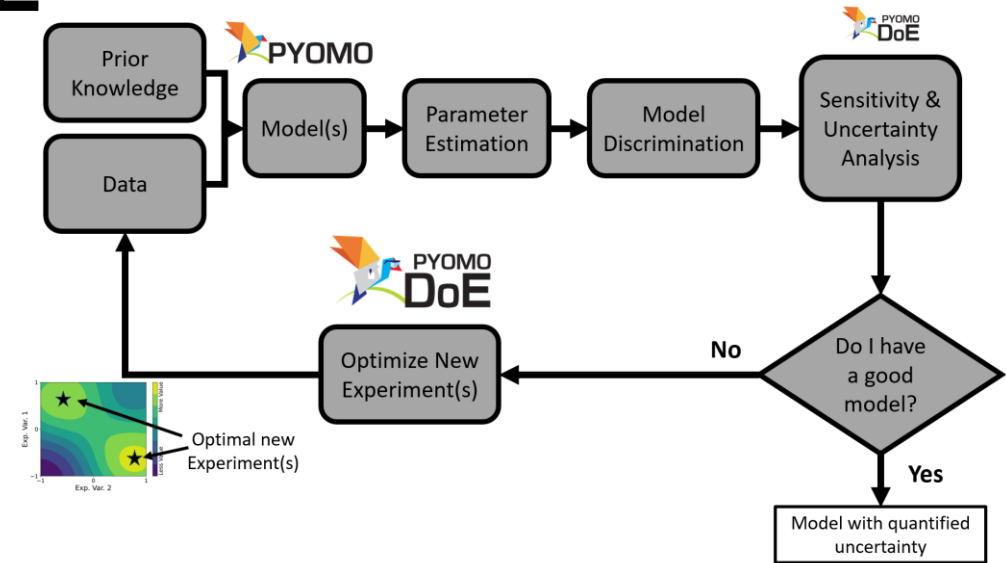


D-optimal



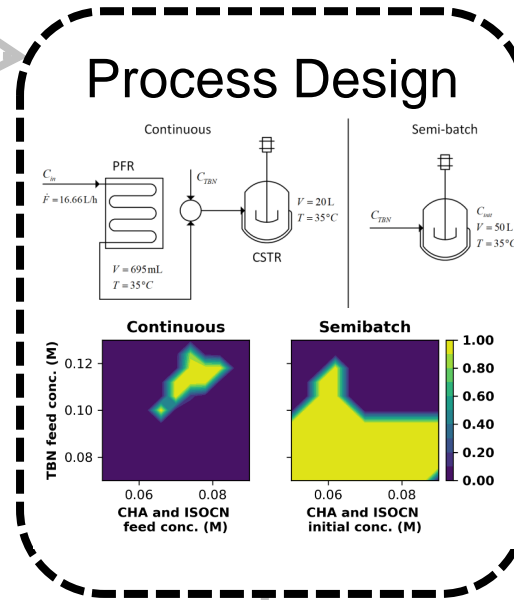
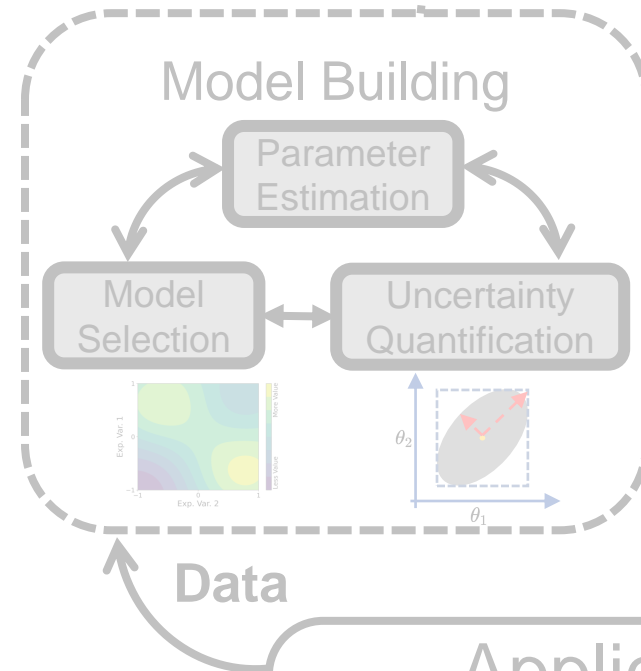
Summary and Future Work in SBDoE

- Pyomo.DoE is a capable and easy-to-use tool for implementing science-based design of experiments (SBDoE) to build digital twins
- Abundant opportunities for future collaboration
- In the future we want to move past individual subsystems toward “system-of-systems” digital twins
 - Uncertainty propagation
 - Subsystem interactions

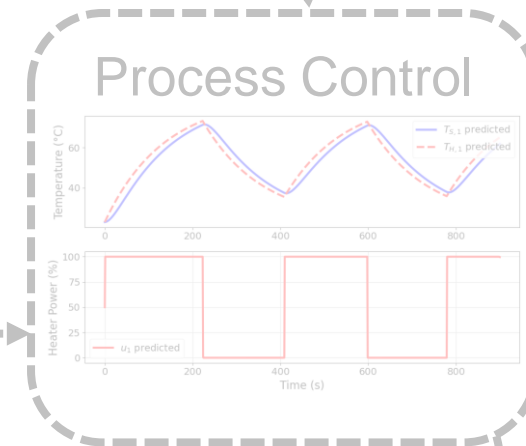


Topic 2

Topic 1

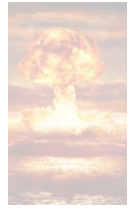


Topic 3



Applications (Physical Systems)

Energetic Materials (DoD)



Pharmaceuticals



Bioprocesses



Power Systems (DoE)

Optimal Process Design (Pharmaceuticals and Energy Systems)

Purdue University
University of Notre Dame



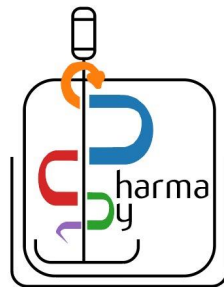
**Dr. Daniel
Casas-Orozco**



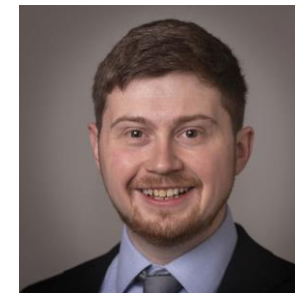
**Prof. Gintaras
Reklaitis**



**Prof. Zoltan
Nagy**



IDAES
Institute for the Design of
Advanced Energy Systems



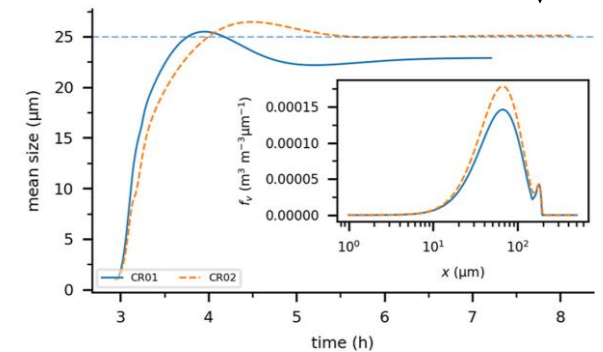
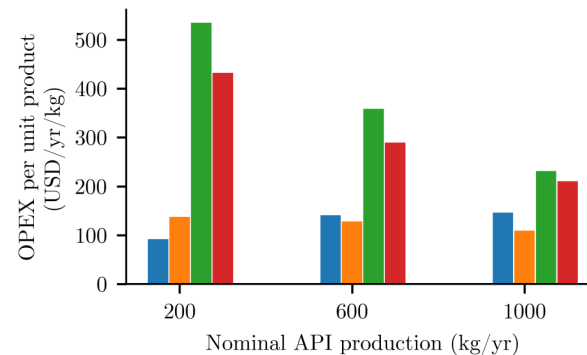
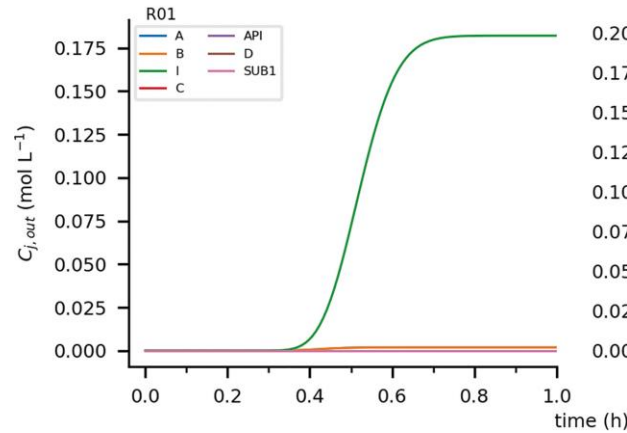
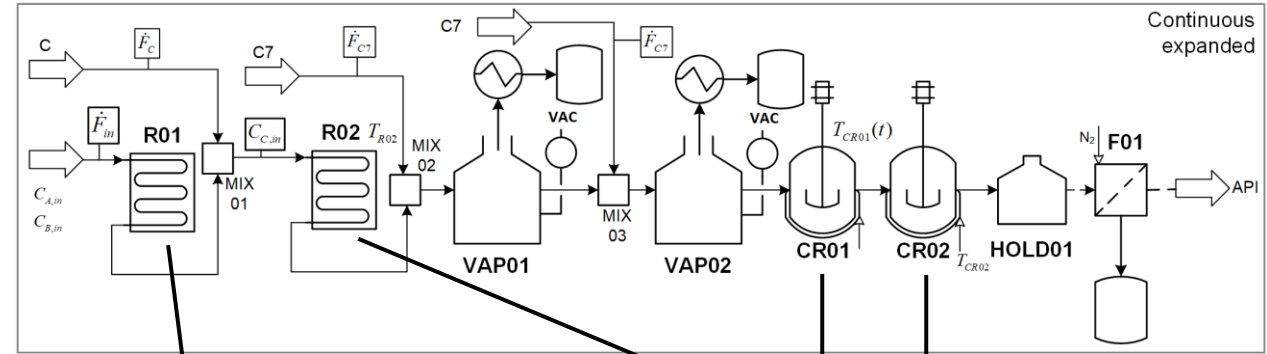
**Prof. Alexander
Dowling**



Nicole Cortes

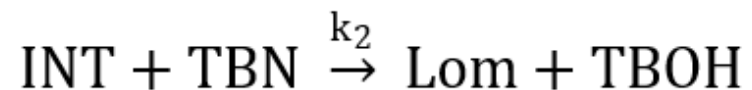
PharmaPy: An Open-Source Process Simulator in Python

- Python-based modeling
 - Code-based UI
- Optimal process design and redesign
 - Modeling and simulation of manufacturing processes
- Evaluate economics for manufacturing processes



Motivating Example: Brain Cancer Treatment – Lomustine

- Lomustine is an anti-cancer drug, began being produced by Bristol-Myers Squibb in 1976
- Was re-released as Gleostine in 2013 by NextSource, after which the price dramatically increased over the next years [27]
- Our colleagues at Purdue have developed a new pathway for a two-step flow synthesis of Lomustine [28] in 2019



NEXTSOURCE
PHARMACEUTICALS

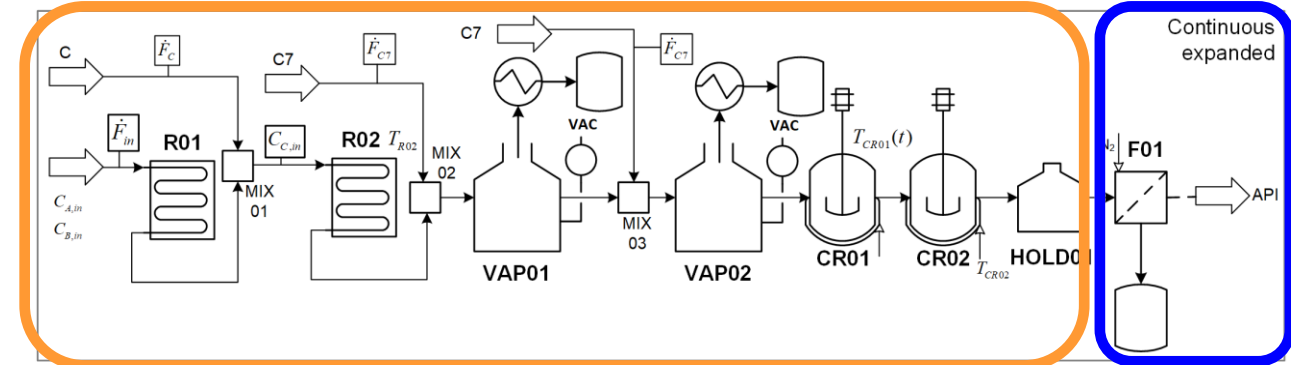
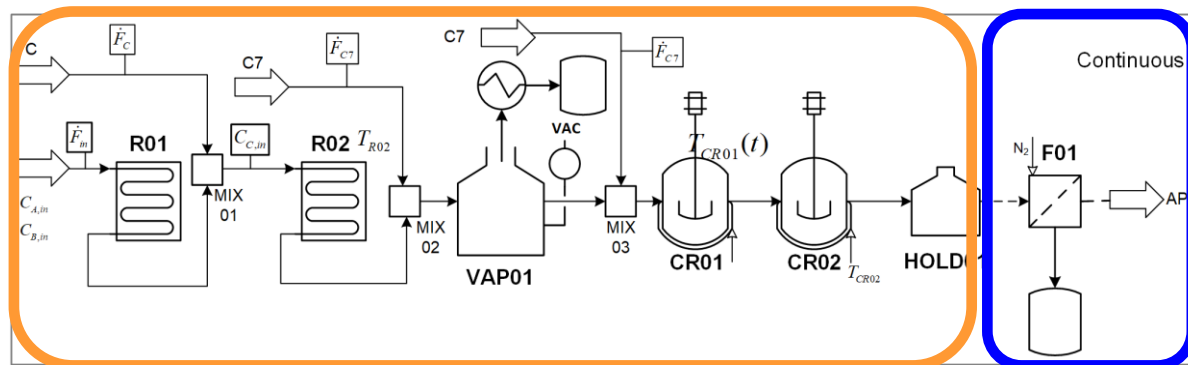
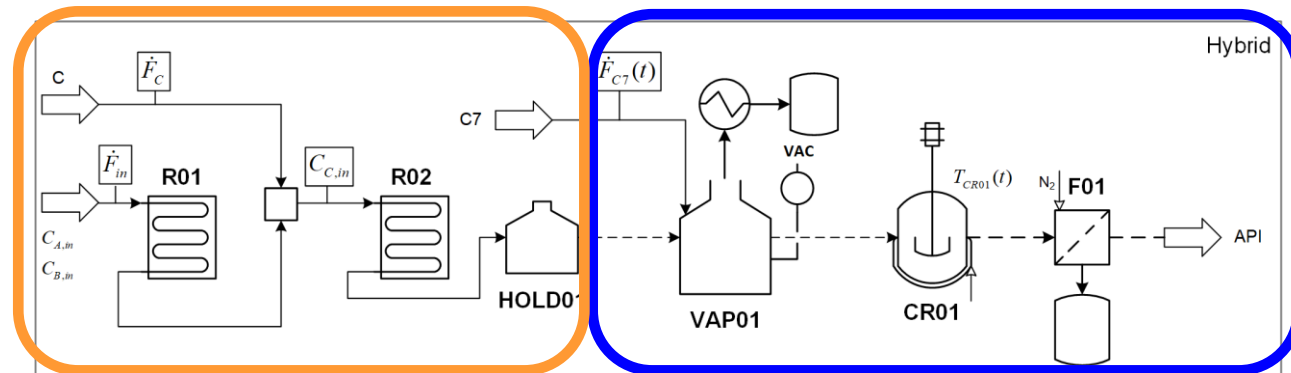
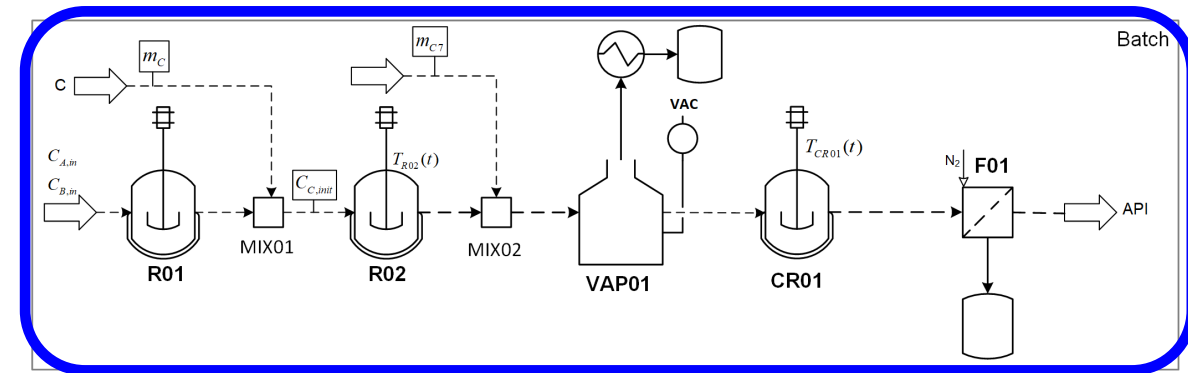
- Reaction 1 occurs very quickly when compared to Reaction 2
- Goals:
 - Determine an efficient manufacturing route for Lomustine
 - Refine kinetic models for Lomustine synthesis
 - Identify flexible operating conditions for Lomustine synthesis under uncertainty

