PURDUE UNIVERSITY
Davidson School of Chemical Engineering

Electrified Heat Exchanger Network Synthesis

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PSE Seminar

October 06, 2023

Outline

01 Introduction & Motivation

- Heat integration
- Heat exchanger network synthesis (HENS)
- Motivation

02 Electrified Heat Exchanger Network

- Proposed optimization model
- Preliminary results

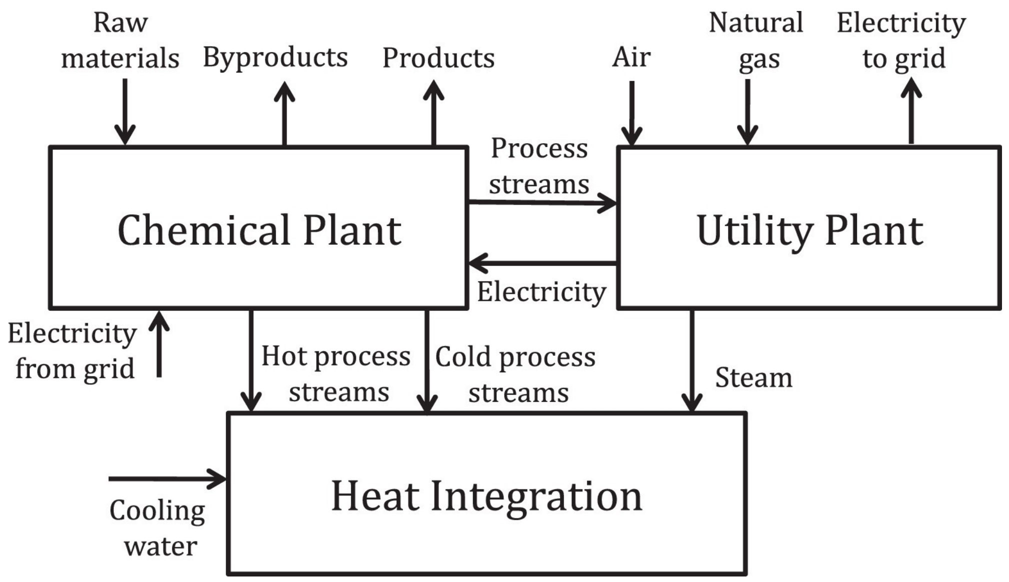
03 Challenges & Next Steps

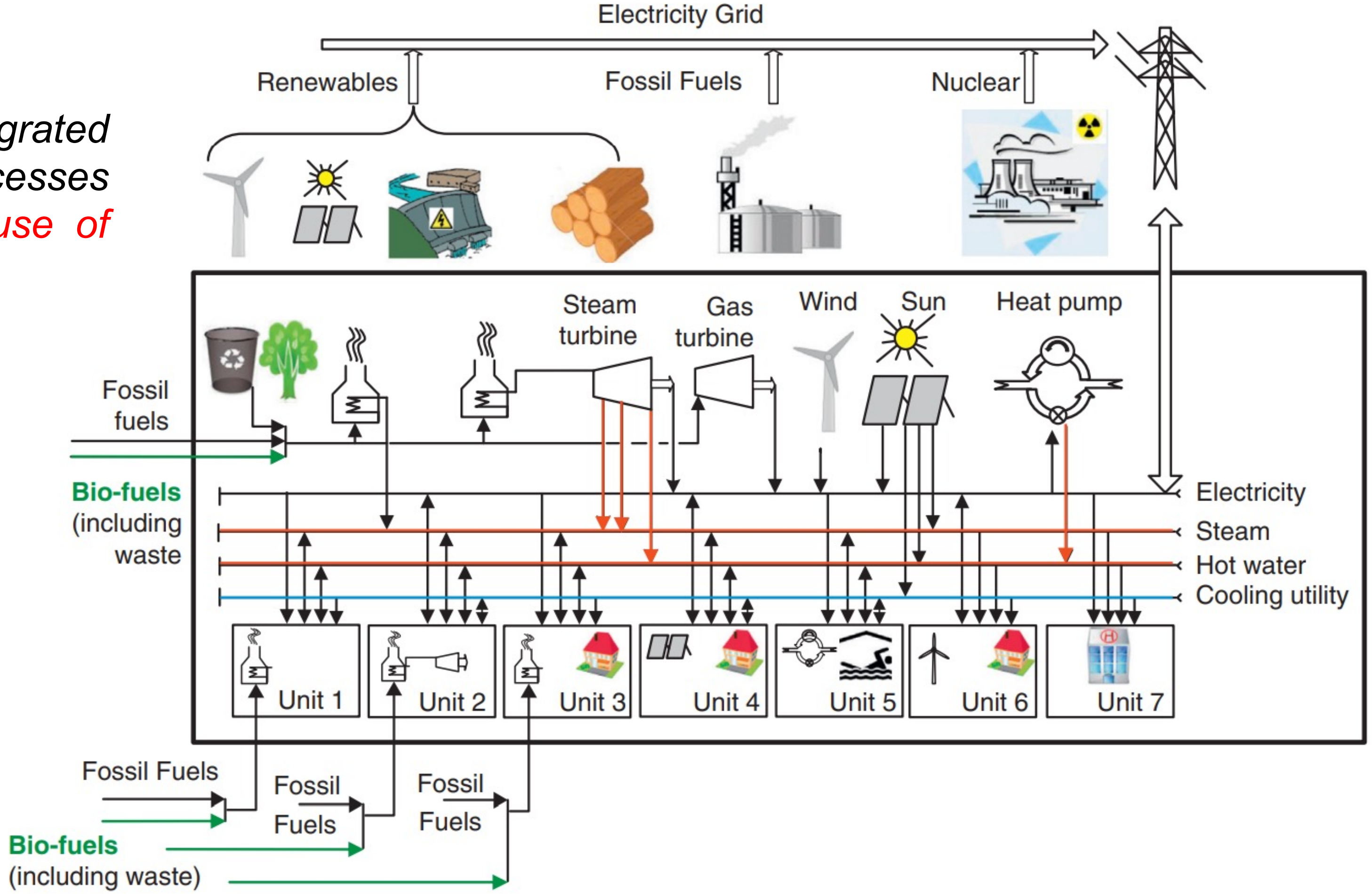
Heat integration

➢ ***Heat integration could result in energy and cost savings up to 15~45%***

❑ Definition

Systematic methods for designing integrated production systems ranging from Individual processes to total sites, with emphasis on the efficient use of energy and reducing environmental effects





Heat exchanger network synthesis (HENS)

➢ *HENS is an important aspect of process design to improve energy efficiency*

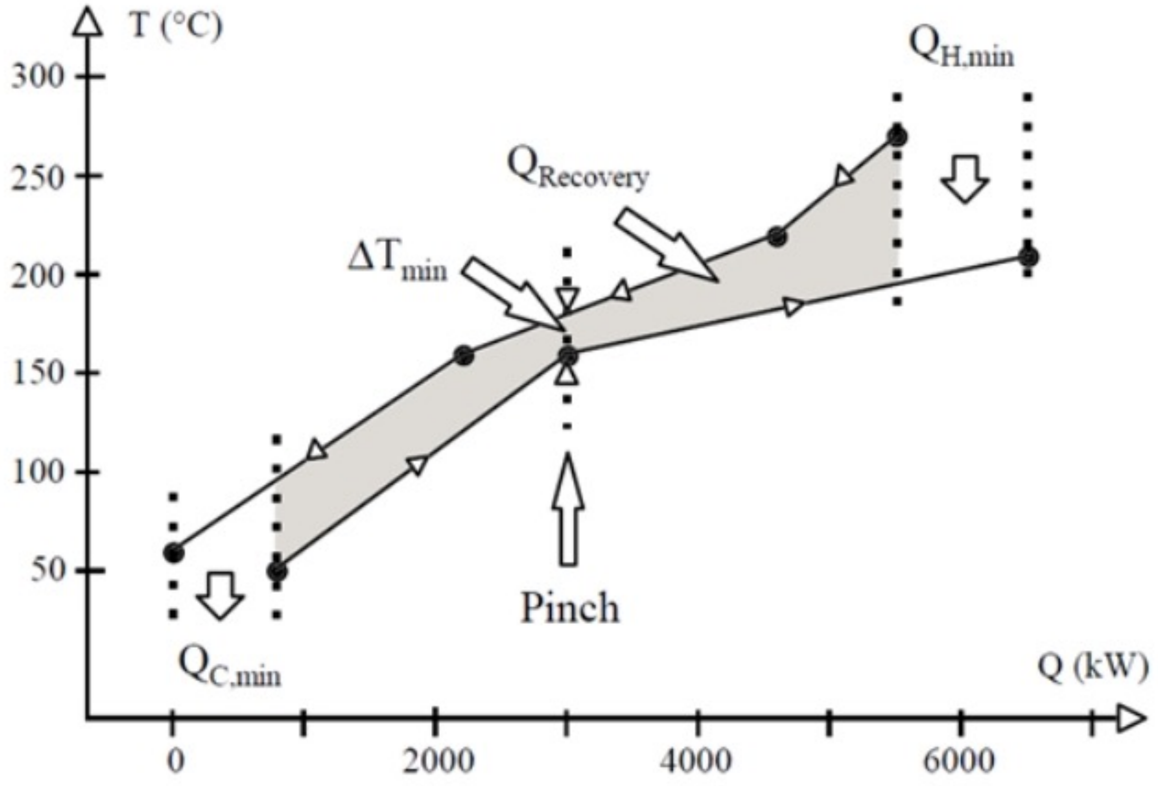
❑ General problem statement

Given:

- a set H of hot process streams to be cooled from the supply temperatures to the target temperatures
- a set C of cold process streams to be heated from the supply temperatures to the target temperatures
- heat capacities and flow rates of the hot and cold process streams,
- utilities available and the temperatures or temperature ranges and the costs for these utilities
- heat-exchanger cost data

Design:

An optimum network of heat exchangers with minimum annualized investment costs and operating costs (and emissions).



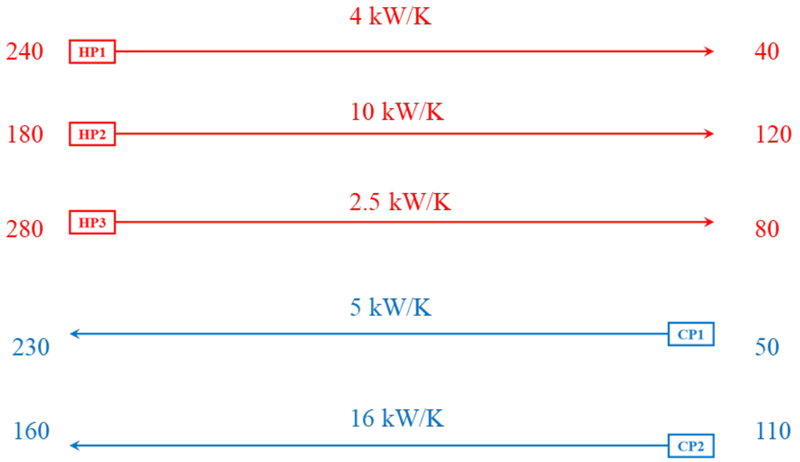
Heat exchanger network synthesis (HENS)

