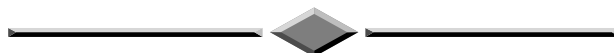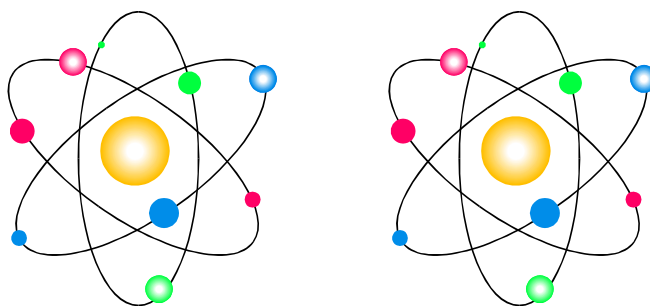The Academy of Sciences of Belarus

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THEORETICAL MODEL FOR NUMERICAL SIMULATION OF ENERGY DEPOSITION BY ROTATING ELECTRONS IN A MAGNETIZED PLASMA HAVING DENSITY, TEMPERATURE AND MAGNETIC FIELD PROFILES.

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The physical and mathematical models of the electron passage through a magnetized plasma having arbitrary density, temperature and magnetic field profiles are presented. The base of the physical model is division of all electron collisions into two groups: "near" and "far". The "near" electron collisions with the plasma nuclei and electrons are described on the base of the theory of quantum electrodynamics. The "far" electron collisions are described by means of the multiple scattering theory. The mathematical model is based on the Monte Carlo method. The developed model was used for numerical simulation of the divertor disruption under action of the electron flux. In the result of numerical simulation the energy deposition profiles of the rotating electron beam in a magnetized carbon plasma are obtained.

The problem of energy deposition by the rotating electrons in a magnetized plasma having density, temperature and magnetic field gradients is of considered. The energy range of electrons is from 1 keV up to 100 keV. Plasma density is from 10^{15} cm^{-3} up to 10^{20} cm^{-3}. Plasma temperature is from 0.5 eV up to 250 eV. The electron energy in the beam is distributed with the maxwellian function. Density, temperature and magnetic field profiles are given in an arbitrary form. The electrons have different longitudinal and transverse energy components by motion in a magnetized plasma. Magnetic field direction is changed from 0 up to 90 degrees with the target surface.

Physical model of the electron passage through a plasma

Physical model of the investigated phenomenon is in following:

- The charge electrons are moving in a plasma and interacting with scattering centers (electrons and nuclei) which are placed by chance. There are density, temperature and magnetic field gradients.

- The incoming electron interacts simultaneously with one scattering center.

- The incident electrons do not interact one with another.

Using this suggestions one can consider spatial localization of interaction and introduce the notion of the electron trajectory. The electron trajectory is some spiral line. Interaction takes place in inflection points. In these points the state (propagation direction and energy) of the incident electrons and plasma particles is changed. The detail of interaction processes is the next step in the physical model.

The interaction of electrons with a plasma is a complex process. The following physical processes are taken into account:

1. The elastic electron-nucleus and electron-electron collisions.

2. The energy losses of electrons for excitation and ionization.

The electrons within given energy range in most cases are fast enough to be investigated in the standard Born approximation. It is supposed the electrons are nonrelativistic. The main feature of electron interaction with the plasma electrons and nuclei is deflection of its propagation direction. It originates mainly from the elastic collisions with the plasma nuclei. This process is Z times more efficient than the elastic scattering on the plasma electrons. Ratio of the mean energy transferred the plasma electrons to one transferred the plasma nuclei is $2M_n/m_e \approx 4000$, where m_e, M_n - the electron and nucleus masses. Hence, the electron energy losses on the nuclei are negligible comparing with that on the plasma electrons. The scattering of the electrons on the plasma nuclei is Z times more efficient than the energy loss is due to collisions with the plasma electrons (bounded and free). Hence, the electrons change many times its direction before stop and one has to take accurately into account the both processes to obtain spatial energy distribution in a plasma.

The long-range coulomb interaction of the electrons with the plasma electrons and nuclei causes the high scattering probability at small angles with small energy transfers ("far" collisions). The scatterings at large angles with large energy transfers ("near" collisions) are low probable events. Therefore there is the specified sense to consider separately "far" and "near" collisions. The great number of "far" collisions is occurred on some electron trajectory length δl_i limited by coordinates of two "near" collisions. "Far" collisions are de-

scribed by the multiple scattering theory. In the end of which length δl_i the summary scattering angle and energy loss of an electron are sampled from the angle distribution in the multiple scattering and from the energy loss distribution. "Near" collisions are treated as individual ones. The characteristics of particles after a "near" collision are calculated from the microscopic cross sections of interaction.

1. Electron-nuclear collisions.

All electron-nuclear collisions are divided into two groups: "near" collisions with deflection angle $\theta > \theta^*$ and "far" collisions with $\theta < \theta^*$. For "near" collisions one can use the Rutherford cross section [1]

$$d\sigma_{en} = \left| \frac{Z\,e^2}{2m_e v^2} \right|^2 \frac{d\Omega}{\sin^4 \theta/2}, \tag{1}$$

if θ^* is chosen in the range $v_0/v \ll \theta^* \ll 1$, where v is the probe electron velocity, v_0 is the character bounded electron velocity, Z is the atomic number of a plasma, e is the electronic charge, m_e is the electron mass, $d\Omega = 2\pi \sin\theta\,d\theta$ is the element of solid angle. From this formula a deflection angle θ is sampled in the range $\theta^* \div \pi$. The value of energy transferred the nucleus in the result of a "near" collision is calculated from the corresponding kinematic relationship. This energy is very small.

The effect of "far" collisions on the electron propagation direction accumulated on the length δl_i can be considered in the following probabilistic approach. Due to the long-range coulomb interaction the electron undergoes a great number of statistically independent collisions whose bulk effect is fully determined in terms of the mean deflection angle square

$$\langle \theta^2 \rangle = N_n \delta l_i \int_{\theta < \theta^*} \theta^2 d\sigma_{en} \approx 8\pi \left| \frac{Z\,e^2}{m_e v^2} \right|^2 \delta l_i \sum_i N_i \Lambda_i,$$

where

$$\Lambda_i = \ln \frac{\theta^* m_e v a_i}{Z} + \left| \frac{i}{Z} \right|^2 \ln \frac{0.607 a_d}{a_i},$$

where i is the ionization degree, N_i is the density of ions in that state, N_n is the density of nuclei in a plasma, is the Plank constant, a_d is the Debay radius, a_i is the character ion size evaluated through the ion form-factor $F(q)$

$$(Z^2 - i^2) \ln a_i = -\int_0^\infty \ln q \frac{\partial}{\partial q} [Z - F(q)]^2 dq,$$

where $q = 2 m_e v \sin(\theta/2)$ is the momentum transferred the nucleus. The probability that the electron due to "far" collisions accumulates a deflection angle θ on the length δl_i is equal [2,3]

$$P(\theta) d\theta = \frac{2\theta}{\langle \theta^2 \rangle} \exp \left| -\frac{\theta^2}{\langle \theta^2 \rangle} \right| d\theta. \qquad (2)$$

The loss of energy because off "far" collisions with the plasma nuclei on the length δl_i is negligible small.