[27] Loftus, P., (2017) *The Wall Street Journal*,

[28] Jaman, Z. et. al. (2019) *Org. Process Res. Dev.*

PharmaPy Digital Twins Can Help Compare Manufacturing Candidates²

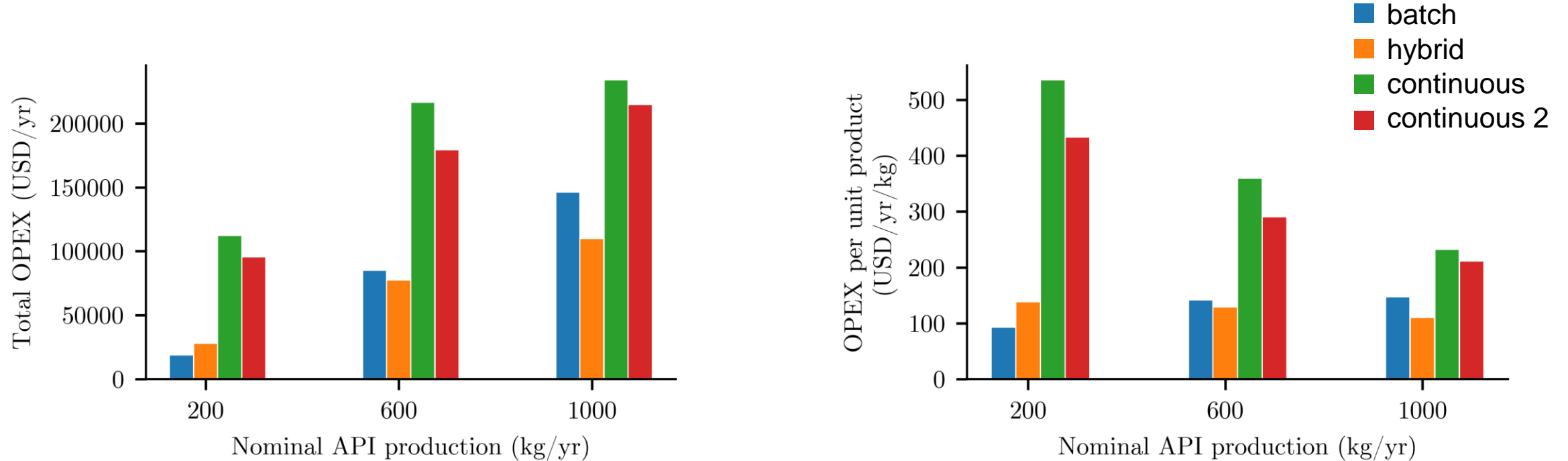
- Analyze production of Lomustine for different routes and scales
 - Scales: 200kg, 600kg, 1000kg
 - Modes: Batch, Hybrid, Continuous



 Continuous unit operations

 Batchwise unit operations

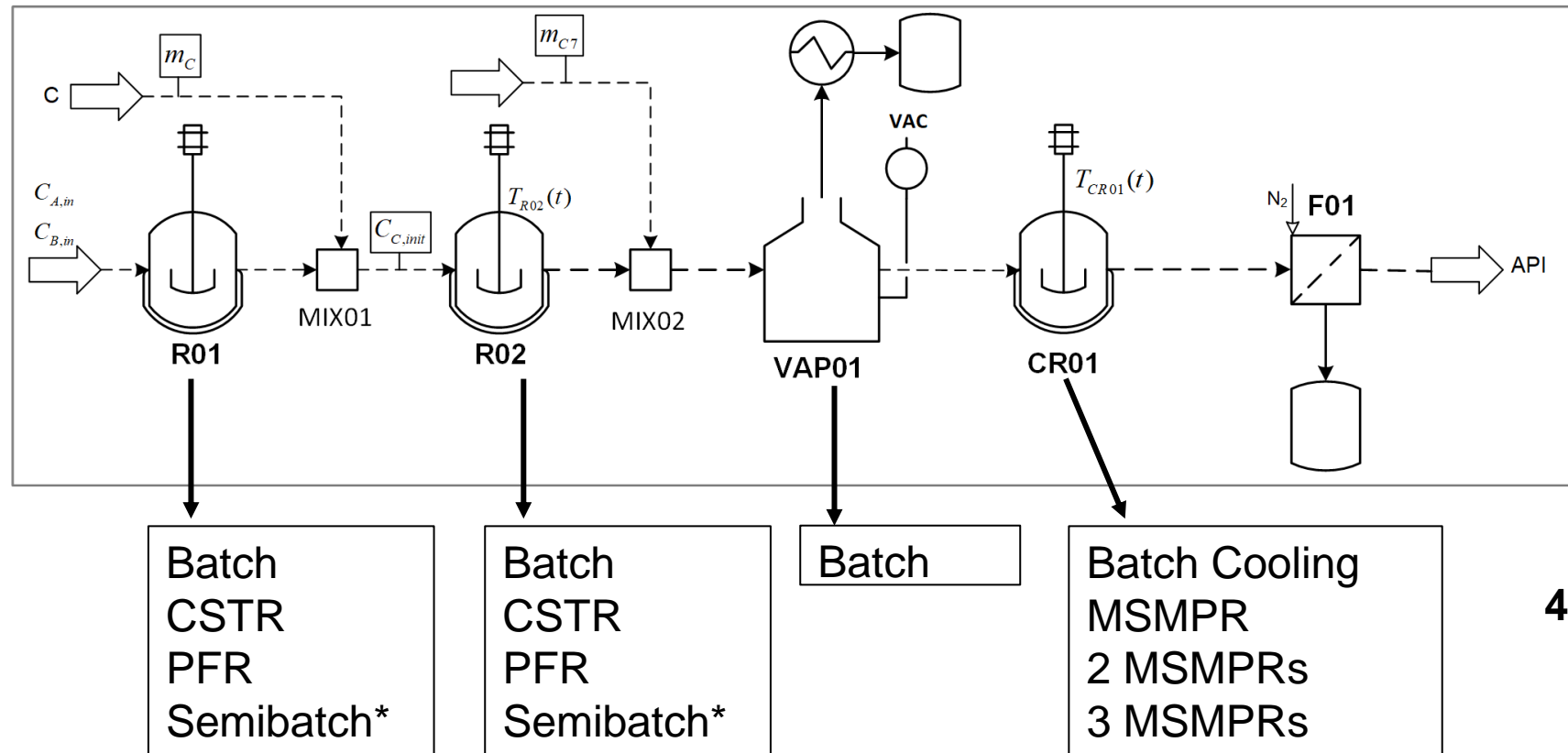
PharmaPy Digital Twins Can Help Compare Manufacturing Candidates²



- Derivative free optimization (Nelder-Mead in scipy with penalties for constraint violation)
- Batch or Hybrid operating modes have the lowest cost for all production scenarios
- As scale increases, continuous is becoming more desirable

PharmaPy Digital Twins Can Help Identify Optimal Manufacturing Routes¹⁵

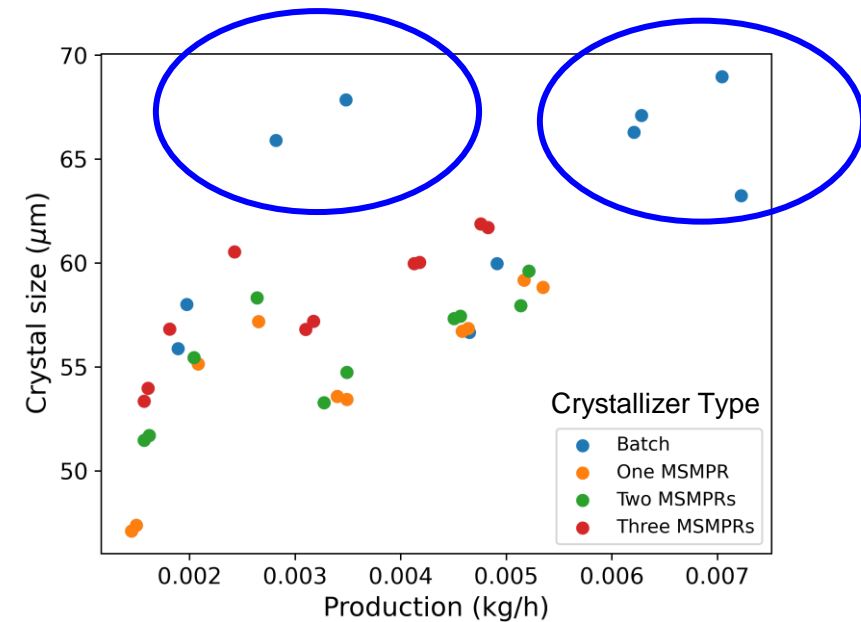
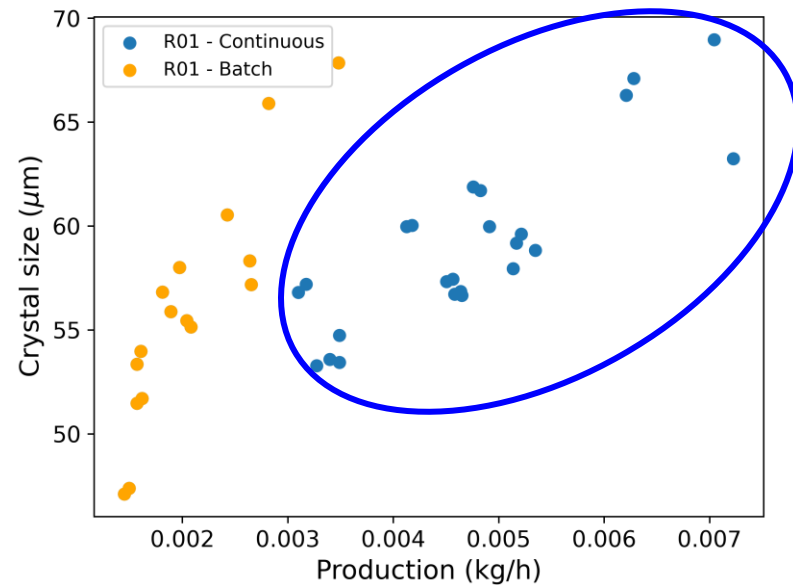
- Generate API with the optimal manufacturing route
- Process includes: (1) Synthesis, (2) Solvent Switch (vaporization), (3) Crystallization, (4) Filtration



40 Manufacturing Pathways

PharmaPy Digital Twins Can Help Identify Optimal Manufacturing Routes¹⁵

- Major observations from pareto-optimal curves:
 1. First reactor should not be batch
 2. Batch cooling crystallization yields much better size and production than the continuous MSMPRs

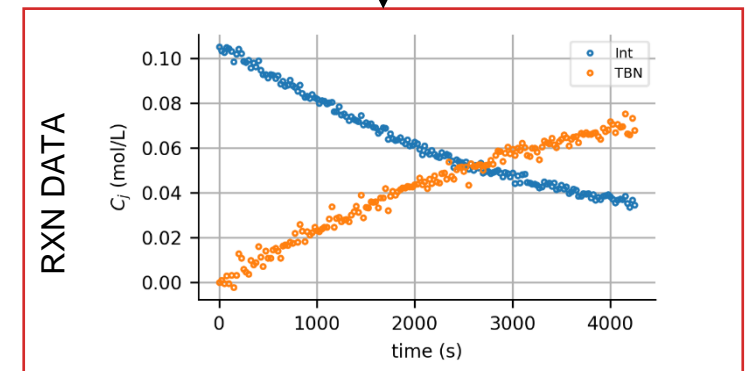
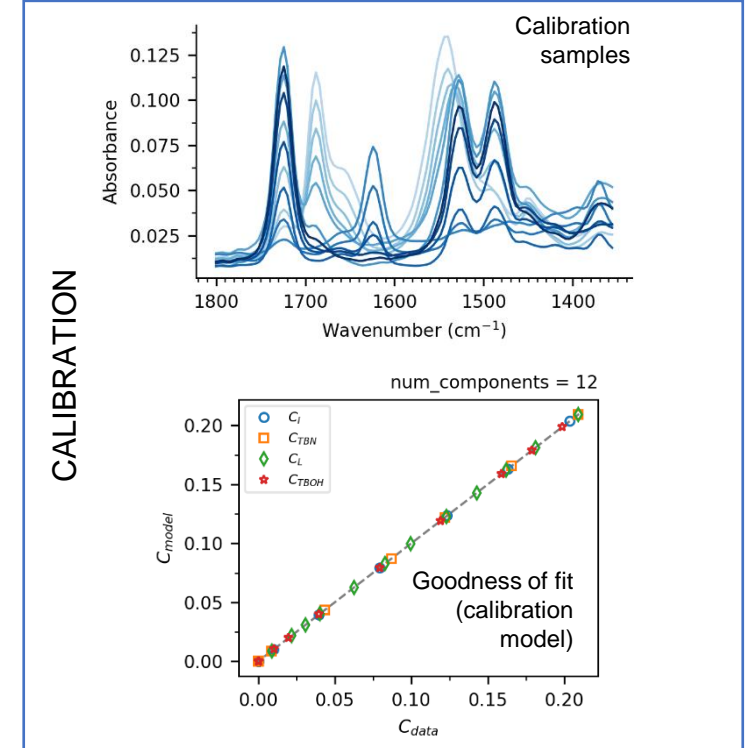


- Heuristic potentially reduces computational time:
 - We know reaction 1 is fast, so choose continuous reactor 2 if reactor 1 is continuous (reduces computational requirement by 25%)

PharmaPy Digital Twins Can Aid in Uncertainty Identification¹⁴

- Estimate kinetic parameters Lomustine reaction mechanism
- **Lomustine** (L) synthesis:
$$I + TBN \xrightarrow{THF} \textcolor{red}{L} + TBOH$$
- Partial Factorial DoE:

Experiment	$C_I : C_{TBN}$ ratio	T (°C)
1	1	15
2	2	15
3	1	25
4	2	25
5	1	35
6	2	35



