Chapters 1 – 7: Overview

- Chapter 1: Introduction
- Chapters 2 – 4: Data acquisition
- Chapters 5 – 7: Data manipulation
  - Chapter 5: Vertical imagery
  - Chapter 6: Image coordinate measurements and refinements
  - Chapter 7: Mathematical model and bundle block adjustment
    - Collinearity Equations
    - Alternative approaches for representing rotation in space
    - Projective Transformation
    - Direct Linear Transformation (DLT)
- So far, we covered the necessary tools for the manipulation of photogrammetric data for the purpose of object space reconstruction from imagery.

CE 59700: Chapter 8

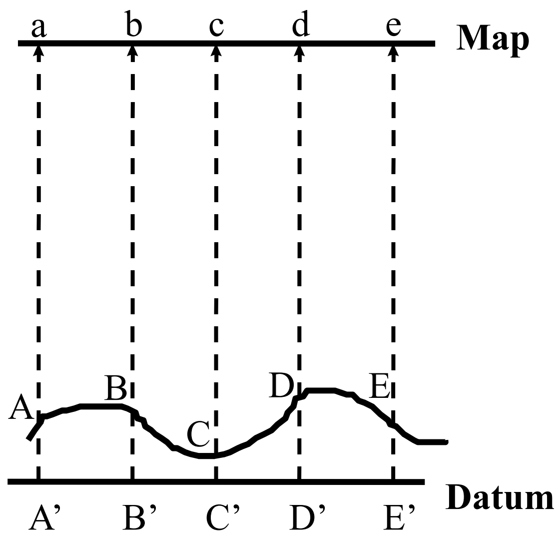
Theory of Orientation & Photogrammetric Triangulation

Overview

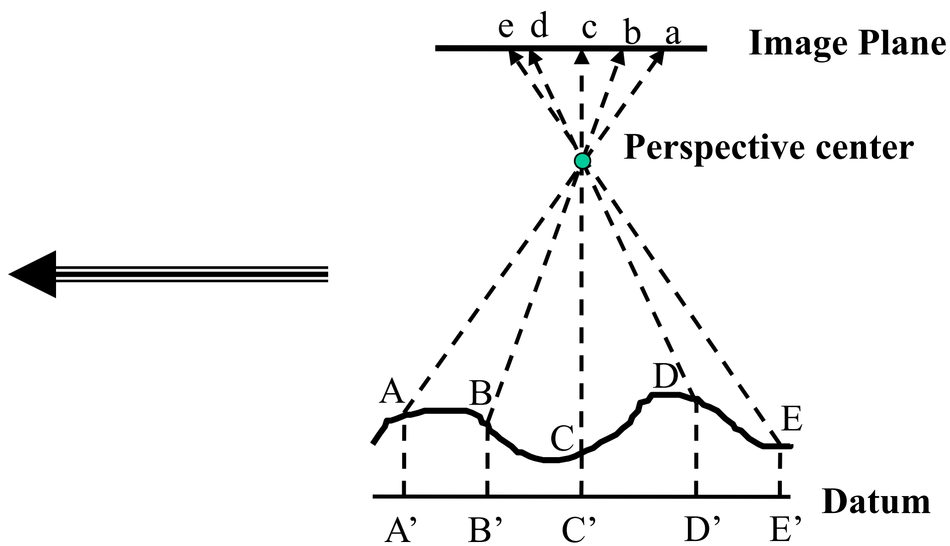
- Objective: Transform centrally projected images into a three-dimensional model, which we can use to plot an orthogonal map
- Interior orientation
- Exterior orientation:
  - Relative orientation
    - x versus y-parallax
  - Absolute orientation
- Aerial Triangulation: Strip and block triangulation