2. Electron-electron collisions.

The electron-electron collisions are considered in the same manner. "Near" collisions, when the recoil electron gets energy that exceeds E_{min}, are described by the cross section [1]

$$d\sigma_{ee} = \pi Z e^2 \left| \frac{1}{\varepsilon^2} + \frac{1}{(E - \varepsilon)^2} - \frac{1}{\varepsilon(E - \varepsilon)} \right| \frac{d\varepsilon}{E}, \qquad (3)$$

where $E = m_e v^2/2$ is the kinetic energy of the incident electron, ε is the kinetic energy of the recoil electron. In this case all plasma electrons are considered as free and to be at rest. The expression (3) takes into account the identity of colliding electrons and the exchange effects. According to this formula the energy of the incident electron is

reduced on the value ε and the new electron arises. The value ε is within the range $E_{min} \div E - E_{min}$. The recoil electrons may have a considerable energy and in this case should be treated as incident ones to obtain their energy distribution as well. The scattering angles of the incident and recoil electrons after collision is calculated from the kinematic relationships.

The loss of energy in "far" collisions accumulated on the length δl_i is equal

$$\delta \varepsilon = \frac{2\pi e^4}{m_e v^2} \delta l_i \sum_i N_e^i \ln \frac{2E_{min} m_e v^2}{\hbar^2 \omega_i^2},$$ (4)

where the summation is performed over all electrons in a plasma, N_e^p is the density of free plasma electrons, N_e^i is the density of i-th bounded electrons, ω_i is the plasma frequency $\omega_p = \left(4\pi N_e^p e^2 / m_e\right)^{1/2}$ for free electrons and the character ion frequency for bounded electrons that may be calculated through the use of oscillator strengths f_{on} and energy levels E_n of the ion

$$\ln \omega_i = \frac{1}{Z - i} \sum_n f_{0n} \ln\left(E_n - E_0\right).$$

An exact calculation of ω_i from first principles is difficult, and experimental data which would allow its determination for highly ionized atoms do not exist. Therefore, these characteristics were found and tabulated from quantum-mechanical calculations [4,5] in accordance with the Hartree-Fock-Sleter model. The energy E_{min} must satisfy the condition $\omega_i \ll E_{min} \ll E$.

Thus, the electron state (propagation direction and energy) in the end of the length δl_i is determined by the distributions (2,4).

Numerical simulation by means of the Monte Carlo method

The Monte Carlo method [6] makes it possible to simulate the random trajectories of the electrons in a plasma as the real ones. This method is efficient for an inhomogeneous plasma and by presence of external fields. It gives the possibility to take into account the secondary particles. The random numbers are used in the Monte Carlo method for sampling of the trajectory elements (length, scattering angle) from corresponding distributions. This method is discrete one. From this point of view it is applicable to the utmost for a wide adoption on a computer. It is important that the Monte Carlo method doesn't require formulation of the discrete model of the corresponding transfer equation.

The plasma target in our model is presented in the form of a cube or parallelepiped. The coordinate system is chosen in the following manner. It is supposed the electron beam is incoming in the center of the plane (Y, Z) and propagating along the axis X. The lengths along the axes X, Y, Z are descretized and represented by the $I + 1$, $J + 1$, $K + 1$ mesh points x_i, y_j, z_k. It is considered a spatial grid where the points x_i, y_j, z_k form the cell boundaries. Thus, the cell is described by the numbers (i, j, k). The target characteristics such as the density, temperature and magnetic field strength are assumed constant within the boundaries of the cell (i, j, k). The characteristics of the separate cells may be different.

The vector B of magnetic intensity is in the plane (Z, X). An arbitrary angle α may be given between directions of the vector B and axis X. The Lorenz force affects on the electron moving in the magnetic field of intensity B and having the velocity v. This force is di-

rected perpendicularly both the vector *B* and the velocity *v*. Under action of this force the electron is rotating around a magnetic line with the Larmor frequency $\omega_L = ecB/(m_e c^2 + E)$, where *c* is the light velocity, *E* is the electron kinetic energy. In the plane which is perpendicular to the magnetic field direction the electron is moving along a circle with the Larmor radius $R_L = v_t/\omega_L$, where v_t is the transverse velocity component. The center of this circle is moved along a magnetic line. The resulting trajectory is a spiral line with the step $h_l = 2\pi v_l/\omega_L$, where v_l is the longitudinal velocity component. This spiral line is winded on a magnetic line. By entrance into a plasma such electron undergoes the collisions with plasma particles and it trajectory is deflected. The values and directions of the magnitudes *v*, v_t and v_l are changed in the result of a collision. After a collision the electron is moved along a new magnetic line with the new values of velocity, Larmor radius and spiral step before a next collision is occurred.

The algorithm of construction of the electron trajectory includes the following stages:

I. The sampling of the initial parameters of an incoming electron.

The initial energy of the incoming electron may be given as a monoenergetic value or sampled from the maxwellian function of energy distribution in the electron beam. For velocity components the two-dimensional maxwellian function is written in the form:

$$f(v_x)dv_x = \sqrt{\frac{m_e}{2\pi T_b}} \exp\left|-\frac{m_e v_x^2}{2T_b}\right| dv_x; \ f(v_z)dv_z = \sqrt{\frac{m_e}{2\pi T_b}} \exp\left(-\frac{m_e v_z^2}{2T_b}\right) dv_z, \ (5)$$

where T_b is the beam temperature, v_x and v_y are the velocity components along corresponding axes. Using the substitutions

$$x^2 = m_e v_x^2/2T_b, \qquad z^2 = m_e v_z^2/2T_b \tag{6}$$

and multiplying the equations (5) one can obtain the following expression $f(x,z)dxdz = \pi^{-1}\exp(-x^2 - z^2)dxdz$. This expression may be written in the polar coordinate system in the form: $f(r,\phi)drd\phi = \pi^{-1}\exp(-r^2)dr^2d\phi$, where the substitutions

