PURDUE ENGINEERING

CE 59700: Digital Photogrammetric Systems

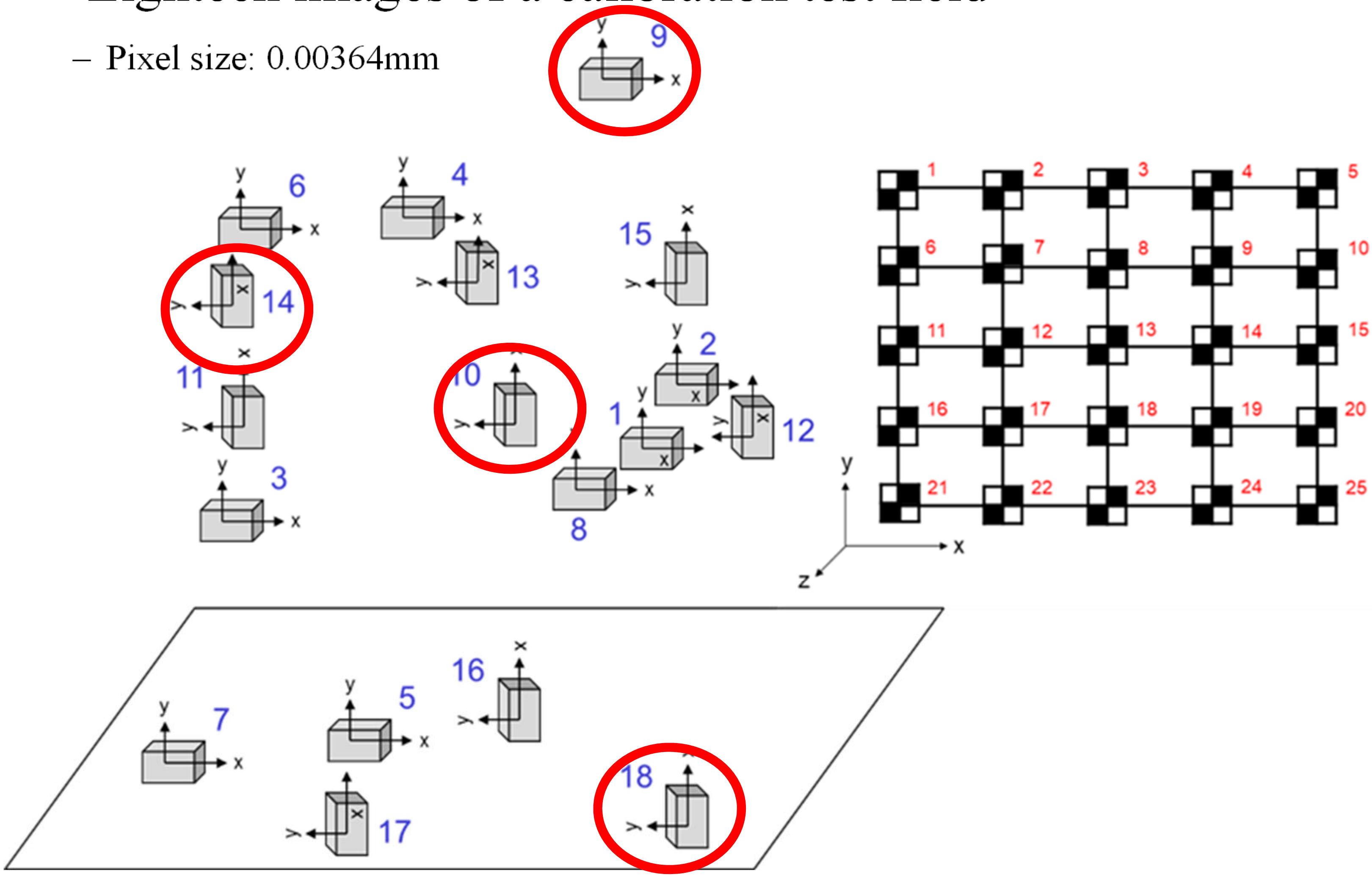
Lab2: Single Photo Resection

Objectives

- Determine the Exterior Orientation Parameters (EOP) of a single photo using least squares adjustment procedure.