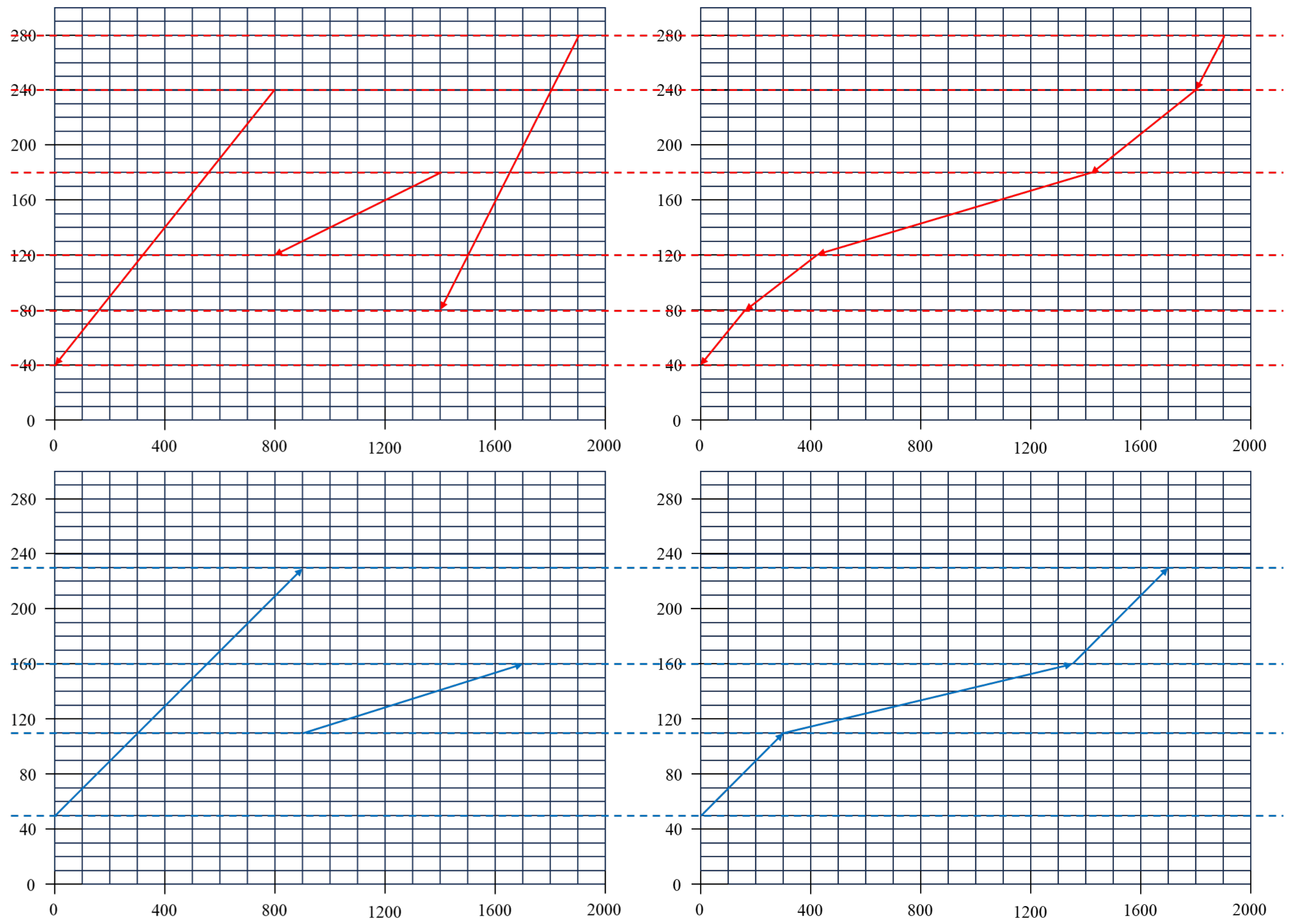
➢ *Method1: Pinch Analysis (PA) based on thermodynamics*

$$slope = \frac{dT}{dQ} = \frac{1}{FCp}$$

❑ General steps

1. Thermal data extraction
2. Minimum approach temperature selection
3. (Grand) composite curve construction
4. Minimum utility targeting
5. Feasible heat exchanger network design
6. Network evaluation and optimization

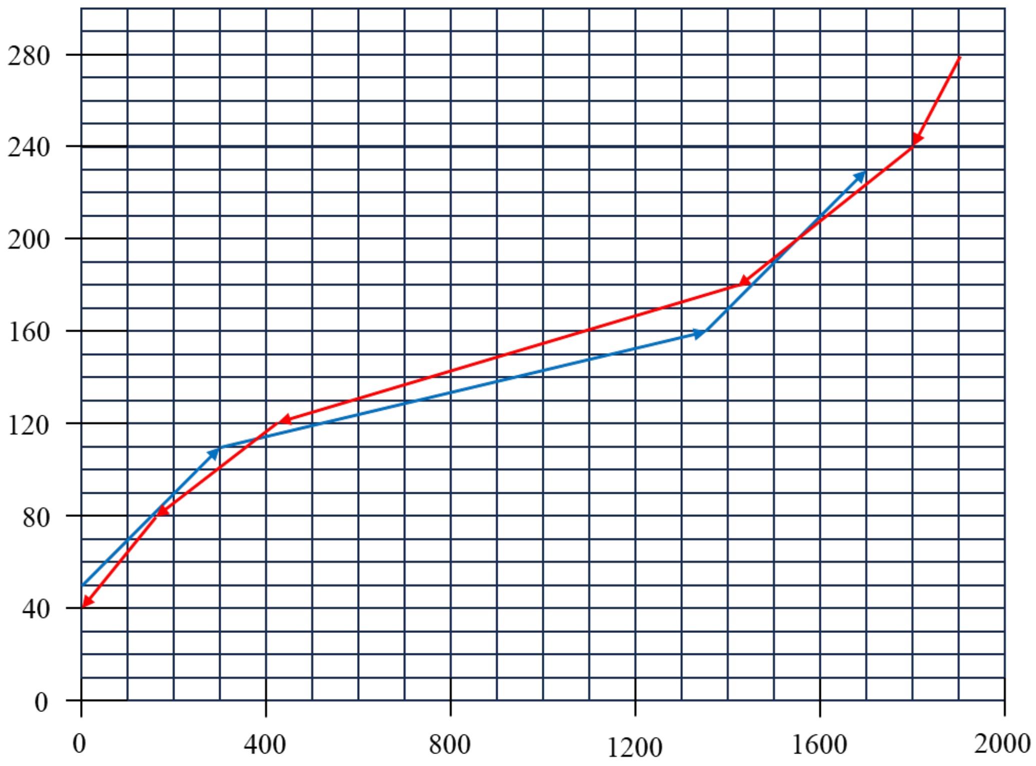


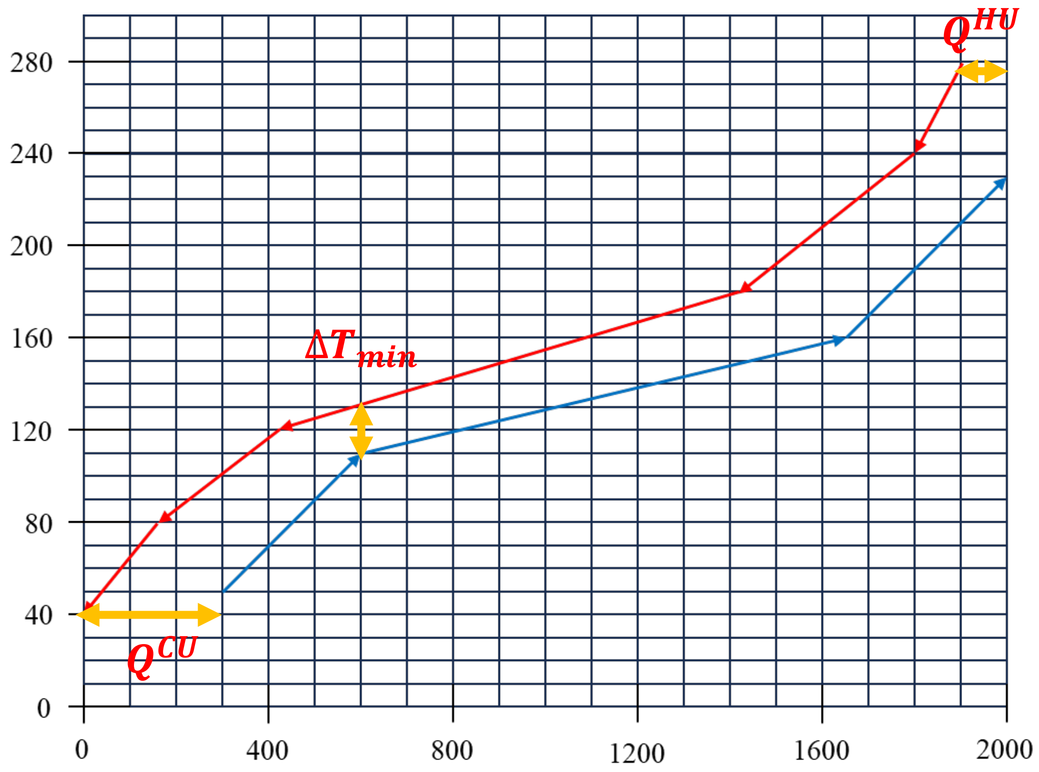


Heat exchanger network synthesis (HENS)

➢ *Method1: Pinch Analysis (PA) based on thermodynamics*

❑ Composite curve



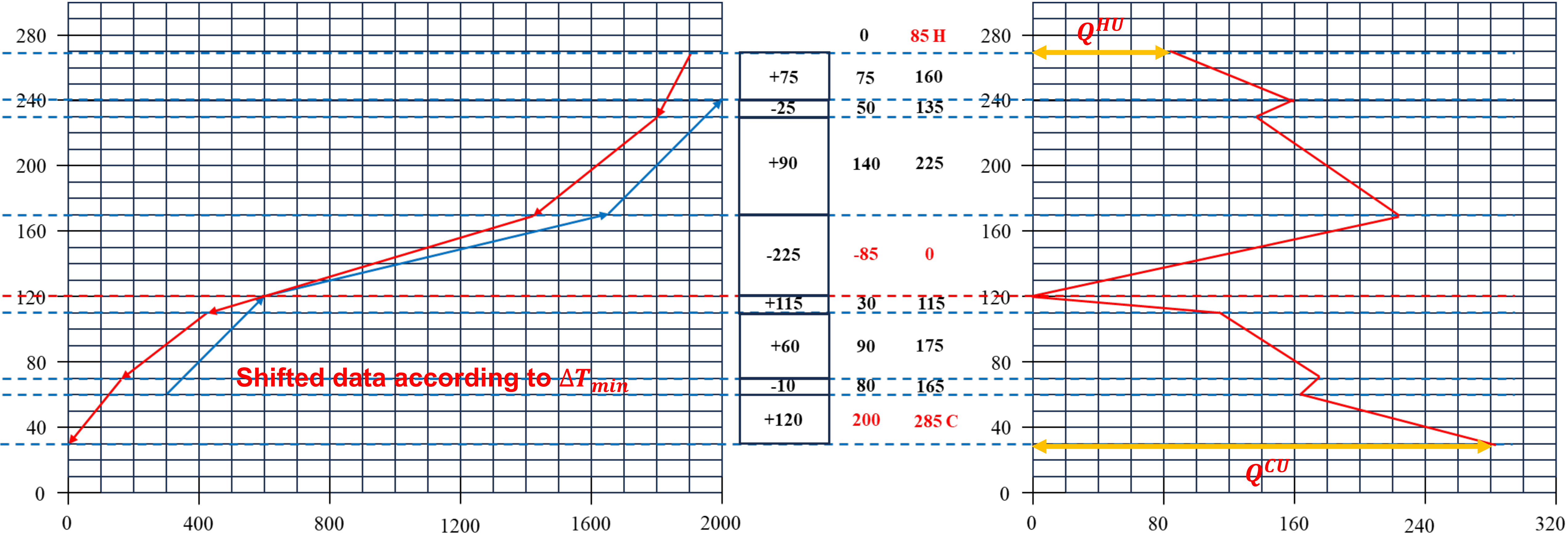


ΔT_{min} represents a bottleneck for heat-recovery and is referred to as the Heat Recovery Pinch.