$$x = r\cos\phi, \quad z = r\sin\phi \tag{7}$$

were used. The coordinates r and ϕ are independent. Therefore the values r and ϕ may be sampled according to

$$\frac{\int_0^\phi f(\phi)d\phi}{\int_0^{2\pi} f(\phi)d\phi} = \xi_1, \qquad \frac{\int_0^r f(r)dr}{\int_0^\infty f(r)dr} = \xi_2, \tag{8}$$

where ξ_1 and ξ_2 are the random numbers distributed uniformly in the interval [0,1]. Solving Eqs.(8) with respect to r and ϕ we have

$$\phi = 2\pi\xi_1, \qquad r = \sqrt{-\ln\xi_2}. \tag{9}$$

This sampling method of the values r and ϕ from Eq.(8) is the reverse function method. Substitution of Eqs.(9) into Eqs.(7) gives

$$x = \sqrt{-\ln\xi_2}\cos(2\pi\xi_1), \quad z = \sqrt{-\ln\xi_2}\sin(2\pi\xi_1).$$

In the case of the three-dimensional distribution function it is necessary to generate the new random numbers ξ_3, ξ_4 and calculate the value

$$y = \sqrt{-\ln\xi_3}\cos(2\pi\xi_4).$$

The sampled energy components of the electron are obtained from Eqs.(6): $\varepsilon_x = T_bx^2$, $\varepsilon_z = T_bz^2$, and similarly $\varepsilon_y = T_by^2$. The sampled initial electron energy is $\varepsilon_0 = \varepsilon_x$, $\varepsilon_0 = \varepsilon_x + \varepsilon_z$, and $\varepsilon_0 = \varepsilon_x + \varepsilon_z + \varepsilon_y$ for one, two and three dimensional maxwellian functions, respectively.

The electrons with high energy ("tail" of maxwellian distribution) are occurred with lower probability. But such electrons influence essentially on the "tail" of the absorbed energy distribution in a plasma. To increase the quota of the height energy electrons sampled from

the maxwellian distribution the scheme with "weight" is used. The electron energy components $\varepsilon_x, \varepsilon_z, \varepsilon_y$ are sampled equipprobably from the interval $[\varepsilon_1, \varepsilon_2]$ in the accordance with formulas: $\varepsilon_x = \varepsilon_1 + \xi_1(\varepsilon_2 - \varepsilon_1)$, $\varepsilon_z = \varepsilon_1 + \xi_2(\varepsilon_2 - \varepsilon_1)$, $\varepsilon_y = \varepsilon_1 + \xi_3(\varepsilon_2 - \varepsilon_1)$. The initial energy is $\varepsilon_0 = \varepsilon_x + \varepsilon_z + \varepsilon_y$. The "weight" of the electron is determined by the following expressions:

- for one-dimensional maxwellian function

$$P(\varepsilon_x) = \frac{2(\varepsilon_2 - \varepsilon_1)}{\sqrt{\pi \varepsilon_x T_b}} \exp\left[-\frac{\varepsilon_x}{T_b}\right].$$

- for two-dimensional maxwellian function

$$P(\varepsilon_x, \varepsilon_z) = \frac{4(\varepsilon_2 - \varepsilon_1)}{\pi T_b \sqrt{\varepsilon_x \varepsilon_z}} \exp\left[-\frac{\varepsilon_x}{T_b} - \frac{\varepsilon_z}{T_b}\right].$$

- for three-dimensional maxwellian function

$$P(\varepsilon_x, \varepsilon_z, \varepsilon_y) = \frac{8(\varepsilon_2 - \varepsilon_1)}{\left(\pi T_b \varepsilon_x \varepsilon_z \varepsilon_y\right)^{3/2}} \exp\left[-\frac{\varepsilon_x}{T_b} - \frac{\varepsilon_z}{T_b} - \frac{\varepsilon_y}{T_b}\right].$$

In the result of collisions and stopping the electron loses its energy $\Delta\varepsilon$. This value of energy is placed into the cell in which collisions and stopping were occurred with calculated "weight" P, i.e. the absorbed energy is $\Delta\varepsilon P$. The sum of the value $\varepsilon_n P_n$ over all histories gives the mean energy of the electron beam: $\sum_n \varepsilon_n P_n = \langle\varepsilon\rangle$, where ε_n is the sampled electron energy on the n-th history, P_n is the electron "weight" on the n-th history.

The initial angle of the incoming electron with surface may be fixed or sampled from an arbitrary distribution function. In the case of the magnetized plasma the electron trajectory is a spiral line with the Larmor radius R_L. The trajectory start is sampled randomly on a circle of the radius R_L. The initial electron energy may be presented as the

sum of the longitudinal and transverse energy components: $\varepsilon = \varepsilon_l + \varepsilon_t$. If the electron has the longitudinal and transverse energy components, then there is some angle β between the magnetic field and electron velocity directions. This angle is defined from the condition $\cos\beta = \sqrt{\varepsilon_l/\varepsilon}$. The electron moves along a spiral path at this angle until an entrance point into the target. Therefore the angle between the velocity and axis X directions may be found in the range from $\alpha - \beta$ up to $\alpha + \beta$, where α is the angle between the magnetic field and axis X directions. The initial angle of the incoming electron with the surface is concluded in the range from $90 - (\alpha + \beta)$ up to $90 - (\alpha - \beta)$.

The center coordinates of the Larmor circle may be fixed or sampled randomly on some plasma square.

II. Calculation of the macroscopic cross sections.

The full microscopic electron-nucleus σ_{en} and electron-electron σ_{ee} cross sections in "near" collisions are obtained by integration of the corresponding differential cross sections (1,3). The macroscopic cross sections of interaction of the electrons will be given then by

$$\Sigma_{en} = N_n\sigma_{en}, \qquad \Sigma_{ee} = ZN_n\sigma_{ee},$$

where N_n is the plasma ion number density, Z is the atomic number.

