THEORETICAL MODEL FOR COMPUTATION OF THE ENERGY RELEASE OF AN ELECTRON BEAM IN A MAGNETIZED PLASMA

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Physical and mathematical models for computation of the energy release of an electron beam in a homogeneous magnetized plasma are developed. Profiles of the energy release in a carbon plasma as functions of various parameters of the beam and the plasma state are obtained.

Introduction. An electron that moves with the velocity **v** in a magnetic field with the induction vector **B** is acted upon by a Lorentz force directed perpendicular to both the vector **B** and the velocity. Under the action of this force the electron rotates about a line of the magnetic field with the cyclotron frequency $\omega = eB/m$. The electron rotation in a plane perpendicular to the direction of the magnetic field occurs in a circle with the cyclotron radius $R = mv_{\perp}/eB$ whose center travels parallel or antiparallel to the vector **B** with a constant velocity $v_{\parallel}$. The resultant trajectory is spiral motion along a line of force of the magnetic field with the step $H = 2\pi v_{\parallel}/\omega$. On entering a plasma these electrons will collide with its particles and come off the magnetic lines, as a result of which the magnitude and direction of **v**, $\mathbf{v}_{\perp}$, and $\mathbf{v}_{\parallel}$ change. After this the electron moves along a new line of force with new values of the velocity, radius, and step until it collides again. Using this model of single-particle electron drift in a magnetized plasma, we can investigate the energy release of an electron beam as a function of various parameters and the state of the plasma.

1. Physical Model of Passage of an Electron in a Plasma. Let us make the following assumptions:

1) the electrons of the beam move in the plasma and interact with scattering centers (plasma electrons and nuclei) that are disposed in a random manner;

2) each electron of the beam interacts with just one scattering center at a time (the approximation of binary collisions);

3) the electrons of the beam do not interact with each other.

Based on these assumptions we can introduce the concept of the trajectory of an electron and describe its motion between collisions using the following expressions: $m d\mathbf{v}/dt = -e[\mathbf{v}\times\mathbf{B}]$ and $d\mathbf{r}/dt = \mathbf{v}$. The trajectory of the motion, determined from simultaneous solution of these equations, is a spiral line. At collision points, the direction of the electron and its kinetic energy change.

Electron scattering in a plasma is considered in the first Born approximation. The electrons of the beam are assumed to be nonrelativistic. In describing their passage through the plasma we allow for the following physical processes of interaction: a) elastic scattering on plasma electrons and nuclei; b) energy losses through interaction with free and bound electrons of the plasma. The probability of elastic scattering on nuclei is Z times larger than on electrons. The energy losses on nuclei, because of the large difference in mass, are negligibly small compared to the losses on electrons. The first process is Z times more efficient than the second. Consequently, a characteristic feature is that an electron changes its direction of motion many times before it loses its entire energy.

1.1. Elastic scattering of electrons on nuclei. All electron scatterings on nuclei are divided into two groups: scattering at small angles $\theta < \theta^*$ and scattering at large angles $\theta > \theta^*$. The magnitude of the angle θ^* is chosen in the interval $v_0/v \ll \theta^* \ll 1$. The differential cross section of elastic scattering $d\sigma_{en}$ at large angles is described by the Rutherford formula [1]

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$$d\sigma_{en} = \left(\frac{Ze^2}{2mv^2}\right)^2 \frac{d\Omega}{\sin^4\theta/2}$$

The total cross section for scattering at large angles is

$$\left(\frac{Ze^2}{mv^2}\right)^2 \frac{1+\cos\theta^*}{1-\cos\theta^*} \tag{1}$$

Scattering at small angles $\theta < \theta^*$ is allowed for in the following manner. Because of the effect of long-range Coulomb forces an electron, having traversed a distance l, will experience numerous statistically independent weak collisions. The probability of its scattering in the interval of angles θ–$\theta + d\theta$ after traversing the distance l is prescribed by the Gaussian distribution [2]

$$P(\theta)\, d\theta = \frac{2\theta}{\langle\theta^2\rangle} \exp\left(-\frac{\theta^2}{\langle\theta^2\rangle}\right) d\theta, \tag{2}$$

where the mean square of the scattering angle is

$$\langle\theta^2\rangle = N_n l \int\limits_{\theta<\theta^*} \theta^2\, d\sigma_{en} \approx 8\pi l \left(\frac{Ze^2}{mv^2}\right)^2 \sum_i N_i \Lambda_i$$

Summation is performed over all the kinds of ions that exist in the plasma. The Coulomb logarithm Λ_i of an i-fold ion is determined by the following expression:

$$\Lambda_i = \ln\frac{\theta^* mva_i}{\hbar} + \left(\frac{i}{Z}\right)^2 \ln\frac{0.607D}{}$$

The characteristic dimension of the i-th ion a_i is computed in terms of the atomic form factor $F(q)$

$$(Z^2 - i^2)\ln a_i = -\int_0^\infty \ln q \frac{\partial}{\partial q}[Z - F(q)]^2\, dq,$$

where $\hbar q = 2mv\sin\theta/2$ is the momentum transferred to the nucleus, q is the wave number.

