Quantum Electromagnetics: An Introduction

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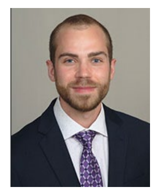

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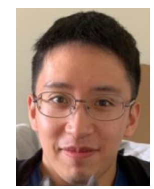

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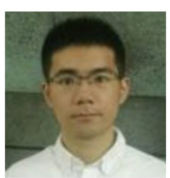

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Why Quantum Electromagnetics/Optics?

Importance of Quantum Electromagnetics
When wavelength much longer than atomic spacings—E.g. microwave photons (macroscopic quantum electromagnetics)

- Leaps and bounds progress of quantum technologies:
  - Quantum communications—security and encryption
  - Quantum computers—quantum parallelism
  - Quantum sensing—enhanced sensitivity
- These are going to be transformative technologies

Classical EM knowledge has been with us for over 150 years.
Wish for a gradual transition from classical to quantum!

Articulation, articulation, articulation!

Classical CEM and Quantum CEM

Both Classical and Quantum Maxwell's equations are derivable from energy conservation!

Classical Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$H = \int \left(\frac{\mathbf{D}^2}{\varepsilon} + \frac{\mathbf{B}^2}{\mu} \right) dV$$

Quantum Maxwell's equations

$$\nabla \times \hat{\mathbf{E}} = -\frac{\partial \hat{\mathbf{B}}}{\partial t}$$

$$\nabla \times \hat{\mathbf{H}} = \frac{\partial \hat{\mathbf{D}}}{\partial t} + \hat{\mathbf{J}}$$

$$\nabla \cdot \hat{\mathbf{D}} = \hat{\rho}$$

$$\nabla \cdot \hat{\mathbf{B}} = 0$$

Fields become quantum operators. $\hat{H} = \int \left(\frac{\hat{\mathbf{D}}^2}{\varepsilon} + \frac{\hat{\mathbf{B}}^2}{\mu} \right) dV$

An operator must act on a state vector (quantum state function).

$$\hat{H}|\psi\rangle = i\hbar\partial_t|\psi\rangle$$

Cohen-Tannoudji et al (1986), Mendel and Wolf (1995).

Quantization of EM field: Glauber (1991), Huttner-Barnett (1992), Milonni (1995), Welsch (1996), Suttorp and Wub (2004), Scheel and Buhmann (2008), Philbin (2010), A. Drezet (2017), Jauslin {2019).

Chew et al (2016).

Chew et al (2019).

Chew et al (Feb 2021, IEEE AP Magazine).

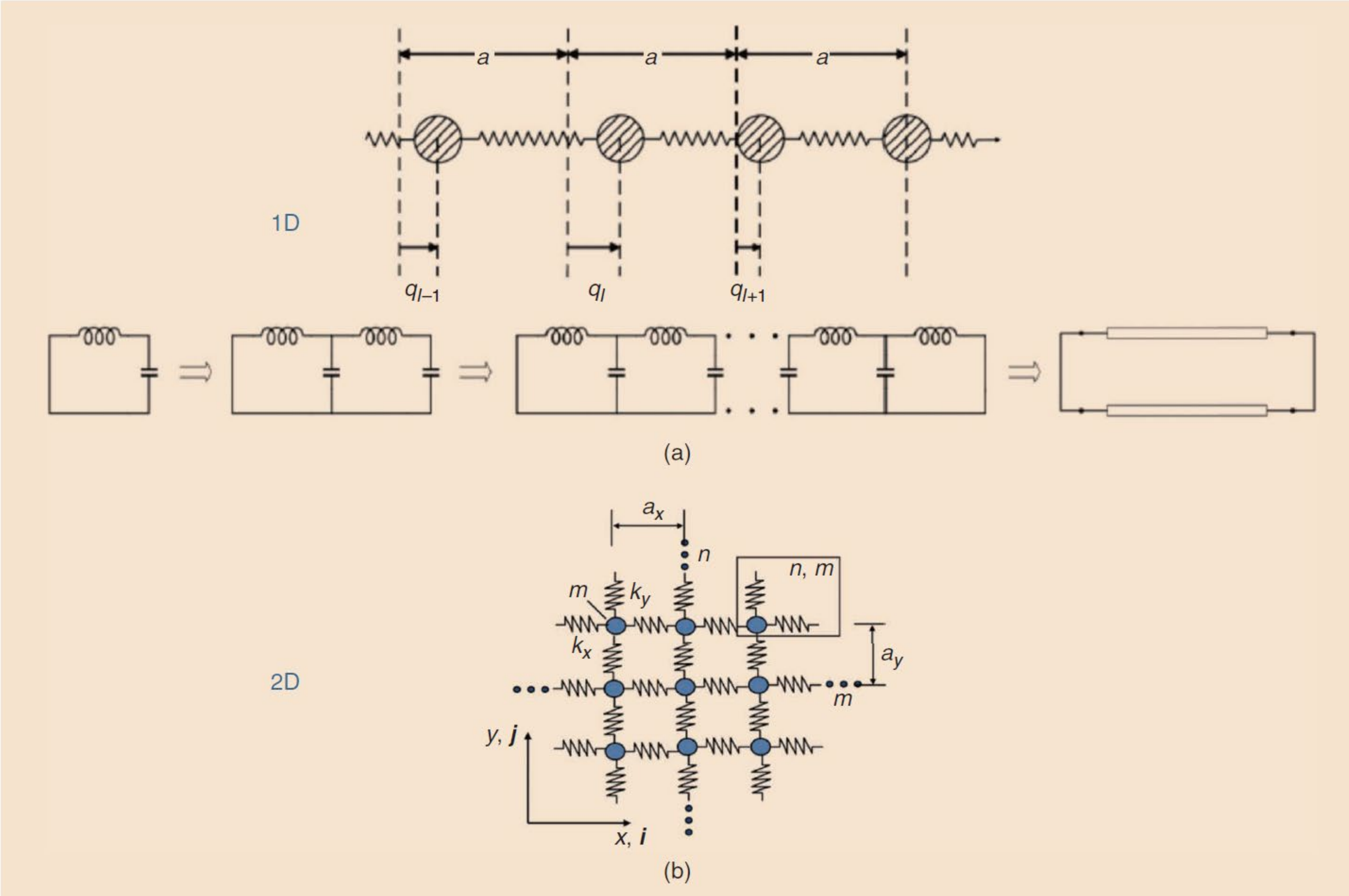


FIGURE 5. Maxwell's equations describe the coupling of harmonic oscillators in a 3D space. This figure illustrates waves propagating on (a) a string or a 1D transmission line, or (b) a 2D array of coupled oscillators. The sawtooth symbol in the figure represents a spring.

