phys. stat. sol. (b) **182**, 171 (1994)

Subject classification: 72.90 and 73.40; 72.30; S7.12

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Beam Instability and Space-Charge Wave Amplification in a Semiconductor Plasma

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The amplification of a space-charge wave in a semiconductor induced by the beam instability is considered. The electron beam is regarded as being injected out of the quantum well by means of resonant tunneling. The plasma–beam interaction is treated in terms of the hydrodynamic equations of a two-component liquid and the Poisson equation. The increment of SCW is calculated and the dependence of the increment on the different parameters such as temperature, carrier concentration, spacing of grating, and so on is also considered.

Es wird die Verstärkung von Raumladungswellen als Folge einer Instabilität des Elektronenstrahls im Halbleiter betrachtet. Der Elektronenstrahl wird mittels Resonanztunneln aus der Quantenmulde im Halbleiter injiziert. Die Wechselwirkung des Strahls mit dem Elektronenplasma des Halbleiters wird durch Gleichungen der Hydrodynamik für zwei Flüssigkeiten beschrieben. Das Anwachsen der Raumladungswellen wird berechnet und seine Abhängigkeit von verschiedenen Parametern, wie Temperatur, Konzentration der Ladungsträger, Periodizität des Gitters usw., wird untersucht.

1. Introduction

The different kinds of plasma instabilities in a semiconductor were the object of intense investigations during many years of research [1, 2]. The reason is that such instabilities can be the basis for the development of a novel type of solid state generators and amplifiers in the submillimeter and far-infrared regions of the spectrum.

While the beam instability of a gaseous plasma is a well-known phenomenon [3, 4] which is widely used for the excitation and amplification of waves, the beam instability of a semiconductor plasma has not received similar attention. To our mind, the main reason for the lack of experimental studies of this effect in solid state systems so far is the difficulty in realizing appropriate electron beams in a semiconductor.

In [5, 6] one of the authors considered the space-charge wave amplification induced by the beam instability in a semiconductor plasma of a multielectrode MIS microstructure. There it was employed substantially that, as follows from [7, 8], the electron distribution function of GaAs in a strong electric field of about $100\,\mathrm{kV\,cm^{-1}}$ is inverted. This means that for electron wave vectors perpendicular to the external field the derivative of the distribution function with respect to the electron velocity is greater than zero, $\mathrm{d}f/\mathrm{d}v > 0$, for some electron velocities v satisfying the condition $v_0 - \Delta v < v < v_0$. In other words, the electron distribution function $f(v)$ has an extra maximum at $v = v_0$ which is equivalent to the existence of a beam with mean velocity v_0 and velocity spread Δv. This leads to the amplification of the slow waves (i.e. the space-charge waves) moving through the semi-

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conductor plasma, if the frequency ω and the wave vector k_0 of the wave satisfy to the condition $\omega = k_0 v_0$.

It is clear, however, that the way of beam instability excitation proposed in [5, 6] has some weak points. One of them is the necessity to use only gallium arsenide as semiconductor, since the inversion of the distribution function in a strong electric field has been established hitherto only for this material. The second one is the field intensity itself, because only for an intensity of the order of 100 kV cm^{-1} the inversion of the electron distribution function becomes possible. However, in such high field electric breakdown of the material also becomes possible. Thus, in this paper a quite new scheme of beam instability excitation is proposed which in our opinion gives the possibility to avoid these drawbacks.

2. Model Description and Main Equations

The remarkable achievements of modern semiconductor fabrication technology gives the possibility to create quantum-size-effect heterostructures which employ the wave nature of charge carriers for working out quite novel devices. One of such phenomena is the electron resonant tunneling (RT) through a sequence of quantum wells (QW) or out of a single one.