The full macroscopic cross section Σ_t is calculated as the sum of the electron-electron Σ_{ee} and electron-nucleus Σ_{en} ones. The cross section Σ_t determines the mean length δl_i between two "near" collisions. The length δl_i must not be less some minimal value below which the multiple scattering theory is invalid. This value may not be very large such that the energy losses for ionization will compared with the electron energy at the beginning of this length. It is necessary to choose the length δl_i in such manner that the difference in the interaction cross section was negligible on this length. To take into ac-

count this situation the fictitious collisions are introduced. The fictitious macroscopic cross section Σ_f is equal to $\left(\delta\varepsilon/\delta l_i\right)/\left(0.1\varepsilon_i\right)$, where ε_i is the kinetic electron energy at the beginning of the length δl_i, $\delta\varepsilon$ is described by Eq.(4). The mean value of δl_i is sampled from the condition that the energy losses on the length δl_i would not higher 10 % of the initial electron energy at the beginning of this length. Thus, the full macroscopic cross section Σ_t is calculated as the sum of the electron-electron Σ_{ee}, electron-nucleus Σ_{en} and fictitious Σ_f ones.

III. Simulation of the path length δl_i between two "near" collisions

In our investigation a plasma is inhomogeneous, i.e. there are the gradients of density, temperature and magnetic field. To simulate the path length in such medium the approximation of a plasma with segment-constant characteristics is used. It is supposed that which plasma cell (i, j, k) has the constant values of density, temperature and magnetic field strength. But these characteristics may be different in the neighboring cells. Let us take it that the electron propagates from the point r_0 in the direction Ω. Plasma is magnetized. The electron path length is a spiral line. The simulation algorithm of the path length in a plasma with segment-constant characteristics is in the following:

1. The random value of the optical path length is sampled according to $s = -\ln\xi$.

2. The full macroscopic cross section $\Sigma_t^{i,j,k}$ is calculated as described above for the (i, j, k)-th cell in which the point r_0 is located.

3. The spiral length d to the nearest cell boundary is determined in the direction Ω.

4. The optical thickness d^{opt} is evaluated from the expression $d^{opt} = \Sigma_t^{i,j,k} d$.

5. If $s \leq d^{opt}$, then the coordinates of the collision point is calculated:

$$x_1 = c_x + h_l \delta l_i \cos\alpha/l_t - R_L\left[\cos\left(2\pi\delta l_i/l_t\right)\cos\varphi_z - \sin\left(2\pi\delta l_i/l_t\right)\sin\varphi_y\right]\sin\alpha,$$

$$z_1 = c_z + h_l \delta l_i \sin\alpha/l_t + R_L\left[\cos\left(2\pi\delta l_i/l_t\right)\cos\varphi_z - \sin\left(2\pi\delta l_i/l_t\right)\sin\varphi_y\right]\cos\alpha,$$

$$y_1 = c_y + R_L\left[\sin\left(2\pi\delta l_i/l_t\right)\cos\varphi_z + \cos\left(2\pi\delta l_i/l_t\right)\sin\varphi_y\right], \qquad (10)$$

where c_x, c_y, c_z are the center coordinates of the Larmor circle, $\delta l_i = s/\Sigma_t^{i,j,k}$ is the spiral length up to the collision point, $l_t = 2\pi v/\omega_L$ is the spiral period length, α is the angle between the magnetic field and axis X directions, φ_z is the initial phase of the electron with the axis Z, φ_y is the initial phase of the electron with the axis Y. Eqs.(10) are written for the case when the vector B of magnetic intensity is in the plane (Z, X) and has the angle α with direction of the axis X. On the length δl_i the energy loss is taken into account in the (i, j, k)-th cell in accordance with $\Delta E_{i,j,k} = \delta\varepsilon$, where $\delta\varepsilon$ is described by Eq.(4). In the end of the length δl_i the new direction cosines $\alpha_1, \beta_1, \gamma_1$ are calculated with account of a small deflection angle θ by the following formulas:

$$\alpha_1 = \alpha_0 \cos\theta + \left(\alpha_r - \alpha_0 \cos\theta_r\right)\sqrt{\left(1 - \cos^2\theta\right)/\left(1 - \cos^2\theta_r\right)},$$

$$\beta_1 = \beta_0 \cos\theta + \left(\beta_r - \beta_0 \cos\theta_r\right)\sqrt{\left(1 - \cos^2\theta\right)/\left(1 - \cos^2\theta_r\right)},$$

$$\gamma_1 = \gamma_0 \cos\theta + \left(\gamma_r - \gamma_0 \cos\theta_r\right)\sqrt{\left(1 - \cos^2\theta\right)/\left(1 - \cos^2\theta_r\right)},$$

where $\alpha_0, \beta_0, \gamma_0$ are the initial direction cosines with coordinate axes in the collision point, $\alpha_r, \beta_r, \gamma_r$ are the direction cosines of the random vector in space which has the uniform probability of distribution, $\cos\theta_r$ is the angle cosine between the initial direction and the random vector in the collision point. The simulation of the spiral path length is finished.

6. If $s > d^{opt}$ and the electron is within the boundaries of the target, then the new values of s and Σ_t are calculated: $s = s - d^{opt}$, $\Sigma_t = \Sigma_t^{i',j',k'}$, where $\Sigma_t^{i',j',k'}$ is the macroscopic cross section for a new cell which the electron is crossing. The new coordinates of the point on the cell boundary is evaluated by means of Eqs.(10) where instead δl_i is used the value d. The energy loss and correction of the direction cosines are performed on the length d as described above. After that the simulation is continued from the item 3.

7. If $s > d^{opt}$ and the electron crossed any target boundary, then the energy loss is taken into account on the length d and the electron trajectory is stopped.

The length d to the nearest cell boundaries in the magnetized plasma is calculated from the joint solving of the spiral equations (10) and the plane ones.

IV. Simulation of the process type in a "near" collision

The process type in the collision point may be following: the elastic electron-nucleus collision, the elastic electron-electron collision and the fictitious process to take into account the large path lengths δl_i. The selection of the process type is made randomly according to the probabilities Σ_{en}/Σ_t, Σ_{ee}/Σ_t, Σ_f/Σ_t.

1). "Near" electron-nucleus collision.

This process gives main contribution into the scattering of electrons in a plasma. The differential cross section of this process is described by Eq.(1). The scattering angle θ in the laboratory coordinate system is sampled from this expression by means of the reverse function method described above. The energy passed to the nucleus is calculated from the kinematic relationship:

$$\Delta E_n = 4\frac{m_e M_n}{(m_e + M_n)^2} E \sin^2\frac{\theta}{2},$$

where E is the kinetic electron energy before a collision, M_n is the nucleus mass. The electron energy will be less by this value after scattering. The criterion of the trajectory end in the energy is checked. If the electron energy is less $\varepsilon_{min} = 1\ keV$, then the electron trajectory is stopped and the new trajectory is started. In opposite way, the electron trajectory is traced with the new electron energy and direction cosines. The simulation procedure is continued from the stage **II**.