1.2. Electron scattering on electrons. For a high-energy incident electron, plasma electrons (including bound electrons) can be considered to be free electrons. In this case, the differential cross section $d\sigma_{ee}$ of the energy transfer to a plasma electron [1] is equal to $d\sigma_{ee} = (\pi Ze^4/E)d\varepsilon/\varepsilon^2$, where $E = mv^2/2$ is the kinetic energy of the incident electron; ε is the integration variable. It is very large in the case of small energy transfers. Therefore all electron-electron collisions are divided into strong and weak collisions. Strong collisions, when the energy transferred exceeds some value E^*, are described by the cross section $d\sigma_{ee}$. Their total cross section is

$$\sigma_{ee} = \int\limits_{\varepsilon>E^*} d\sigma_{ee} = \left(\frac{\pi Ze^4}{E}\right)\frac{E-E^*}{EE^*} \tag{3}$$

Weak electron-electron collisions are allowed for by decrease in the energy of the electron, traversing the distance l, by the amount

$$\Delta E = \frac{2\pi e^4}{mv^2}\left(N_e^p \ln\frac{2E^* mv^2}{\hbar^2\omega_p^2} + \sum_i N_e^i \ln\frac{2E^* mv^2}{I_i^2}\right) \tag{4}$$

where the summation is performed over all the kinds of ions that exist in the plasma; $\omega_p = (4\pi N_e^p e^2/m)^{1/2}$ is the plasma frequency of the free electrons; I_i is the characteristic ionization potential for the bound electrons of the i-th ion. The magnitude of I_i was determined from quantum-mechanical atomic computations by the Hartree–Fock–Slater model [3, 4] using oscillator strengths and energy levels of the ion by means of the formula [1]

$$\ln I_i = \frac{1}{Z - i} \sum_n N_{0n} \ln (E_n - E_0) .$$

Here, the summation is performed over all possible states of the electron. The energy E^* must satisfy the condition $I_i \ll E^* \ll mv^2/2$.

2. Modeling by the Monte-Carlo Method. The standard Monte-Carlo method enables us to determine all characteristics of the primary and secondary electrons after a collision. Their trajectories are traced until they lose their entire energy. The plasma is assumed to have a constant density and temperature. A coordinate system is introduced in the following manner. The electron beam is incident on the plane (Z, Y) and propagates along the X axis. The induction vector of the magnetic field **B** lies in the plane (Z, X) and can make an arbitrary angle α with the axis X. The electrons of the beam enter the plasma, spiraling on the magnetic lines of force.

When the trajectories of the electrons are constructed in the magnetized plasma all collisions are divided into "near" and "distant" collisions. The latter are related to small energy transfers and small scattering angles and are described by multiple-scattering theory [2]. In this theory, the distributions of the scattering angles and the energy losses are assumed to be uncorrelated, which is true for small segments l, when the change in the interaction cross section during electron retardation in the plasma can be disregarded. The "near" collisions are related to large energy transfers and are considered to be individual events. The parameters that govern the division of all collisions into "distant" and "near" are the angle θ^* and the energy E^*. Thus, the trajectory of an electron is constructed in the form of a broken line, on each link of which there are numerous "distant" collisions.

The Monte-Carlo algorithm for constructing an electron trajectory in a magnetized plasma involves the following steps:

a) *Determination of the total macroscopic cross section at the interaction point.* The total macroscopic cross section Σ of the interaction of the electron with the plasma is equal to the sum of the macroscopic cross sections of the individual processes $\Sigma = \Sigma_{en} + \Sigma_{ee} + \Sigma_f$, where $\Sigma_{en} = N_n\sigma_{en}$ is the macroscopic cross section of the electron-nucleus interaction in "near" collisions; $\Sigma_{ee} = ZN_n\sigma_{ee}$ is the macroscopic cross section of the electron-electron interaction in "near" collisions; $\Sigma_f = (\Delta E/l)/(0.01E)$ is the macroscopic cross section of a fictitious collision. The quantities of σ_{en}, σ_{ee}, and ΔE are determined, respectively, by formulas (1), (3), and (4). The quantity Σ_f is introduced to provide automatic choice of the length of the segment l. The cross section Σ_f describes scattering of the electron without a change in its energy or direction of motion. The random length of the trajectory l between collisions can be rather large, which leads to violation of the condition of the applicability of multiple-scattering theory. Introduction of fictitious collisions enables us to allow for this fact. The value of Σ_f is prescribed in such a way that the energy loss on the segment l is about 1% of the kinetic energy of the electron at the beginning of this segment.