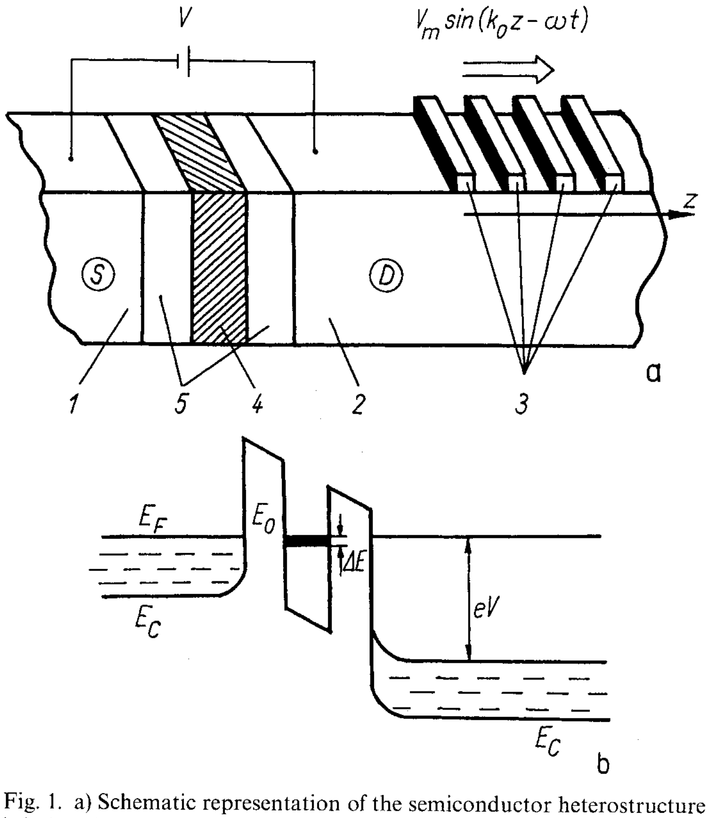

Fig. 1. a) Schematic representation of the semiconductor heterostructure in which the beam instability is induced. 1 source, 2 drain, 3 gratings, 4 GaAs, 5 AlGaAs, V is the applied voltage. b) Energy diagram of the heterostructure (when the voltage V is applied between source and drain). E_F is the Fermi level, E_0 the quantization level in the QW, E_C the bottom of the conduction band

The electron RT is used for the development of elements with negative differential conductivity, energy filters, and monoenergetic hot electron injectors.

Let us consider the microstructure schematically represented in Fig. 1a. It is composed of a GaAs/AlGaAs heterostructure which forms QWs and of the bulk semiconductor which is adjusted to the heterostructure.

Let us suppose further that a voltage V is applied between the two semiconductor domains of the structure which are labelled as source (S) and drain (D) and let the voltage be sufficient for the Fermi level to intersect with the quantization level in the QW (Fig. 1b). Then the tunnel current which can be considered as the injected electrons in the drain, will flow through the microstructure. The electron velocity spread Δv will be very small and corresponds to the width of the quantization level ΔE which is approximately 1 meV.

An obvious estimate shows that the unperturbed velocity of the electrons injected into the semiconductor (D) is greater than the mean thermal velocity of the electrons of the semiconductor plasma and also greater than the thermal velocity of the beam electrons.

So the electrons of the beam and of the semiconductor plasma could be considered as two reciprocally penetrating fluids which can be treated in terms of hydrodynamic equations together with the Poisson equation. In the absence of an external magnetic field they are [9]

$$
\begin{aligned}
&\frac{\partial \boldsymbol{v}_{1j}}{\partial t} + (\boldsymbol{v}_{1j}\nabla)\,\boldsymbol{v}_{1j} = \frac{e}{m_e^*}\nabla\Phi - \frac{\nabla p_j}{m_e^* n_{1j}} - \nu_j(\boldsymbol{v}_{1j} - \boldsymbol{v}_{0j})\,,\\
&\frac{\partial n_{1j}}{\partial t} + \operatorname{div}(n_{1j}\boldsymbol{v}_{1j}) = 0\,, \qquad\qquad (1)\\
&\operatorname{div}\boldsymbol{E} = -\nabla\Phi = -4\pi e\sum_j (n_{1j} - n_{0j})\,.
\end{aligned}
$$

Here $\boldsymbol{v}_{1j}$ and n_{1j} are the hydrodynamic velocity and the concentration of the particles of the j-th kind, respectively, subscript j becomes equal to e (plasma electrons) or b (beam electrons); n_{0j} is the equilibrium concentration, p_j the pressure, Φ the potential, and $\boldsymbol{E}$ the electric field of the external perturbation: $E = -\nabla\Phi$.

As usual we believe that the charge of the plasma electrons and the beam electrons is compensated by the positive charge of the lattice ions. Also we believe that

$$
n_{1j} = n_{0j} + n_j\,, \qquad \boldsymbol{v}_{1j} = \boldsymbol{v}_{0j} + \boldsymbol{v}_j\,, \qquad |n_j| \ll n_{0j}\,, \qquad |v_j| \ll v_{0j}\,, \qquad (2)
$$

where n_j and v_j are the deviations from the corresponding equilibrium values.