The Fundamental Building Block of EM Medium is a Dipole

Sir William Rowan Hamilton
(1805–1865)

Classical Dipole

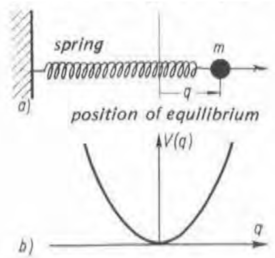


Fig. 10
The harmonic oscillator
a) example from mechanics
b) potential energy as a function of the displacement.

A Simple Classical Pendulum

Hooke's law, Newton's law

$$m\frac{d^2q}{dt^2} + \kappa q = 0$$

$$q = q_0 e^{-i\omega t}$$

$$-m\omega^2 q_0 + \kappa q_0 = 0$$

$$\omega = \omega_0$$

$$\omega_0 = \sqrt{\frac{\kappa}{m}}$$

Hamiltonian Theory

$$H = T + V$$

$$T = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

$$V = \frac{1}{2}\kappa q^2 = \frac{1}{2}m\omega_0^2 q^2$$

$$H = T + V = \frac{p^2}{2m} + \frac{1}{2}m\omega_0^2 q^2$$

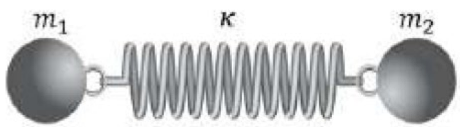
$$\frac{d}{dt}H\left(p(t), q(t)\right) = 0 = \frac{dp}{dt}\frac{\partial H}{\partial p} + \frac{dq}{dt}\frac{\partial H}{\partial q}$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q}, \qquad \frac{dq}{dt} = \frac{\partial H}{\partial p}$$

$$m\frac{d^2q}{dt^2} = -m\omega_0^2 q = -\kappa q$$

Wave-Particle Duality Needed (De Broglie, 1923)

Quantum Dipole



$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega_0^2 q^2 = E$$

$$\hat{p} = -i\hbar\frac{\partial}{\partial q} \qquad \hat{q} = \hat{I}q$$

$$\frac{\hat{p}^2}{2m}\psi(q,t) + \frac{1}{2}m\omega_0^2\hat{q}^2\psi(q,t) = i\hbar\frac{\partial}{\partial t}\psi(q,t)$$

$$\hat{H}\psi(q,t) = i\hbar\frac{\partial}{\partial t}\psi(q,t)$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0^2\hat{q}^2$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2}\psi(q,t) + \frac{1}{2}m\omega_0^2 q^2\psi(q,t) = i\hbar\frac{\partial}{\partial t}\psi(q,t)$$

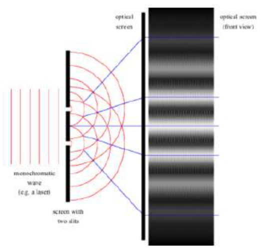
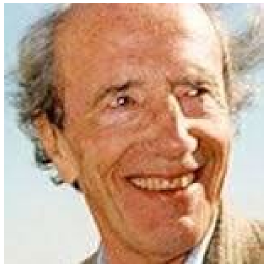


Figure 38.1: A Young's double-slit experiment. Again, the interference pattern reveals the wave nature of light (Figure By: https://www.shmoop.com).

Zen paradoxes

Solutions to Schrodinger Equation



Francis Low of MIT

using separation of variables

$$\psi(q,t) = \psi_n(q) e^{-i\omega_n t}$$

Eigenfunction

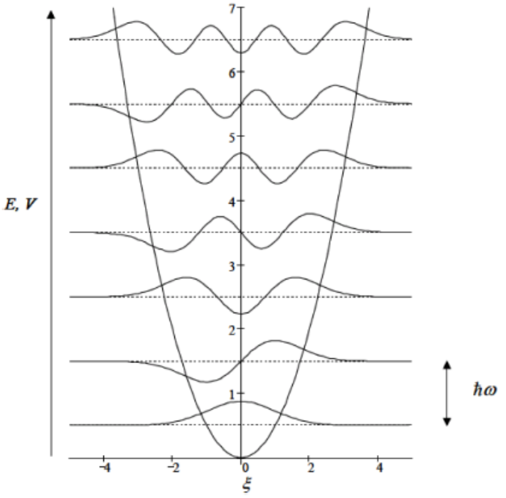
$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dq^2} + \frac{1}{2}m\omega_0^2 q^2\right]\psi_n(q) = E_n \psi_n(q)$$

$$E_n = \hbar\omega_n = \left(n + \frac{1}{2}\right)\hbar\omega_0$$

Photon Number State

Hermite-Gaussian functions (1864)

$$\psi_n(q) = \sqrt{\frac{1}{2^n n!}\sqrt{\frac{m\omega_0}{\pi\hbar}}} e^{-\frac{m\omega_0}{2\hbar}q^2} H_n\left(\sqrt{\frac{m\omega_0}{\hbar}}q\right)$$



using separation of variables

Number state or Fock state analogous to Fourier modes

Beautifying Schrödinger's Equation

$$\hat{H}\psi_n(q) = \left[-\frac{\hbar^2}{2m}\frac{d^2}{dq^2} + \frac{1}{2}m\omega_0^2 q^2\right]\psi_n(q) = E_n\psi_n(q)$$