Heat exchanger network synthesis (HENS)

- *Method1: Pinch Analysis (PA) based on thermodynamics*

❑ Problem table method and Grand composite curve



Heat exchanger network design

➢ *Method2: Mathematical Programming (MP) based on optimization techniques*

❑ Sequential method

Decomposed into a series of target subproblems which are solved successively:

1. Given heat recovery approach temperature, the goal is to find the minimum utility consumption/cost

   (Expanded) LP transshipment model $\rightarrow Q^H$ *and* Q^C

2. Fixed the utility consumption (from the LP model), the goal is to find the minimum number of heat exchanger units (matches between hot and cold streams) required and their corresponding heat load

   MILP model $\rightarrow y_{i,j}$ *and* $\sum_k Q_{i,j,k}$

3. Fixed the number of HEs as the number of matches as well as their heat loads (from the previous MILP model), the goal is to find the heat exchanger network configuration with minimum area/capital cost

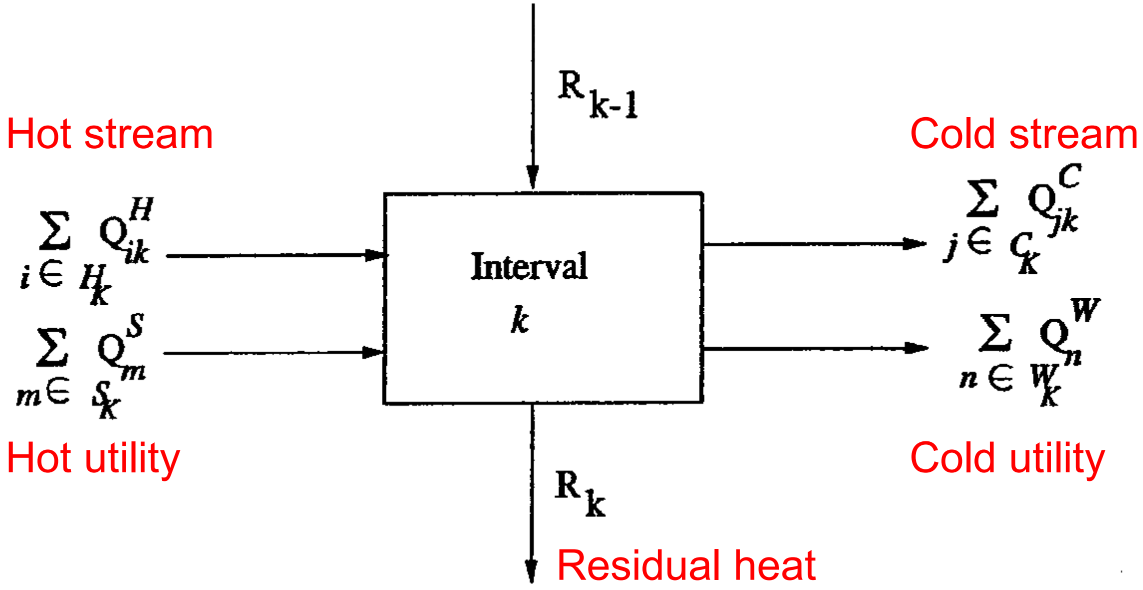
   NLP superstructure model $\rightarrow F, T, A$

Heat exchanger network design

➢ <u>*Method2: Mathematical Programming (MP) based on optimization techniques*</u>

❑ Sequential method

<u>Step1: LP transshipment model</u>



$$\min \quad \sum_{m \in S} C_m Q_m^S + \sum_{n \in W} C_n Q_n^W$$

$$s.t. \quad R_{k-1} + \sum_{i \in H_k} Q_{ik}^H + \sum_{m \in S_k} Q_m^S = R_k + \sum_{j \in C_k} Q_{jk}^C + \sum_{n \in W_k} Q_n^W, \forall k$$

$$Q_m^S, Q_n^W, R_k \geq 0$$

$$R_0 = 0$$

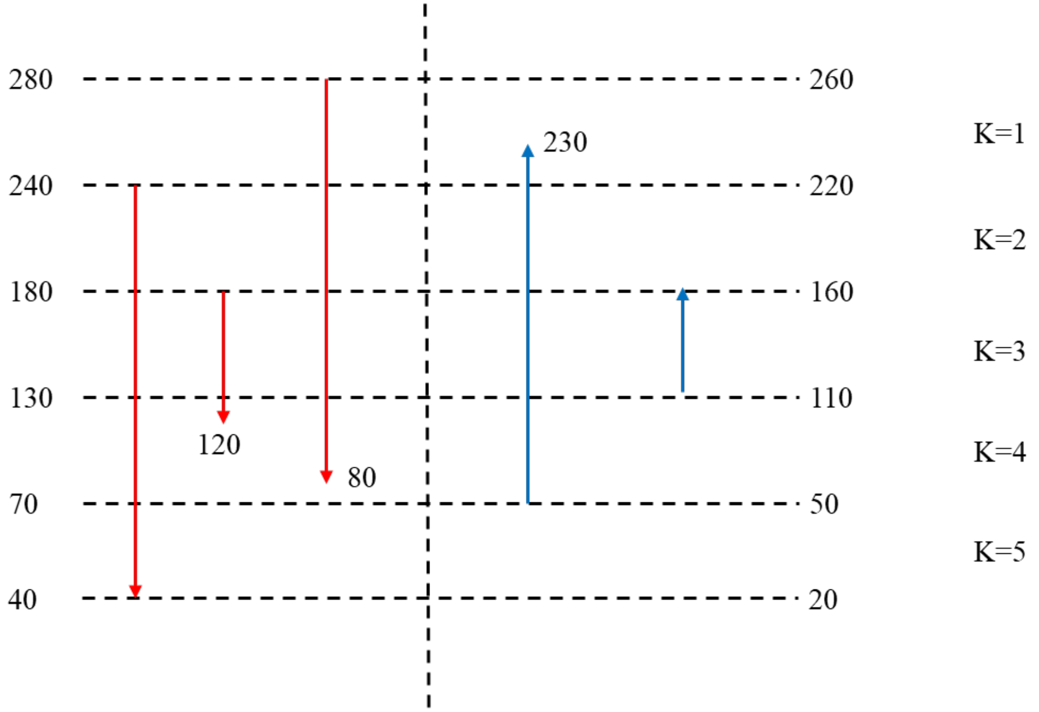
$$R_K = 0$$

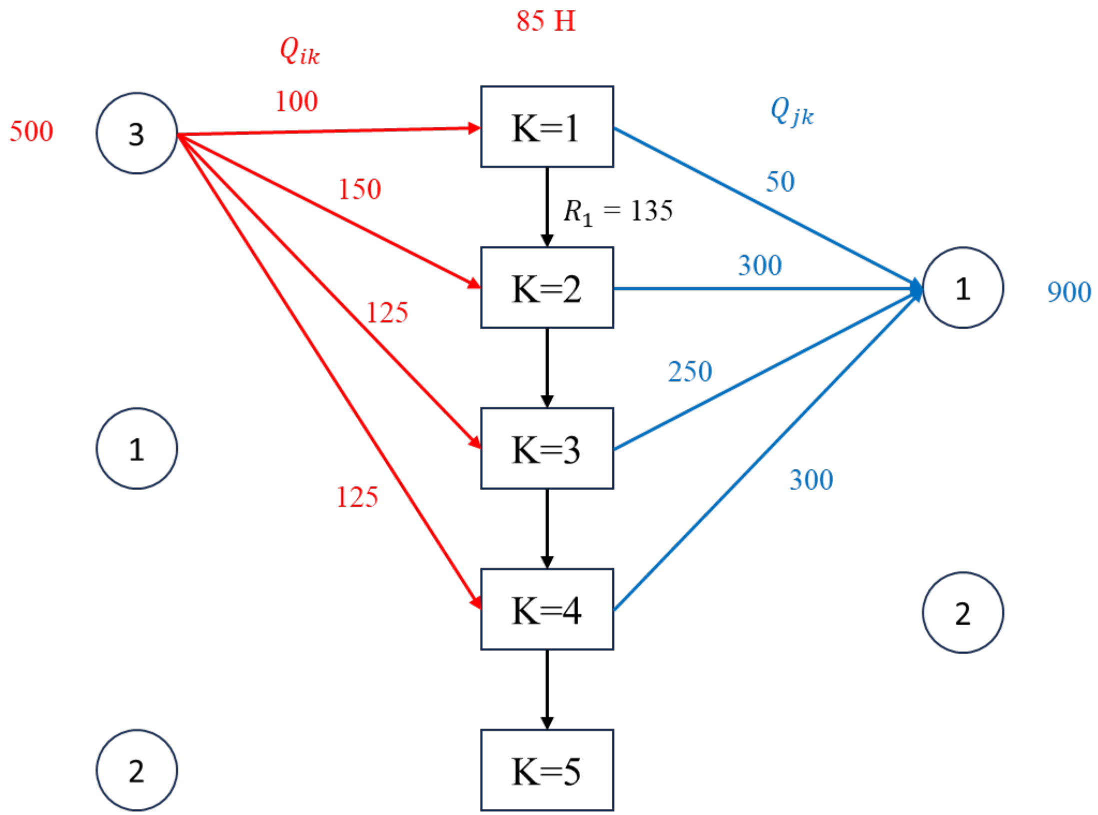
Heat exchanger network design

➢ *Method2: Mathematical Programming (MP) based on optimization techniques*

❑ Sequential method

Step1: LP transshipment model



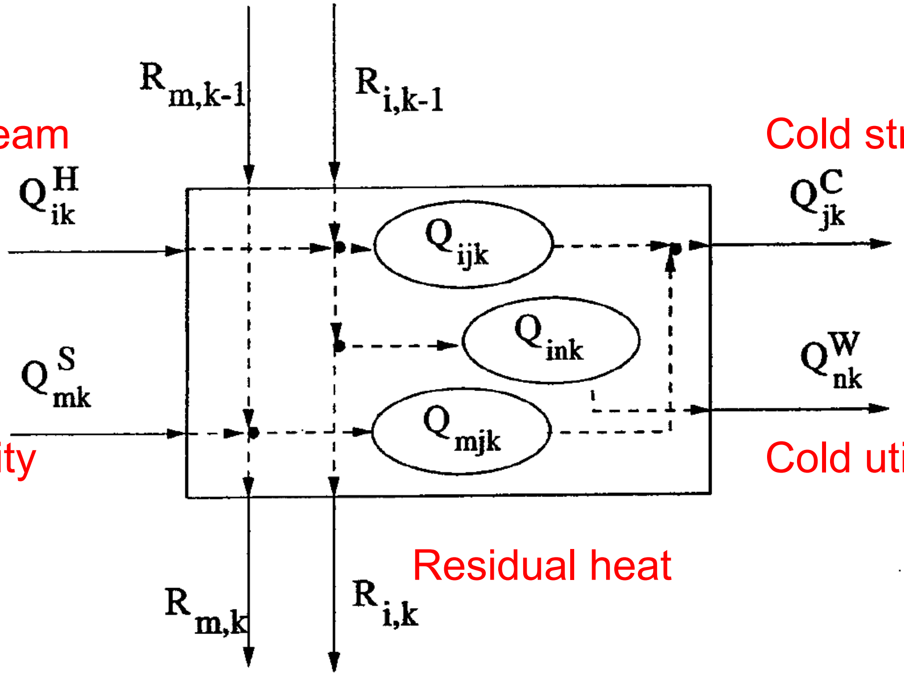


Heat exchanger network design

➢ *Method2: Mathematical Programming (MP) based on optimization techniques*

❑ Sequential method

Step1: Expanded LP transshipment model



$$\min \sum_k \left(\sum_{m \in S} C_S Q^S_{mk} + \sum_{n \in W} C_n Q^W_{nk} \right)$$

$$s.t.$$

$$\left.\begin{array}{l}
R_{i,k} - R_{i,k-1} + \sum_{j \in C_k} Q_{ijk} + \sum_{n \in W_k} Q_{ink} = Q^H_{ik}, \forall i \in H_k \\
R_{m,k} - R_{m,k-1} + \sum_{j \in C_k} Q_{mjk} = Q^S_m, \forall m \in S_k \\
\sum_{i \in H_k} Q_{ijk} + \sum_{m \in S_k} Q_{mjk} = Q^C_{jk}, \forall j \in C_k \\
\sum_{i \in H_k} Q_{ink} = Q^W_n, \forall n \in W_k \\
R_{mk}, R_{ik}, Q_{ijk}, Q_{mjk}, Q_{ink}, Q^S_m, Q^W_n \geq 0
\end{array}\right\} k = 1, \ldots, K$$

$$R_{i0} = 0, \forall i \in H_0, \quad R_{m0} = 0 \forall m \in S_0$$

$$R_{iK} = 0, \forall i \in H_K \quad R_{mK} = 0 \forall m \in S_K$$

This can be reduced to compact LP model by defining:

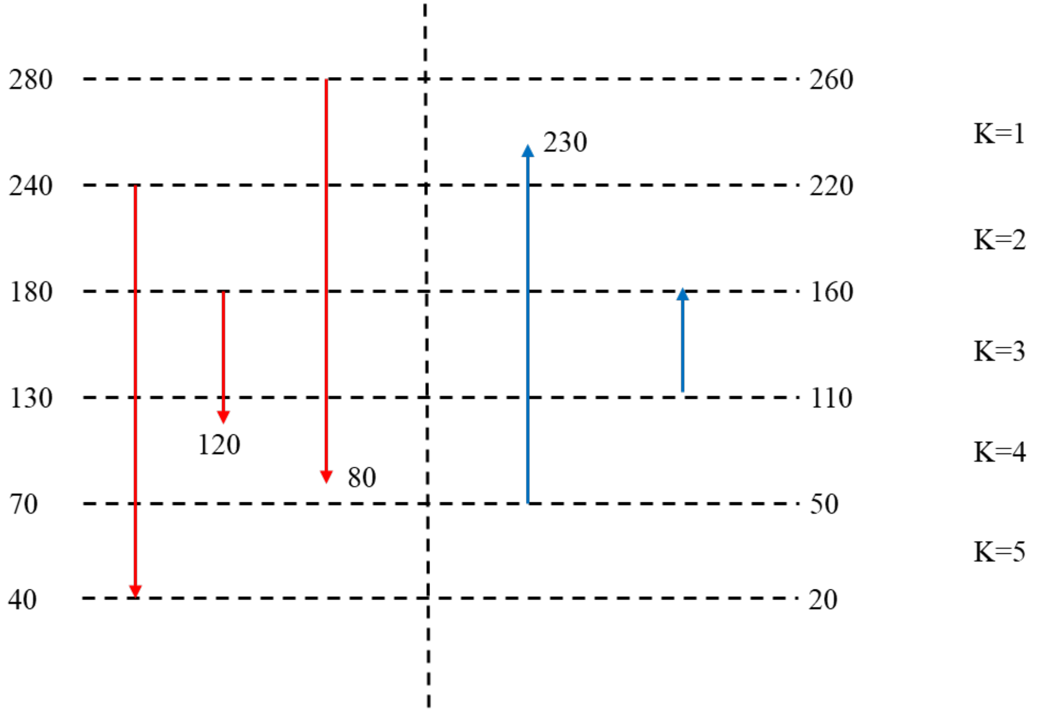
$$R_k = \sum_i R_{i,k} + \sum_m R_{m,k}$$