Supposing the pressure to be linearly dependent on the temperature ($p_j = T_j n_{1j}$, T_j is the temperature of the particles of the j-th kind) and neglecting the terms which consist of products of small factors, we obtain a system of linear equations

$$
\begin{aligned}
&\frac{\partial \boldsymbol{v}_j}{\partial t} + (\boldsymbol{v}_{0j}\nabla)\,\boldsymbol{v}_j = -\frac{e}{m_e^*}\boldsymbol{E} - \frac{v_{Tj}^2}{n_{0j}}\nabla n_j - \nu_j\boldsymbol{v}_j\,,\\
&\frac{\partial n_j}{\partial t} + \operatorname{div}(n_{0j}\boldsymbol{v}_j + n_j\boldsymbol{v}_{0j}) = 0\,, \qquad\qquad (3)\\
&\nabla\Phi = 4\pi e\sum_j n_j\,,
\end{aligned}
$$

where we also introduce the notation $v_{Tj} = (2T_j/m_e^*)^{1/2}$.

Let the wave vector of the plasma perturbation be of the form $\boldsymbol{k} = (0, 0, k_z)$. Then introducing as usual the time- and space-dependent perturbation in the form

$$n_j, v_j, \Phi \sim \exp\left[i(k_z z - \omega t)\right], \tag{4}$$

we have the following dispersion equation for the collective vibrations in the semiconductor plasma with the beam:

$$\begin{gathered} 1 - \frac{\Omega_e^2}{\omega(\omega + i\nu_e)} - \frac{\Omega_b^2}{(\omega - k_0 v_0)^2} - \frac{k^2 v_{Te}^2}{\omega^2} = 0\,, \\ \Omega_j = \frac{(4\pi e^2 n_{0j})^{1/2}}{(\varepsilon_0 m_j^*)^{1/2}}\,; \qquad j = \mathrm{e}, \mathrm{b}\,. \end{gathered} \tag{5}$$

Here we also believe that the hydrodynamic velocity of the plasma electrons is $v_{0e} = 0$, the unperturbed velocity of the beam electrons v_{0b} directed along the z-axis and neglect the particle collisions in the beam.

3. Increment of the Space-Charge Wave

Let the frequency ω be complex and equal to $\omega = k_0 v_0 + \Delta + i\gamma$, where $k_0 v_0 = \omega_0$ is the frequency corresponding to the exact phase resonance, k_0 is the wave number of random or induced perturbation (in our case $k_0 = 2\pi/l$, where l is the spacing of the grating), Δ is the detuning, and γ is the increment. Separating in (5) the real and imaginary parts we obtain a system of equations for the dimensionless detuning $x = \Delta/\omega_0$ and for the increment $y = \gamma/\omega_0$.

$$\begin{aligned} &\{[(1 + x)^2 - y(y + a)]^2 + (1 + x)^2\,(2y + a)^2\}\,[(1 + x)^2 + y^2]\,(x^2 + y^2)^2 \\ &\qquad - \beta_1[(1 + x^2) - y(y + a)]\,[(1 + x)^2 + y^2]\,(x^2 + y^2)^2 \\ &\qquad - \beta_2[(1 + x)^2 - y^2]\,\{[(1 + x)^2 - y(y + a)]^2 \\ &\qquad + (1 + x)^2\,(2y + a)^2\}\,(x^2 + y^2) \\ &\qquad - \beta_3(x^2 - y^2)\,\{[(1 + x)^2 - y(y + a)]^2 \\ &\qquad + (1 + x)^2\,(2y + a)^2\}\,[(1 + x)^2 + y^2]^2 = 0\,, \\ &\beta_1(1 + x)\,(2y + a)\,[(1 + x)^2 + y^2]^2\,[x^2 + y^2]^2 \\ &+ 2\beta_2 y(1 + x)\,\{[(1 + x)^2 - y(y + a)]^2 + (1 + x)^2\,(2y + a)^2\}\,[x^2 + y^2]^2 \\ &+ 2\beta_3 xy\{[(1 + x)^2 - y(y + a)]^2 + (1 + x)^2\,(2y + a)^2\}\,[(1 + x)^2 + y^2]^2 = 0\,, \end{aligned} \tag{6}$$

where we also introduce the denotions $\beta_1 = \Omega_e^2/\omega_0^2$, $\beta_2 = v_{Te}^2/v_0^2$, $\beta_3 = \Omega_b^2/\omega_0^2$, $a = \nu_e/\omega_0$.