2). "Near" electron-electron collision.

It was mentioned that the probability of the electron scattering on the plasma electrons is Z times less efficient that on the nuclei. The energy ε passed to the recoil electron in a "near" collision is sampled from Eq.(3) by means of the reverse function method. Cosine of a scattering angle θ in the laboratory coordinate system is evaluated from the following kinematic expression: $\cos\theta = \sqrt{1 - \varepsilon/E}$, where E is the kinetic energy of the incident electron. The recoil electrons may have the energy higher $\varepsilon_{min} = 1\ keV$. In this case their trajectories are traced in the similar way. When the electron energy is smaller $\varepsilon_{min} = 1\ keV$ the trajectory is stopped. In the opposite way the simulation procedure is repeated from the stage **II** with new values of the electron energy and angle.

3). The fictitious process.

The fictitious collision is occurred with probability Σ_f/Σ_t. If one takes place, then the electron energy and angle are not changed, i.e. after collision the electron energy and direction are the same. The simulation is continued from the stage **II**.

Thus, Monte Carlo simulation is performed by caring out the above calculation on many test electrons.

Numerical results

In the result of Monte Carlo calculations one can obtain the following characteristics:

1) The energy deposition profiles of the electrons in a plasma.

In the result of N histories we have some energies $\Delta E_{i,j,k}$ $[keV]$ in which cell (i, j, k). In order to calculate the energy $\Delta \varepsilon_{i,j,k}$ $[keV]$ per one electron it is necessary to divide the energy $\Delta E_{i,j,k}$ $[keV]$ in which cell by the number of histories N. Let us consider the case of slab geometry. In this case the lengths along the axes Y and Z are assumed as placed at infinity. The energy $\Delta \varepsilon_i$ $[keV]$ in the i-th layer is calculated by taking the summation with respect to all values $\Delta \varepsilon_{i,j,k}$ $[keV]$ for the given i-th index, i.e. the summation is performed over all j-th and k-th indexes. Dividing each value $\Delta \varepsilon_i$ $[keV]$ by the given size of mesh h_i $[cm]$ we obtain the deposited energy (dE/dx) $[keV/cm]$ per one electron along the axis X. This value was averaged over different collision histories. Therefore it can be treated as the power density (dQ/dx) $[keV/(cm^3 \, sec)]$ for the unit flux $F = 1$ $[el/(cm^2 \, sec)]$ of electrons before the target. In the case of monoenergetic beam the energy flux P $[keV/(cm^2 \, sec)]$ of the impinging beam and the electron energy E $[keV]$ are given as initial parameters. The initial flux of electrons before the target is determined as $F = P/E$ $[el/(cm^2 \, sec)]$. For hydrodynamic problems the power density (dQ/dx) $[keV/(cm^3 \, sec)]$ is required. This value can be easily obtained. It is necessary to multiply the value (dE/dx) $[keV/cm]$ by the flux F $[el/(cm^2 \, sec)]$ before the target.

In the case of maxwellian beam the initial electron energy E is sampled from the maxwellian function $f_m(v)$. The average flux is given by $F = \int_0^\infty n v f_m(v)\, dv$, where $n = P/vE$ is the number density of electrons in the beam. To obtain the power density $(dQ/dx)\ [keV/(cm^3\ sec)]$ for the maxwellian distributed electrons it is necessary to multiply $(dE/dx)\ [keV/cm]$ by the calculated flux F.

2) The angular and energetic spectrums of past and reflected electrons.

When the electron is crossing any target boundary its energy and angle are fixed. The averaging over all electron histories gives the angular and energetic spectrums of electrons for each target boundary.

The presented physical model was used for calculation of divertor material erosion under plasma disruptive conditions. Pulse durations larger than 100 μs and beam power densities around 5 MW/cm^2 are considered. It is supposed that the electron beam has the maxwellian distributed energy. Temperature of the electron beam is equal to 10 keV. Two-dimensional maxwellian function corresponding to the investigated case is shown in Fig.(1). The energy of the incoming electron is sampled from this function according to the procedure described above. The scheme with "weight" was used to calculate the energy deposition of the tail of maxwellian electrons. The magnetized carbon plasma has the density and temperature gradients. These profiles are shown in Figs.(2,3) for different time moments. The solid part of target is also taken into account. Magnetic field lines have the angle about 5^0 with the plasma surface. The electrons under guidance of ones enter a plasma under small angles.

It is necessary to calculate the deposited energy of the electron beam in this inhomogeneous medium. First three calculated variants are shown in Figs.(4,5,6). In this case it is supposed that the magnetic intensity is constant and has the components $B_x(x) = 0.5\ T$ and $B_z(x) = 5\ T$. Thus, the magnetic field direction is constant and has the angle about 5^0 with the plasma surface. Figs.(4,5,6) correspond the different longitudinal energy components $E_l = E_{tot}$, $E_l = 0.5E_{tot}$, $E_l = 0.1E_{tot}$ of the electron beam, respectively. We see that the deposition profiles have two peaks. First peak is the result of the energy deposition on the shock wave. Second peak is coupled with the energy deposition in the solid part of carbon target. In these figures we can notice that a considerable part of energy is deposited in the thin layer of the solid carbon. Mean percentage of absorbed energy is 97%, 90% and 70% for Figs.(4,5,6), respectively. If the electron beam is strongly rotating then a considerable part of its energy is reflected from a plasma.

Second three variants are shown in Figs.(7,8,9). In this case there are the gaussian magnetic field profiles along the axis X for the magnetic field component B_z. The range of B_z is from $0.5\ T$ up to $5\ T$. These profiles are shown in Fig.(10). Magnetic field component B_x is constant and equal to $0.5\ T$. In this case the magnetic field value and direction are changed along the target. Initially magnetic field lines have the angle about 5^0 with the plasma surface. Further the direction of magnetic field lines is turned and the value of magnetic field is changed as $B = \sqrt{B_x^2 + B_z^2}$ with the plasma depth. This value is about $0.7\ T$ near the boundary of solid carbon. Three variants were also calculated for the different longitudinal components E_l. The influence of

the gaussian profile of magnetic field on the deposited energy is seen in these figures. Some part of the electron beam energy is deposited near the plasma surface. Mean percentage of absorbed energy is 99.3%, 97.5% and 93% for Figs.(7,8,9), respectively.