Theory of Orientation

Theory of Orientation



Map

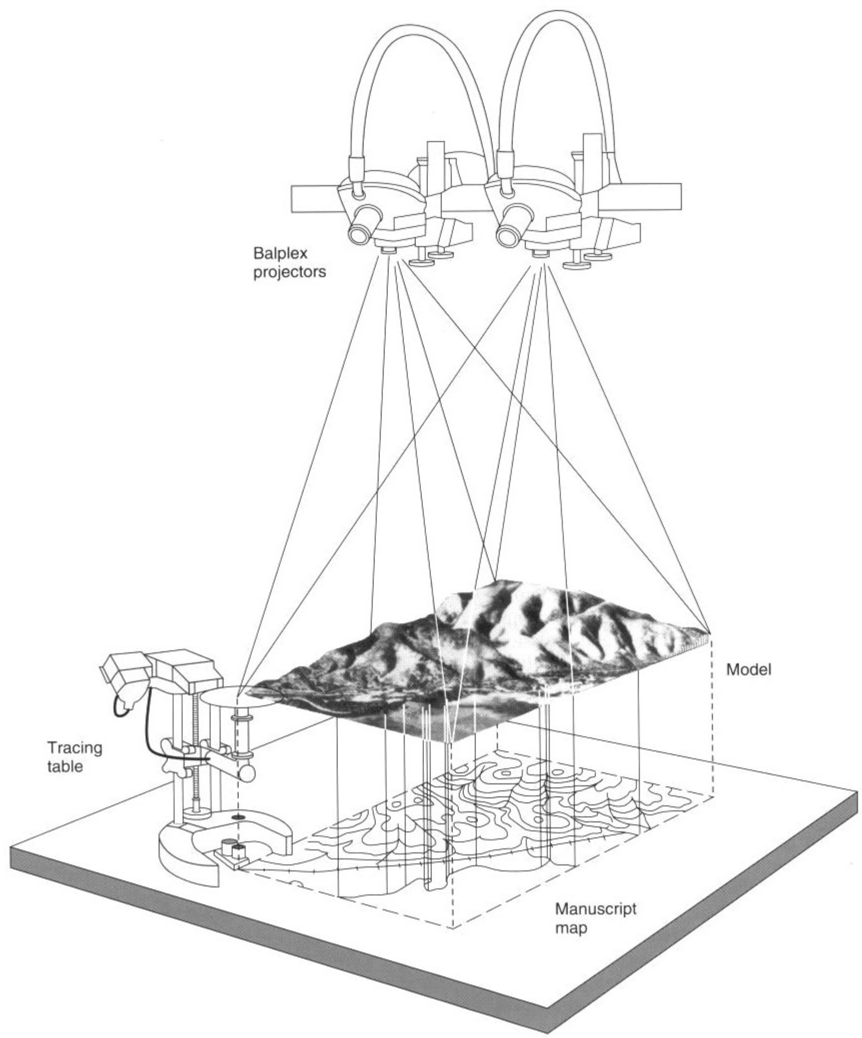


Image

Theory of Orientation

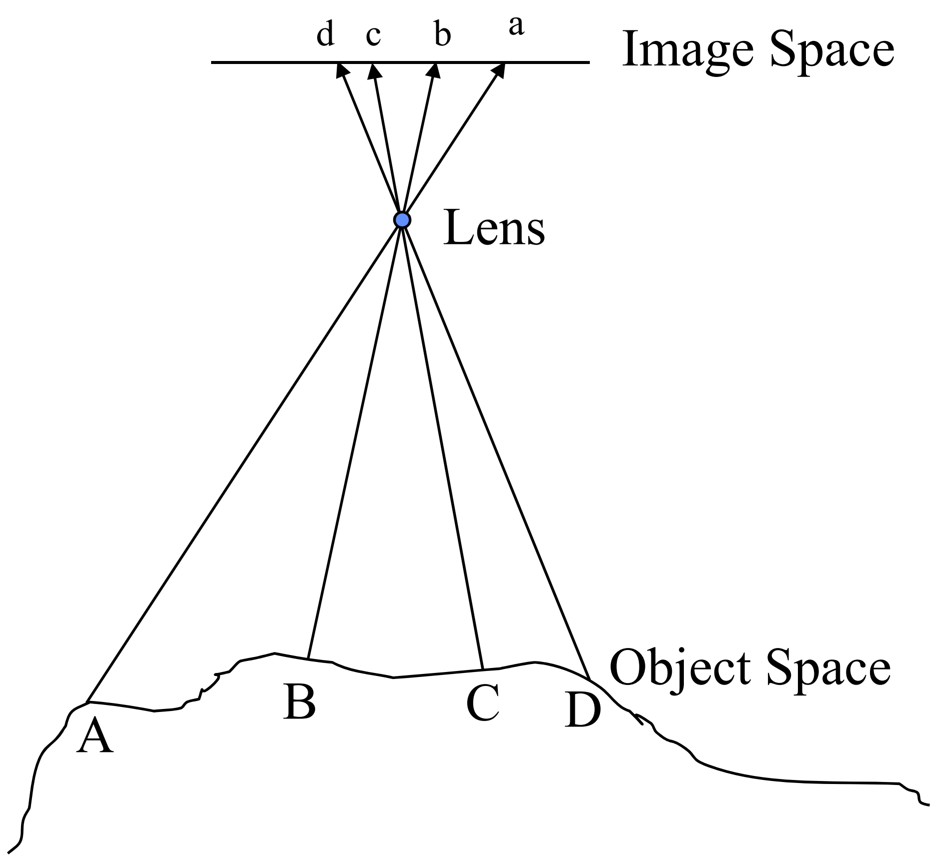
- The objective of orientation is to transform centrally projected images into a three-dimensional model, which we can use to plot an orthogonal map.
- The three-dimensional model can be obtained through:
  - Interior Orientation
  - Exterior Orientation