$$\frac{1}{2}\left(-\frac{d^2}{d\xi^2} + \xi^2\right)\Psi_n(\xi) = \frac{E_n}{\hbar\omega_0}\Psi_n(\xi)$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{2}}\left(-\frac{d}{d\xi} + \xi\right), \qquad \hat{a} = \frac{1}{\sqrt{2}}\left(\frac{d}{d\xi} + \xi\right)$$

$$\hat{\xi} = \frac{1}{\sqrt{2}}\left(\hat{a}^\dagger + \hat{a}\right) = \xi\hat{I}$$

$$\hat{\pi} = \frac{i}{\sqrt{2}}\left(\hat{a}^\dagger - \hat{a}\right) = -i\frac{d}{d\xi}$$

$$\frac{1}{2}\left(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger\right)\Psi_n(\xi) = \left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)\Psi_n(\xi) = \frac{E_n}{\hbar\omega_0}\Psi_n(\xi)$$

$$\left[\hat{a}, \hat{a}^\dagger\right] = \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = \hat{I}$$

Annihilation and creation operators (ladder operators)

$$\hat{a}\Psi_n(\xi) = \sqrt{n}\Psi_{n-1}(\xi)$$
$$\hat{a}^\dagger\Psi_n(\xi) = \sqrt{n+1}\Psi_{n+1}(\xi)$$

$$\Psi_n(\xi) \to |\Psi_n\rangle$$

$$\hat{a}\Psi_n(\xi) = \sqrt{n}\Psi_{n-1}(\xi) \to \hat{a}|\Psi_n\rangle = \sqrt{n}|\Psi_{n-1}\rangle$$

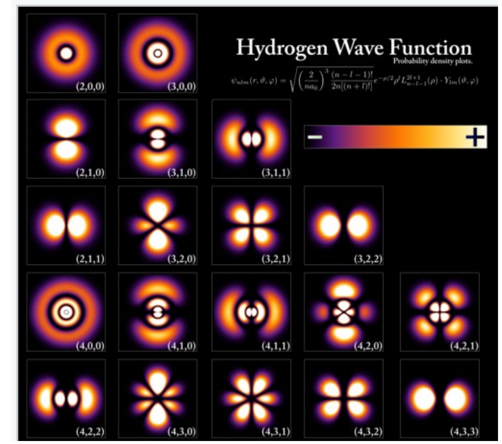
$$\hat{a}^\dagger\Psi_n(\xi) = \sqrt{n+1}\Psi_{n+1}(\xi) \to \hat{a}^\dagger|\Psi_n\rangle = \sqrt{n+1}|\Psi_{n+1}\rangle$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$
$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

Generalization to 3D (1925)

Schrodinger Equation in 3D (Hydrogen Atom)

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t)$$



Erwin Schrodinger

Max Born
(1954)

Physical Meaning of the Wave Function (Max Born)

$$\int_{-\infty}^{\infty} dq|\psi(q,t)|^2 = 1 \Leftrightarrow \underbrace{\langle\psi(t)|\psi(t)\rangle}_{\text{Dirac}} = 1 \Leftrightarrow \underbrace{\boldsymbol{\psi}(t)^\dagger \cdot \boldsymbol{\psi}(t)}_{\text{Matrix}} = 1$$

$$\hat{q} = \hat{I}q$$

$$\int_{-\infty}^{\infty} dqq|\psi(q,t)|^2 = \langle q(t)\rangle = \bar{q}(t) \Leftrightarrow \underbrace{\langle\psi(t)|\hat{q}|\psi(t)\rangle} \Leftrightarrow \underbrace{\boldsymbol{\psi}(t)^\dagger \cdot \overline{\mathbf{q}} \cdot \boldsymbol{\psi}(t)}$$

$$\hat{p} = -i\hbar\partial/(\partial q)$$

$$-i\hbar\int_{-\infty}^{\infty} dq\psi^*(q,t)\frac{\partial}{\partial q}\psi(q,t) = \langle p(t)\rangle = \bar{p}(t)$$

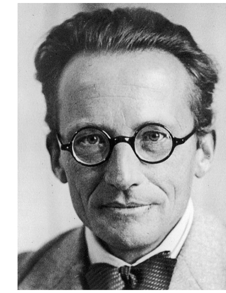
$$\Leftrightarrow \underbrace{\langle\psi(t)|\hat{p}|\psi(t)\rangle}_{\text{Dirac}} \Leftrightarrow \underbrace{\boldsymbol{\psi}(t)^\dagger \cdot \overline{\mathbf{p}} \cdot \boldsymbol{\psi}(t)}_{\text{Matrix}}$$

$$[\hat{p},\hat{q}] = \hat{p}\hat{q} - \hat{q}\hat{p} = -i\hbar\hat{I}$$

Heisenberg Picture versus Schrödinger Picture

$$\hat{H}|\Psi(t)\rangle = i\hbar\partial_t|\Psi(t)\rangle$$

$$|\Psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\Psi(0)\rangle$$



$$\bar{O}(t) = \langle\Psi(t)|\hat{O}_s(0)|\Psi(t)\rangle$$

Schrodinger Picture

$$\bar{O}(t) = \langle\Psi(0)|e^{i\hat{H}t/\hbar}\hat{O}_s(0)e^{-i\hat{H}t/\hbar}|\Psi(0)\rangle$$

$$\bar{O}(t) = \langle\Psi(0)|\hat{O}_h(t)|\Psi(0)\rangle$$

Heisenberg Picture

$$\hat{O}_h(t) = e^{i\hat{H}t/\hbar}\hat{O}_s(0)e^{-i\hat{H}t/\hbar}$$

Equations of Motion of Operators in Heisenberg Picture

$$\frac{d\hat{O}_h}{dt} = \frac{i}{\hbar}\left(\hat{H}\hat{O}_h - \hat{O}_h\hat{H}\right) = \frac{i}{\hbar}\left[\hat{H}, \hat{O}_h\right]$$

$$\frac{d\hat{p}}{dt} = \frac{i}{\hbar}\left[\hat{H}, \hat{p}\right], \qquad \frac{d\hat{q}}{dt} = \frac{i}{\hbar}\left[\hat{H}, \hat{q}\right]$$