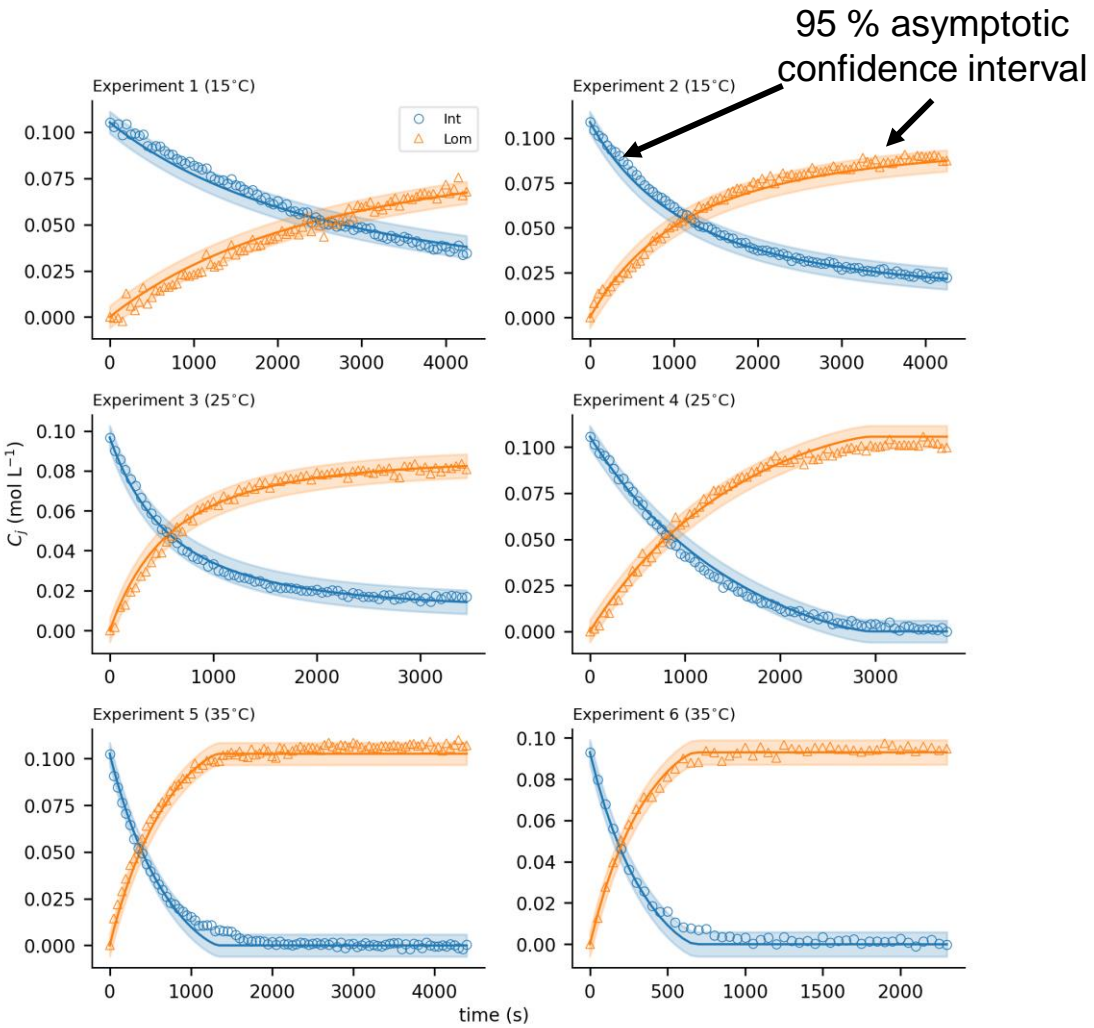
PharmaPy Digital Twins Can Aid in Uncertainty Identification¹⁴

Best of 5 reaction mechanisms considered [14]

$$r = k (C_I)^{\alpha_I} (C_{TBN})^{\alpha_{TBN}}$$
$$k = \exp \left(\phi_1 + \exp(\phi_2) \left(\frac{1}{T_{mean}} - \frac{1}{T} \right) \right) \quad \phi_1 = \ln(A) - \frac{E_a}{RT_{mean}} \quad \phi_2 = \ln \left(\frac{E_a}{R} \right)$$

• Partial Factorial DoE:

Experiment	$C_I : C_{TBN}$ ratio	T (°C)
1	1	15
2	2	15
3	1	25
4	2	25
5	1	35
6	2	35



14. Casas-Orozco et al. (2023), Chem. Eng. Science

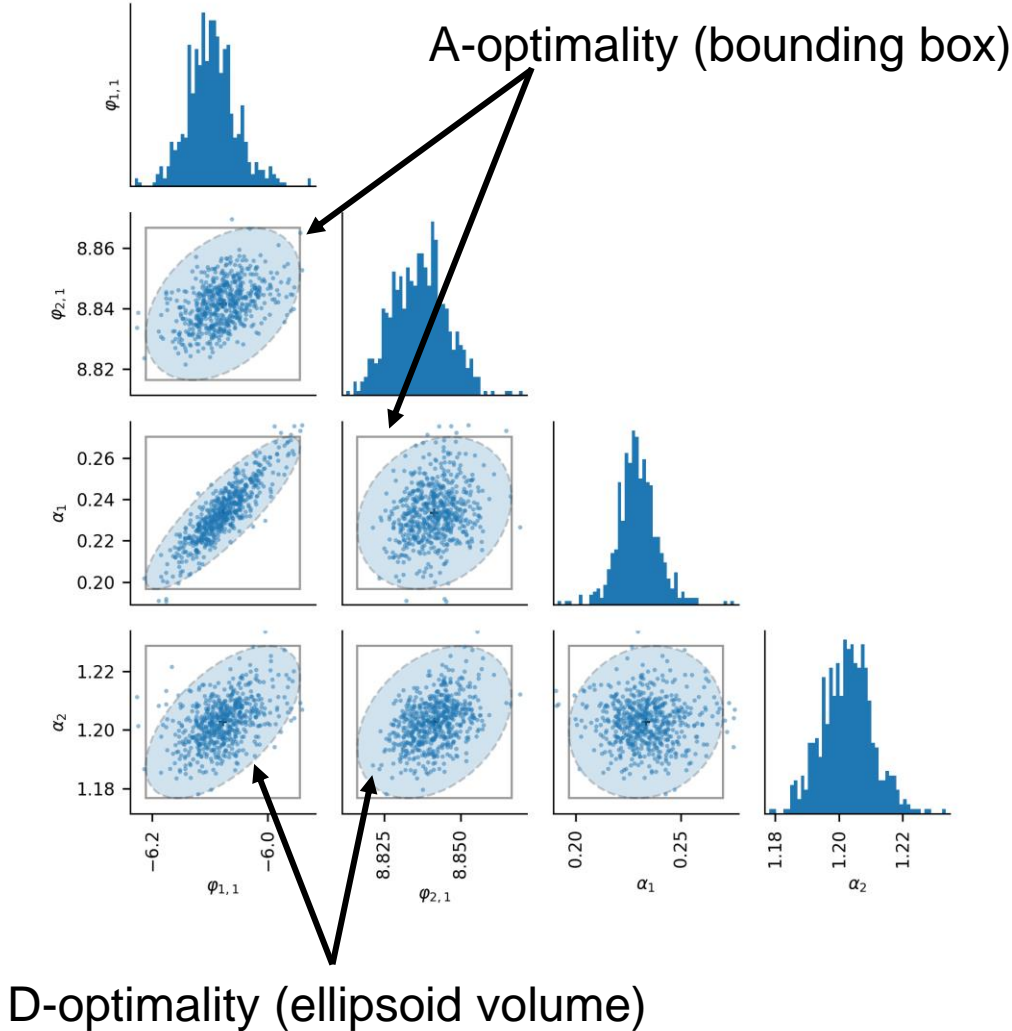
PharmaPy Digital Twins Can Aid in Uncertainty Identification¹⁴

- Bootstrapping with PharmaPy:

$$\mathbf{y}_{boot,k} = \mathbf{y}(\boldsymbol{\theta}^*) + \boldsymbol{\varepsilon}_{boot,k}(\boldsymbol{\theta}^*), \quad k = \{1, \dots, n_{samples}\}$$

Final residual $\boldsymbol{\varepsilon}(\boldsymbol{\theta}^*)$ is sampled **with replacement** to produce a set of $\boldsymbol{\varepsilon}_{boot,k}(\boldsymbol{\theta}^*)$

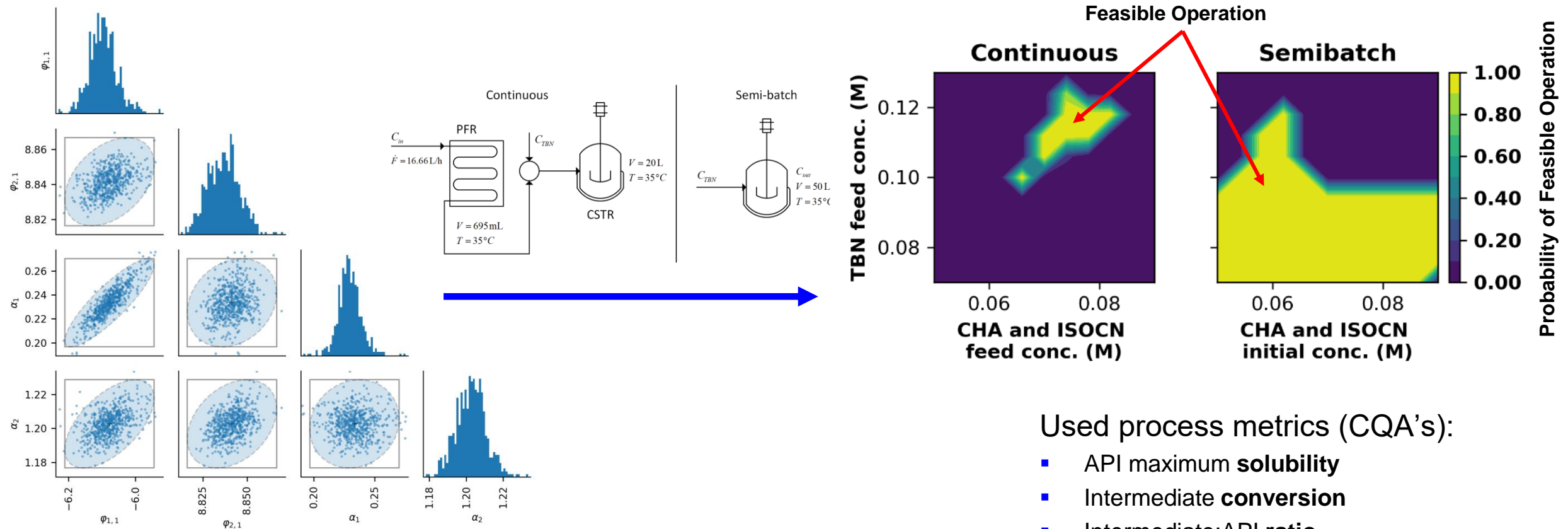
Parameter	Estimate	95% CI (asymptotic)
φ_1	-6.073	± 1.39
φ_2	8.818	± 0.18
α_I	0.246	± 9.52
α_{TBN}	1.189	± 1.38



14. Casas-Orozco et al. (2023), Chem. Eng. Science

PharmaPy Digital Twins Can Incorporate Uncertainty for More Robust Optimization and Design Space Analysis¹⁵

- Using covariance information/confidence intervals, identify what operating regions are feasible (probabilistic design space)
- Semi-batch has a larger region of feasibility than the continuous process



Optimal Process Design (Pharmaceuticals and Energy Systems)

Purdue University
University of Notre Dame



Dr. Daniel
Casas-Orozco



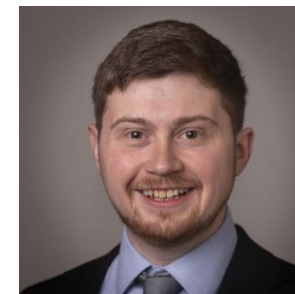
Prof. Gintaras
Reklaitis



Prof. Zoltan
Nagy



IDAES
Institute for the Design of
Advanced Energy Systems



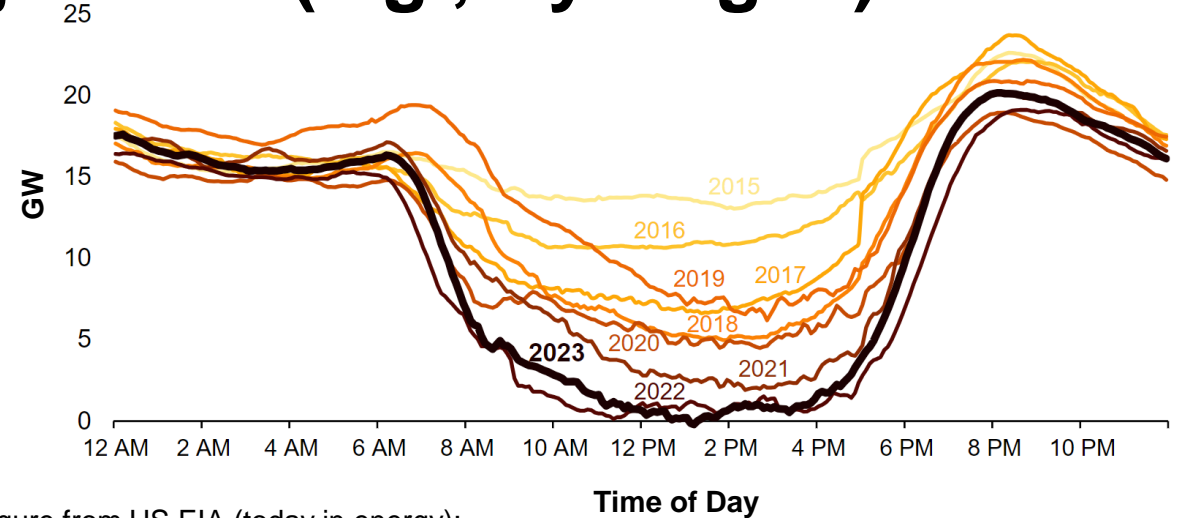
Prof. Alexander
Dowling



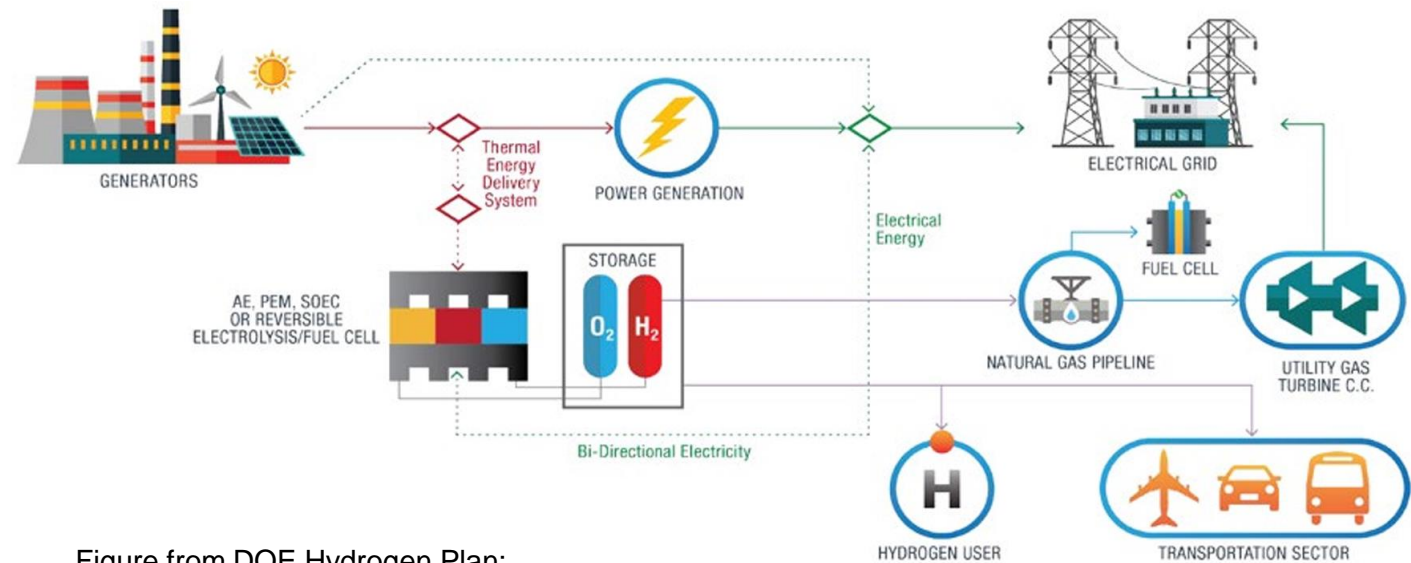
Nicole Cortes

Energy Systems Require Increased Flexibility and Must Adapt to Modern Sustainable Energy Objectives (e.g., Hydrogen)