Theory of Orientation

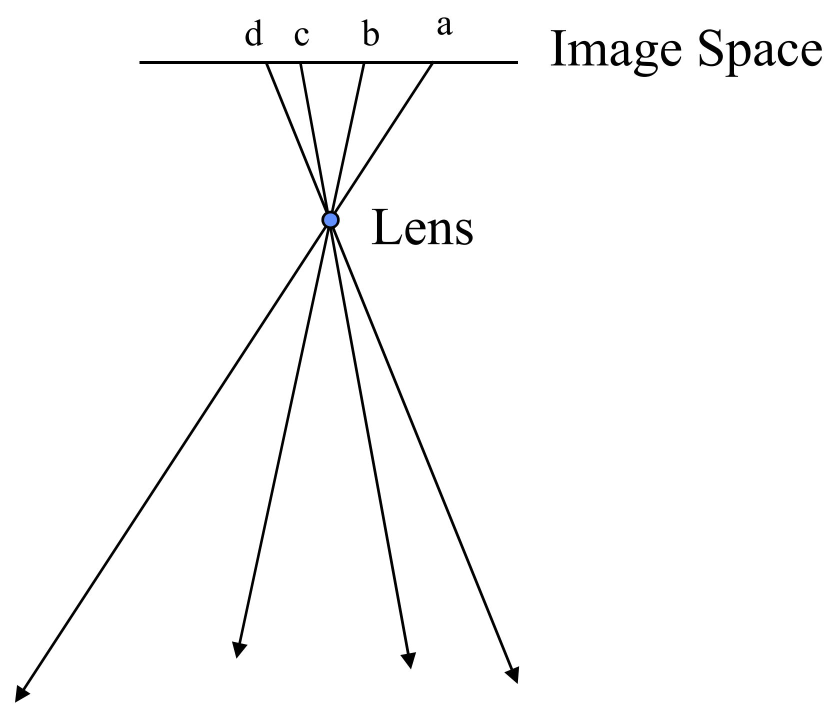


Interior Orientation