In Figs.(11,12) are shown mean energetic and angular spectrums of reflected electrons from the plasma surface. The form of spectrums is the same for all calculated variants.
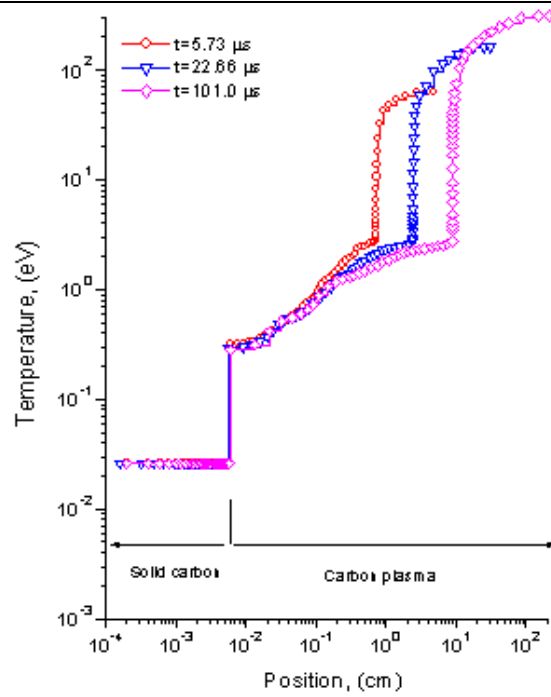

Fig.1 Two-dimensional maxwellian function of the energy distribution in the electron beam.

Fig.2 Density profiles for different time moments.
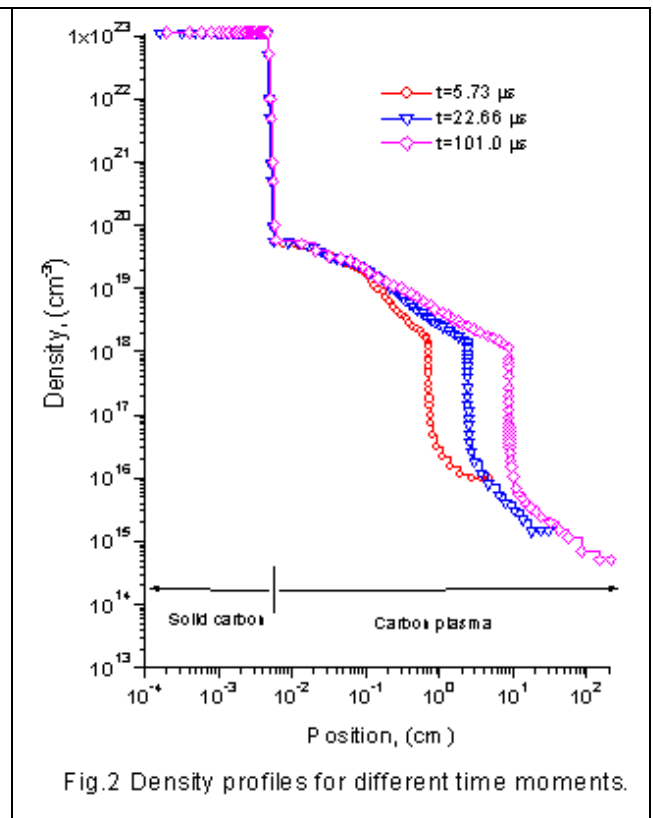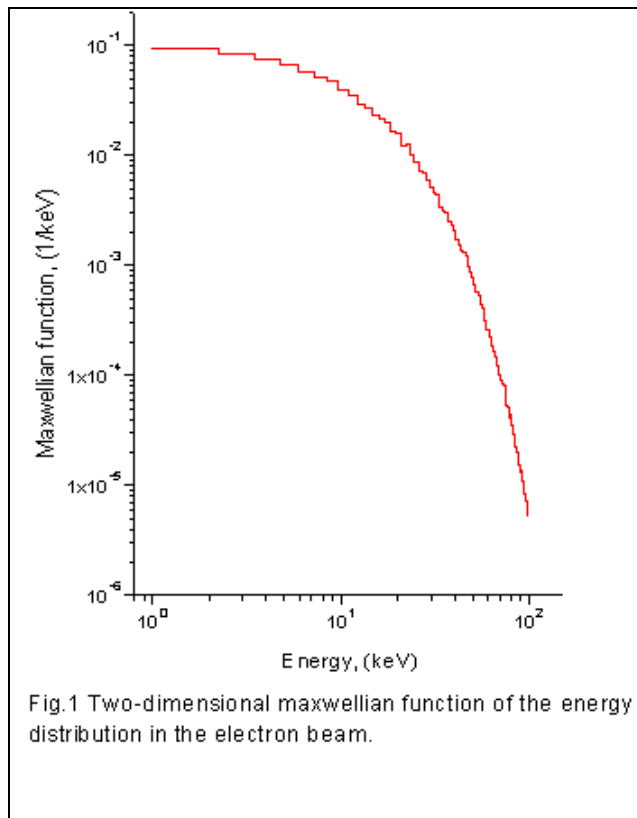
Fig.3 Temperature profiles for different time moments.
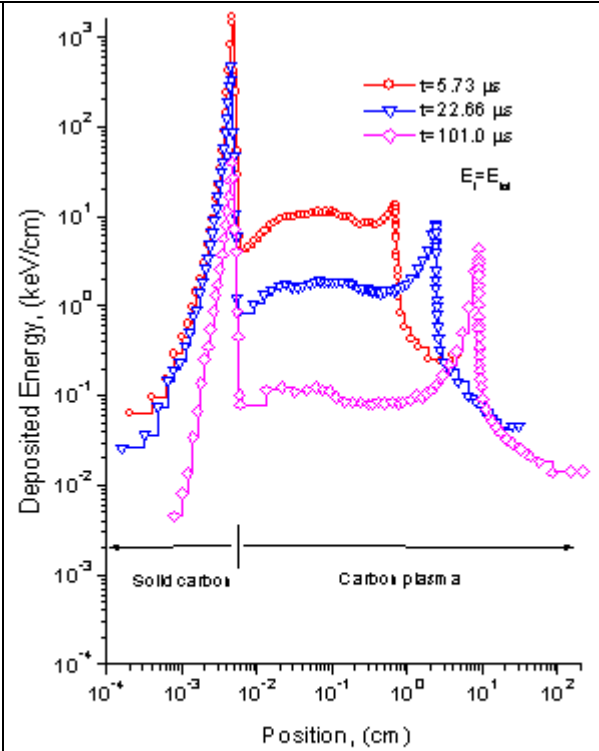
Fig.4 Energy deposition profiles of the rotating electron beam into a magnetezed carbon plasma for different time moments.
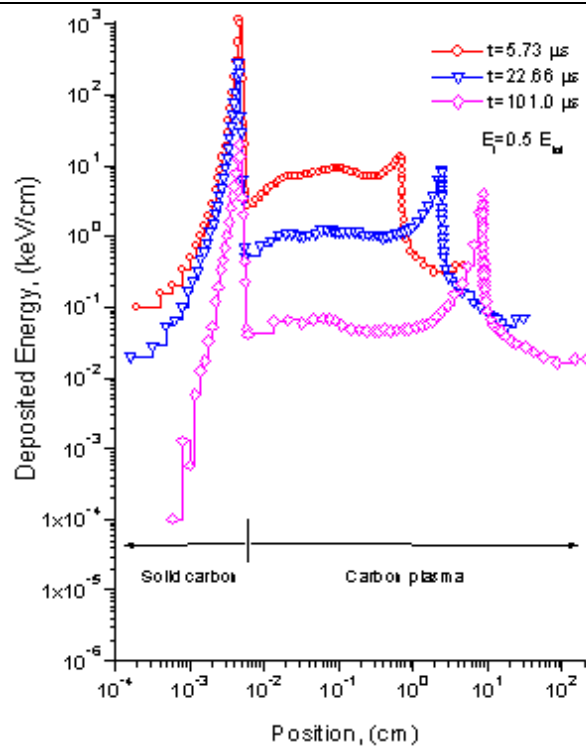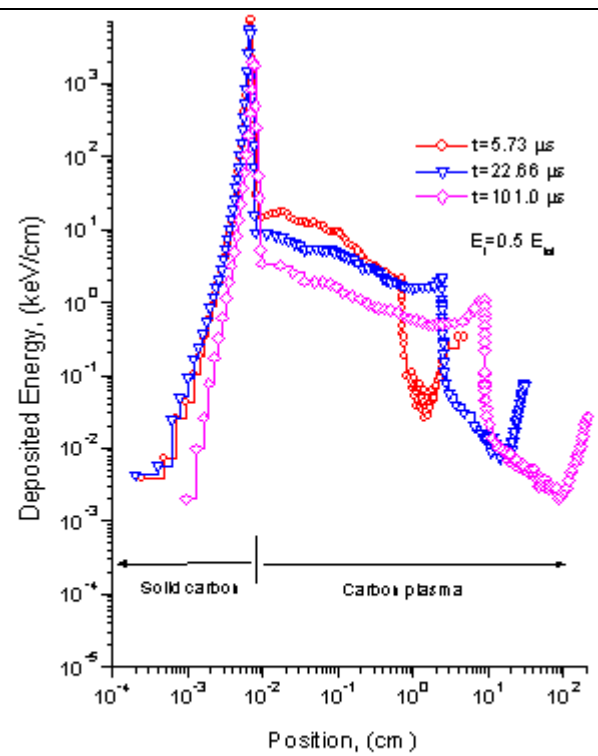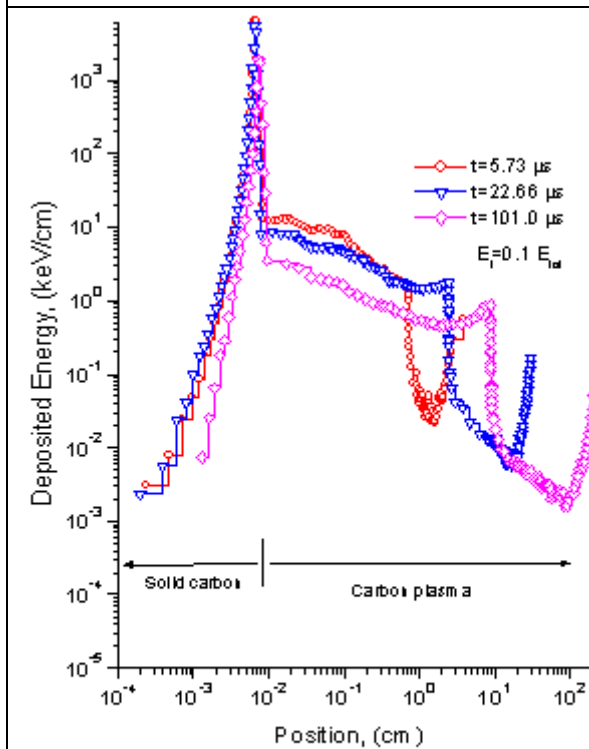