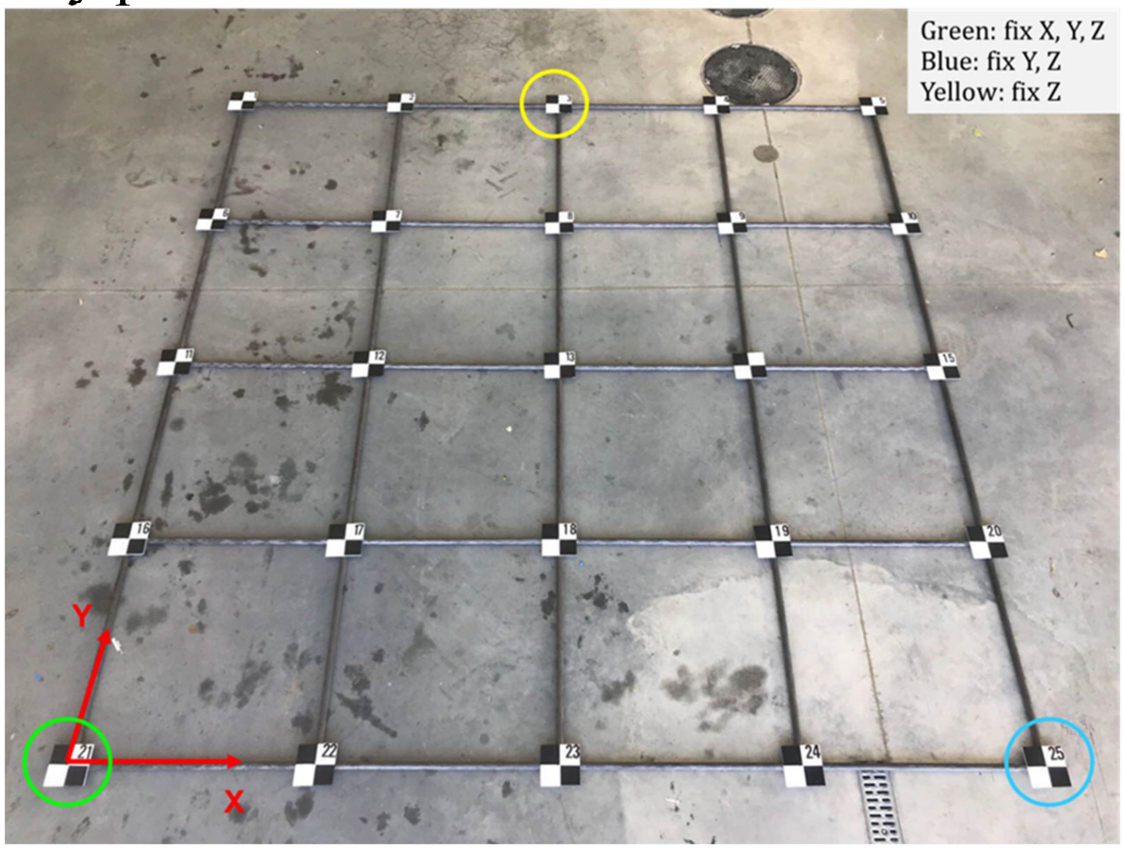
Given

- Eighteen images of a calibration test field
  - Pixel size: 0.00364mm



Given

- Necessary parameters for datum definition



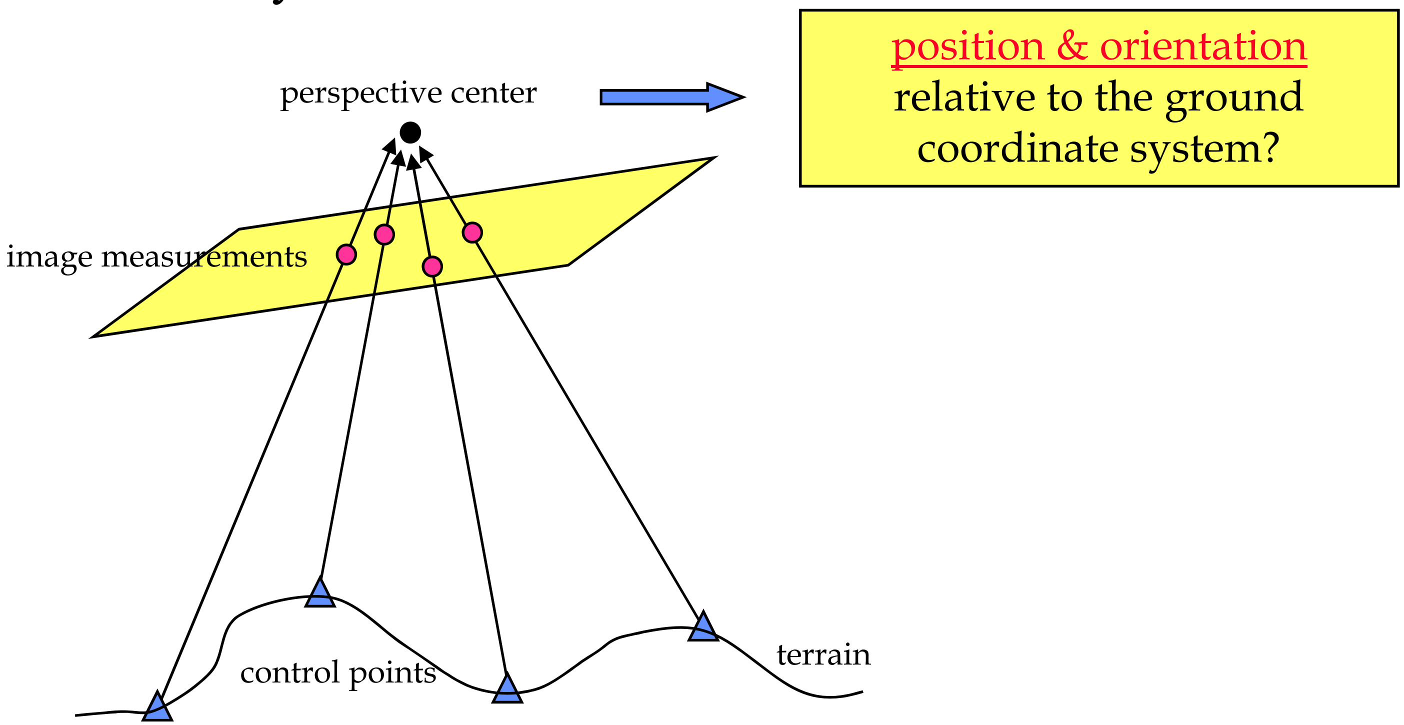
Fixed coordinates are shown.
Some distances have been also measured.

Given

- Image coordinates of the calibration targets as measured/provided in Lab 1 for images 65_09_20180803.jpg, 65_10_20180803.jpg, 65_14_20180803.jpg, and 65_18_20180803.jpg;
- Estimated IOP from lab # 1 (principal point coordinates, principal distance, and lens distortion parameters); and
- Estimated ground coordinates of the different targets in the calibration test field.

Single Photo Resection (SPR)

- The objective of single photo resection is to determine the position of the perspective center and the orientation of the image coordinate system (i.e., the EOP for a given image) relative to the ground coordinate system.