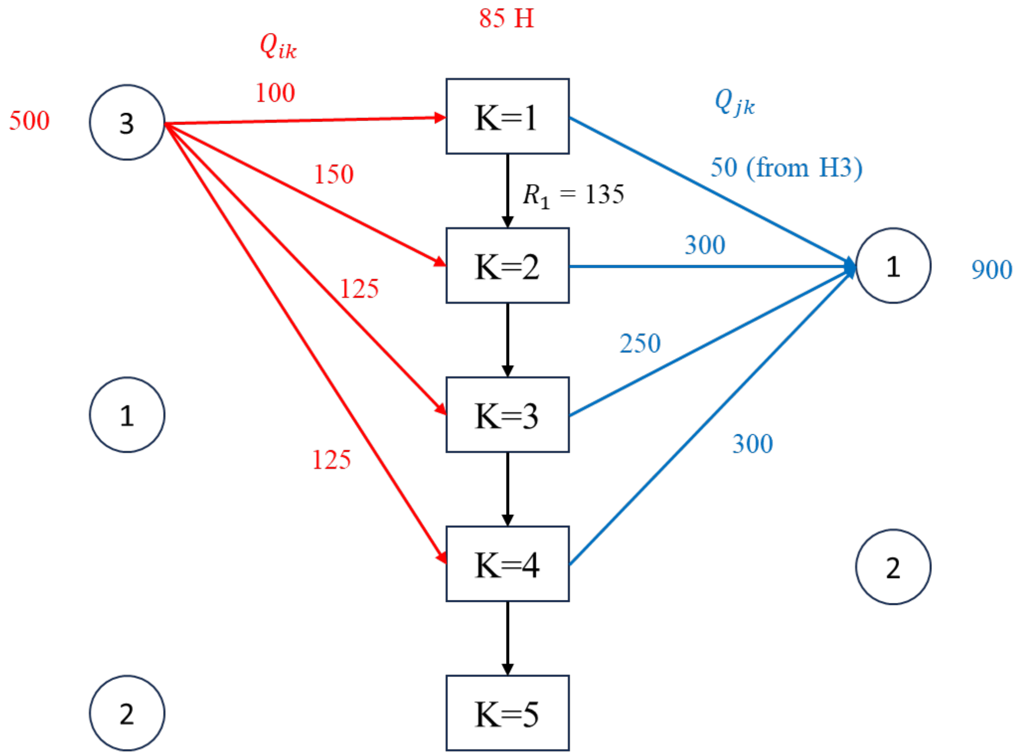
Heat exchanger network design

➢ *Method2: Mathematical Programming (MP) based on optimization techniques*

❑ Sequential method

Step1: Expanded LP transshipment model





$R_1 = 135 = 85\text{H} + 50$ (from H3)

Heat exchanger network design

➢ *Method2: Mathematical Programming (MP) based on optimization techniques*

❑ Sequential method

Step2: MILP model for minimum number of matches

$$\min \quad \sum_{i\in H_k}\sum_{j\in C_k} y_{ij}^q$$

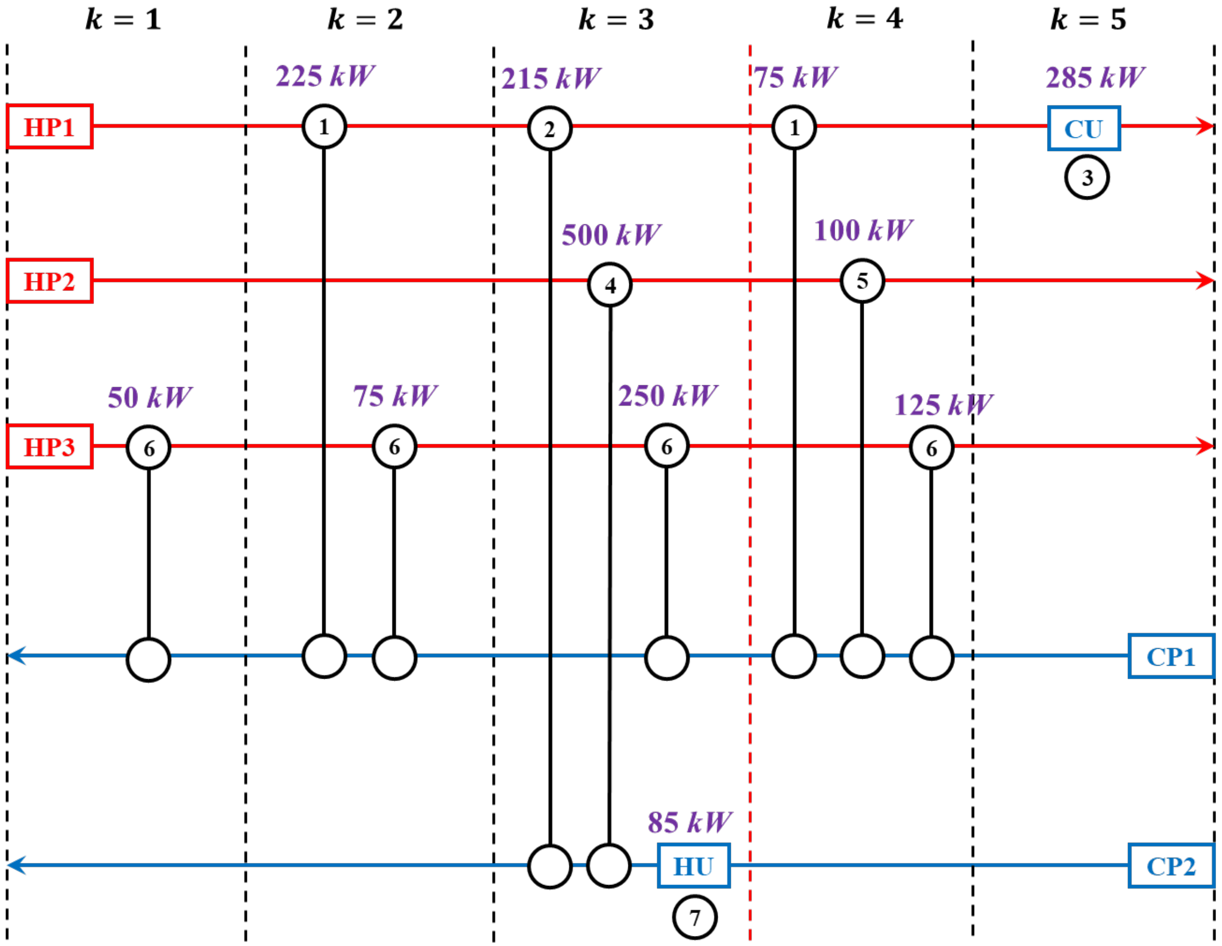
$$\text{s.t.} \quad R_{i,k} - R_{i,k-1} + \sum_{j\in C_k} Q_{ijk} = Q_{ik}^H,\ i\in H_k,\ k=1,\ldots,K_q$$

$$\sum_{i\in H_k} Q_{ijk} = Q_{jk}^C,\ j\in C_k,\ k=1,\ldots,K_q$$

$$\sum_{k\in K_q} Q_{ijk} - Q_{ij}^U y_{ij}^q \le 0,\ i\in H,\ j\in C$$

$$R_{ik}, Q_{ijk} \ge 0,\ \forall i,\ \forall j,\ \forall k$$

$$y_{ij}^q \in \{0,1\},\ \forall i,\ \forall j$$

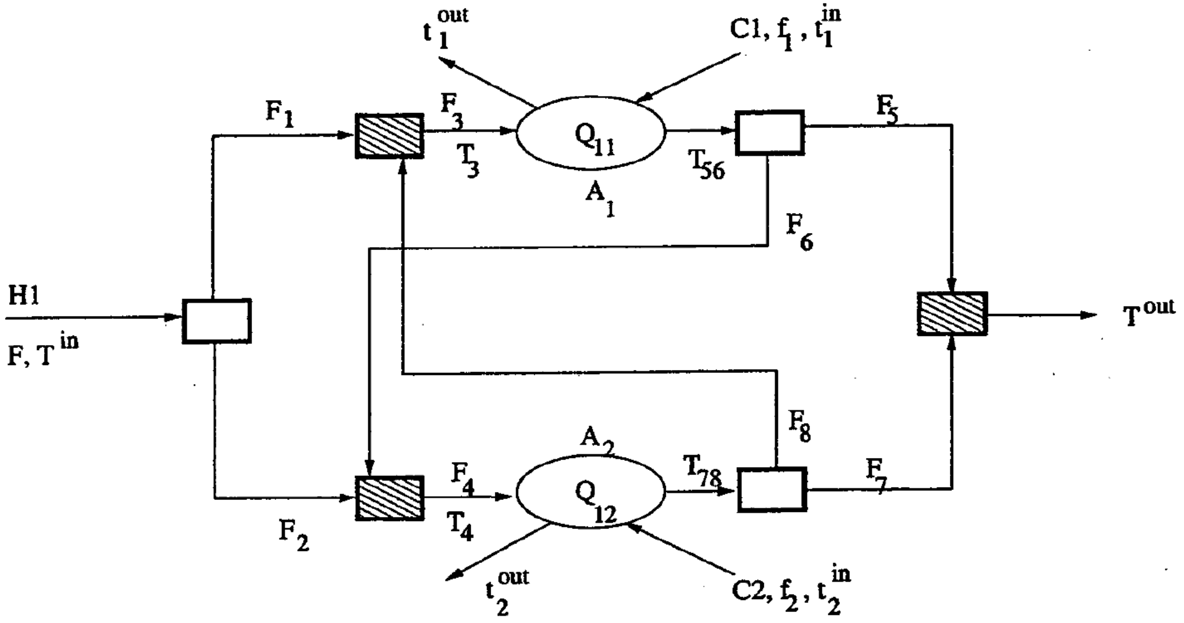


Heat exchanger network design

➢ <u>*Method2: Mathematical Programming (MP) based on optimization techniques*</u>

❑ Sequential method

<u>Step3: NLP model for network generation</u>



Objective function

$$\min \text{cost} = c_1 A_1^{0.6} + c_2 A_2^{0.6}$$

Mass and energy balance for splitter and mixer

$$F_1 + F_2 = F$$

$$\text{Mixer 1} \begin{cases} F_1 + F_8 - F_3 = 0 \\ F_1 T^{IN} + F_8 T_{78} - F_3 T_3 = 0 \end{cases}$$

$$\text{Mixer 2} \begin{cases} F_2 + F - 6 - F_4 = 0 \\ F_2 T^{IN} + F_6 T_{56} - F_4 T_4 = 0 \end{cases}$$

$$F_3 - F_6 - F_5 = 0$$
$$F_4 - F_7 - F_8 = 0$$

Heat load of heat exchanger

$$F_3 (T_3 - T_{56}) = Q_{11}$$
$$F_4 (T_4 - T_{78}) = Q_{12}$$

Feasibility

$$\begin{aligned} T_3 - t_1^{OUT} &\geq \Delta T_{min} \\ T_{56} - t_1^{IN} &\geq \Delta T_{min} \\ T_4 - t_2^{OUT} &\geq \Delta T_{min} \\ T_{78} - t_2^{IN} &\geq \Delta T_{min} \\ F_j &\geq 0, \ j = 1, \ldots, 8 \end{aligned}$$

Area calculation

$$Q_{11} = U A_1 \frac{(T_3 - t_1^{out}) - (T_{56} - t_1^{in})}{\ln\left(\frac{T_3 - t_1^{out}}{T_{56} - t_1^{in}}\right)}$$

$$Q_{12} = U A_2 \frac{(T_4 - t_2^{out}) - (T_{78} - t_2^{in})}{\ln\left(\frac{T_4 - t_2^{out}}{T_{78} - t_2^{in}}\right)}$$