b) *Drawing the length of the trajectory l.* The mean free path of the electron to the next "near" collision in a homogeneous magnetized plasma with the total macroscopic cross section of interaction Σ is a random quantity with the probability density $P(l) = \Sigma \exp(-\Sigma l)$. The random length of the trajectory l that corresponds to this probability density is drawn from the expression $l = -\ln \xi/\Sigma$, where ξ is a random number uniformly distributed in the interval [0, 1].

c) *Drawing the type of process.* At a collision point, we observe one of the following processes of interaction: elastic scattering of the electron on a nucleus or electron, fictitious collision. "Near" electron-nucleus and electron-electron collisions occur with the probabilities Σ_{en}/Σ and Σ_{ee}/Σ. The probability of a fictitious collision is equal to Σ_f/Σ. The type of process of interaction is drawn with allowance for these three possible events: for $\xi < \Sigma_{en}/\Sigma$, electron-nucleus scattering occurs; for $\Sigma_{en}/\Sigma \le \xi < (\Sigma_{en}/\Sigma + \Sigma_{ee}/\Sigma)$, electron-electron scattering occurs, for

$\Sigma_{en}/\Sigma + \Sigma_{ee}/\Sigma) \leq \xi$, a fictitious collision occurs. If scattering on a nucleus or electron is drawn, from the corresponding differential cross sections of the processes, we draw the scattering angle θ, and from kinematic relations we determine the kinetic energy transferred to the nucleus or electron of the plasma. If a fictitious collision is drawn, the direction and energy of the electron do not change.

d) *Determination of the coordinates of the collision point.* The Cartesian coordinates x_1, y_1, z_1 of the interaction point for the electron are determined by the following expressions:

$$x_1 = x_0 + Hl \cos \alpha/L - R\,[\cos(2\pi l/L) \cos \varphi_z - \sin(2\pi l/L) \sin \varphi_y] \sin \alpha\,;$$

$$z_1 = z_0 + Hl \sin \alpha/L + R\,[\cos(2\pi l/L) \cos \varphi_z - \sin(2\pi l/L) \sin \varphi_y] \cos \alpha\,;$$

$$y_1 = y_0 + R\,[\sin(2\pi l/L) \cos \varphi_z + \cos(2\pi l/L) \sin \varphi_y]\,,$$

where x_0, y_0, z_0 are the initial coordinates of the center of the Larmor circle; $L = 2\pi v/\omega$ is the length of the spiral period; α is the angle between the directions of the coordinate axis X and the induction vector B; φ_y and φ_z are the initial phases of the electron with the Y and Z axes. As a result of the collision the electron is deflected through an angle θ, determined from differential cross sections. The direction cosines α_1, β_1, γ_1 of the scattered electron with the coordinate axes X, Y, Z are computed by the following formulas:

$$\alpha_1 = \alpha_0 \cos \theta + (\alpha_s - \alpha_0 \cos \theta_s) \sqrt{(1 - \cos^2 \theta)/(1 - \cos^2 \theta_s)}\,;$$

$$\beta_1 = \beta_0 \cos \theta + (\beta_s - \beta_0 \cos \theta_s) \sqrt{(1 - \cos^2 \theta)/(1 - \cos^2 \theta_s)}\,; \qquad (5)$$

$$\gamma_1 = \gamma_0 \cos \theta + (\gamma_s - \gamma_0 \cos \theta_s) \sqrt{(1 - \cos^2 \theta)/(1 - \cos^2 \theta_s)}\,,$$

where α_0, β_0, γ_0 are the direction cosines of the electron before the collision; α_s, β_s, γ_s are the direction cosines of the random vector S uniformly distributed in space; $\cos \theta_s$ is the cosine of the angle between the initial direction of the electron and the direction of the random vector S.

e) *Allowance for "distant" collisions.* After the electron traverses the length of the trajectory l, its kinetic energy is decreased by ΔE, which is determined by formula (4). The total angle of deflection acquired by the electron on the length l due to "distant" collisions is drawn from distribution (2). The direction cosines are determined from relations (5).

Subsequent modeling of the electron trajectory is continued paragraph a. If the kinetic energy of the electron becomes lower than the prescribed boundary energy below which it can be considered to be absorbed, or it falls outside the limits of the region in question, its trajectory terminates. We need to trace a large number of these trajectories to obtain a spatial distribution of the energy release of the electron beam.