- Increased renewable generation requires innovative energy demand solutions



- Hydrogen as an emerging fuel source needs technological evaluation paradigms to understand benefit

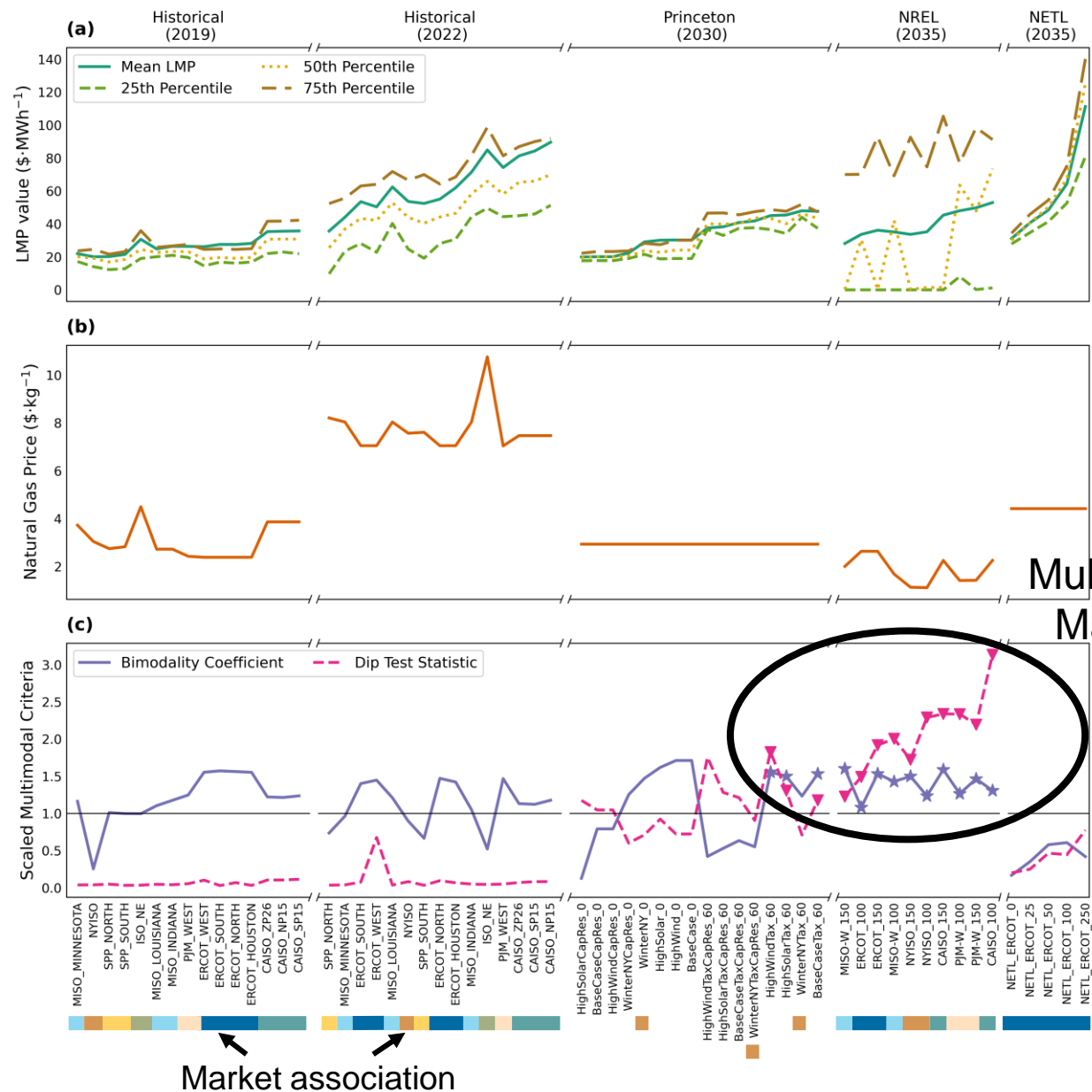


61 Markets Used to Evaluate Emerging Technologies



Figure from FERC (<https://www.ferc.gov/electric-power-markets>)

- 15 historical markets (2019)
- 15 “current” markets (2022)
- 16 forecasted scenarios (2030)
 - “Princeton”
- 10 forecasted scenarios (2035)
 - “NREL”
- 5 forecasted scenarios (2035)
 - “NETL”



Emerging Co-production Technologies Make a Profit in Most Scenarios

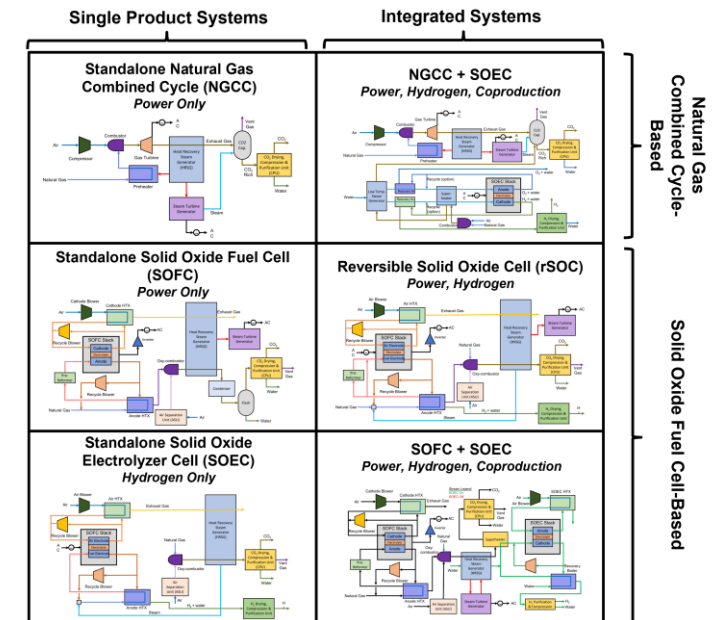
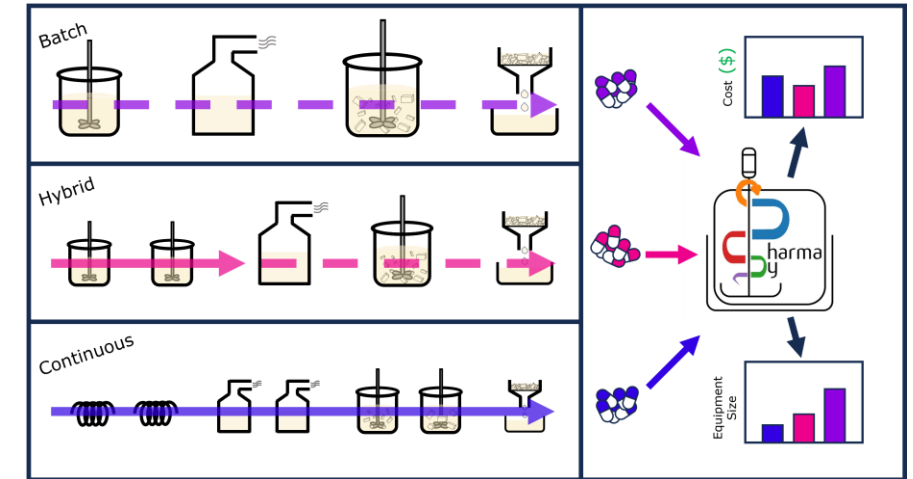
Percentage of scenarios that make profit at each hydrogen selling price

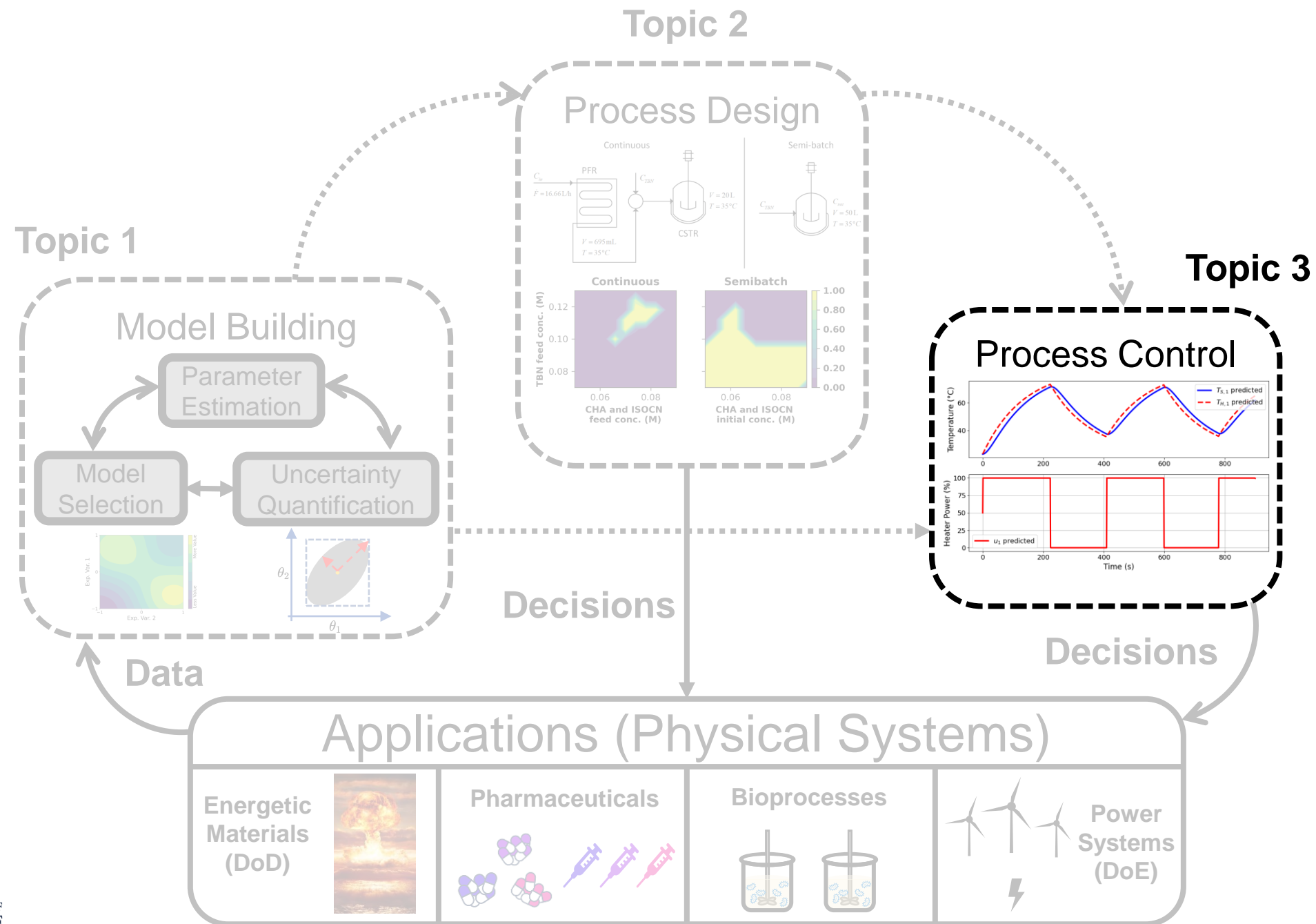
Process Concept	1.00\$.kg ⁻¹	1.50\$.kg ⁻¹	2.00\$.kg ⁻¹	2.50\$.kg ⁻¹	3.00\$.kg ⁻¹
NGCC	13%	13%	13%	13%	13%
SOFC	54%	54%	54%	54%	54%
NGCC + SOEC	8%	11%	16%	62%	80%
rSOC	54%	77%	97%	100%	100%
SOFC + SOEC	46%	79%	98%	100%	100%
SOEC	10%	49%	74%	87%	98%

- At sufficient hydrogen price (\$2.50 +), even the existing thermal generation technology with co-production (**NGCC +SOEC**) sees profit in over half of the market scenarios

Optimal Design Conclusions

- PharmaPy can be leveraged to develop impactful digital twins
 1. Simulate process models
 2. Optimize process conditions
 3. Compare candidate models
 4. Generate and utilize model uncertainty
 5. Evaluate process superstructure to inform potential manufacturing routes
- Data-driven and mechanistic models can be combined to evaluate emerging energy technologies using digital twins



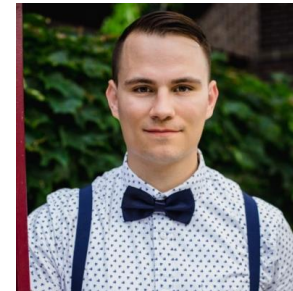


Data Reduction using Topological Data Analysis

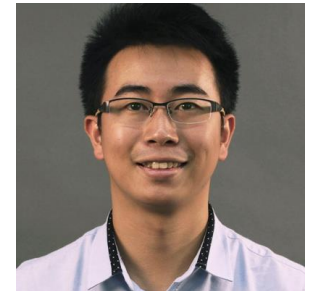
University of Wisconsin-Madison



**Prof. Victor
Zavala**



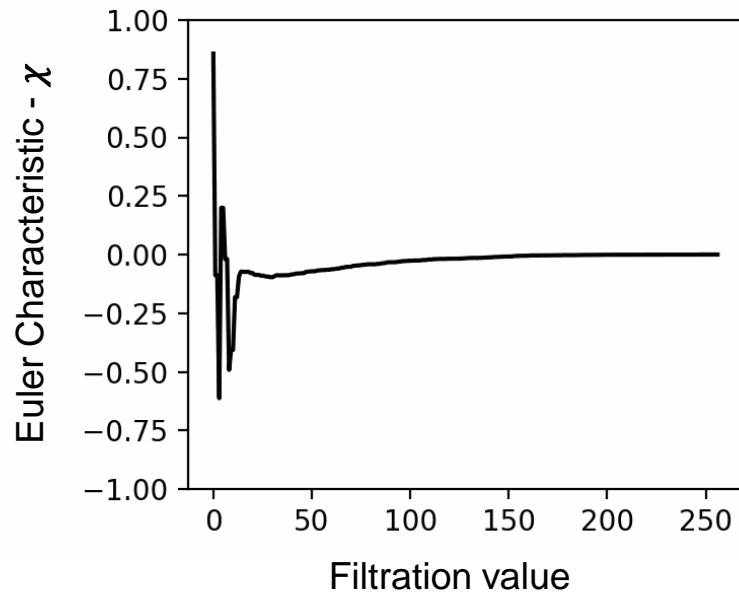
**Dr. Alexander
Smith**



**Dr. Shengli
(Bruce) Jiang**

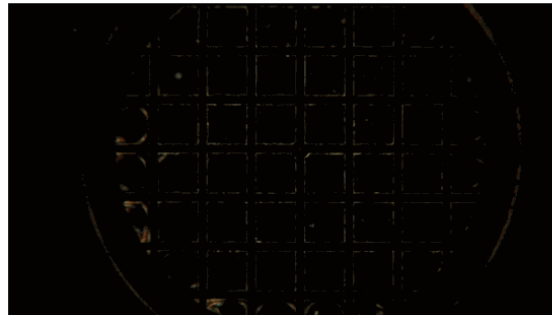
Topological Data Analysis for Classification and Quality Control

