THE DESIGN OF A LIQUID LITHIUM LENS FOR A MUON COLLIDER*

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Abstract

The last stage of ionization cooling for the muon collider requires a multistage liquid lithium lens. This system uses a large (~0.5 MA) pulsed current through liquid lithium to focus the beam while energy loss in the lithium removes momentum which will be replaced by lines. The beam optics are designed to maximize the 6 dimensional transmission from one lens to the next while minimizing emittance growth. The mechanical design of the lithium vessel is constrained by a pressure pulse due to sudden ohmic heating, and the resulting stress on the Be window. We describe beam optics, the liquid lithium pressure vessel, pump options, power supplies, as well as the overall optimization of the system.

1 INTRODUCTION

The muon collider is being studied as a way of looking at particle interactions at the energy frontier[1]. The event rate in such a machine is goes like $e^{1}$, so the muon emittance, $\varepsilon$, must be minimized.

An attractive configuration for final stages of $\mu$ cooling is obtained by passing the beam through a conducting light metal rod which acts simultaneously as a focusing element and as an energy loss absorber. The magnetic field produced is given by $B_{p} = \mu_{0} j r / 2$, where $r$ is the current density, $r$ the radius and $\mu_{0}$ the permeability constant. This azimuthal magnetic field combined with the longitudinal velocity produces a radial focusing force with the equation of motion, $d^{2}r/dz^{2} + B_{0}^{2}r / Bp = 0$. The beta function for a Li lens is then $\beta_{l} = (r \rho \beta)^{1/2} = \rho (\rho)_{\infty}^{1/2} r / \beta$, where the field gradient, $B_{0}$, can be very large. For a lens with $r_{\text{ext}} = 2 \text{ mm}$ and the muon momentum in the lens, $p = 300 \text{ MeV}/c$, one can achieve $\beta_{l} = 1 \text{ cm}$, which is difficult to achieve with solenoidal focusing systems.

Multiple scattering sets a lower limit to the emittance that can be obtained using this method. This limit is the equilibrium emittance, $\varepsilon_{\text{eq,N}}$, where ionization cooling is balanced by multiple scattering,

$$\varepsilon_{\text{eq,N}} = \frac{0.014^{2} \beta}{2 \rho_{\text{m}}^{2} L_{R} dE/dx} \beta_{l} C_{l} \beta \approx (p / \rho)_{\infty}^{1/2} C_{l} \beta,$$

where $L_{R}$ is the radiation length, $dE/dx$ is the energy loss, $\beta_{l}$ and $\rho_{m}$ are the relativistic, velocity and muon mass, and constant $C$ depends on the material. For Hydrogen, Lithium and Beryllium this constant is 42, 79 and 103 mm-mass/MeV respectively. Thus the lowest emittances are obtained with low momenta, high current densities, and low $Z$ materials. The lower momentum limit is determined by the muon velocity spread and the slope of $dE/dx$ at low momentum, both of which increase the emittance.

For muon cooling, relatively long lengths are needed to obtain large energy losses (total loss should be a few hundred MeV, at $dE/dx = 87.5 \text{ MeV}/\text{cm}$), and the high ohmic power deposition associated with higher frequency operation would melt the Li.

A complete Lithium lens system would consist of a number of lithium lenses, matching optics required to transmit the beam from one lens to another and lines used to replace the momentum lost by ionization. Since the performance of the muon collider is related to the current density in the lithium lens, one must look at the structural and mechanical limits of the design of the lens.

The present plan is to produce a conceptual design of lenses with $r = 0.5 \text{ cm}$, $B_{\text{ext}} = 15 \& 20 \text{ T}$, $L = 1 \text{ m}$ for testing as part of the MUCOOL experiment[2].

2 THE PRESSURE PULSE

The lithium lens is powered by a capacitor bank which produces a voltage proportional to a half cosine wave which produces a half sine wave current pulse. The length of the current pulse, $\tau$, must be sufficient to allow the current to penetrate the lens uniformly. The skin depth $\delta$ is proportional to $(\rho_{\beta} \delta)^{1/2}$, for resistivity, $\rho_{\beta}$, of Lithium, (which melts at 180 °C).

![Figure 1, Resistivity of Lithium.](image)

The ohmic power produces heating, thermal expansion, and subsequently pressure pulses which will ultimately limit the performance of the lens. Power deposited in the

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lithium must be removed by circulating it, and we assume this would be done at the beam repetition rate.

The lithium lens can be modeled as a cylindrical tube of internal radius 4.5 mm contained in a thin steel shell 0.5 mm thick. In this example, 0.5 MA is drawn through the lens by a voltage pulse which is a half sine wave 50 µs long. The hydrodynamic equations, as well as the response of the envelope are considered in the HEIGHTS package [3]. Lithium is heated by Joule heating as well as energy loss from the muon beam, which is introduced 25 µs into the 50 µs pulse, producing a discontinuity in the temperature. The temperature pulse is shown in Figure 3.