3. Numerical Results. The theoretical model presented was used to calculate the energy release of an electron beam in a magnetized carbon plasma. The ionization composition of the plasma was determined on the basis of quantum-mechanical calculations [3, 4]. Figure 1a shows the energy release in the carbon plasma as a function of the degree of twisting $\eta = v_\perp/v$ of the electrons of the beam. Consideration was given to a plasma with a density of 10^{25} m^{-3} and a temperature of 232,100 K. The induction vector of the magnetic field B is directed along the X axis and is equal to 5 T in magnitude. The kinetic energy of the electrons in the beam is $8 \cdot 10^{-15}$ J. The density of the incident electron flux is $6.25 \cdot 10^{24}$ m^{-2}·sec. The power density of the energy flux is $5 \cdot 10^4$ MW/m^2. We calculated four energy-release profiles as a function of the longitudinal component of the kinetic energy $E_\parallel = mv_\parallel^2/2$. It is evident from Fig. 1a that the larger the transverse component of the kinetic energy $E_\perp = mv_\perp^2/2$ of the beam electrons, the closer the energy-release region is to the plasma surface. A strongly twisted electron beam penetrates a small distance into the plasma.

Figure 1b gives the energy release as a function of the angle of incidence of the beam electrons on the plasma. The characteristics of the beam and the plasma are the same as in the previous case. Initially, the electrons of the beam are not twisted, i.e., $v_\perp = 0$. The velocity v of the electrons is directed along the vector B. The calculation

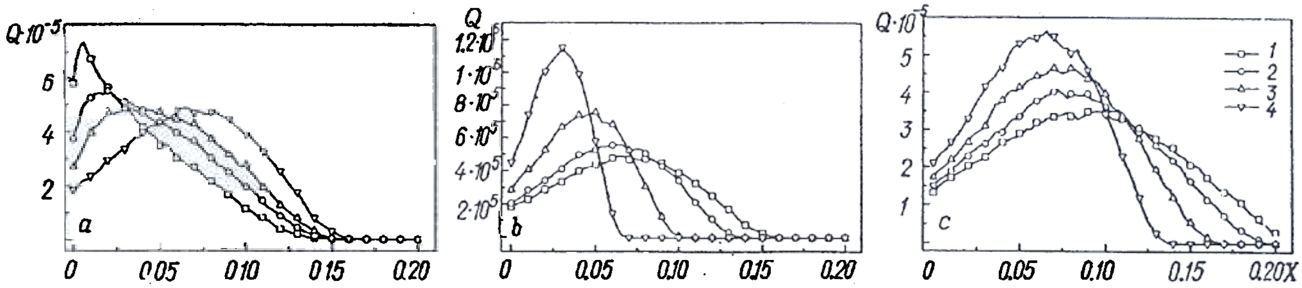

Fig. 1. Profiles of the energy release of an electron beam in a magnetized carbon plasma for various longitudinal energies $E_{\parallel}$ (a), for various angles α of entry of the electrons into the plasma (b), and for various temperatures T_p of the plasma (c): a: 1) $E_{\parallel} = 1.6\cdot10^{-15}$, 2) $3.2\cdot10^{-15}$, 3) $4.8\cdot10^{-15}$, 4) $8\cdot10^{-15}$; b: 1) $\alpha = 0$, 2) 30, 3) 50, 4) 65; c: 1) $T_p = 11{,}605$; 2) 58,025; 3) 116,050; 4) 1,160,500. Q, MW/m^3; X, m; E, J; α, deg; T, K.

is performed for different angles α between the directions of the vector **B** and the X axis. It is evident from Fig. 1b that the larger the angle α, the closer the energy-release profile is to the plasma surface.

Figure 1c shows profiles of the energy release as a function of the plasma temperature. The characteristics of the beam and the plasma are the same as in the previous cases. The induction vector **B** is directed along the X axis. Initially, the velocity **v** of the beam electrons is directed along **B**, i.e., the electrons are not twisted. It is evident that as the temperature increases, the maximum of the energy release shifts toward the plasma surface. This is explained by the increase in the degree of plasma ionization and the larger contribution of energy losses on free electrons.

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NOTATION

e, electron charge; m, electron mass; $v_{\perp}$and $v_{\parallel}$, electron-velocity components perpendicular and parallel to the direction **B**; **r**, radius vector of the electron; t, time; Z, charge of the ion nucleus; v_0, characteristic velocity of the atomic electron; $d\Omega$, solid angle; N_n, number density of nuclei in the plasma; N_i, number density of i-fold ions; $\hbar$, Planck constant; D, Debye radius; N_e^p, number density of free electrons of the plasma; N_e^i, number density of bound electrons of the i-fold ions; N_{0n}, oscillator strengths; E_0 and E_n, ion energies before and after the collision; Q, power density of the energy release ; X, depth of electron penetration into the plasma; α, angle of entry the beam electrons into the plasma; $E_{\parallel}$, longitudinal energy of the electrons of the beam; T_p, plasma temperature. Subscripts: en, electron scattering on a nucleus; i, ionic multiplicity; ee, electron scattering on an electron; f, fictitious scattering.

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