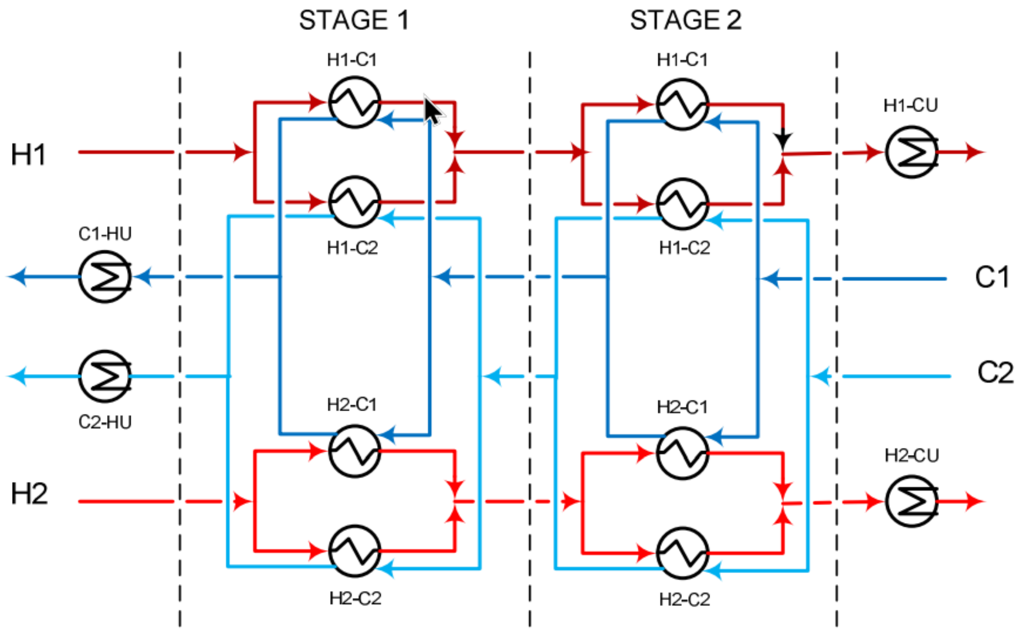
Heat exchanger network design

➢ *Method2: Mathematical Programming (MP) based on optimization techniques*

❑ Simultaneous method

❖ Previous sequential method is based on the assumption that the optimal network has a min utility cost and a min number of units for that utility cost, and a min area cost for that number of units

→ *capital cost and operating cost should be considered simultaneously*



Assumptions:

- isothermal mixing
- no split stream flowing through more than one exchanger
- utilities at the end of the superstructure
- no stream bypass

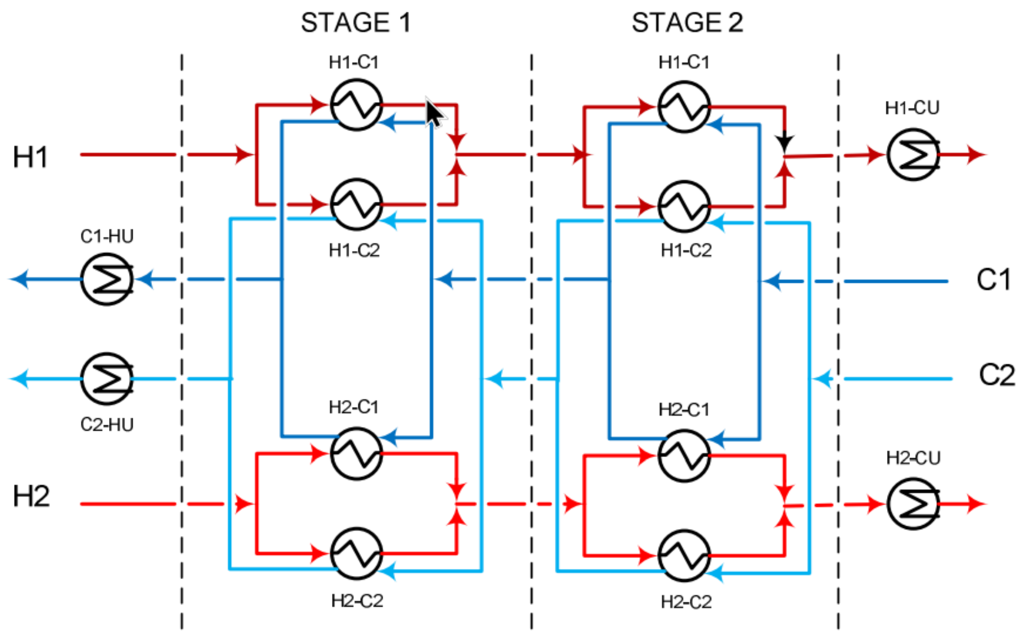
Nonconvex MINLP model:

- Only objective function is nonlinear and nonconvex
- All constraints are linear
- Much less temperature variables; no nonlinear constraints for mixing; no need to define split stream flowrate

Heat exchanger network design

➢ *Method2: Mathematical Programming (MP) based on optimization techniques*

❑ Simultaneous method



Overall heat balance

$$\sum_{k\in ST}\sum_{j\in CP} q_{ijk} + q_{cui} = F_i\left(T_i^{in} - T_i^{out}\right), i \in HP$$

$$\sum_{k\in ST}\sum_{i\in HP} q_{ijk} + q_{huj} = F_j\left(T_j^{out} - T_j^{in}\right), j \in CP$$

Heat balance at each temperature stage

$$\sum_{j\in CP} q_{ijk} = F_i(t_{ik} - t_{ik+1}), i \in HP, k \in ST$$

$$\sum_{i\in HP} q_{ijk} = F_j\left(t_{jk} - t_{jk+1}\right), j \in CP, k \in ST$$

Assignment of Inlet temperature

$$t_{i1} = T_i^{in}, i \in HP$$

$$t_{j,NT+1} = T_j^{in}, j \in CP$$

Energy balance for utility load

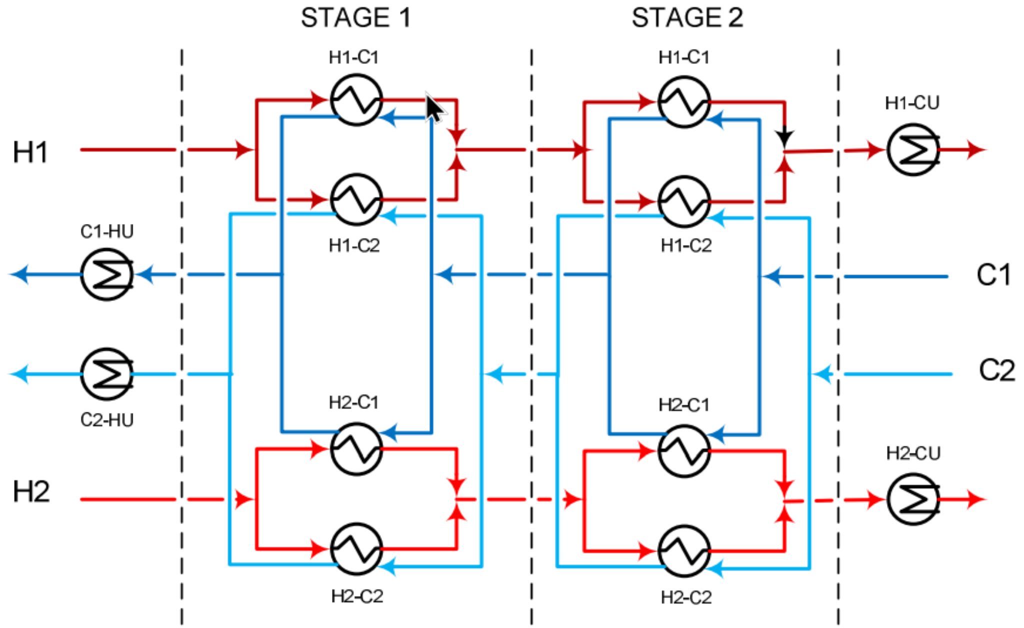
$$q_{cui} = F_i\left(t_{i,NT+1} - T_i^{out}\right), i \in HP$$

$$q_{huj} = F_j\left(T_j^{out} - t_{j1}\right), j \in CP$$

Heat exchanger network design

➢ *Method2: Mathematical Programming (MP) based on optimization techniques*

❑ Simultaneous method



Feasibility of temperature

$$t_{ik+1} \le t_{ik}, i \in HP, k \in ST$$
$$t_{jk+1} \le t_{jk}, j \in CP, k \in ST$$
$$t_{iNT+1} \le T_i^{out}, i \in HP$$
$$t_{j1} \ge T_j^{out}, j \in CP$$

Logical constraints

$$q_{ijk} - \Omega_{ij} z_{ijk} \le 0, i \in HP, j \in CP, k \in ST$$
$$q_{cui} - \Omega_i z_{cui} \le 0, i \in HP$$
$$q_{huj} \le \Omega_j z_{huj}, j \in CP$$

Calculation of temperature difference

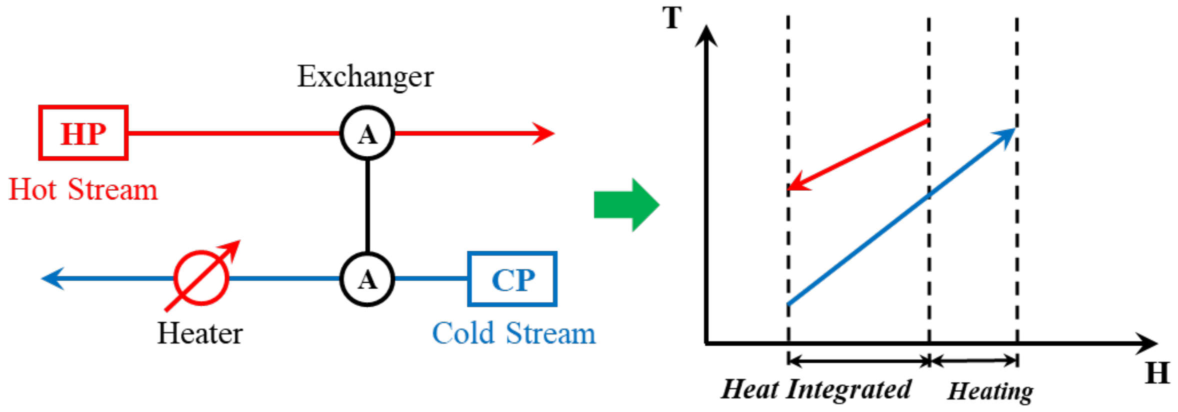
$$dt_{ijk} \le t_{ik} - t_{jk} + \Gamma_{ij}(1 - z_{ijk}), i \in HP, j \in CP, k \in ST$$
$$dt_{ijk+1} \le t_{ik+1} - t_{jk+1} + \Gamma_{ij}(1 - z_{ijk}), i \in HP, j \in CP, k \in ST$$