100x slower than real time

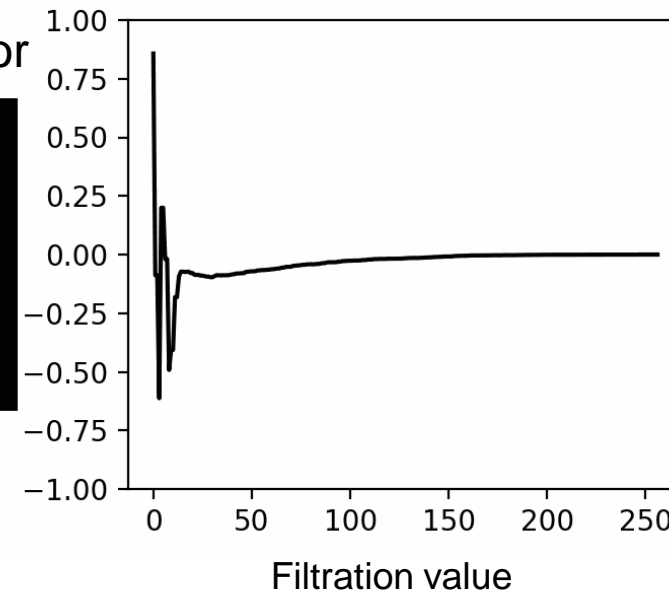


Computation with GUDHI
(state-of-the-art tool)

Liquid crystal chemical sensor



Real-time data processing

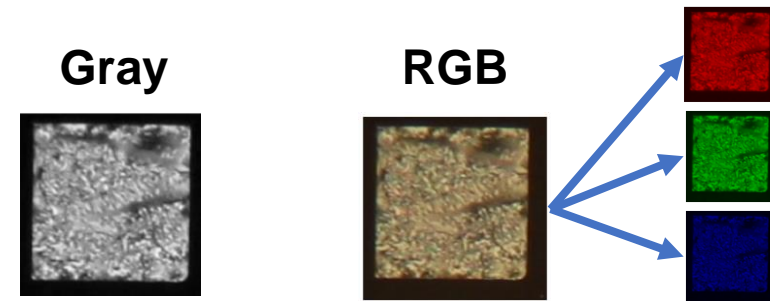


Computation with vertex contributions (**VC**)
(our implementation)

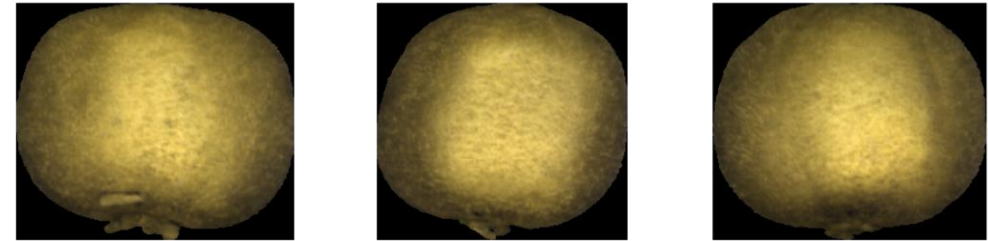
Data in Engineering is Often Represented as a Field

- Images

- Grayscale (2D field)
- RGB (~3D field)
- Hyperspectral (3D field)



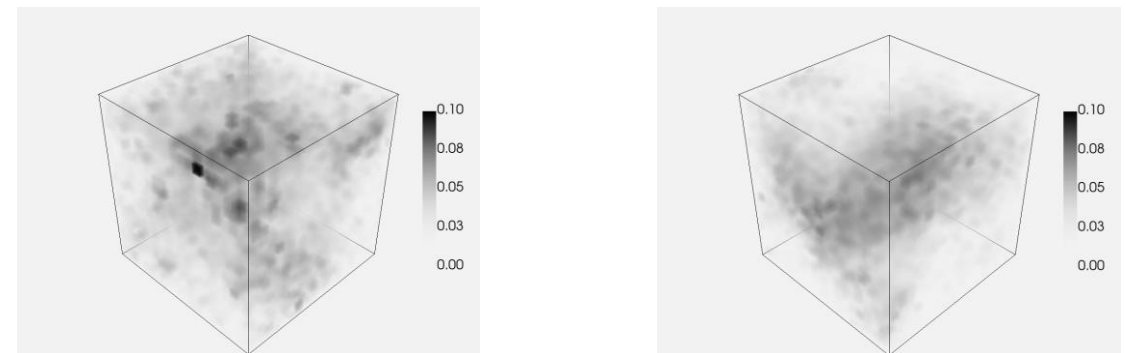
Hyperspectral



- Space-Time Data

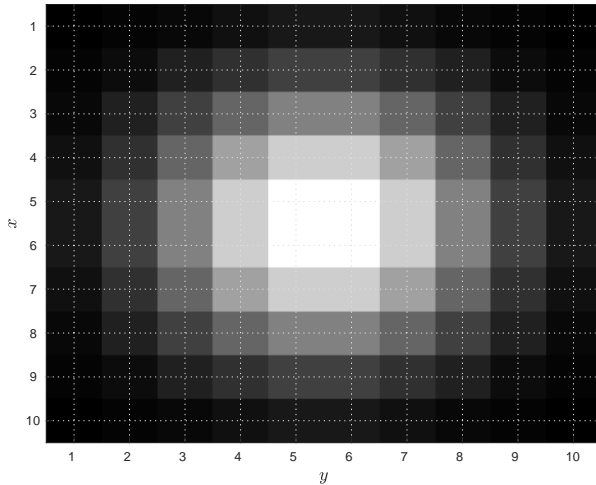
- Molecular dynamics
- Computational fluid dynamics
- *Any spatial data (e.g., GIS)*

Molecular Dynamics



Field Data – Representation, Transformation, Information

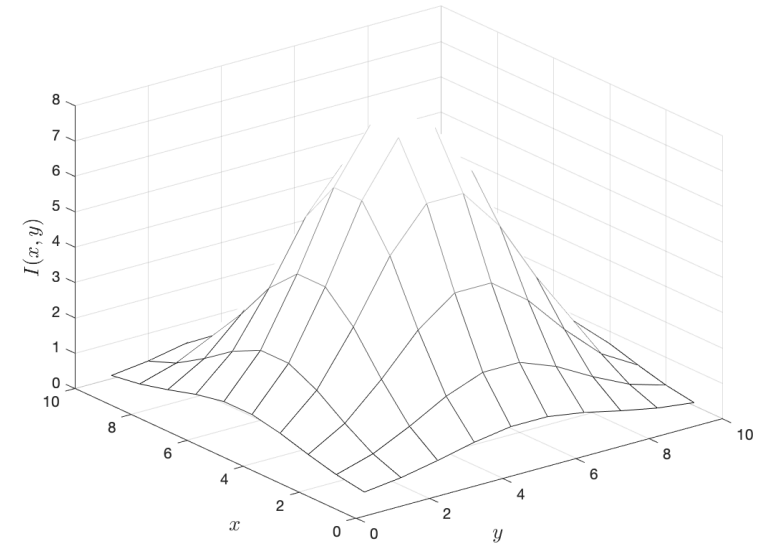
Image



Matrix

0	0	0	1	1	1	1	0	0	0
0	1	1	2	2	2	2	1	1	0
0	1	2	3	4	4	3	2	1	0
1	2	3	5	6	6	5	3	2	1
1	2	4	6	8	8	6	4	2	1
1	2	4	6	8	8	6	4	2	1
1	2	3	5	6	6	5	3	2	1
0	1	2	3	4	4	3	2	1	0
0	1	1	2	2	2	2	1	1	0
0	0	0	1	1	1	1	0	0	0

Function/Manifold



SVD $\mathbf{T} = \mathbf{X} \cdot \mathbf{W}$

Convolution $\mathbf{T} = \mathbf{W} * \mathbf{X}$

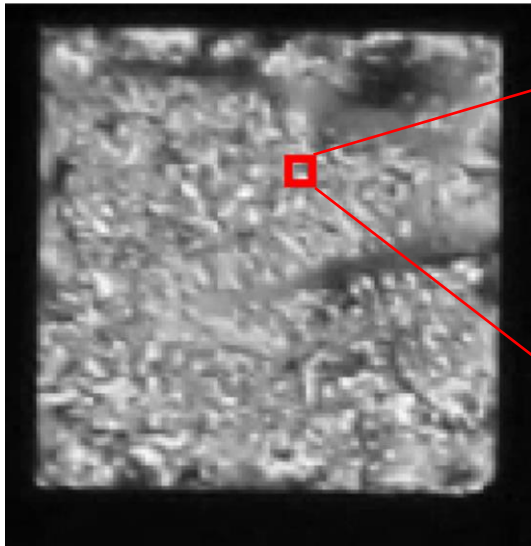
Fourier $\mathbf{T} = \mathcal{F}(\mathbf{X})$



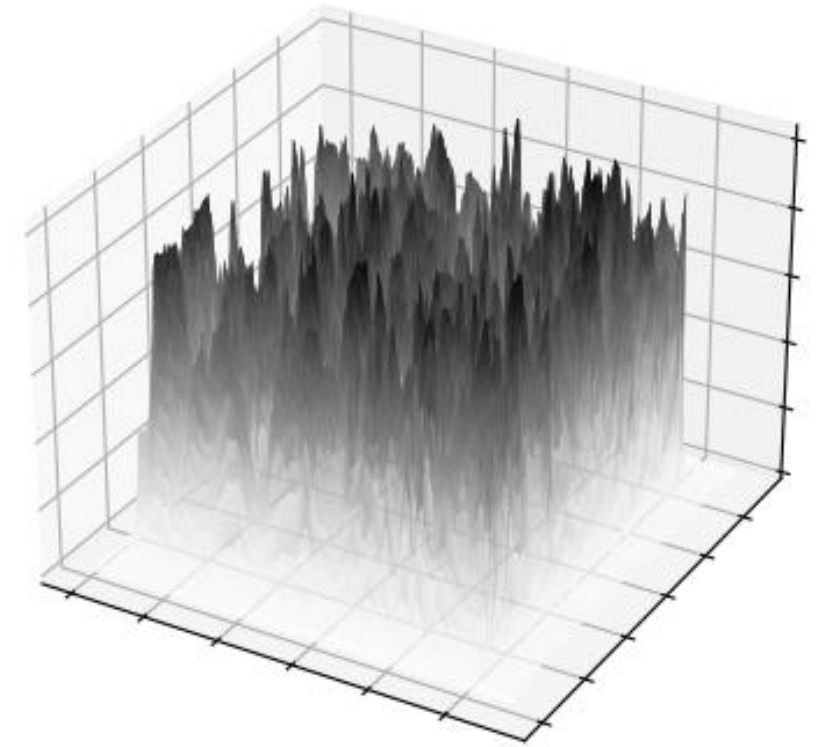
Different representations lead to different transformations

Different transformations extract different types of information

Field Data – Topological Data Analysis (TDA)



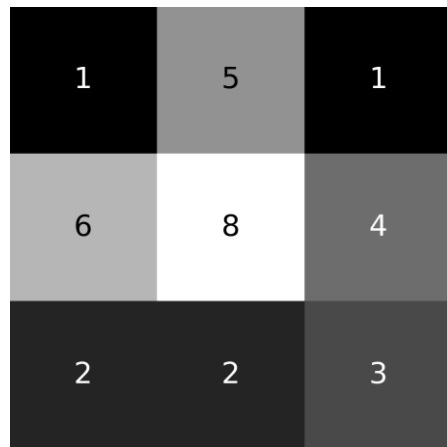
174	211	210	176	162	159
183	186	217	199	158	118
179	145	120	118	108	101
167	152	141	135	125	99
161	151	169	192	193	125
167	141	189	230	247	190



2D fields (i.e., images) reveal topographical features when represented as a 3D surface

Filtration – Euler Characteristic Curve

$$g_c^-(f) = \{(x_w, x_h) | f(x_w, x_h) \leq c\}$$



Euler Characteristic (EC)

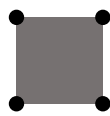
$$\chi = V - E + F - C$$



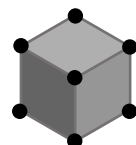
0-simplex
(vertex)



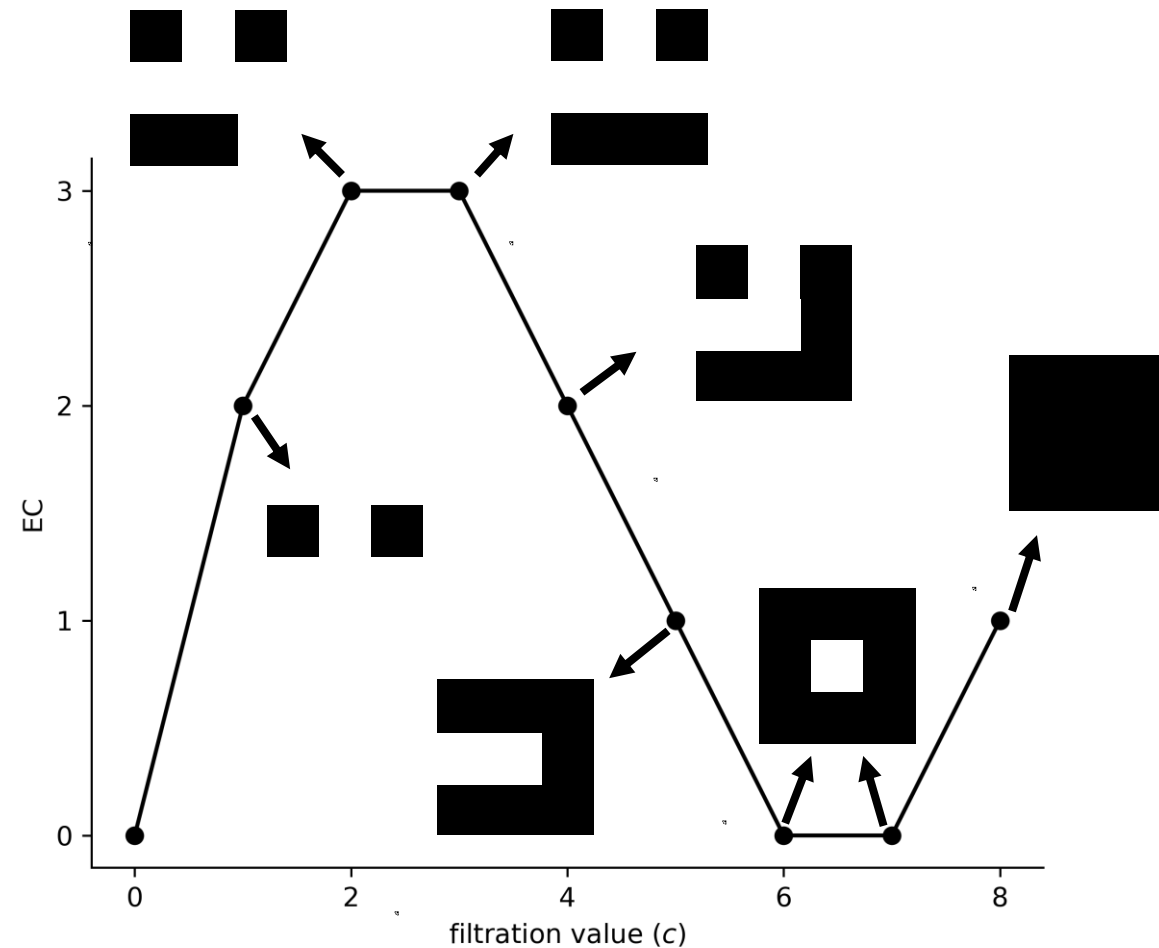
1-simplex
(edge)



2-simplex
(face)