Fig.5 Energy deposition profiles of the rotating electron beam into a magnetezed carbon plasma for different time moments.
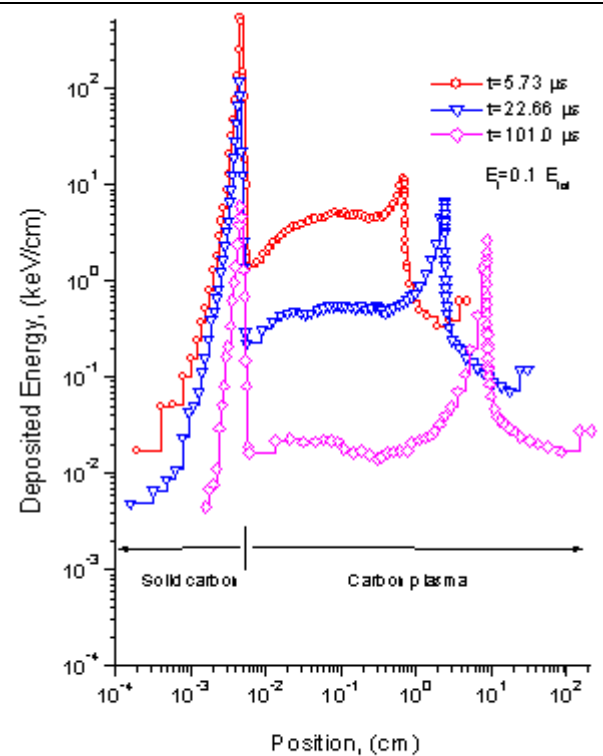
Fig.6 Energy deposition profiles of the rotating electron beam into a magnetezed carbon plasma for different time moments.
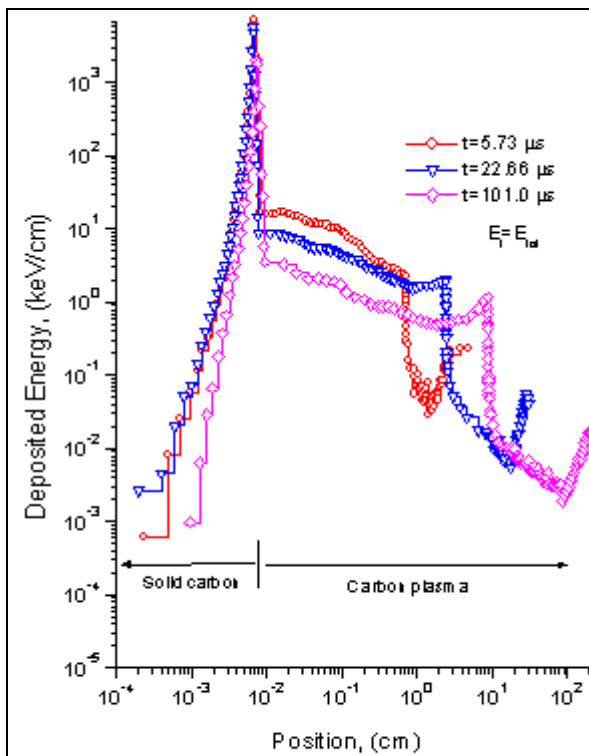
Fig.7 Energy deposition profiles of the rotating electron beam into a magnetezed carbon plasma for different time moments.