Derivation of Quantum Hamilton Equations of Motion

Repeated application of commutation relation:

$$[\hat{p}, \hat{q}] = \hat{p}\hat{q} - \hat{q}\hat{p} = -i\hbar\hat{I}$$

$$[\hat{p}, \hat{q}^n] = -in\hat{q}^{n-1}\hbar.$$

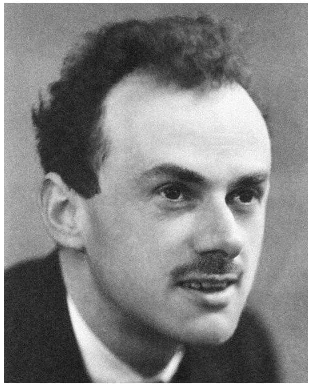
$$\frac{d\hat{p}}{dt} = -\frac{\partial \hat{H}(\hat{p}, \hat{q})}{\partial \hat{q}}, \qquad \frac{d\hat{q}}{dt} = \frac{\partial \hat{H}(\hat{p}, \hat{q})}{\partial \hat{p}}$$

Masood Nekoei

Luis Gomez

are tasked with providing a simpler derivation

Random Nature of Observables



Paul Dirac

- When dynamic conjugate variables are elevated to become quantum operators, the variables become a random variables, called quantum observables.

$$p, q \Rightarrow \hat{p}, \hat{q} \qquad \hat{p} = -i\hbar\frac{\partial}{\partial q}, \qquad \hat{q} = \hat{I}q$$

$$p \Leftrightarrow \{\hat{p}, \psi(q,t)\}\,, \qquad q \Leftrightarrow \{\hat{q}, \psi(q,t)\}$$

p and q are deterministic variables in the classical world, but random variables in the quantum world.

Source of Uncertainty Principle

Rooted in the non-commuting algebra of the operators:

$$[\hat{q}, \hat{p}] = i\hbar\hat{I}, \qquad [\hat{q}, \hat{p}] = 0, \quad \text{when} \quad \hbar \to 0$$

Correspondence principle

When two operators commute, they are "homomorphic" to scalar numbers,

$$\hat{p} = p\hat{I}, \qquad \hat{q} = q\hat{I}$$

Quantum theory becomes classical theory.

Random Nature of Observables

Entanglement-Based Quantum Radar: From Myth to Reality

David Luong, Defence Research and Development Canada; Carleton University
Sreeraman Rajan, Carleton University
Bhashyam Balaji, Defence Research and Development Canada

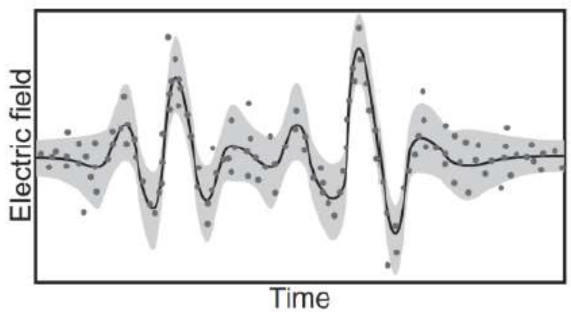


Figure 38.4: Schematic representation of the randomness of the measured electric field. The electric field amplitude, analogous to the amplitude of the quantum pendulum, is a quantum observable, and hence, is a random variable. The electric field amplitude maps to the displacement (position) of the quantum pendulum, which is also a random variable and a quantum observable. (Figure By Kira and Koch [345]).

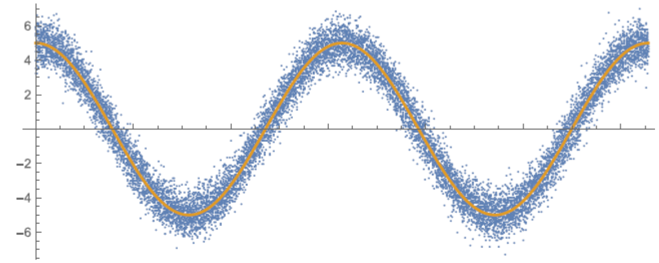


Figure 1.
Solid orange curve is a sinusoidal waveform as a function of time (or, equivalently, of phase). If classical physics were true, a measurement of the waveform under theoretically ideal conditions would give the solid curve exactly. Quantum physics predicts that there will *always* be noise in the measurement result. The blue dots depict what the measurement result would look like according to quantum theory, again under ideal conditions. Units are arbitrary.

In Summary:
Two Ways to Derive Quantum Hamilton Equations

$$\boxed{\frac{d\hat{p}}{dt} = -\frac{\partial \hat{H}(\hat{p},\hat{q})}{\partial \hat{q}}, \qquad \frac{d\hat{q}}{dt} = \frac{\partial \hat{H}(\hat{p},\hat{q})}{\partial \hat{p}}}$$

Quantum Hamilton equations

$$[\hat{q},\hat{p}] = i\hbar\hat{I}, \qquad [\hat{q},\hat{p}] = 0, \quad \text{when} \quad \hbar \to 0$$

Correspondence principle

- Repeated use of the commutator and Heisenberg equations of motion;
- Energy conservation; Chew, W. C., Liu, A. Y., Salazar-Lazaro, C., Na, D. Y., & Sha, W. E. (2019). Hamilton equations, commutator, and energy conservation. *Quantum Reports*, *1*(2), 295-303.