![Temperature profile](image)

Figure 3. The temperature profile of the center and edge of the lens, showing the contribution from the muon beam at the center. The edge heats first, and the shell stays cooler due to the higher specific heat.

The stress and strain generated by the pressure pulse, and the expansion of the Li is shown in Figure 4 for a current of 0.5 MA. The initial compression due to the magnetic field pressure is followed by the expansion due to the heating of the Li.

![Pressure pulse](image)

Figure 4. The pressure pulse in the lens.

### 3 THE PRESSURE VESSEL

There are a number of severe requirements on the pressure vessel holding the liquid Lithium. The pressure vessel must confine the liquid lithium flowing through it while requiring the pulsed current to pass longitudinally down the Lithium. The large pressure pulses must be supported without exceeding fatigue limits, and the insulators, which confine the electrical current, must survive the large mechanical shocks. The emittance of the muon beams must be increased as little as possible by the structure and the windows, which must be thin Beryllium.

Two options are under consideration for the pressure vessel itself, one which would absorb as much of the pressure pulse as possible radially, the other would provide a solid surface which would direct the pressure pulse down the length, and utilize a buffer volume at the ends of the lens to absorb the increased volume of Lithium [4],[5].

Pumping the liquid Lithium has been studied using two techniques: electromagnetic pumps, which have been used with great success at BINP, and centrifugal pumps, which have been used for years at ANL after modification. Either designs might be modified to be more compatible with the large pressure pulses from the lens.

### 4 BEAM OPTICS BETWEEN LENSES

Because the energy loss required for cooling muons is greater than that provided by a single lens, a number of Li Lenses are required. The beam must be reaccelerated and transported between lenses with minimum loss [6][7]. In addition, periodic, \( \varepsilon_n \rightarrow \varepsilon_0 \), emittance exchange sections are required to confine and cool the beam longitudinally. Modulation of the beam envelope is unavoidable in such a section because the Li lens provides a stronger focusing force than any other component. This modulation leads to significant chromatic effects which increase \( \varepsilon_n \). It is not possible to use dispersion and sextupoles to control this modulation because of attendant nonlinear effects.

The chromatic effects can be attenuated to tolerable levels by varying the energy deviation, \( \Delta E \), along the length of the beamline. This is done by requiring a synchrotron phase advance of at least \( \pi \). When this phase advance is used, each particle traverses approximately half the beamline below the median energy and half above it, thus averaging the chromatic contributions to zero. This requires at about 5 m of linac, with a frequency \( f = 805 \) MHz, and 30 MV/m gradients. At high momenta, approximately \( p = 300 \) MeV/c, this works comparatively well because \( \Delta E \propto \Delta p \), but at lower momenta nonlinearities are more significant.

Emittance exchange is another problem, since the energy spread of the beam is large. Lower energy particles tend to lose more energy due to ionization, and straggling widens the energy distribution of the beam, both effects increase the longitudinal emittance. The emittance exchange systems consist of an element which produces dispersion, combined with wedge absorbers which reduce the energy spread of the beam. We are primarily looking at bent solenoids to provide the momentum dispersion. The emittance exchange should be done at the end of the transport section to avoid complicating the chromatic matching[7].

The proposed system which accomplishes these goals consists of a strong solenoid to match the strongly divergent beam coming out of the Li lens, followed by a
weaker solenoid containing a linac section. If the solenoid is bent, the dispersion is sufficient to do the emittance exchange with wedges. At the end of the section is another strong solenoid which focuses the beam on the input of the next Li lens. The long solenoid has a field of 3.1 T, and the short solenoids, used to match to the Lithium lens, have a field of about 12-20 T. The total length of the section is about 10 - 12 m. The optics of this system and more detailed information about its components are described elsewhere [6][7][8]. Studies are being made to determine the optimum cooling energy [8]. Although lower momentum seems desirable, this seems to involve shorter Li absorbers and more tightly constrained beam optics.

5 CONTAINMENT AND SAFETY

Although liquid Lithium freezes at fairly high temperatures, it is reactive and should be confined. The lens under construction at BINP for Fermilab uses an outer vacuum system filled with Argon to isolate the Li system from the environment.

7 CONCLUSIONS

The design of the liquid lithium lens involves both mechanical and electrical issues associated with the pressure vessel as well as beam optics considerations related to the transport lines between lenses. The mechanical problems are dominated by the requirements of coping with the very large pressure pulse produced by Joule and beam heating of the liquid lithium. The design of the beam optics is concerned with the efficient transport of large six dimensional emittance beams between the small lenses. A liquid Lithium lens is being constructed and the problems are going to be the subject of an experimental program.

REFERENCES

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