Each of the equations (6) defines some curve $f_n(x, y) = 0$ ($n = 1, 2$) on the (x, y)-plane and, therefore, if two curves intersect in some point Q with the coordinates (x_0, y_0), this pair of numbers satisfies the system (6).

The numerical solution of (6) is achieved by the special program plotting f_n for some $x \in [x_{in}, x_{fin}]$. For this purpose the program ZEROIN [10] is used which calculates the zeros of $f_n(x_1, y)$ at the point $x_1 \in [x_{in}, x_{fin}]$ on some interval $[y_{in}, y_{fin}]$. In other words, the program finds at a given x_1 a value y such that $f_n(x_1, y) = 0$.

Table 1

Increment value depending on different values of dopant concentration, spacing of grating, temperature, and n_{0b}/n_{0e} ratio

semi-conductor	T (K)	dopant concentr. (cm^{-3})	v_0 ($cm\ s^{-1}$)	l (cm)	$\frac{n_{0b}}{n_{0e}}$	Δ (s^{-1})	γ (s^{-1})
Si	150	1.22×10^{17}	1×10^7	10^{-3}	0.01	-3.52×10^{10}	1.86×10^{10}
	60	1.22×10^{17}	1×10^7	10^{-3}	0.01	-2.22×10^{10}	1.42×10^{10}
	300	1.22×10^{17}	2×10^7	10^{-4}	0.01	-4.16×10^{10}	4.55×10^{10}
	60	1.22×10^{17}	2×10^7	10^{-4}	0.01	-8.57×10^{10}	1.36×10^{11}
	150	1.22×10^{17}	1×10^7	10^{-3}	0.001	-1.22×10^{10}	0.99×10^{10}
	60	1.22×10^{17}	1×10^7	10^{-3}	0.001	-7.55×10^9	5.89×10^9
Ge	300	7.0×10^{15}	1×10^7	10^{-3}	0.01	-6.63×10^9	5.25×10^9
	17	7.0×10^{15}	1×10^7	10^{-3}	0.01	-1.27×10^{10}	6.47×10^9

The next assumptions were approved by the calculations. The effective masses of the beam electrons m_b^* and the plasma ones m_e^* were believed to be equal to each other; the collision frequency ν_e was believed to be equal to the reciprocal moment relaxation time. The latter was extracted from the experimental data for the Hall mobility [11, 12]: $\tau_P = \mu m_\sigma/e$, where μ is the mobility, m_σ the so-called conductivity effective mass. We also believed the anisotropy of the semiconductor to be absent (that is, the semiconductor was believed to be a polycrystal); the carrier concentration was dependent on the temperature. The unperturbed velocity of the beam v_0 was defined by the voltage applied between the source and the drain and was equal to 10^7 or 2×10^7 cm s^{-1}. The spacing of the grating also had two values: 10^{-3} or 10^{-4} cm. The most interesting results are summarized in Table 1 and in Fig. 2. It is obvious that the increment of the wave $\gamma > 0$ and therefore amplification of the SCW induced by the beam instability in such a way indeed is possible.

If we assume the beam electron concentration depending only on the

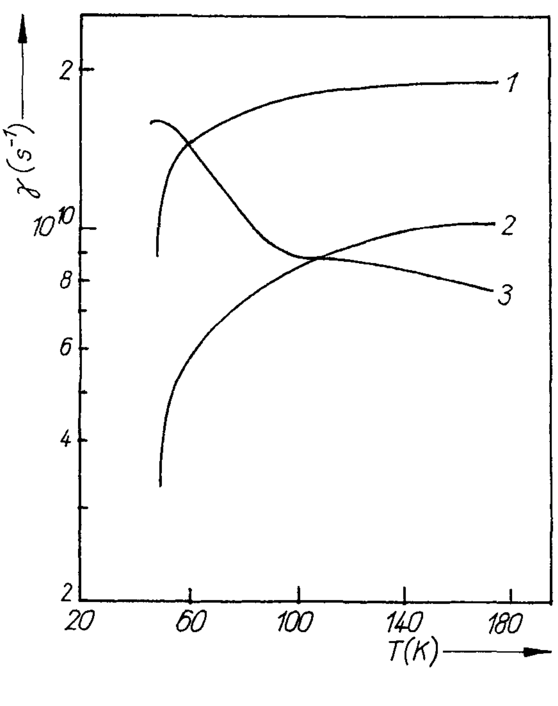

Fig. 2. Temperature dependence of the increment in silicon. (1) $n_{0b}/n_{0e} = 10^{-2}$, (2) 10^{-3}, (3) n_{0b} = const. (does not depend on temperature)