The above is “homomorphic” to the classical Hamilton equations

$$\boxed{\frac{dp}{dt} = -\frac{\partial H}{\partial q}, \qquad \frac{dq}{dt} = \frac{\partial H}{\partial p}}$$

Classical Hamilton equations
No derivative wrt to operators

“Homomorphism” between Classical and Quantum

Hamiltonian—Classical Picture

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q}, \qquad \frac{dq}{dt} = \frac{\partial H}{\partial p}$$

$$H = \frac{1}{2}\int_V d\mathbf{r}\left[\varepsilon \mathbf{E}^2(\mathbf{r},t) + \frac{\mathbf{B}^2(\mathbf{r},t)}{\mu}\right].$$

$$\nabla \cdot \mathbf{A} = 0 \text{ with } \Phi = 0$$

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{E} = -\dot{\mathbf{A}}$$

$$H = \frac{1}{2}\int_V d\mathbf{r}\left[\varepsilon \dot{\mathbf{A}}^2(\mathbf{r},t) + \frac{(\nabla \times \mathbf{A}(\mathbf{r},t))^2}{\mu}\right]$$

$$\mathbf{\Pi} = \varepsilon \dot{\mathbf{A}}$$

$$H = \frac{1}{2}\int_V d\mathbf{r}\left[\frac{\mathbf{\Pi}^2(\mathbf{r},t)}{\varepsilon} + \frac{(\nabla \times \mathbf{A}(\mathbf{r},t))^2}{\mu}\right]$$

Generalization of Hamilton Equations to the Continuum Case

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q}, \qquad \frac{dq}{dt} = \frac{\partial H}{\partial p}$$

$$\frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t} = \frac{\delta H}{\delta \mathbf{\Pi}(\mathbf{r},t)}, \qquad \frac{\partial \mathbf{\Pi}(\mathbf{r},t)}{\partial t} = -\frac{\delta H}{\delta \mathbf{A}(\mathbf{r},t)}$$

$$\frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t} = \frac{\mathbf{\Pi}(\mathbf{r},t)}{\varepsilon}, \qquad \frac{\partial \mathbf{\Pi}(\mathbf{r},t)}{\partial t} = -\frac{1}{\mu}\nabla \times \nabla \times \mathbf{A}(\mathbf{r},t)$$

$$\mu\varepsilon\partial_t^2 \mathbf{A}(\mathbf{r},t) + \nabla \times \nabla \times \mathbf{A}(\mathbf{r},t) = 0$$

$$\mathbf{E} = -\dot{\mathbf{A}} \qquad \mu\mathbf{H} = \nabla \times \mathbf{A},$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \varepsilon\partial_t \mathbf{E}(\mathbf{r},t)$$
$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\mu\partial_t \mathbf{H}(\mathbf{r},t)$$

Chew, W. C., Liu, A. Y., Salazar-Lazaro, C., & Sha, W. E. (2016). Quantum electromagnetics: A new look—Part I. *IEEE Journal on Multiscale and Multiphysics Computational Techniques*, *1*, 73-84.

Chew, W. C., Liu, A. Y., Salazar-Lazaro, C., & Sha, W. E. (2016). Quantum electromagnetics: A new look—Part II. *IEEE Journal on Multiscale and Multiphysics Computational Techniques*, *1*, 85-97.

Hamiltonian—Quantum Picture

$$\hat{H} = \frac{1}{2}\int_V d\mathbf{r}\left[\varepsilon\hat{\mathbf{E}}^2(\mathbf{r},t) + \frac{\hat{\mathbf{B}}^2(\mathbf{r},t)}{\mu}\right] = \frac{1}{2}\int_V d\mathbf{r}\left[\frac{\hat{\mathbf{\Pi}}^2(\mathbf{r},t)}{\varepsilon} + \frac{1}{\mu}\left(\nabla\times\hat{\mathbf{A}}(\mathbf{r},t)\right)^2\right]$$

$$\hat{\mathbf{\Pi}}(\mathbf{r},t) \;=\; \varepsilon\dot{\hat{\mathbf{A}}}(\mathbf{r},t)$$

$$\frac{d\hat{p}}{dt} = -\frac{\partial\hat{H}(\hat{p},\hat{q})}{\partial\hat{q}}, \qquad \frac{d\hat{q}}{dt} = \frac{\partial\hat{H}(\hat{p},\hat{q})}{\partial\hat{p}}$$

$$\left[\hat{\mathbf{\Pi}}(\mathbf{r}',t),\hat{H}\right] = -i\hbar\frac{\delta\hat{H}}{\delta\hat{\mathbf{A}}(\mathbf{r}',t)} = i\hbar\partial_t\hat{\mathbf{\Pi}}(\mathbf{r}',t)$$

$$\left[\hat{\mathbf{A}}(\mathbf{r}',t),\hat{H}\right] = i\hbar\frac{\delta H}{\delta\hat{\mathbf{\Pi}}(\mathbf{r}',t)} = i\hbar\partial_t\hat{\mathbf{A}}(\mathbf{r}',t)$$

$$\frac{\partial\hat{\mathbf{A}}(\mathbf{r},t)}{\partial t} = \frac{\hat{\mathbf{\Pi}}(\mathbf{r},t)}{\varepsilon}, \qquad \frac{\partial\hat{\mathbf{\Pi}}(\mathbf{r},t)}{\partial t} = -\frac{1}{\mu}\nabla\times\nabla\times\hat{\mathbf{A}}(\mathbf{r},t)$$

$$\mu\varepsilon\partial_t^2\hat{\mathbf{A}}(\mathbf{r},t) + \nabla\times\nabla\times\hat{\mathbf{A}}(\mathbf{r},t) = 0$$

$$\nabla \times \hat{\mathbf{H}}(\mathbf{r},t) = \varepsilon \partial_t \hat{\mathbf{E}}(\mathbf{r},t)$$
$$\nabla \times \hat{\mathbf{E}}(\mathbf{r},t) = -\mu \partial_t \hat{\mathbf{H}}(\mathbf{r},t)$$

$$\hat{H}|\Psi(t)\rangle = i\hbar \frac{\partial |\Psi(t)\rangle}{\partial t}$$

$$\nabla \times \hat{\mathbf{H}}(\mathbf{r},t)|\Psi(0)\rangle = \varepsilon \partial_t \hat{\mathbf{E}}(\mathbf{r},t)|\Psi(0)\rangle$$
$$\nabla \times \hat{\mathbf{E}}(\mathbf{r},t)|\Psi(0)\rangle = -\mu \partial_t \hat{\mathbf{H}}(\mathbf{r},t)|\Psi(0)\rangle$$