Step1: Math. Model (Collinearity Equations)

$$x = x_p - c\frac{r_{11}\cdot(X - X_o) + r_{21}\cdot(Y - Y_o) + r_{31}\cdot(Z - Z_o)}{r_{13}\cdot(X - X_o) + r_{23}\cdot(Y - Y_o) + r_{33}\cdot(Z - Z_o)} + dist_x = x_p - c\,\frac{N_x}{D} + dist_x$$

$$y = y_p - c\frac{r_{12}\cdot(X - X_o) + r_{22}\cdot(Y - Y_o) + r_{32}\cdot(Z - Z_o)}{r_{13}\cdot(X - X_o) + r_{23}\cdot(Y - Y_o) + r_{33}\cdot(Z - Z_o)} + dist_y = y_p - c\,\frac{N_y}{D} + dist_y$$

$$N_x = r_{11}\cdot(X - X_o) + r_{21}\cdot(Y - Y_o) + r_{31}\cdot(Z - Z_o)$$

$$N_y = r_{12}\cdot(X - X_o) + r_{22}\cdot(Y - Y_o) + r_{32}\cdot(Z - Z_o)$$

$$D = r_{13}\cdot(X - X_o) + r_{23}\cdot(Y - Y_o) + r_{33}\cdot(Z - Z_o)$$

What are the known and unknown quantities?

Known/Observed Quantities:

1. image measurements
2. interior orientation parameters
3. ground coordinates of control points

Unknown Quantities:

1. position of perspective center
2. orientation of image coordinate system

Step2: Linearization of Collinearity Equations

To implement a least square adjustment procedure

$$x = x_p - c\frac{r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)} + dist_x = x_p - c\,\frac{N_x}{D} + dist_x$$

$$y = y_p - c\frac{r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)} + dist_y = y_p - c\,\frac{N_y}{D} + dist_y$$

$$N_x = r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)$$

$$N_y = r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)$$

$$D = r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)$$

Linearization →

Why?

How?

<u>Non linear model</u> with respect to the unknown parameters

Taylor's theorem

Step2: Linearization of Collinearity Equations

$$x = x_p - c\frac{r_{11}\cdot(X-X_o)+r_{21}\cdot(Y-Y_o)+r_{31}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)} + dist_x = x_p - c\frac{N_x}{D} + dist_x$$

$$y = y_p - c\frac{r_{12}\cdot(X-X_o)+r_{22}\cdot(Y-Y_o)+r_{32}\cdot(Z-Z_o)}{r_{13}\cdot(X-X_o)+r_{23}\cdot(Y-Y_o)+r_{33}\cdot(Z-Z_o)} + dist_y = y_p - c\frac{N_y}{D} + dist_y$$

Linearization

$$x = x_o + \left(\frac{\partial x}{\partial X_o}\right)_o dX_o + \left(\frac{\partial x}{\partial Y_o}\right)_o dY_o + \left(\frac{\partial x}{\partial Z_o}\right)_o dZ_o + \left(\frac{\partial x}{\partial \omega}\right)_o d\omega + \left(\frac{\partial x}{\partial \phi}\right)_o d\phi + \left(\frac{\partial x}{\partial \kappa}\right)_o d\kappa$$

$$y = y_o + \left(\frac{\partial y}{\partial X_o}\right)_o dX_o + \left(\frac{\partial y}{\partial Y_o}\right)_o dY_o + \left(\frac{\partial y}{\partial Z_o}\right)_o dZ_o + \left(\frac{\partial y}{\partial \omega}\right)_o d\omega + \left(\frac{\partial y}{\partial \phi}\right)_o d\phi + \left(\frac{\partial y}{\partial \kappa}\right)_o d\kappa$$

1. *xo* and *yo* are the evaluated *x* and *y* image coordinates using the initial approximations of the unknowns parameters.
2. $\left(\frac{\partial x}{\partial X_o}\right)_o, \left(\frac{\partial x}{\partial Y_o}\right)_o$… etc., are the partial derivatives of *x* and *y* with respect to the unknowns evaluated at the initial approximations of these parameters.
3. dX_o, dY_o … etc., are the unknown corrections to be applied to the initial approximations.

Step3: Initialization of EOPs

- Use the sketch in lab # 1 (the one showing the approximate location and orientation of the different images relative to the calibration test field coordinate system) to come up with approximations for the EOP of the different images.