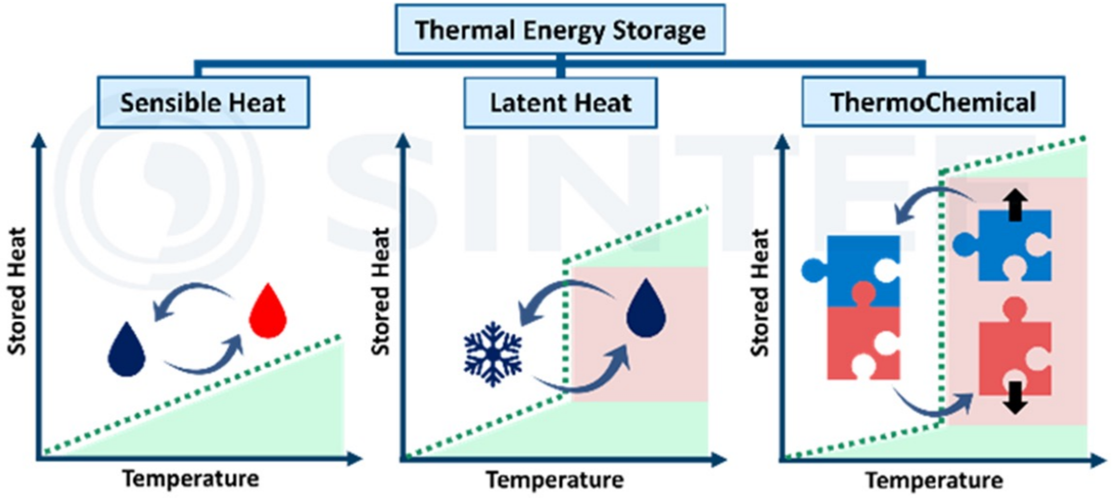
Calculation of HEX area

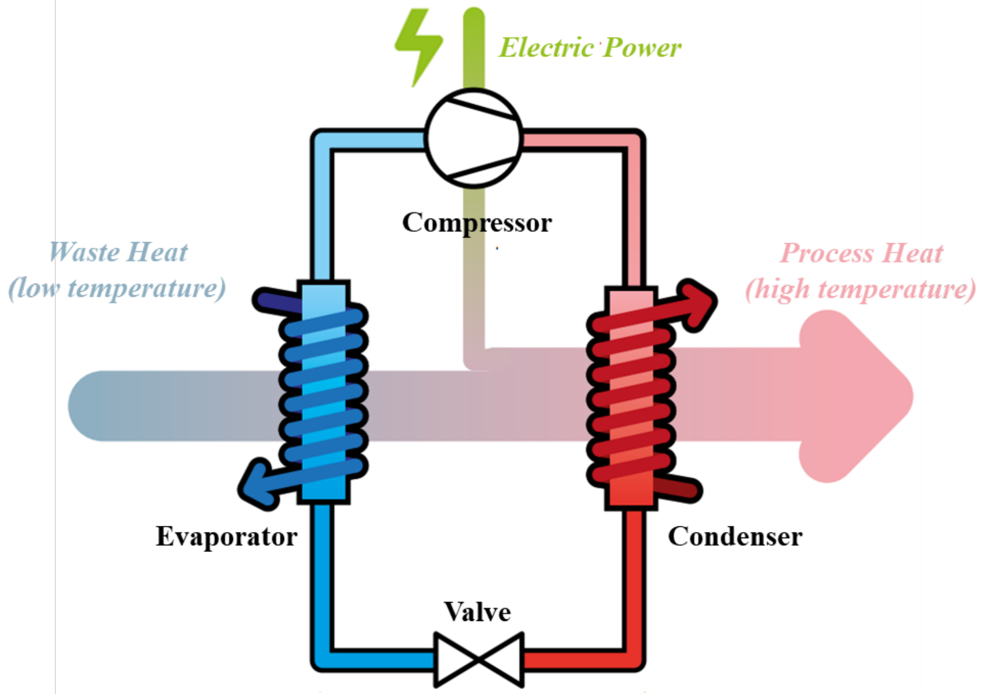
$$LMTD_{ijk} = \left[dt_{ijk} dt_{ijk+1} \frac{dt_{ijk} + dt_{ijk+1}}{2}\right]^{1/3}, i \in HP, j \in CP, k \in ST$$
$$A_{ijk} = q_{ijk}\left(h_i^{-1} + h_j^{-1}\right)/LMTD_{ijk}, i \in HP, j \in CP, k \in ST$$

Motivation

- ➢ *Electrified heat exchanger network is required*







The **GOAL** is to discern the optimal design and operation of heat exchanger network (HEN), utility system, electrified heat pump (EHP), and thermal energy storage (TES)

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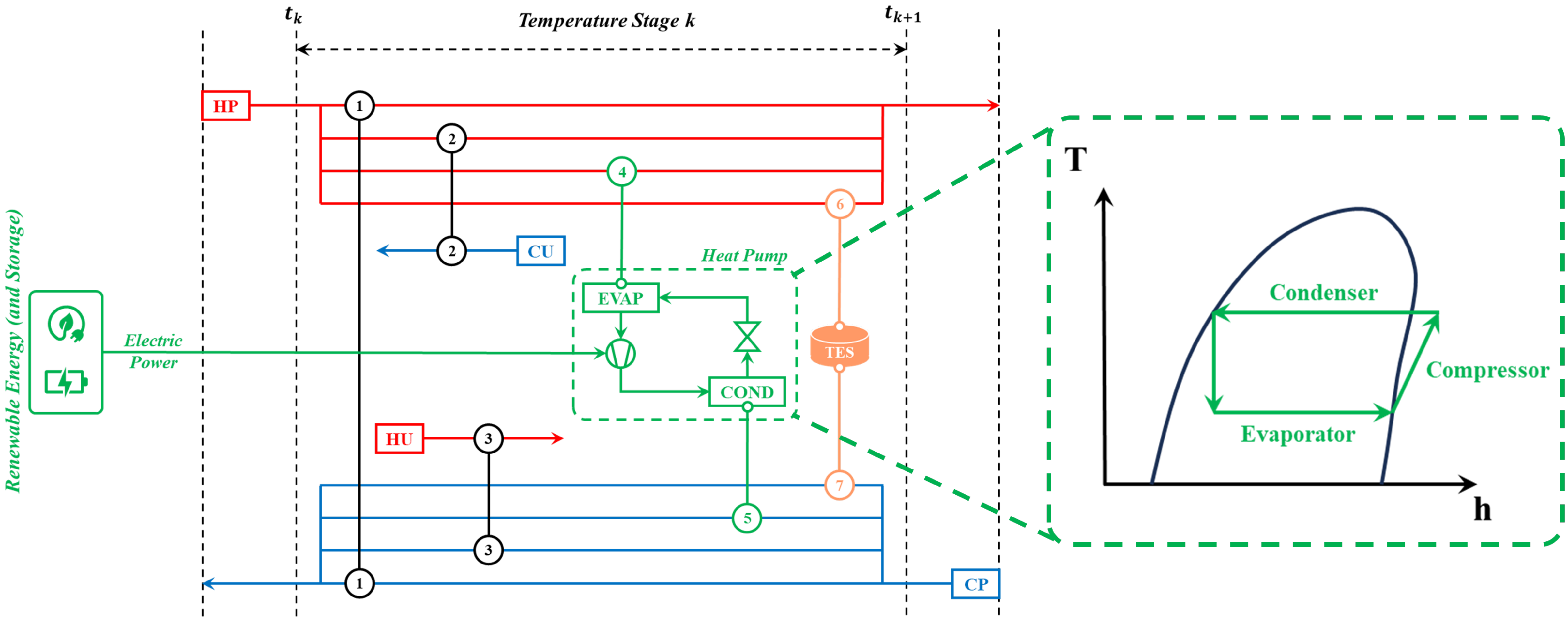
Electrified Heat Exchanger Network

➢ *A superstructure of the electrified heat exchanger network*



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Electrified Heat Exchanger Network

➢ *Optimization model is developed by extending the simultaneous model*

❑ Proposed optimization model

$$Min\ TAC = utility\ cost\ (\boxed{HU}\ \boxed{CU})$$
$$+\ heat\ exchanger\ cost\ (①②③④⑤⑥⑦)$$
$$+\ heat\ pump\ cost\ (mainly\ )+\ storage\ cost\ (TES)$$

$$s.t.\quad B_1\begin{bmatrix} z \\ y \end{bmatrix} + B_2\begin{bmatrix} t \\ dt \\ q \\ A \\ h \\ F^{wf} \\ W_{comp} \end{bmatrix} - b \leq 0$$

Eqs. → heat exchanger area calculation

Eqs. → heat load of heat pump components calculation

(heat-pump capital cost is a function of W_{comp}…)

- Overall heat balance
- Heat balance at each temperature stage
- Assignment of Inlet and outlet temperature
- Feasibility of temperature
- Logical constraints
- Calculation of temperature difference

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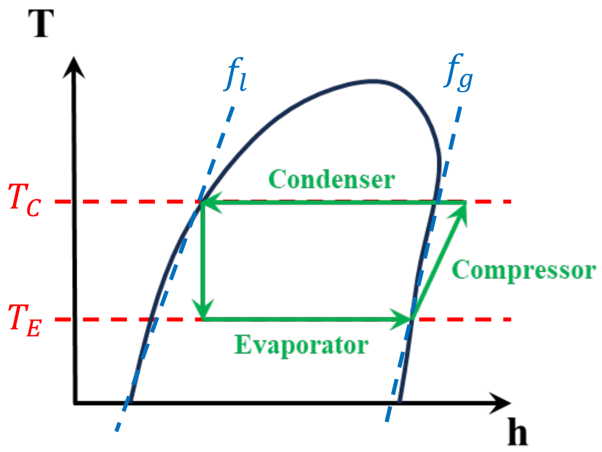
Electrified Heat Exchanger Network

➢ *Optimization model is developed by extending the simultaneous model*

❑ Heat pump model



Approximation of enthalpy (linearization)

$$h_{evap}^{out} = f_g(T_E) \qquad h_{evap}^{in} = h_{cond}^{out}$$

$$h_{cond}^{in} = f_g(T_c) \qquad h_{cond}^{out} = f_l(T_c)$$

Calculation of electric power

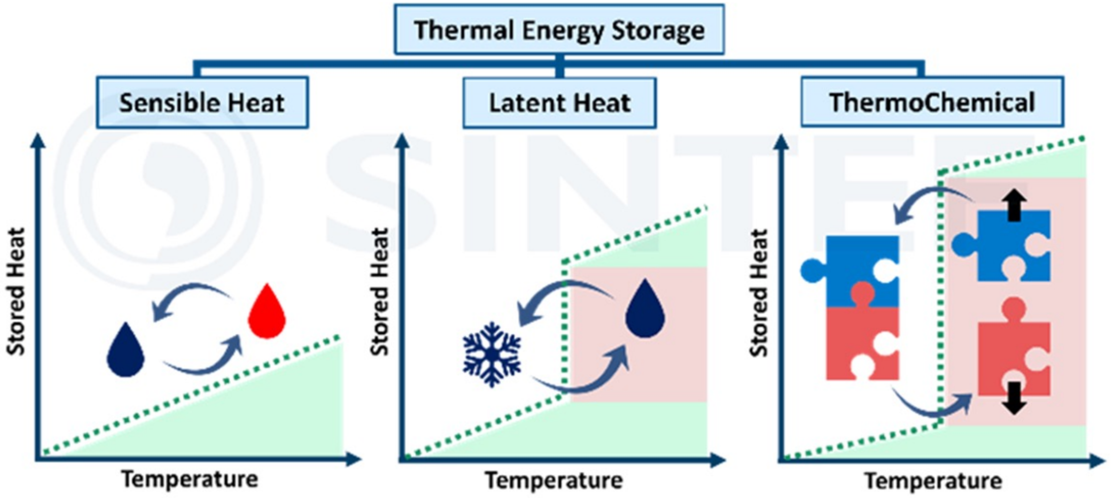
$$W^{elec}\eta = F^{wf}(h_{cond}^{in} - h_{evap}^{out})$$