The Tenet of Quantum Interpretations and Measurements

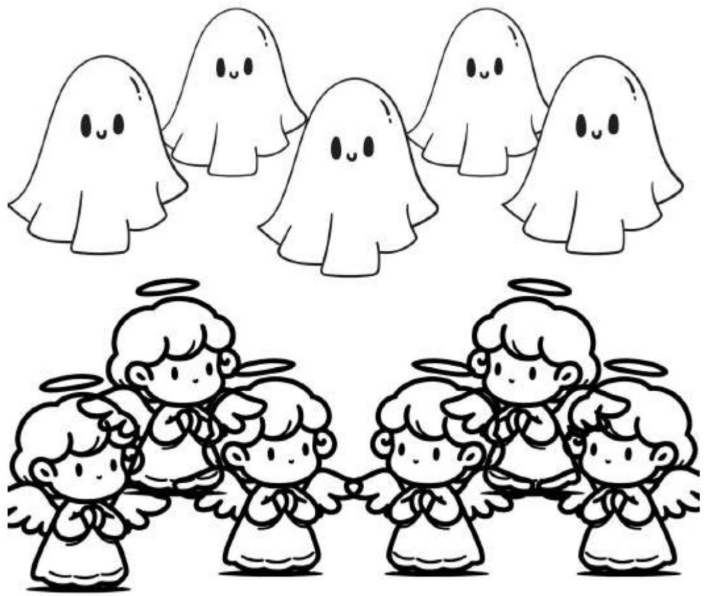
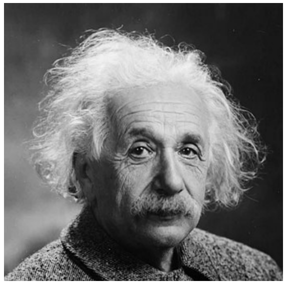


Figure 38.6: The quantum linear superposition of states in the quantum world can be interpreted as "ghosts-and-angels". Before the measurement, we do not know the quantum state of the quantum system. After the measurement, the quantum state is said to have collapsed to the state that has been measured.

Min Chen of MIT

The Tenet of Quantum Measurement, contd.

- A quantum system is in a linear superposition of states before a measurement;
- A measurement collapses a quantum system to one state, the one that is measured;
- It is unknown which state a quantum system will collapse to before the measurement.
  - **The almighty is still involved.**



Albert Einstein

Niels Bohr

Alain Aspect

Classical Hamiltonian

$$H = \frac{1}{2}\int\limits_V d\mathbf{r}\left[\varepsilon\mathbf{E}^2(\mathbf{r},t) + \frac{\mathbf{B}^2(\mathbf{r},t)}{\mu}\right]$$

$$\mathbf{B} = \nabla\times\mathbf{A} \text{ and } \mathbf{E} = -\dot{\mathbf{A}},$$

$$H = \frac{1}{2}\int\limits_V d\mathbf{r}\left[\varepsilon\dot{\mathbf{A}}^2(\mathbf{r},t) + \frac{\left(\nabla\times\mathbf{A}(\mathbf{r},t)\right)^2}{\mu}\right].$$

$$\mathbf{\Pi} = \varepsilon\dot{\mathbf{A}}$$

$$H = \frac{1}{2}\int\limits_V d\mathbf{r}\left[\frac{\mathbf{\Pi}^2(\mathbf{r},t)}{\varepsilon} + \frac{\left(\nabla\times\mathbf{A}(\mathbf{r},t)\right)^2}{\mu}\right]$$

Simple Monochromatic Plane Wave

$$A_{x,l}(z,t) = \frac{1}{2} A_l e^{-i\omega_l t + i k_l z} + c.c.$$

$$H = \frac{1}{2} \int_V d\mathbf{r} \left[\varepsilon \dot{\mathbf{A}}^2(\mathbf{r},t) + \frac{(\nabla \times \mathbf{A}(\mathbf{r},t))^2}{\mu} \right].$$

$$H_l = \frac{V\varepsilon}{2} \omega_l^2 \left| \frac{A_l}{2} \right|^2 + \frac{V}{2\mu} k_l^2 \left| \frac{A_l}{2} \right|^2 = V\varepsilon\omega_l^2 \left| \frac{A_l}{2} \right|^2$$

$$H_l = n_l \hbar \omega_l$$

Photon-Carrying Plane Wave–Quantum Case

$$\hat{H} = \frac{1}{2}\int\limits_V d\mathbf{r}\left[\varepsilon\hat{\mathbf{E}}^2(\mathbf{r},t) + \frac{\hat{\mathbf{B}}^2(\mathbf{r},t)}{\mu}\right] = \frac{1}{2}\int\limits_V d\mathbf{r}\left[\frac{\hat{\mathbf{\Pi}}^2(\mathbf{r},t)}{\varepsilon} + \frac{1}{\mu}\left(\nabla\times\hat{\mathbf{A}}(\mathbf{r},t)\right)^2\right]$$

$$\hat{\mathbf{A}}_l(z,t) = \mathbf{a}_x\hat{A}_{x,l}(z,t) = \mathbf{a}_x\frac{1}{2}\hat{A}_l e^{-i\omega_l t + ik_l z} + h.c.$$

$$\hat{H}_l = \frac{1}{4}V\varepsilon\omega_l^2\left(\tfrac{1}{2}\sqrt{V\varepsilon\omega_l^2}\hat{A}_l = \sqrt{\tfrac{\hbar\omega_l}{2}}\hat{a}_l\ l\right)$$

$$\frac{1}{2}\sqrt{V\varepsilon\omega_l^2}\hat{A}_l = \sqrt{\frac{\hbar\omega_l}{2}}\hat{a}_l$$

$$\hat{H}_l = \frac{1}{2}\hbar\omega_l\left(\hat{a}_l\hat{a}_l^\dagger + \hat{a}_l^\dagger\hat{a}_l\right) = \hbar\omega_l\left(\hat{a}_l^\dagger\hat{a}_l + \frac{1}{2}\right)$$

