THERMOELASTIC RESPONSE OF SUDDENLY HEATED LIQUID TARGETS IN HIGH-POWER COLLIDERS

A. Hassanein, I. Konkashbaev, and J. Norem
Argonne National Laboratory, Argonne, IL, USA

Abstract
Thermoelastic response of liquid metal targets exposed to high-volumetric-energy deposition in times shorter than the target hydrodynamic response time (i.e., sound travel time) is of interest to several research areas, including targets for high-power accelerators such as the Spallation Neutron Source, muon collider targets, etc. Sudden energy deposition causes shock and rarefaction waves of magnitude $\Delta P$ that corresponds to an initial thermal pressure of tens of katm. Nevertheless, a liquid subjected to a negative pressure is metastable. The problem of liquid target oscillations in the presence of large negative pressure, and the mechanism of fragmentation and its consequences, are considered in this paper.

1 INTRODUCTION
Sudden deposition of beam energy in a liquid metal target causes excitation of cylindrical sound waves with pressure oscillation $\pm \Delta P$ of several tens of katm. The fragmentation of liquid at strong negative pressure was investigated in the 1940s and 1970s by several authors who based their work on the Frenkel theory [1-2]. We have developed a new concept of shock wave tensile relaxation that will lead to liquid target fragmentation because of target heating. Most of the numerical calculations, performed with the HEIGHTS package, are for a free liquid mercury jet heated by a 20-GeV proton beam, as described in work on the muon collider project [3]. Spatial energy deposition in the target was calculated with the MARS code. The stationary flow of the mercury jet was studied previously [4].

2 LIQUID JET OSCILLATION
A cylindrical jet with radius $R_0 = 0.7$ cm, length $L_0 = 29$ cm, velocity $V_0 = 20$ m/s is positioned inside a solenoid with a magnetic field $B_0 = 20$ T. Two proton beams of the same 20-GeV energy separately strike this jet at $t_1 = 0$ and $t_2 = 150$ ns. During beam/target interaction, the deposition of proton beam energy is $Q = 50-130$ J/g. Such high-energy deposition leads to instant heating of the target to 1000-2000 K, with a corresponding rise in thermal pressure to 50-100 katm. Pressure relaxation leads to the excitation of both radial and axial oscillations [5].

The radial oscillations of pressure with frequency $f_\phi \approx 100$ kHz have a positive magnitude $\Delta P^+ \approx 50$ katm and negative magnitude $\Delta P^- \approx -40$ katm (due to accumulation in the radial direction); The magnitude of the surface velocity reaches $100$ m/s; the velocity of the axial oscillations is the same but with a corresponding lower frequency $f_z \approx 10$ kHz. Positive pressure has the same magnitude $\Delta P^+ \approx 40$ katm but less negative $\Delta P^- \approx -10$ katm because of absence of accumulation.

3 CAVITY APPEARANCE
Let us consider the consequences of these oscillations in relationship to the problem of target breakage and fragmentation. A liquid that is subjected to a strong negative pressure is known to be metastable. A main mechanism of possible liquid fragmentation, developed in this work, is the tensile relief wave that is spontaneously initiated by the birth of cavities in severely stretched liquids. The formation of cavities under strong negative pressure displays features that differ from those suggested by the theory of void nucleation in the superheated state because the cavity vapor pressure can be neglected when compared with a large negative pressure of several tens of katm. In the case of empty voids, the theory of nucleation [2] predicts the value of the fracture pressure is

$$P_f = \sigma^{3/2}, \quad P_f(Hg) = 22.3 \text{ katm}.$$  \hspace{1cm} (1)

Therefore, at negative pressure $P < -P_f$, mercury can be fractured (broken) because of the growth of cavities. The critical radius $R_{crit} (P = 22.3 \text{ katm}) \approx 4 \text{ Å};$ thus, the probability of voids arising is very high due to thermodynamic fluctuations. Experiments to study liquid fracture under negative pressure have been carried out with liquid mercury ($P_f = 19 \text{ katm}$) in good agreement with theory for well-purified liquid.