Calculation of heat load

$$q_{evap} = F^{wf}(h_{evap}^{out} - h_{evap}^{in})$$

$$q_{cond} = F^{wf}(h_{cond}^{in} - h_{cond}^{out})$$

❑ TES model



Calculation of TES temperature change

$$mC_p(T_{p+1} - T_p) = \tau_p(Q^{in} - Q^{out})$$

Additional constraint

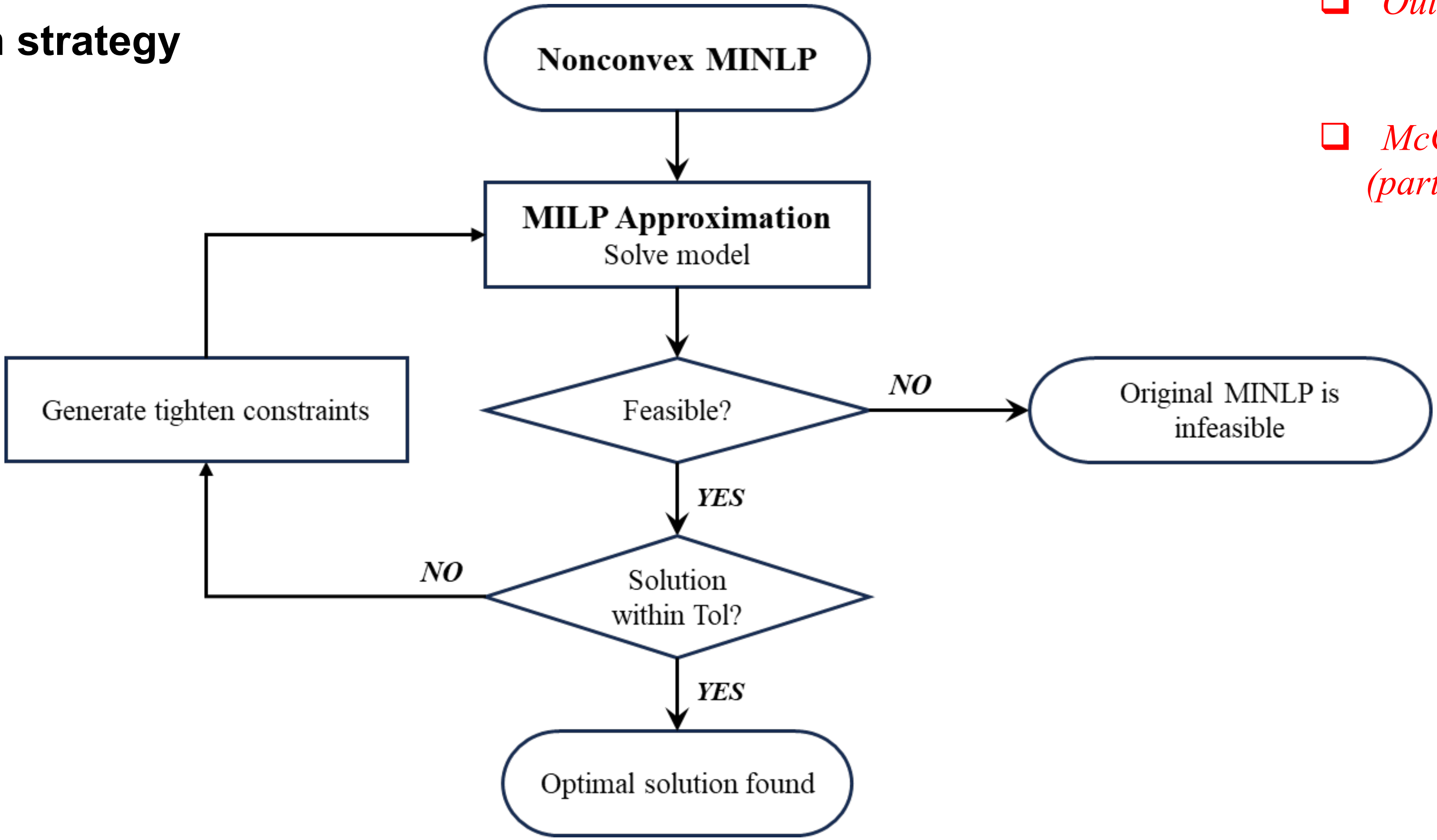
$$T_{p=1} = T_{p=end}$$

$$T_p \leq T^{max}$$

Electrified Heat Exchanger Network

➢ *MILP approximation to guarantee global optimum*

❑ **Solution strategy**

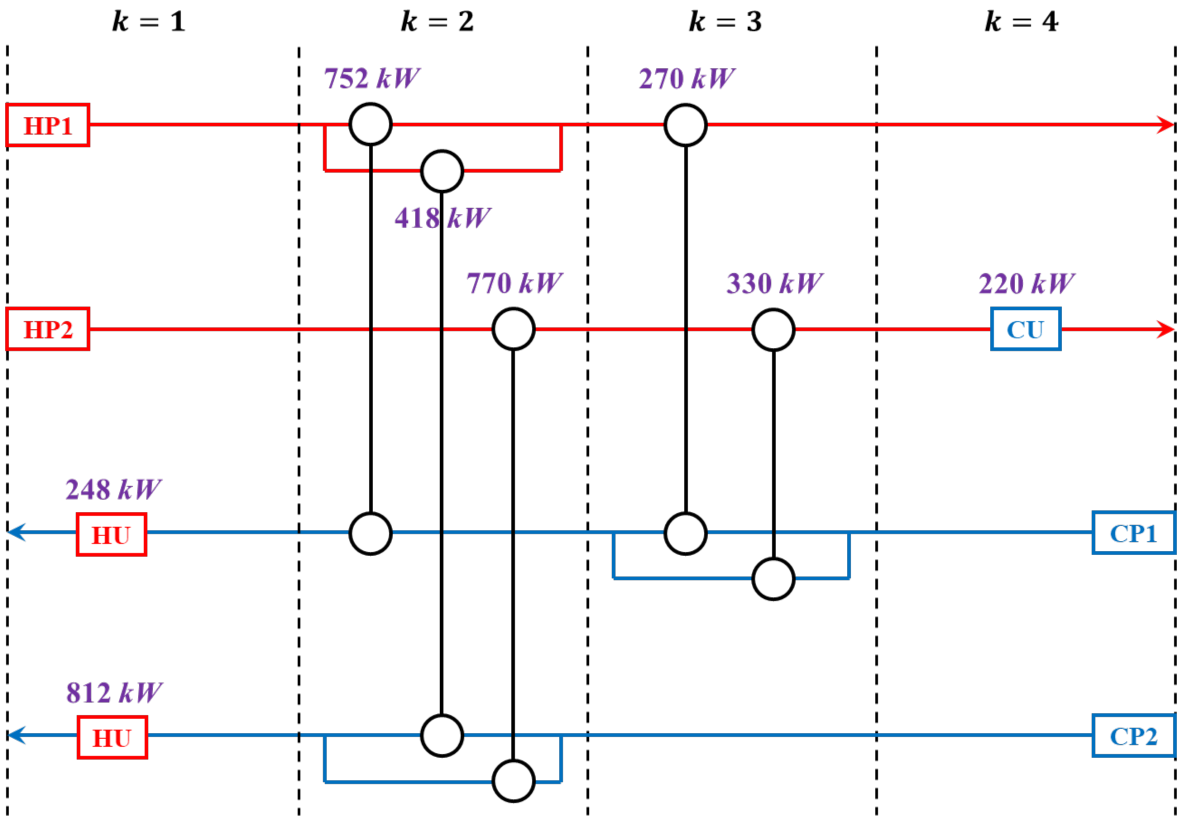


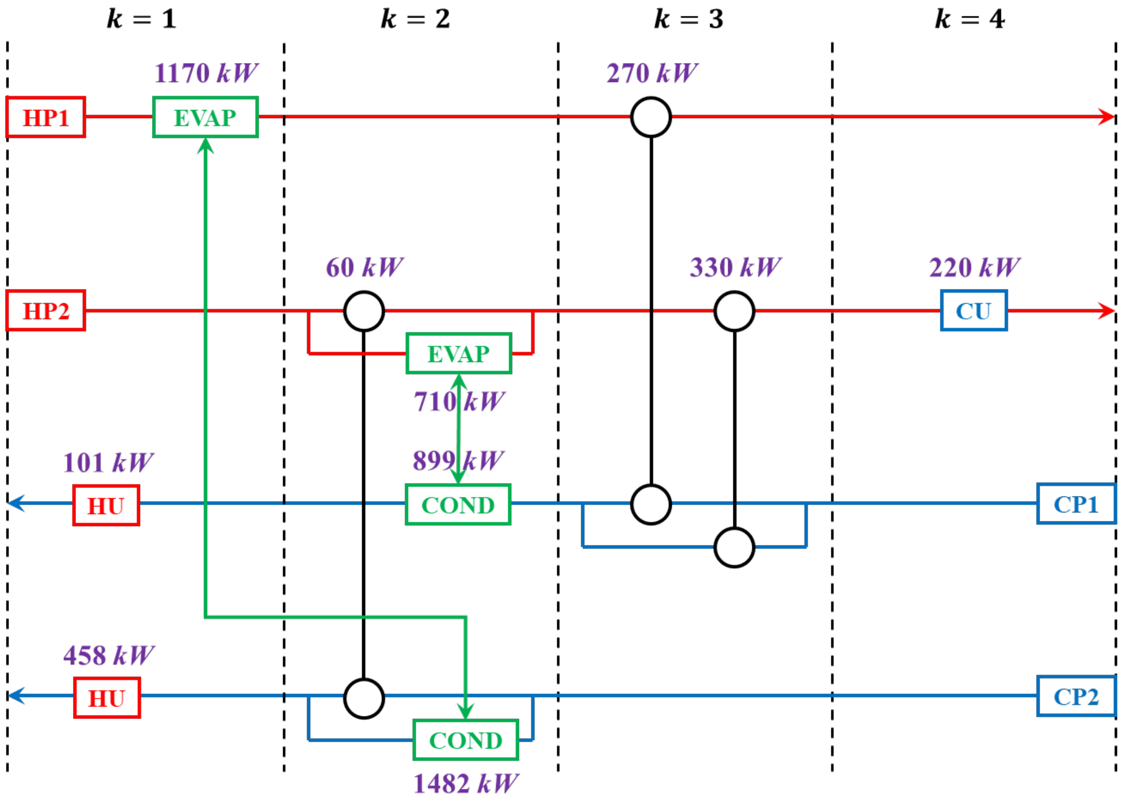
❑ *Outer Approximation*
→ RecLMTD (convex)

❑ *McCormick Convex Hull (partitioning scheme)*
→ bilinear terms (Area, heat load of heat pump components)

Preliminary results

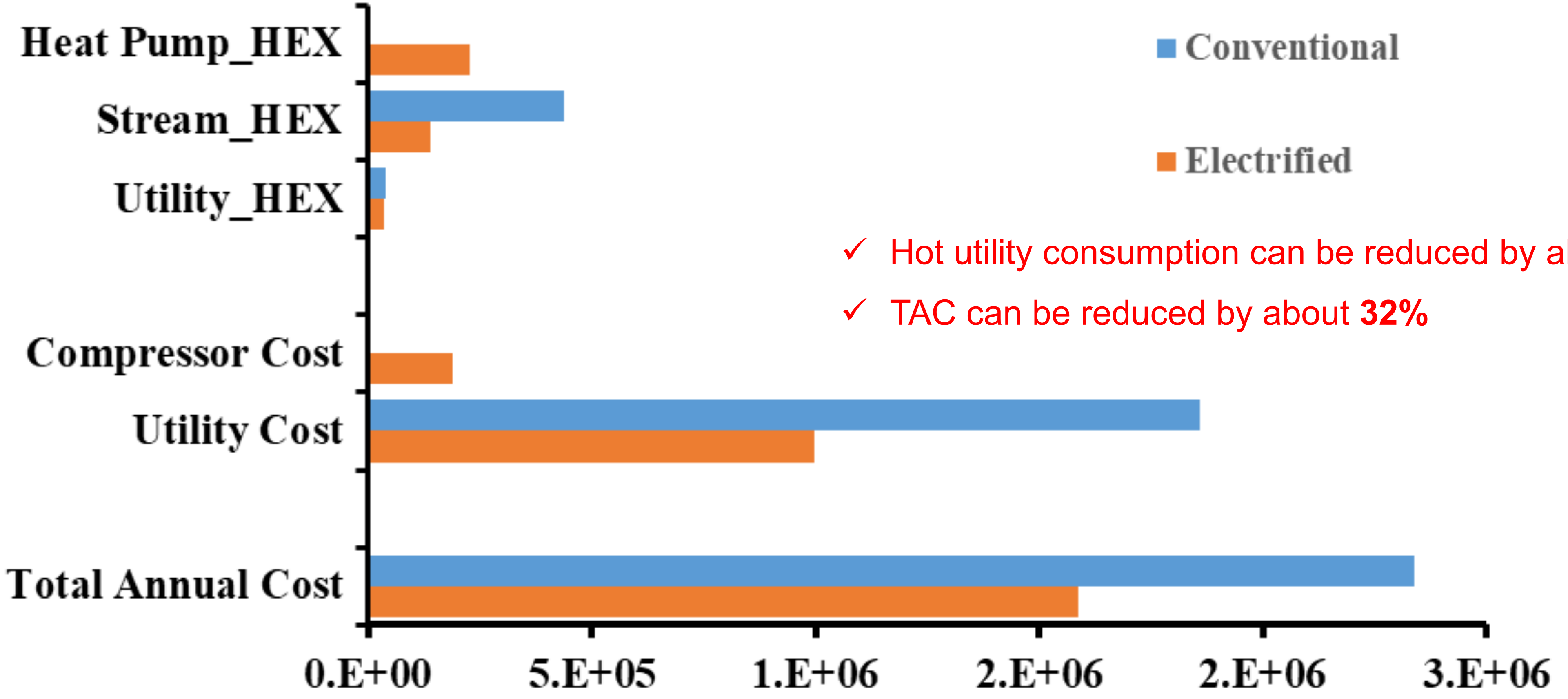
➢ *Case 1: only conventional utility is available; Case 2: electrified heat pump is available*