Need to solve the quantum state equation

$$\hat{H}|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

$$\hat{H}_l|\psi_l(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi_l(t)\rangle$$

$$|\psi_l(t)\rangle = |\Psi_l\rangle e^{-i\omega_l t}$$

$$\hat{H}_l|\Psi_l\rangle = \hbar\omega_l|\Psi_l\rangle$$

$$\hat{H}_l = \frac{1}{2}\hbar\omega_l\left(\hat{a}_l\hat{a}_l^\dagger + \hat{a}_l^\dagger\hat{a}_l\right) = \hbar\omega_l\left(\hat{a}_l^\dagger\hat{a}_l + \frac{1}{2}\right)$$

$$\psi_n(q) = \sqrt{\frac{1}{2^n n!}\sqrt{\frac{m\omega_0}{\pi\hbar}}}\, e^{-\frac{m\omega_0}{2\hbar}q^2} H_n\left(\sqrt{\frac{m\omega_0}{\hbar}}q\right)$$

Monochromatic Photon Case (Unphysical)

$$\hat{A}_{x,l}(z,t) = \sqrt{\frac{\hbar}{2V\varepsilon\omega_l}}\hat{a}_l e^{-i\omega_l t + ik_l z} + h.c.$$

$$\hat{A}_{x,l}(z,t)|\Psi_l\rangle = \sqrt{\frac{\hbar}{2V\varepsilon\omega_l}}\hat{a}_l e^{-i\omega_l t + ik_l z}|\Psi_l\rangle + \sqrt{\frac{\hbar}{2V\varepsilon\omega_l}}\hat{a}_l^\dagger e^{i\omega_l t - ik_l z}|\Psi_l\rangle$$

Using CEM to find a general solution

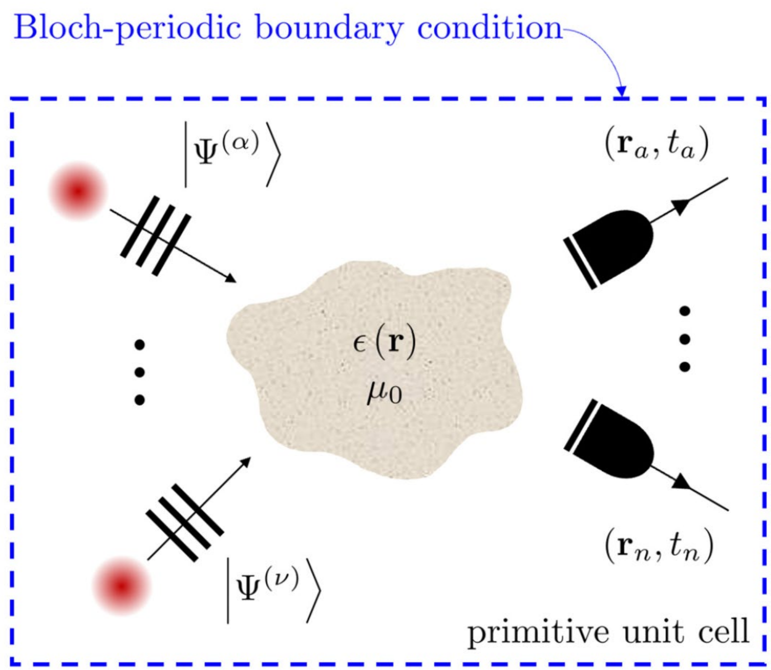


FIG. 1. Two-dimensional illustration of arbitrary passive and lossless quantum-optical systems driven by multiple photons. The systems can be modeled by inhomogeneous dielectric media. The Bloch-periodic boundary condition can be used to extract traveling-wave normal modes and retain the Hermiticity of quantum Maxwell's equations.

Periodic Inhomogeneous Media

$$\epsilon(\mathbf{r}) = \epsilon(\mathbf{r} + \mathbf{R}),$$

$$\hat{\mathbf{A}}(\mathbf{r}, t) = \hat{\mathbf{A}}^{(+)}(\mathbf{r}, t) + \hat{\mathbf{A}}^{(-)}(\mathbf{r}, t),$$

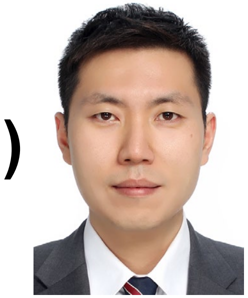
$$\hat{\mathbf{A}}^{(+)}(\mathbf{r}, t) = \sum_p \sqrt{\frac{\hbar}{2\omega_{\kappa_p}}} \mathbf{A}_{\kappa_p}(\mathbf{r}) \hat{a}_{\kappa_p}(t),$$

$$\hat{\mathbf{A}}^{(-)}(\mathbf{r}, t) = \sum_p \sqrt{\frac{\hbar}{2\omega_{\kappa_p}}} \mathbf{A}^*_{\kappa_p}(\mathbf{r}) \hat{a}^\dagger_{\kappa_p}(t)$$