4 RELAXATION SHOCK WAVE
Below, we will consider our new concept of the mechanism of fracture as the consequence of cavities with radius greater than the critical one coming into existence.
Proceedings of the 2001 Particle Accelerator Conference, Chicago

during the negative-pressure phase. When these cavities arise, the pressure inside the cavity is equal to zero, i.e., $P_{\text{cavity}} = 0$, but the liquid outside the cavity is at negative pressure $P_{\text{liquid}} = -P$. At the cavity interface, a jump of pressure given by $\Delta P = P_{\text{cavity}} - P_{\text{liquid}} = P$ takes place ahead of the cavity boundary; therefore, a shock wave with amplitude $P_{\text{shock}} = \Delta P$ is formed. This shock wave is actually a relaxation wave that occurs when the stretched medium reverts back from density $\rho < \rho_0$ to the normal density $\rho_0$. The characteristic of this relaxation shock wave is similar to that of a detonation wave because the energy that initiates the shock is taken from within the medium itself, i.e., it releases the energy used in stretching the target.

The shock wave speed $D$ of medium relaxation, and velocity of the medium $U$ between the shock wave and the cavity boundary, are

$$D = C_{so} \sqrt{\frac{1}{1 - \varepsilon}} > C_{so}, \quad \varepsilon = \frac{\rho_0 - \rho}{\rho_0} \ll 1,$$

$$U = C_{so} \sqrt{\varepsilon^2 - 1 - \varepsilon}, \quad U = D\varepsilon$$

Thus, the shock front moves supersonically as usually observed in such shock waves but the liquid behind the shock wave moves with subsonic velocity. Figure 1 schematically illustrates cavity expansion and the initiated shock wave of relaxation.

![Fig. 1. Relaxation shock wave initiated by appearance of cavity](image)

**5 CAVITY DYNAMICS**

The dynamics of a spherical cavity coming into being at the time of maximum negative pressure are shown in Fig. 2. The velocity of the cavity surface oscillates with decreasing magnitude from a very high value of 1 km/s to $<0.2$ km/s at $t = 50$ μs. A detailed description of the calculations is given in Ref. 6.

From these calculations, one can conclude that after a cavity is initiated during the negative-pressure phase, it expands freely with a decreasing expansion velocity to a value defined by the elastic energy stored in the system. This means that any cavity coming into existence during the negative-pressure stage will continue to grow and will not disappear. This failure to disappear or collapse is a major difference between cavity dynamics in a stretched medium and usual cavitation (wherein vapor bubbles collapse during a phase of increased pressure), and is the result of the discharging or unloading of the medium by a relaxation shock wave that is initiated by the appearance of cavities.

![Fig. 2. Dynamics of cavity growth in Hg target](image)

At longer times, these spherical cavities can join to produce an elongated cavity with an axial size that is much larger than the radial one. The hollow cylindrical shell expands freely with a rather high mean surface velocity of $\approx 50$ m/s that can lead to jet destruction. Compression of the magnetic flux lines will lead to slowing of the liquid metal expansion, i.e., conditions arise for the growth of MHD Rayleigh-Taylor instability, which would lead to the destruction and fragmentation of the liquid jet, with the formation of droplets that splash everywhere with high velocities.

**6 SUMMARY**

The thermoelastic response of liquid metal targets exposed to intense volumetric-energy deposition for short times was studied to define the mechanisms of fragmentation and its consequences. A cavity coming into existence during the negative-pressure stage initiates a relaxation shock wave when the stretched medium returns from low to normal density. This cavity expands permanently and does not disappear or collapse, even during the positive-pressure phase. The failure to disappear or collapse is a major difference between the
cavity dynamics in a stretched medium and usual cavitation, wherein vapor bubbles collapse during a phase of increased pressure, and is the result of the “discharging” or “unloading” of the liquid medium by the relaxation shock wave that is initiated by cavity appearance. Detailed calculations of cavity dynamics are provided for the spherical and cylindrical target cases.

Cavities (voids) that come into existence quickly enough initiate relaxation shock waves that release elastic energy in the form of kinetic energy of liquid motion. These cavities soon join together to form an elongated cylindrical cavity. The expansion of this elongated cavity leads to the transformation of the liquid metal cylinder into one or more cylindrical shells that are expanding with a radial velocity determined by the stored potential energy, which is equal to the deposited energy less the remaining thermal energy. Hollow cylindrical shells will then expand into the outer strong magnetic field of, for example, the muon collider. Compression of the magnetic flux lines will cause the liquid metal to slow down, constituting a condition for the growth of MHD Rayleigh-Taylor instability. Development of MHD instabilities would lead to the destruction and fragmentation of the liquid jet, with the formation of droplets that splash with high velocities. Further studies that take into account surface tension and other effects are required.

7 ACKNOWLEDGMENTS

Work is partially supported by the U.S. Department of Energy, Office of High-Energy Physics.

8 REFERENCES