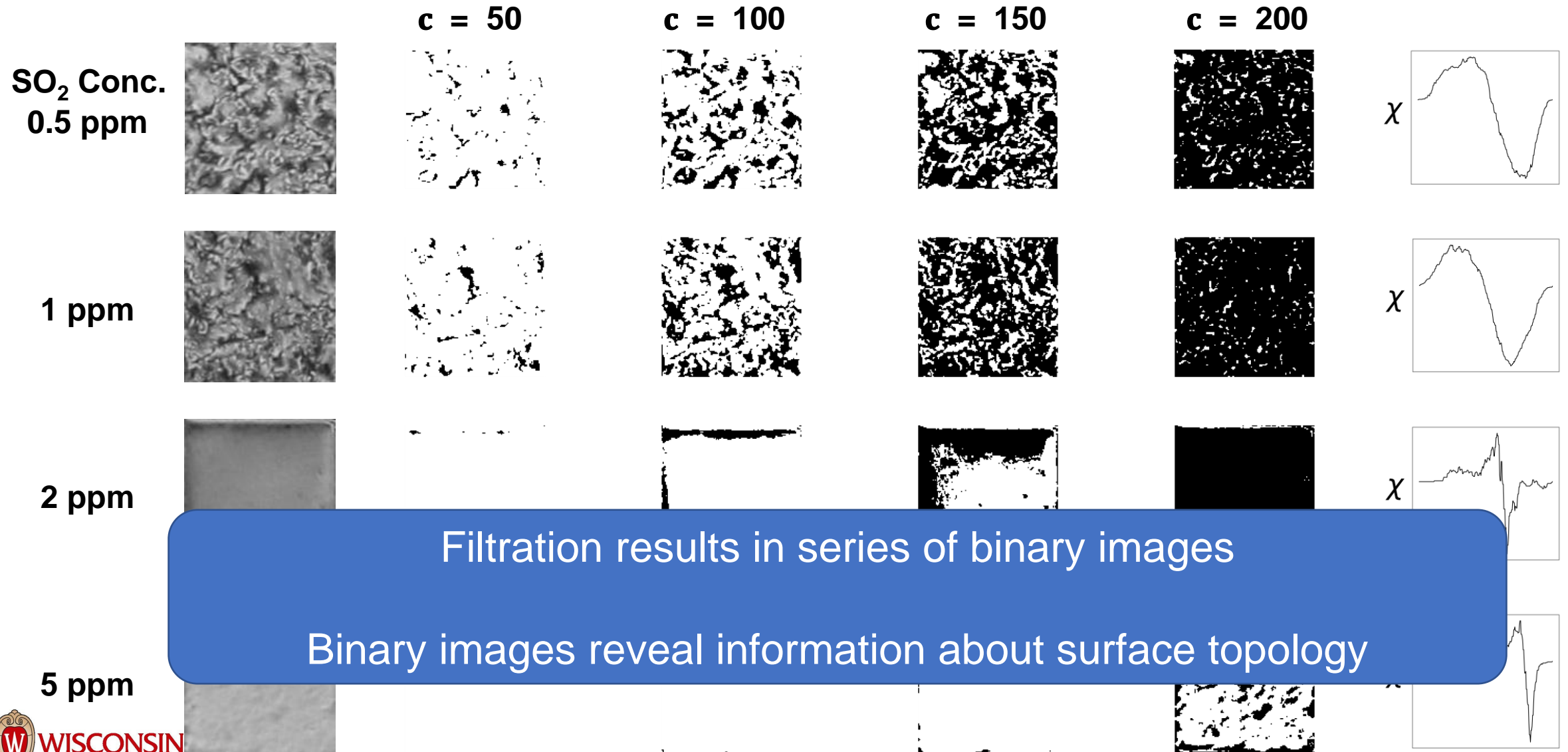


3-simplex
(cell)



Filtration yields a series of binary fields for the EC computation

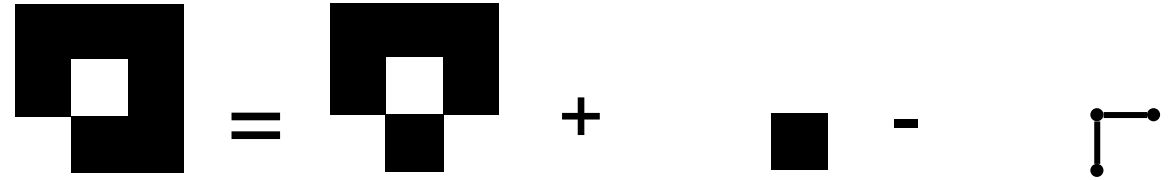
Filtration Example – Liquid Crystal Sensors



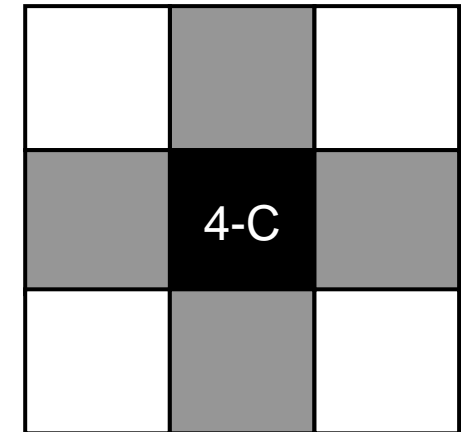
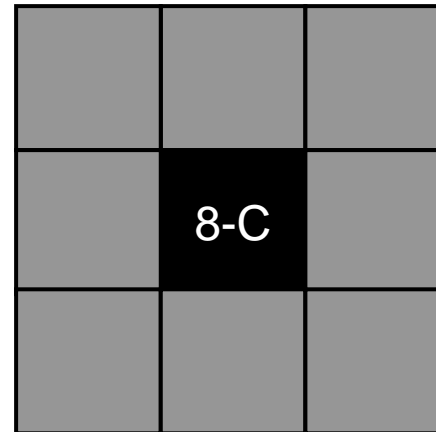
Euler Characteristic Generalization – Problems

- Euler characteristic follows the **inclusion-exclusion principle**

$$\chi(A \cup B) = \chi(A) + \chi(B) - \chi(A \cap B)$$

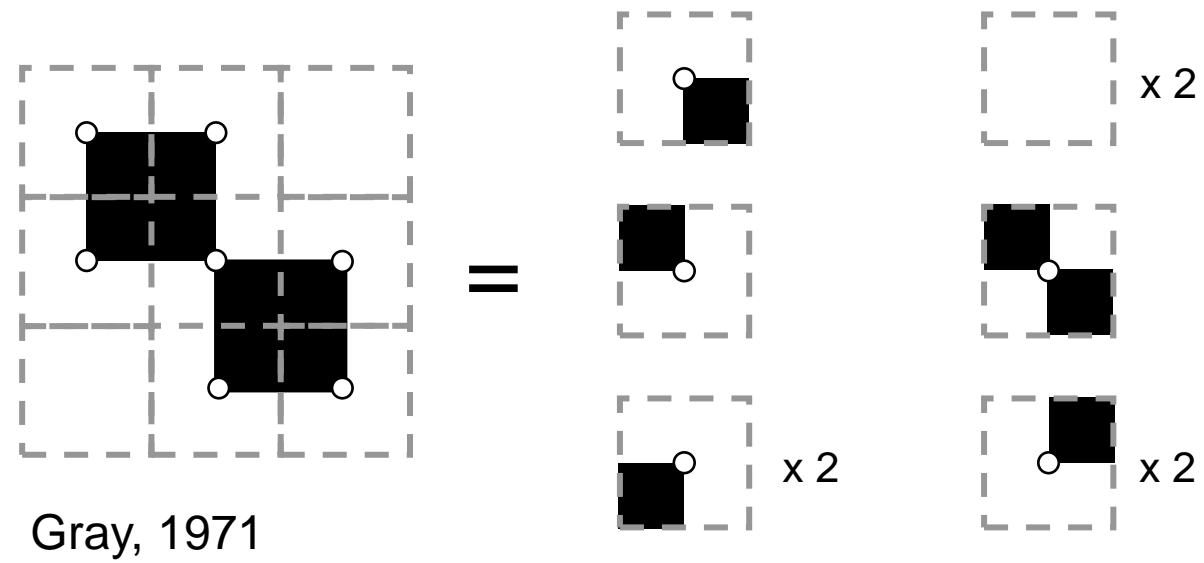


- Should faces be considered connected via edges (4-C), or via vertices (8-C)?



- This principle is key for enabling **parallel computation**

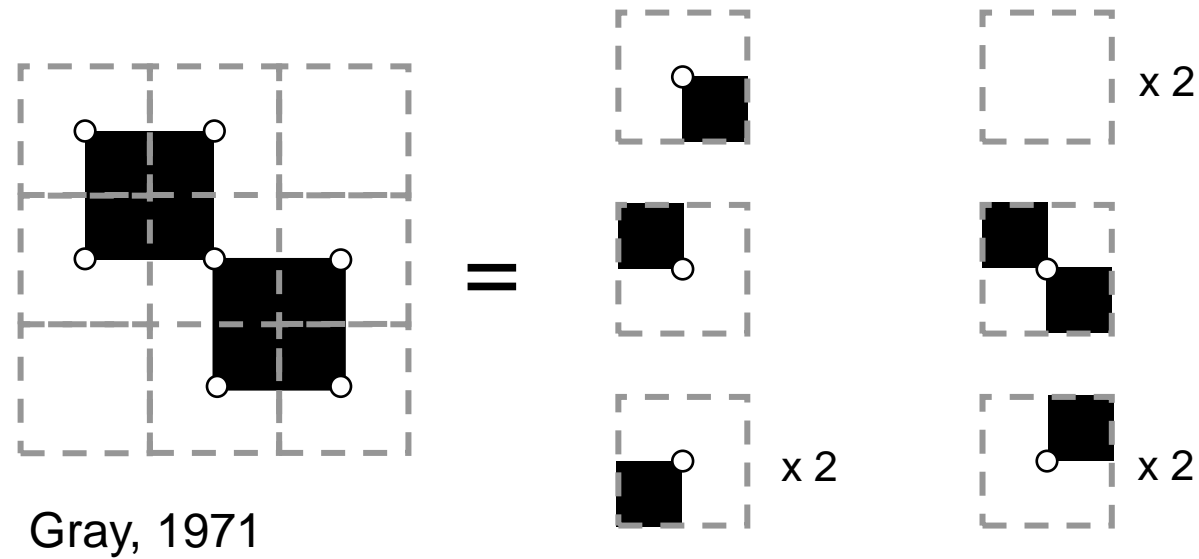
Avoiding Repetitive Evaluation through Abstraction



$$\chi(A \cup B) = \chi(A) + \chi(B) - \chi(A \cap B)$$

0

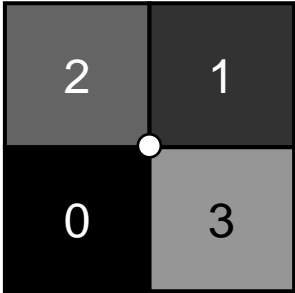
Avoiding Repetitive Evaluation through Abstraction



Type	V (8-C)	E (8-C)	F (8-C)	EC (8-C)	V (4-C)	E (4-C)	F (4-C)	EC (4-C)	Perimeter	Area
	0	0	0	0	0	0	0	0	0	0
	1	2 * (0.5)	1 * (0.25)	0.25	1	2 * (0.5)	1 * (0.25)	0.25	1	0.25
	1	4 * (0.5)	2 * (0.25)	0.5	2	4 * (0.5)	2 * (0.25)	0.5	2	0.5
	1	4 * (0.5)	3 * (0.25)	-0.25	1	4 * (0.5)	3 * (0.25)	-0.25	1	0.75
	1	4 * (0.5)	4 * (0.25)	0	1	4 * (0.5)	4 * (0.25)	0	0	1

Even with this method, we still must compute contributions at *each* filtration value (not scalable)

Avoiding Repetitive Evaluation through Abstraction



	birth – 0 death – 1											
	birth – 1 death – 2											
	birth – 2 death – 3											
	birth – 3 no death											

index	birth	death	birth	death	birth	death	birth	death	birth	death	birth	death
0	0	0	1	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	1	0	0	0	0	0
2	0	0	0	0	0	0	0	1	1	0	0	0
3	0	0	0	0	0	0	0	0	0	1	1	0























Only need local vertex neighborhood values → Highly parallelizable; Low memory requirements

$\mathcal{O}(n_c lw)$
→
 $\mathcal{O}(n_c + lw)$

l - length of the field
 w - width of the field
 n_c - number of filtration values

Abstraction for 3D fields

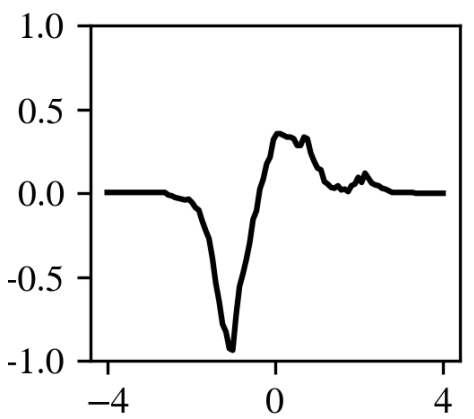
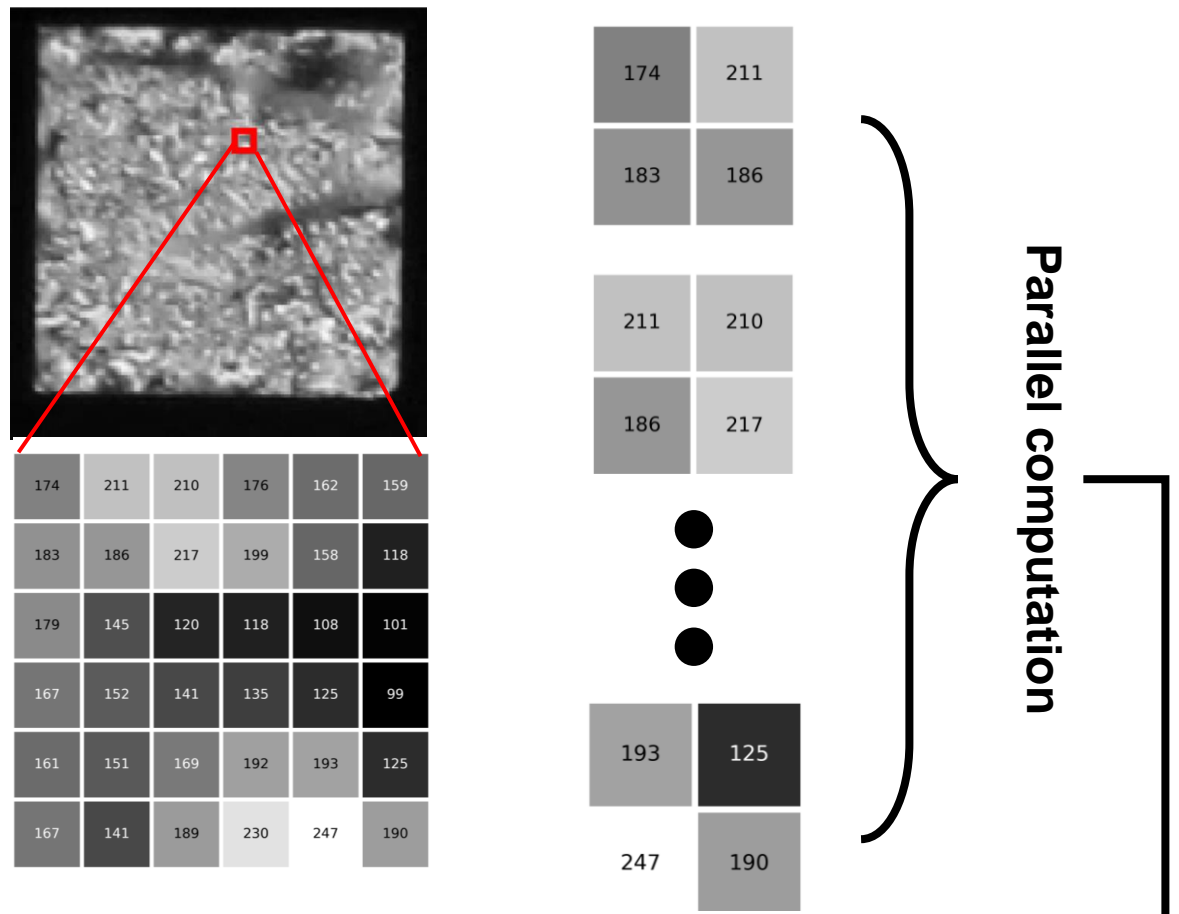
- Voxels in 3D digital images (or cells) [4]
- Voxel-sized neighborhood about a vertex yields the same savings
- More hassle distinguishing which pattern is represented for active cells