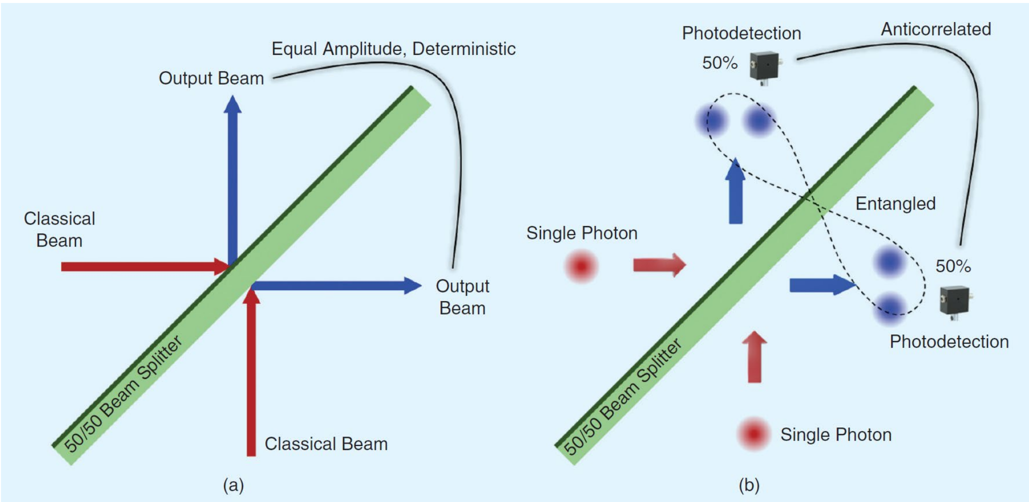
$$\left[\hat{a}_{\kappa_p}(t), \hat{a}^\dagger_{\kappa_{p'}}(t)\right] = \delta_{\kappa_p, \kappa_{p'}},$$

$$\left[\hat{a}_{\kappa_p}(t), \hat{a}_{\kappa_{p'}}(t)\right] = \left[\hat{a}^\dagger_{\kappa_p}(t), \hat{a}^\dagger_{\kappa_{p'}}(t)\right] = 0.$$

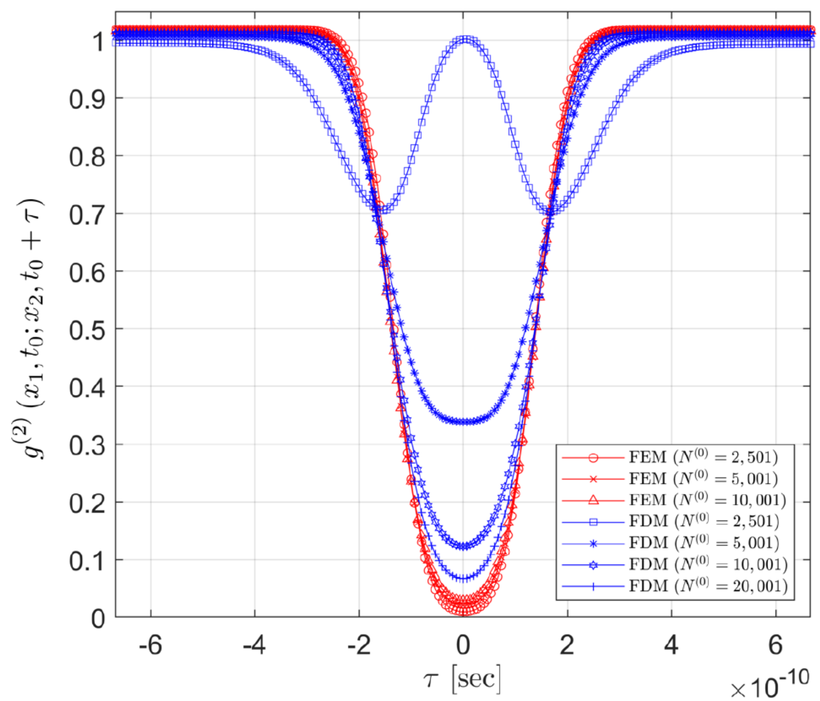
A Quantum Beam Splitter Can Be Modeled Using Mode Decomposition (Bloch-Floquet Modes)



Dr Dong-Yeop Na



$$A = \left\langle \Psi^{(2)} \middle| \hat{A}^{(-)}(x_1, t_0)\, \hat{A}^{(-)}(x_2, t_0 + \tau) \right.$$
$$\left. \times \hat{A}^{(+)}(x_2, t_0 + \tau)\, \hat{A}^{(+)}(x_1, t_0) \middle| \Psi^{(2)} \right\rangle,$$
$$B_1 = \left\langle \Psi^{(2)} \middle| \hat{A}^{(-)}(x_2, t_0)\, \hat{A}^{(+)}(x_2, t_0) \middle| \Psi^{(2)} \right\rangle,$$
$$B_2 = \left\langle \Psi^{(2)} \middle| \hat{A}^{(-)}(x_2, t_0 + \tau)\, \hat{A}^{(+)}(x_2, t_0 + \tau) \middle| \Psi^{(2)} \right\rangle,$$
$$g^{(2)}(x_1, t_0; x_2, t_0 + \tau) = \frac{A}{B_1 B_2}$$



Hong-Ou-Mandel Effect

Conclusions

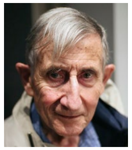
- A brief overview of quantum theory and quantum electromagnetics is given here;
- Much deep contemplation is needed to have myriad of concepts sink into your soul;
- Many out-of-box thinkings are needed to learn quantum theory and quantum electromagnetics;
- Since the operator-vector pair can be regarded as infinite dimensional, they have much more information content than classical systems;
- Quantum “weirdness” should be exploited in future quantum technologies;
- Example of such “weirdness” are linear superposition, entanglement, uncertainty principle, wave-particle duality, squeezed state, and quantum tunnelling.
- These properties cannot be described by the classical theory.

Final Words of Wisdom

The best way to win a war is to have no war
---Sun Zi, 500BC

Use ICT (Internet Communication Technologies) to bring about a more peaceful world!

- Advancing Technology for Humanity—Leah Jamieson (1949-) (Former Purdue Dean of Engineering, IEEE President).



- “Technology is a gift of God. After the gift of life, it is perhaps the greatest of God’s gifts. It is the mother of civilizations, of arts and of sciences.”—Freeman Dyson (1923-2020).
- All Humans Can Be Taught—Confucius (450BC).

- Love your enemy—Buddha (500BC) and Jesus Christ (20 AD).

- Our human mind is the greatest gift of God!
- Peace to this world. Collaborate to solve our global warming problems and save our planet Earth!