Fig.8 Energy deposition profiles of the rotating electron beam into a magnetezed carbon plasma for different time moments.

Fig.9 Energy deposition profiles of the rotating electron beam into a magnetezed carbon plasma for different time moments.
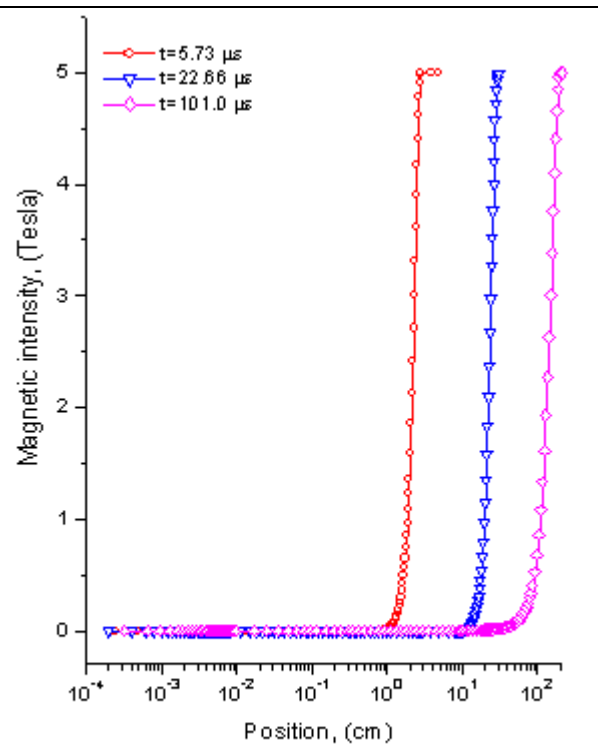
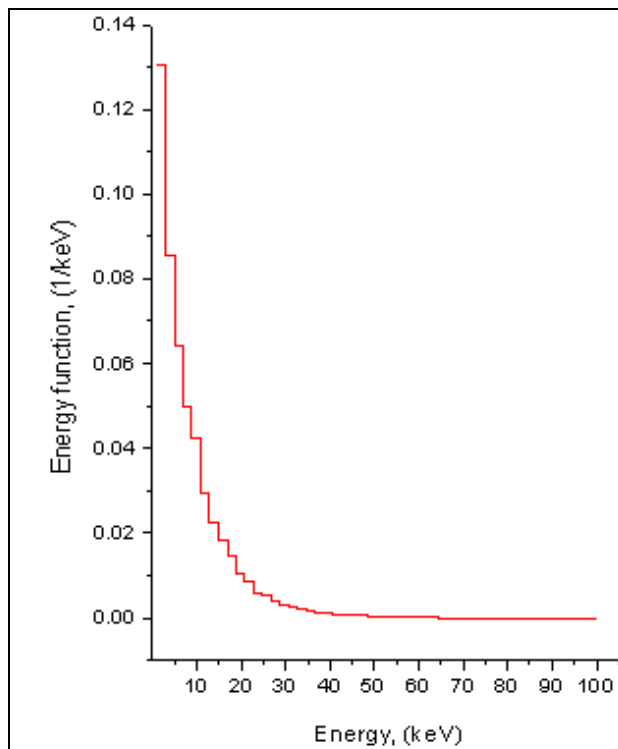Fig.10 Gaussian profiles of the magnetic field component B_y along the axis X.

Fig.11 Energy spectrum of the reflected electrons from the plasma surface.
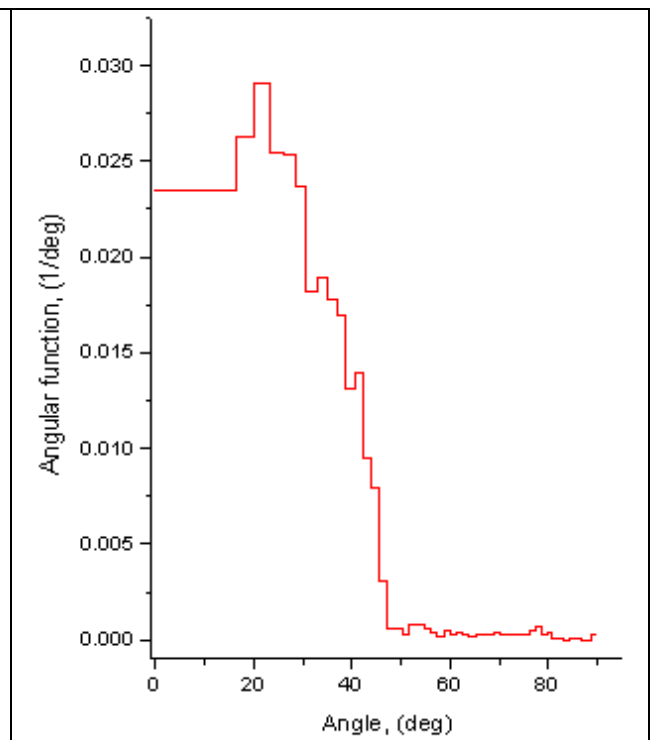

Fig.12 Angular spectrum of the reflected electrons from the plasma surface.

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TEMPERATURE AND MAGNETIC FIELD PROFILES.

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В ЗАМАГНИЧЕННУЮ ПЛАЗМУ, ИМЕЮЩУЮ ПРОФИЛИ
ПЛОТНОСТИ, ТЕМПЕРАТУРЫ И МАГНИТНОГО ПОЛЯ.

Препринт № 4