Step4: Least Squares Adjustment Procedure

$$a_1 = \left(\frac{\partial x}{\partial X_o}\right)_o, a_2 = \left(\frac{\partial x}{\partial Y_o}\right)_o, a_3 = \left(\frac{\partial x}{\partial Z_o}\right)_o, a_4 = \left(\frac{\partial x}{\partial \omega}\right)_o, a_5 = \left(\frac{\partial x}{\partial \phi}\right)_o, a_6 = \left(\frac{\partial x}{\partial \kappa}\right)_o$$

$$b_1 = \left(\frac{\partial y}{\partial X_o}\right)_o, b_2 = \left(\frac{\partial y}{\partial Y_o}\right)_o, b_3 = \left(\frac{\partial y}{\partial Z_o}\right)_o, b_4 = \left(\frac{\partial y}{\partial \omega}\right)_o, b_5 = \left(\frac{\partial y}{\partial \phi}\right)_o, b_6 = \left(\frac{\partial y}{\partial \kappa}\right)_o$$

$$x - x_o = a_1\, dX_o + a_2\, dY_o + a_3\, dZ_o + a_4\, d\omega + a_5\, d\phi + a_6\, d\kappa$$

$$y - y_o = b_1\, dX_o + b_2\, dY_o + b_3\, dZ_o + b_4\, d\omega + b_5\, d\phi + b_6\, d\kappa$$

⇩

$$y = \underbrace{\begin{bmatrix} x_1 - x_{1_o} \\ y_1 - y_{1_o} \\ x_2 - x_{2_o} \\ y_2 - y_{2_o} \\ \vdots \\ \vdots \\ x_{n/2} - x_{n/2_o} \\ y_{n/2} - y_{n/2_o} \end{bmatrix}_{n\times 1}}_{Y} = \underbrace{\begin{bmatrix} a_{1_1} & a_{2_1} & a_{3_1} & a_{4_1} & a_{5_1} & a_{6_1} \\ b_{1_1} & b_{2_1} & b_{3_1} & b_{4_1} & b_{5_1} & b_{6_1} \\ a_{1_2} & a_{2_2} & a_{3_2} & a_{4_2} & a_{5_2} & a_{6_2} \\ b_{1_2} & b_{2_2} & b_{3_2} & b_{4_2} & b_{5_2} & b_{6_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}_{n\times 6}}_{A} \cdot \underbrace{\begin{bmatrix} dX_o \\ dY_o \\ dZ_o \\ d\omega \\ d\phi \\ d\kappa \end{bmatrix}_{6\times 1}}_{X} + \underbrace{\begin{bmatrix} e_{x_1} \\ e_{y_1} \\ e_{x_2} \\ e_{y_2} \\ \vdots \\ \vdots \\ e_{x_{n/2}} \\ e_{y_{n/2}} \end{bmatrix}_{n\times 1}}_{e}$$

Step4: Least Squares Adjustment Procedure

```
Starting from initial values for the 6 exterior parameters;
while(1)
        prepare Y and A matrices using initial values;
        solve for X_hat using the Y and A matrices;
        update initial values using X_hat;

        If(# interaction > 100) or (|X_hat| < 1.0e-12)
                stop iteration;
        end
end
```

- compute final values for the exterior orientation parameters;
- compute the variance component;
- compute the variance-covariance matrix of the estimated unknowns;
- compute the residuals of image coordinate measurements;

Deliverables and Report Preparation

Your lab report should include the following for images (65_09_20180803.jpg, 65_10_20180803.jpg, 65_14_20180803.jpg, and 65_18_20180803.jpg):

- Measured image coordinates and the approximations of the unknown EOP
- The modified Exterior Orientation Parameters after each iteration
- The final adjusted values of the Exterior Orientation Parameters ± standard deviations
- Comparison with the BASC-based EOP for those images
- An estimate of the variance component
- The a-posteriori variance-covariance (dispersion) matrix of the parameters
- The residuals associated with the image coordinate measurements
- Explanation of your results and any problems encountered
- Computer code for the SPR procedure

Deliverables and Report Preparation

- Notes:
  - Units should be explicitly specified,
  - Number of significant digits should be considered, and
  - Sufficient narrative analysis of the results.