Preliminary results

➢ *Economic comparison of the two case studies*



✓ Hot utility consumption can be reduced by about **47%**

✓ TAC can be reduced by about **32%**

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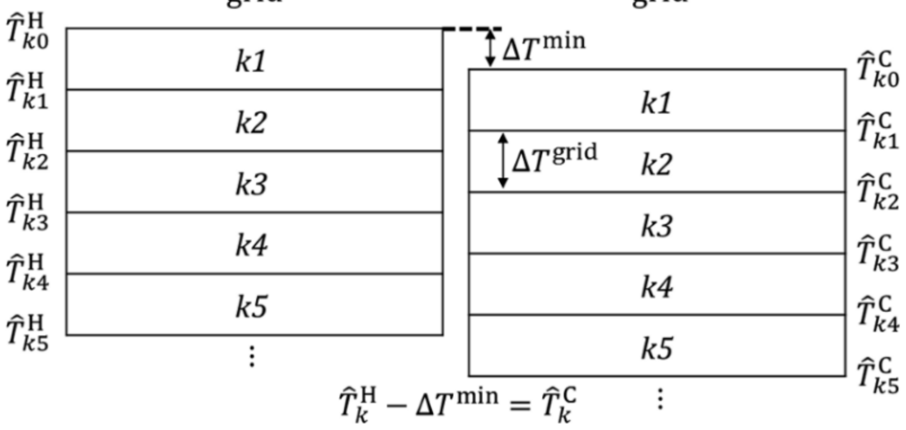
Next steps

➢ ***A different MILP formulation through temperature discretization***



Calculation of heat load

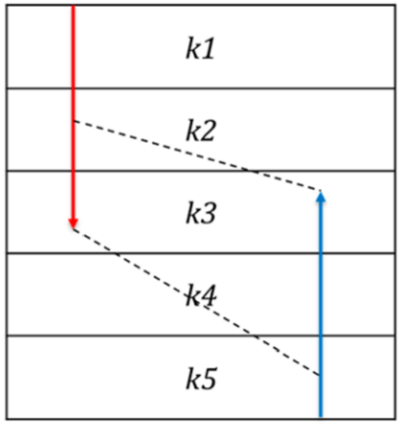
$$H^{H}_{s,k} = (\hat{T}^{H}_{k-1} - \hat{T}^{H}_{k})F^{H}_{s,k} = \Delta T^{\text{grid}} F^{H}_{s,k}$$

$$H^{H}_{s,k} = \sum_{k' \geq k} \sum_{s' \neq s} Q^{\text{EX}}_{s,k,s',k'} \qquad \bar{Q}^{H}_{s,k,s'} = \sum_{k' \geq k} Q^{\text{EX}}_{s,k,s',k'}$$

$$\beta X^{H}_{s,k,s'} \leq \bar{Q}^{H}_{s,k,s'} \leq \alpha_s(\hat{T}^{H}_{k-1} - \hat{T}^{H}_{k})X^{H}_{s,k,s'}$$

$$X^{H}_{s,k,s'} = X^{H}_{s,k-1,s'} + V^{S,H}_{s,k,s'} - V^{E,H}_{s,k-1,s'}$$

$X^{H}_{s,k,s'}$	$V^{S,H}_{s,k,s'}$	$V^{E,H}_{s,k,s'}$
-	-	-
1	1	-
1	-	1
-	-	-
-	-	-



Assignment of temperature

$$T^{\text{IN}}_{s} = \sum_{k \in \mathbf{K}} (Y^{\text{IN,H}}_{s,k} \hat{T}^{H}_{k})$$

$$T^{\text{OUT}}_{s} = \sum_{k \in \mathbf{K}} (Y^{\text{OUT,H}}_{s,k} \hat{T}^{H}_{k})$$

$$Z^{H}_{s,k} = Y^{\text{IN,H}}_{s,k-1} + Z^{H}_{s,k-1} - Y^{\text{OUT,H}}_{s,k-1}$$

Assignment of flowrate

$$F_s = F^{H}_{s,k} + \tilde{F}_{s,k}$$

$$F^{H}_{s,k} \leq \alpha_s Z^{H}_{s,k}$$

$$\tilde{F}_{s,k} \leq \alpha_s(1 - Z^{H}_{s,k})$$

Next steps

➢ ***A different MILP formulation through temperature discretization***

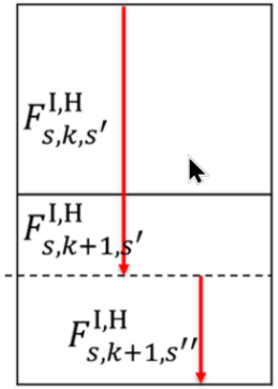
Stream splitting

$$W^{\mathrm{H}}_{s,k,s'} \le X^{\mathrm{H}}_{s,k,s'}$$

$$F^{\mathrm{H}}_{s,k} = F^{\mathrm{I,H}}_{s,k,s'} + \tilde{F}^{\mathrm{SP,H}}_{s,k,s'} \qquad F^{\mathrm{H}}_{s,k} + \tilde{F}^{\mathrm{tot,H}}_{s,k} = \sum_{s'} F^{\mathrm{I,H}}_{s,k,s'}$$

$$F^{\mathrm{I,H}}_{s,k,s'} \le \alpha_s X^{\mathrm{H}}_{s,k,s'}$$

$$\gamma_s W^{\mathrm{H}}_{s,k,s'} \le \tilde{F}^{\mathrm{SP,H}}_{s,k,s'} \le \alpha_s\left(1 - X^{\mathrm{H}}_{s,k,s'} + W^{\mathrm{H}}_{s,k,s'}\right)$$

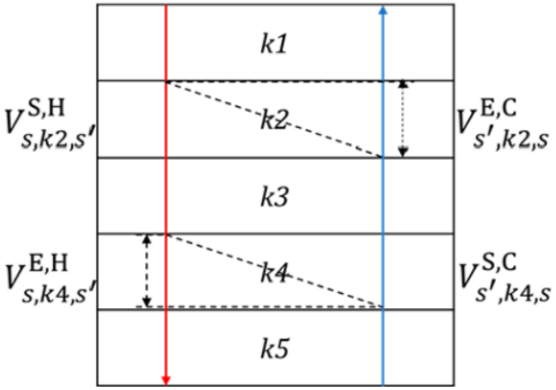


Flowrate change due to start/end of heat exchanger

$$F^{\mathrm{I,H}}_{s,k,s'} = F^{\mathrm{I,H}}_{s,k-1,s'} + \tilde{F}^{\mathrm{S,H}}_{s,k,s'} - \tilde{F}^{\mathrm{E,H}}_{s,k-1,s'}$$

$$\tilde{F}^{\mathrm{S,H}}_{s,k,s'} \le \alpha_s V^{\mathrm{S,H}}_{s,k,s'}$$

$$\tilde{F}^{\mathrm{E,H}}_{s,k,s'} \le \alpha_s V^{\mathrm{E,H}}_{s,k,s'}$$



Heat duty of heat exchanger

$$\bar{Q}^{\mathrm{H}}_{s,k,s'} = F^{\mathrm{I,H}}_{s,k,s'}\left(\hat{T}^{\mathrm{H}}_{k-1} - \hat{T}^{\mathrm{H}}_{k}\right) - \tilde{Q}^{\mathrm{D,H}}_{s,k,s'}$$

$$\tilde{Q}^{\mathrm{D,H}}_{s,k,s'} \le \delta_s\left(V^{\mathrm{S,H}}_{s,k,s'} + V^{\mathrm{E,H}}_{s,k,s'}\right)$$

Feasibility of temperature

$$F^{\mathrm{I,H}}_{s,k,s'}\left(\hat{T}^{\mathrm{H}}_{k-1} - \hat{T}^{\mathrm{H}}_{k}\right) \le \bar{Q}^{\mathrm{H}}_{s,k,s'} + \varepsilon\left(2 - V^{\mathrm{S,H}}_{s,k,s'} - V^{\mathrm{E,C}}_{s',k,s}\right)$$

Calculation of area

$$A_{s,s'} = \sum_{k\in\mathbf{K}^{\mathrm{I}}} \sum_{k'\in\mathbf{K}^{\mathrm{I}}} \frac{Q^{\mathrm{EX}}_{s,k,s',k'}\left(h_{s,k} + h_{s',k'}\right)}{h_{s,k}h_{s',k'}\Delta T^{\mathrm{lm}}_{k,k'}}$$

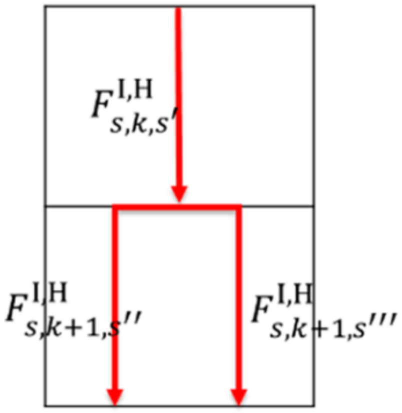
$$N^{\mathrm{EX}}_{s,s'} = \sum_{k\in\mathbf{K}^{\mathrm{I}}} V^{\mathrm{S,H}}_{s,k,s'} = \sum_{k\in\mathbf{K}^{\mathrm{I}}} V^{\mathrm{E,H}}_{s,k,s'}$$

Next steps

➢ ***A different MILP formulation through temperature discretization***

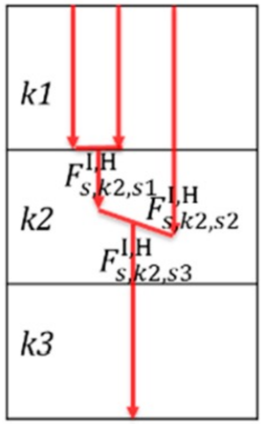
Isothermal mixing

$$\tilde{F}^{\text{tot,H}}_{s,k} \le \alpha_s(1 - W^{\text{H}}_{s,k,s'})$$

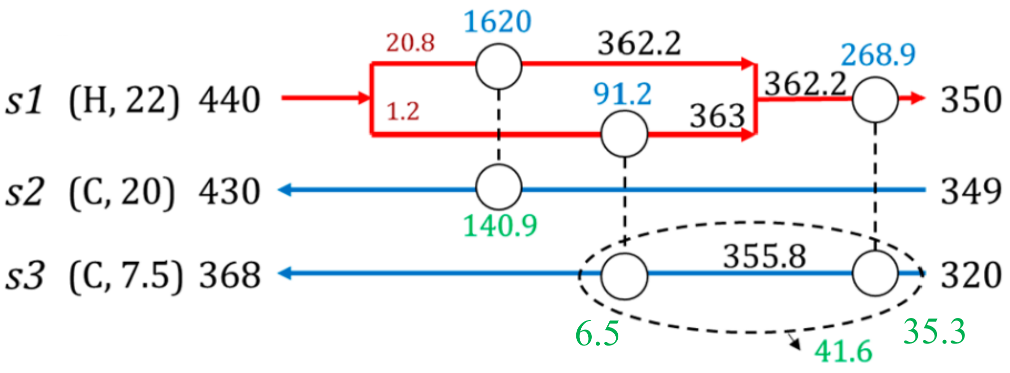


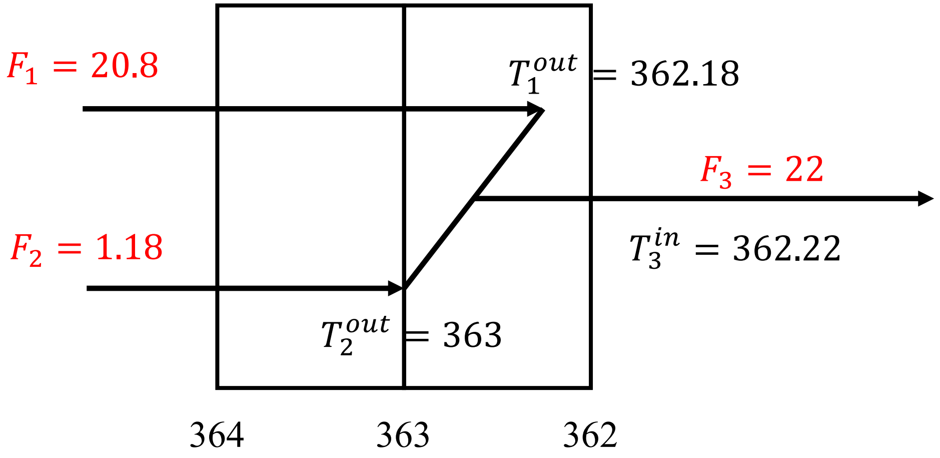
Nonisothermal mixing

$$\tilde{F}^{\text{tot,H}}_{s,k} \le \alpha_s(1 - W^{\text{H}}_{s,k,s'} + V^{\text{E,H}}_{s,k,s'})$$



$$F^{\text{H}}_{s,k} + \tilde{F}^{\text{tot,H}}_{s,k} = \sum_{s'} F^{\text{I,H}}_{s,k,s'}$$





❑ Inclusion of:

1. Utility → stream with variable flowrate
2. Heat pump system → streams with variable flowrate and temperature
3. Multiperiod operation → previous formulation or $Q_{i,j,k,k',p,p'}$

THANK YOU