Type	f_{adj}	V (26-C)	E (26-C)	F (26-C)	Vox (26-C)	EC (26-C)	Perimeter	Area	Volume
	0	0	0	0	0	0	0	0	0
	0	1	3 * (0.5)	3 * (0.25)	1 * (0.125)	0.125	1.5	0.75	0.125
	1	1	4 * (0.5)	5 * (0.25)	2 * (0.125)	0	1	1	0.25
	1.414	1	5 * (0.5)	6 * (0.25)	2 * (0.125)	-0.25	3	1.5	0.25
	1.732	1	6 * (0.5)	6 * (0.25)	2 * (0.125)	-0.75	3	1.5	0.25
	3.414	1	5 * (0.5)	7 * (0.25)	3 * (0.125)	-0.125	1.5	1.25	0.375
	4.146	1	6 * (0.5)	8 * (0.25)	3 * (0.125)	-0.375	2.5	1.75	0.375
	4.243	1	6 * (0.5)	9 * (0.25)	3 * (0.125)	-0.125	4.5	2.25	0.375
	6.828	1	5 * (0.5)	8 * (0.25)	4 * (0.125)	0	0	1	0.5
	7.243	1	6 * (0.5)	9 * (0.25)	4 * (0.125)	-0.25	3	1.75	0.5
	7.560	1	6 * (0.5)	9 * (0.25)	4 * (0.125)	-0.25	2	1.5	0.5
	7.975	1	6 * (0.5)	10 * (0.25)	4 * (0.125)	0	3	2	0.5
	8.293	1	6 * (0.5)	10 * (0.25)	4 * (0.125)	0	2	2	0.5
	8.485	1	6 * (0.5)	12 * (0.25)	4 * (0.125)	0.5	6	3	0.5
	3.414 [†]	1	6 * (0.5)	10 * (0.25)	5 * (0.125)	-0.125	1.5	1.25	0.625
	4.146 [†]	1	6 * (0.5)	11 * (0.25)	5 * (0.125)	0.125	2.5	1.75	0.625
	4.243 [†]	1	6 * (0.5)	12 * (0.25)	5 * (0.125)	0.375	4.5	2.25	0.625
	1 [†]	1	6 * (0.5)	11 * (0.25)	6 * (0.125)	0	1	1	0.75
	1.414 [†]	1	6 * (0.5)	12 * (0.25)	6 * (0.125)	0.25	3	1.5	0.75
	1.732 [†]	1	6 * (0.5)	12 * (0.25)	6 * (0.125)	0.25	3	1.5	0.75
	0 [†]	1	6 * (0.5)	12 * (0.25)	7 * (0.125)	0.125	1.5	0.75	0.875
	0 [†]	1	6 * (0.5)	12 * (0.25)	8 * (0.125)	0	0	0	1



Software Implementation

- All code written in **C++** and called from an interface in Python
- Exploit high-powered computing hardware for parallelization
 - Intel Xeon E5-2697—2.7 GHz
 - 24 Cores
 - 256 GB RAM
- Implemented low-memory version for 2D, bitmap files (.BMP)

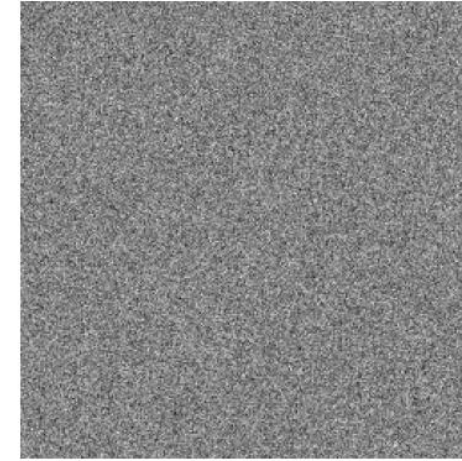


Type	V (8-C)	E (8-C)	F (8-C)	EC (8-C)
	0	0	0	0
	1	2 * (0.5)	1 * (0.25)	0.25
	1	3 * (0.5)	2 * (0.25)	0
	1	4 * (0.5)	2 * (0.25)	-0.5
	1	4 * (0.5)	3 * (0.25)	-0.25
	1	4 * (0.5)	4 * (0.25)	0

Baseline Case - Random Fields (2D and 3D)

- “Standard” resolutions

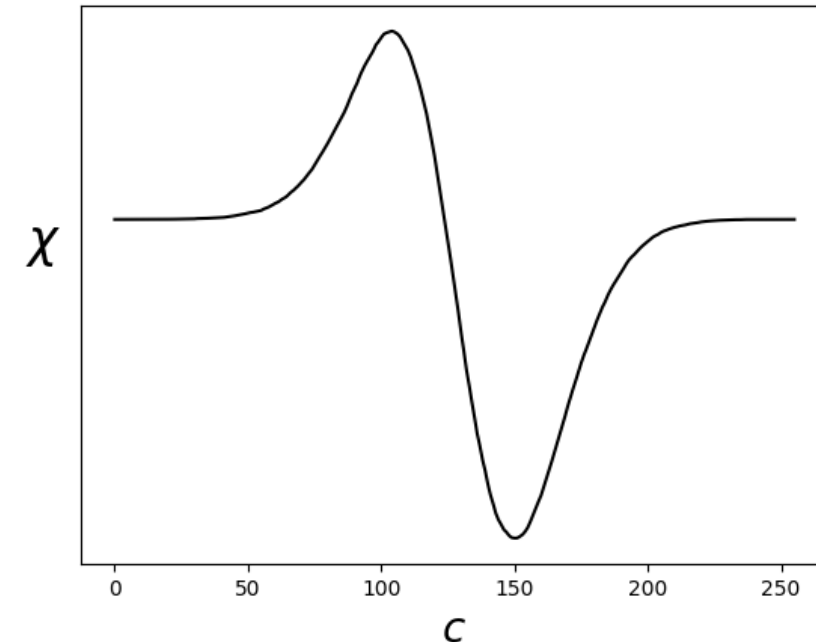
- 1920x1080
- 1280x720
- 128x128x128
- 256x256x256



- Large, square systems

- 2048x2048
- 4096x4096

- All field sizes tested with both **Uniform** and **Normal noise**



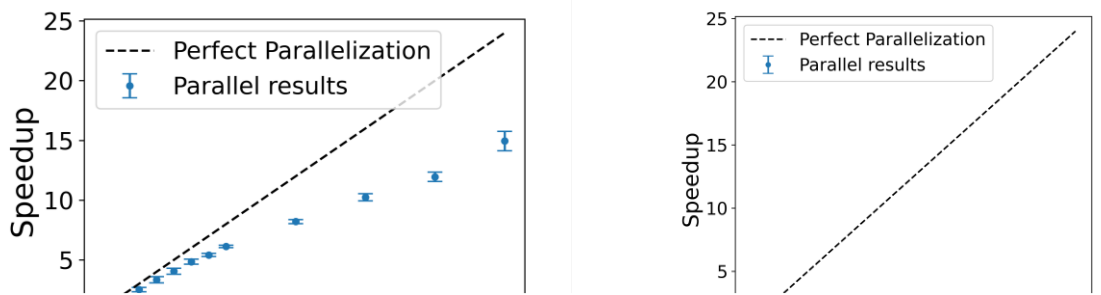
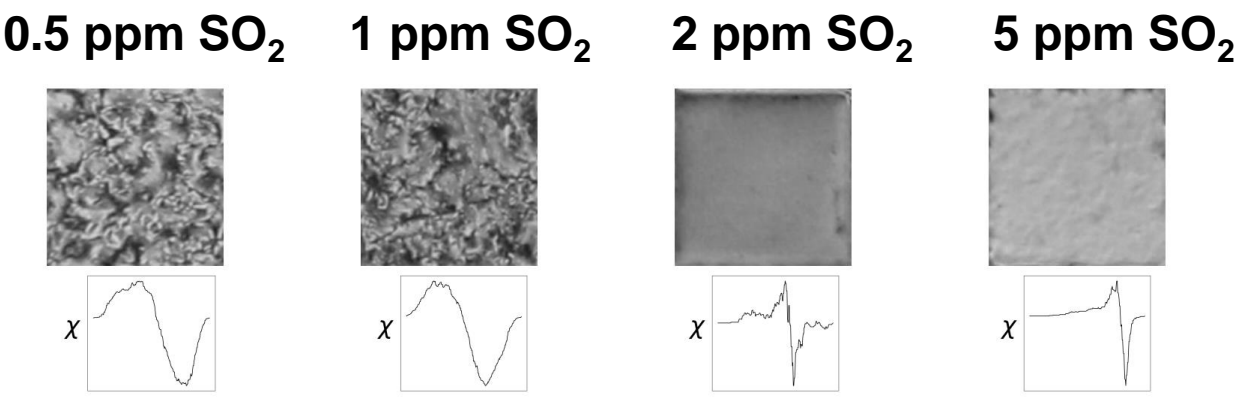
Random Fields (2D and 3D) – Significant Speedup

Type (2D)	GUDHI (s)	GUDHI (MP/s)	VC, 24 cores (s)	VC, 24 cores (MP/s)	Speedup
1280x720	5.1 ± 0.2	0.181 ± 0.005	0.0093 ± 0.0006	99.3 ± 4.9	550
1920x1080	14.6 ± 0.4	0.142 ± 0.004	0.0188 ± 0.0011	110.8 ± 4.6	780
2048x2048	31.6 ± 1.1	0.133 ± 0.004	0.0381 ± 0.0029	110.4 ± 6.0	830
4096x4096	145.2 ± 5.0	0.116 ± 0.003	0.210 ± 0.032	81.95 ± 13.2	706
Type (3D)	GUDHI (s)	GUDHI (MV/s)	VC, 24 cores (s)	VC, 24 cores (MV/s)	Speedup
128x128x128	42.1 ± 1.2	0.050 ± 0.001	0.0765 ± 0.0067	27.5 ± 1.2	550

1. Significant speedup over traditional tools
(2-3 orders of magnitude with 24-core parallelization)
2. Can process ~100Megapixel or ~30Megavoxel images in **one second**

Liquid Crystal Sensor Analysis

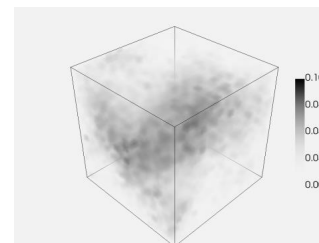
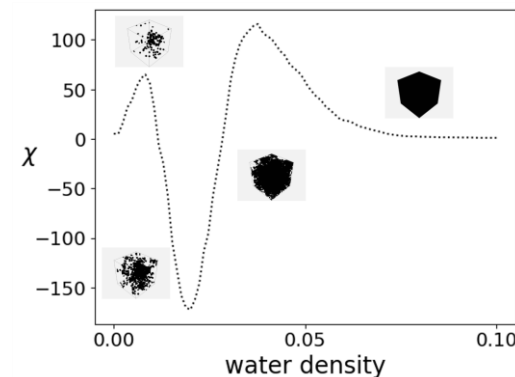
- LC micrograph responses to SO₂ under 40% RH
- Average image size of 134x134
- Small size drastically reduces efficiency of parallelization



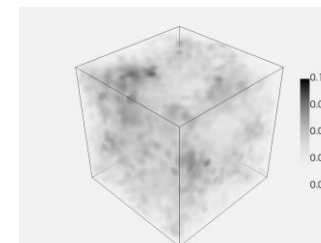
Type	GUDHI (MP/s)	VC, serial (MP/s)	VC, 12 cores (MP/s)	Speedup
SO ₂ Images	0.332 ± 0.009	6.268 ± 0.171	11.33 ± 0.49	34
Random Field	0.271 ± 0.009	6.127 ± 0.132	11.20 ± 0.39	41

Molecular Dynamics Simulations

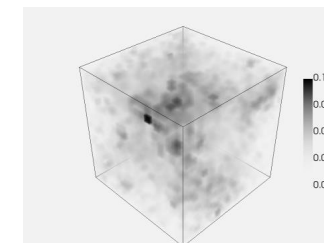
- Biomass (fructose) interaction in water-cosolvent systems
 - dioxane
 - γ -valerolactone
 - tetrahydrofuran
- Water molecule density can be represented as a 3D field



dioxane



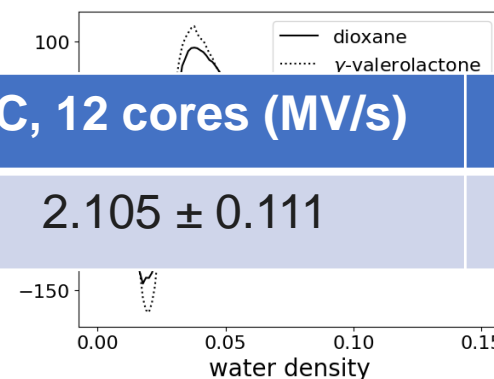
γ -valerolactone



tetrahydrofuran

• Field

Type	GUDHI (MV/s)	VC, serial (MV/s)	VC, 12 cores (MV/s)	Speedup
MD Fields	0.108 ± 0.004	1.065 ± 0.025	2.105 ± 0.111	19.5



Hyperspectral Imaging

- Determining kiwi ripeness without destructive sampling²⁶

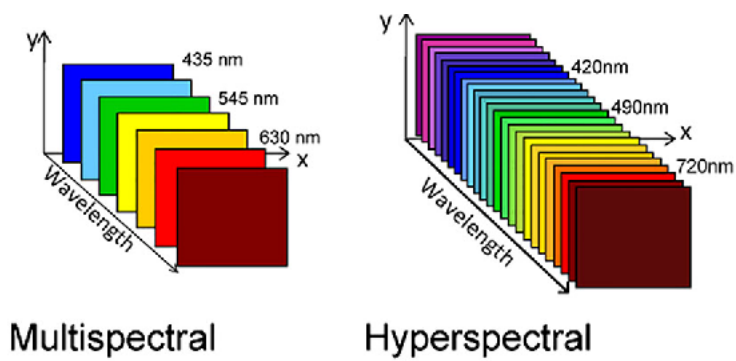
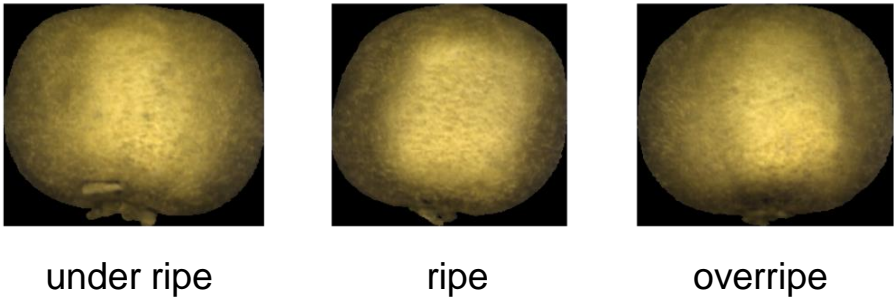