width of the level ΔE in the QW, the increment decreases when the temperature increases (Table 1, Fig. 2, curve 3). However, as a matter of fact the beam electron concentration is probably dependent also on the temperature. Indeed, the electron concentration in the source region is strongly dependent on the temperature and hence we have all reason to think that the number of tunneling electrons is also temperature dependent. Thus, we believe in this case that the beam electron concentration depends on the temperature, but the ratio n_{0b}/n_{0e} is believed to be constant and defined by the energy level width ΔE. The results are represented in Fig. 2, curves 1 and 2. Now the temperature dependence of the increment radically changes: at the initial stage, at lower temperature the increment increases, then at higher temperature the increase is saturated.

At still higher temperatures the increment even slowly decreases but we do not adduce the corresponding data here because the operation temperature of the proposed device is hardly believed to be higher than 100 to 150 K. Indeed, the effect of size quantization itself can be observable only at low temperatures.

Such a behaviour of the increment is explained by the counterplay of two competitive processes: increase of the electron concentration, on the one hand, and decrease of the electron mobility, on the other. The latter slowly decreases at low temperatures and then (approximately at 60 K [11]) decreases abruptly; as a consequence the increase is saturated and then even slowly decreases. Also it is worth to note that as it should be expected the absolute value of the increment is the higher, the larger the n_{0b}/n_{0e} ratio is.

Acknowledgement

We should like to thank the Fund for Advanced Research of the Republic of Belarus for financial support (Grant No. F51 – 300).

References

[1] M. GLICKSMAN, Solid State Phys. **26**, 275 (1971).
B. B. ROBINSON and B. VURAL, RCA Rev. **29**, 270 (1968).
[2] R. N. SUDAN, A. CAVALIERE, and M. N. ROSENBLUTH, Phys. Rev. **158**, 387 (1967).
N. N. BELETSKII, A. A. BULGAKOV, S. I. HANKINA, and V. M. YAKOVENKO, Plasma Instabilities and Nonlinear Phenomena in Semiconductors, Izd. Naukova dumka, Kiev 1984 (in Russian).
[3] R. J. BRIGGS, Electron-Stream Interactions with Plasma, MIT Press, Cambridge (Mass.) 1964.
[4] A. B. MIKHAILOVSKII, Theory of Plasma Instabilities, Vol. 1, Izd. Nauka, Moscow 1975 (in Russian).
[5] I. E. TRALLE, Dokl. Akad. Nauk Belarus. **35**, 132 (1991).
[6] I. E. TRALLE and A. B. FILONOV, phys. stat. sol. (b) **173**, 775 (1992).
[7] W. FAWCETT, A. D. BOARDMAN, and S. SWAIN, J. Phys. Chem. Solids **31**, 1963 (1970).
[8] J. POZELA and A. REKLAITIS, in: Inverted Hot Electron Distributions in Semiconductors, Ed. A. A. ANDRONOV and J. POZELA, Izd. IPFAN USSR, Gorki 1983 (in Russian).
[9] V. L. GINSBURG and A. A. RUCHADZE, The Waves in Magnetoactive Plasma, Izd. Nauka, Moscow 1975; Principles of Plasma Physics, Vol. 1, Ed. R. S. SAGDEEV, M. ROSENBLUTH, A. A. GALEEV, and R. SUDAN, Izd. Energoatomizdat, Moscow 1983 (Soviet–American ed.).
[10] G. E. FORSYTHE, M. A. MALCOLM, and C. B. MOLER, Computer Methods for Mathematical Computations, Prentice-Hall, Inc., Englewood Cliffs (N.J.) 1977.
[11] F. T. MORIN and I. P. MAITA, Phys. Rev. **96**, 28 (1954).
[12] N. B. HANNAY (Ed.), Semiconductors, V. N. Reinhold Publ. Co., Ltd., New York.

(Received August 11, 1993; in revised form November 29, 1993)