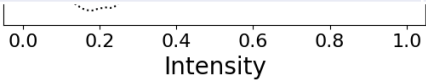
Figure from Mehta et al. 2018

- RGB images look similar



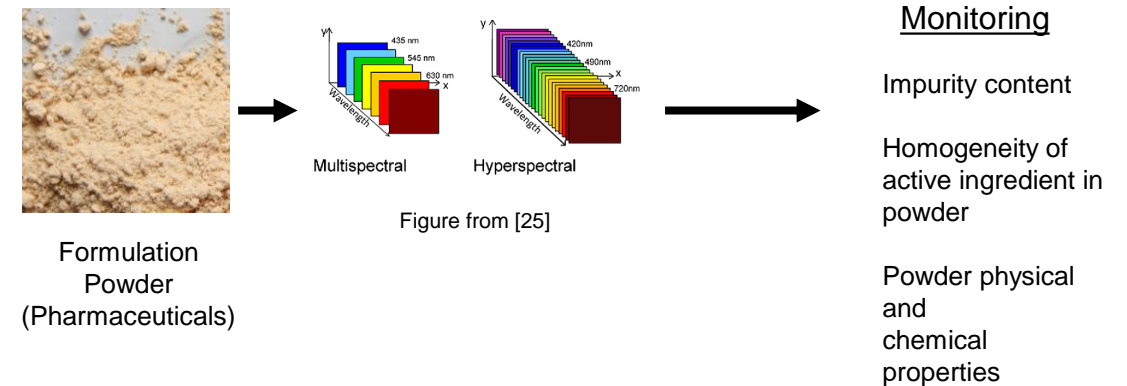
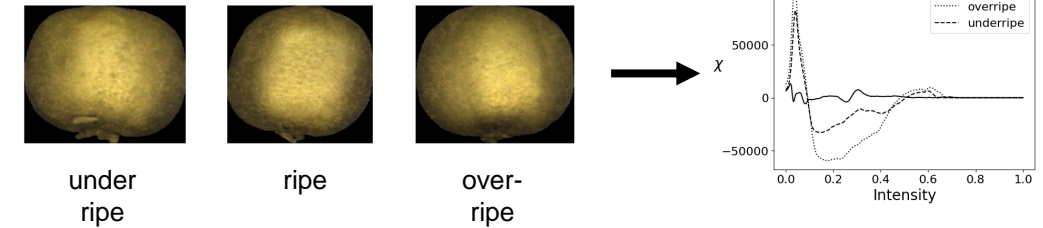
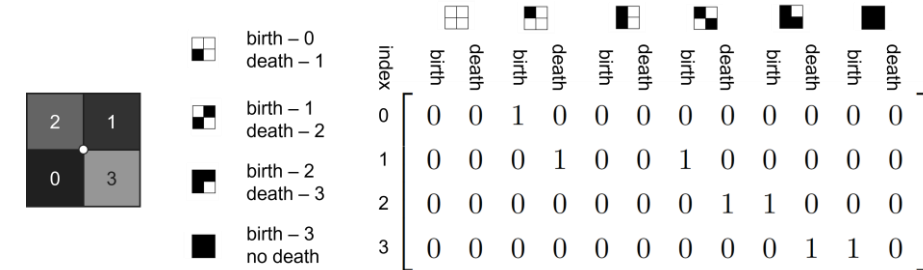
- EC dif

Type	GUDHI (MV/s)	VC, serial (MV/s)	VC, 24 cores (MV/s)	Speedup
Kiwi Images	0.0701 ± 0.0078	1.973 ± 0.112	34.57 ± 2.38	493
Random Field	0.0427 ± 0.0031	1.506 ± 0.006	28.19 ± 1.02	660

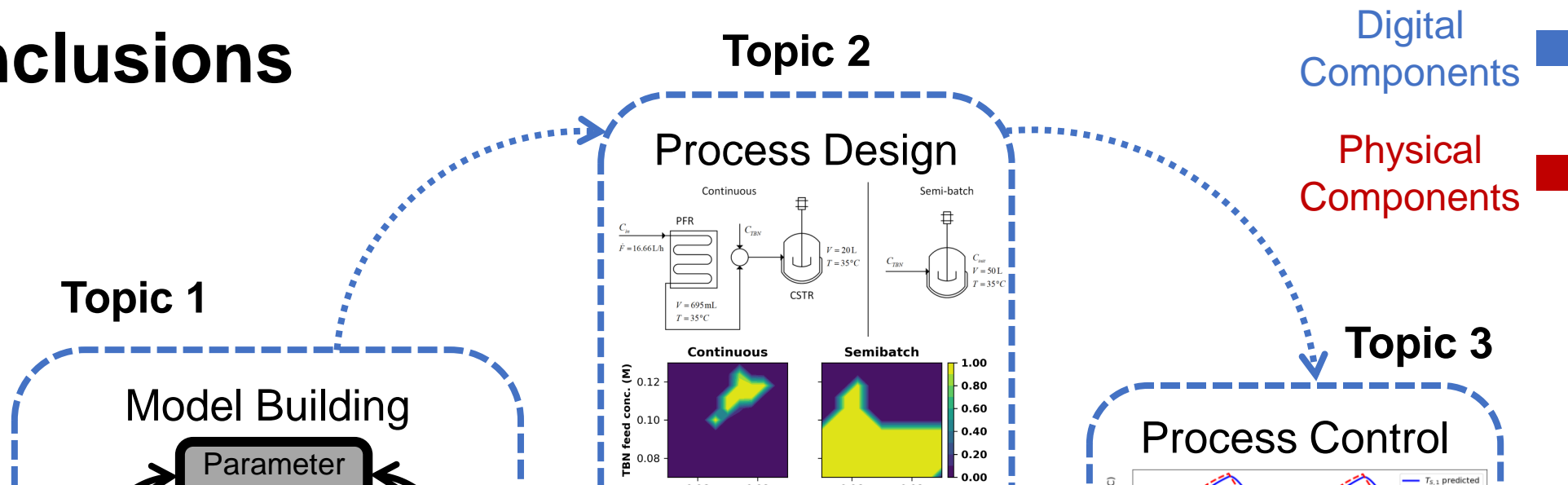


Conclusions

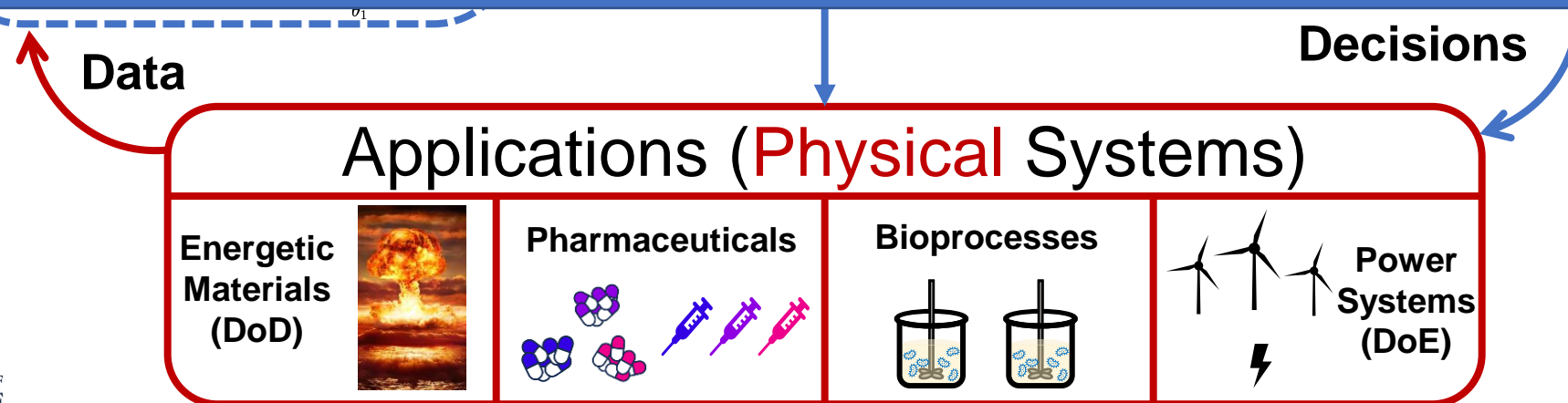
- Topology reveals order and patterns throughout field data
- How we process these patterns drastically impacts computational efficiency
 - Enabling real-time analysis and control
- Extend these methods to other applications
 - Pharmaceuticals
 - Other powder/melt-based processing



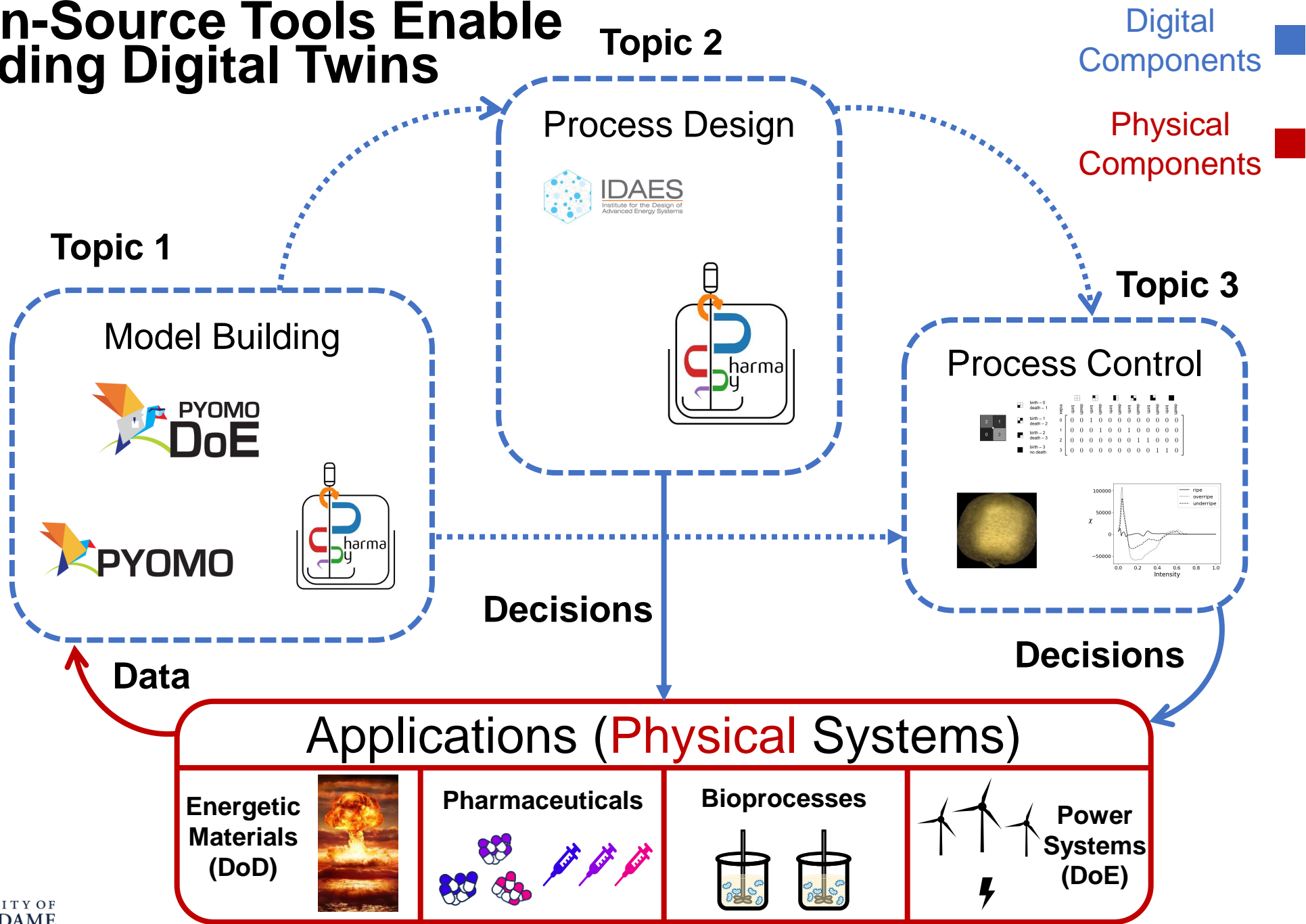
Conclusions



1. Flexible software tools enable building of fit-for-purpose digital twins through:
 - **Functionality** (e.g., model building, optimal process design)
 - **Computational tractability** (real-time algorithms)

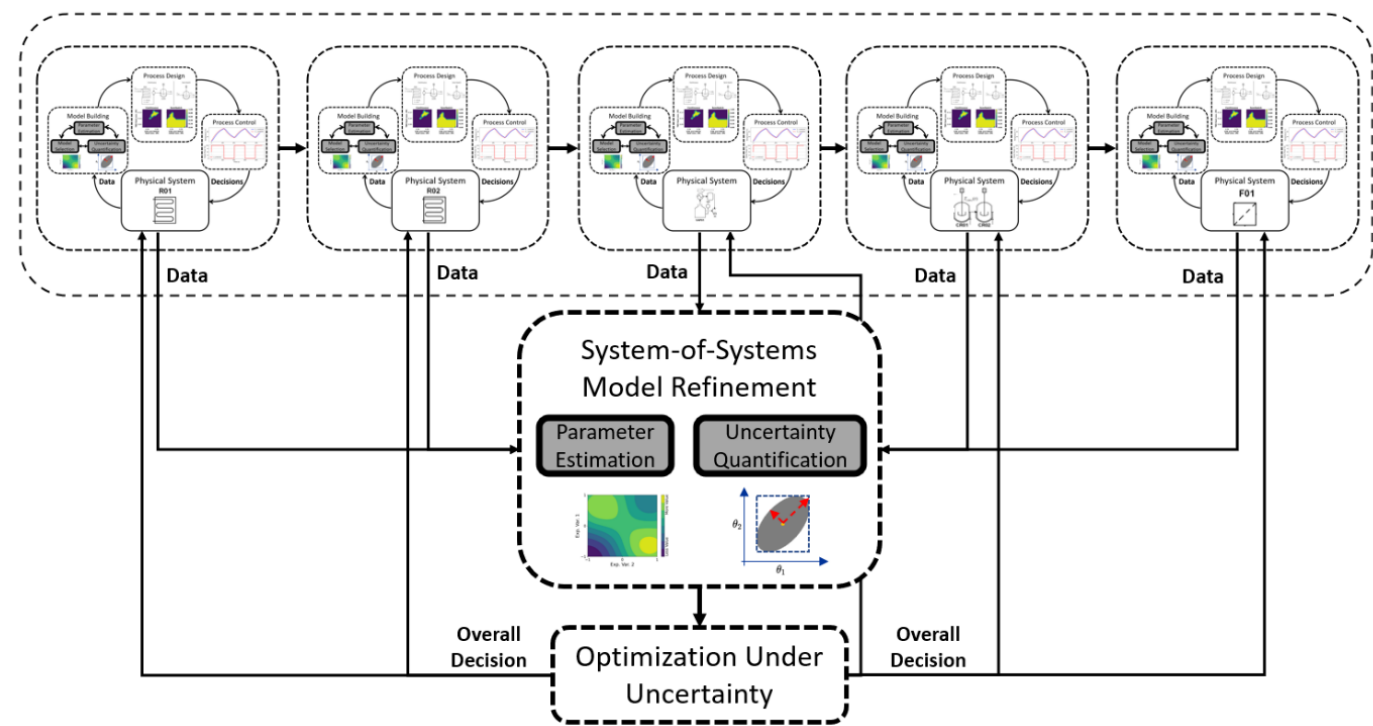


Open-Source Tools Enable Building Digital Twins



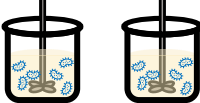
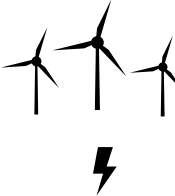


Research Opportunities for Fit-for-Purpose Digital Twins

System-of-Systems Digital Twins



Applications (Physical Systems)

Energetic Materials (DoD) 	Pharmaceuticals 	Bioprocesses 	Power Systems (DoE) 
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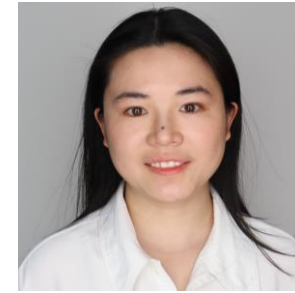


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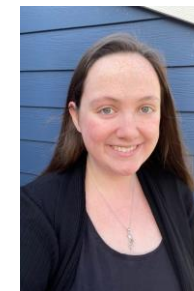
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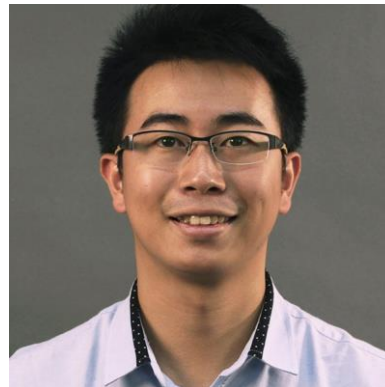


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