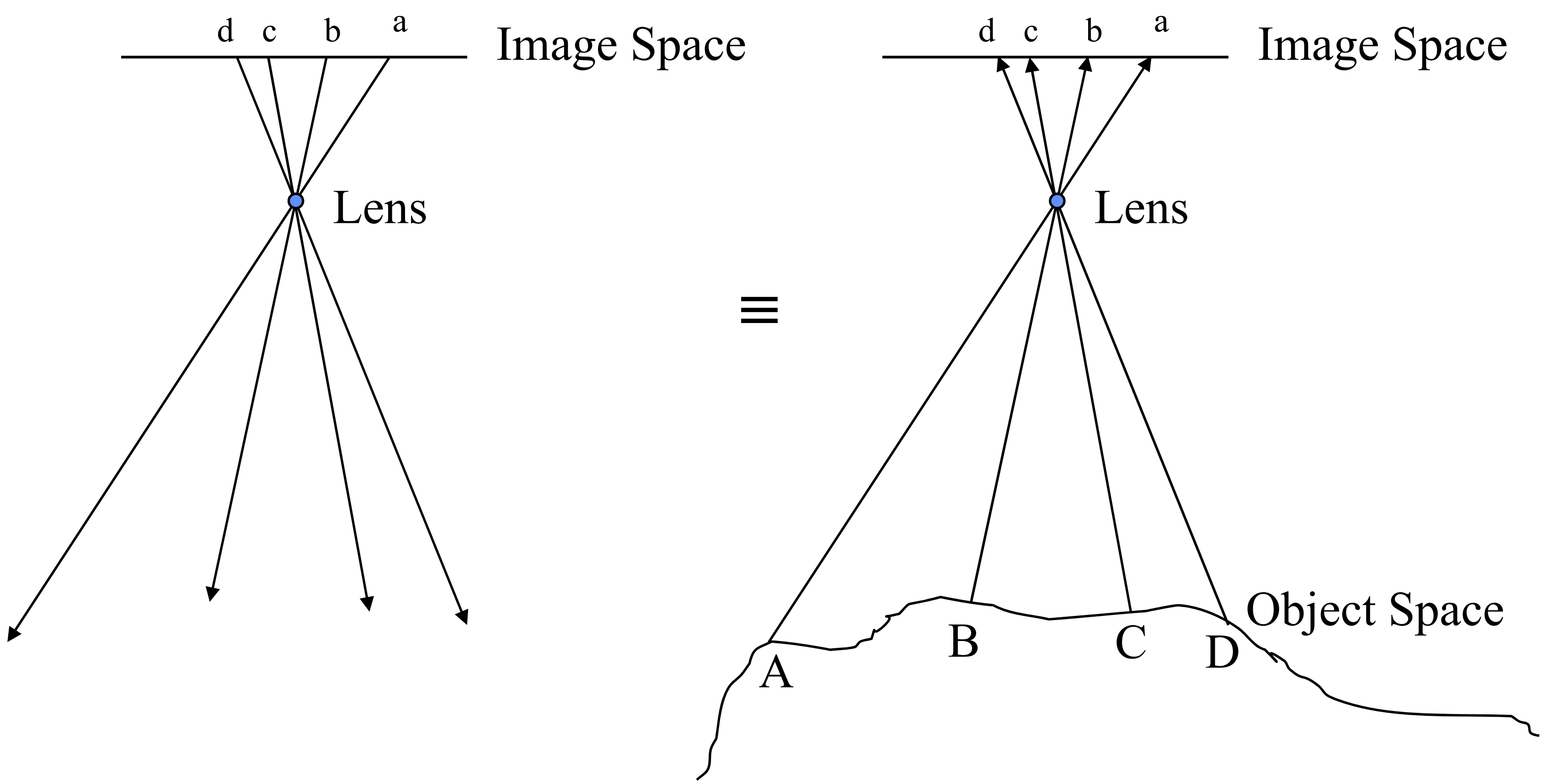
Photography



Interior Orientation



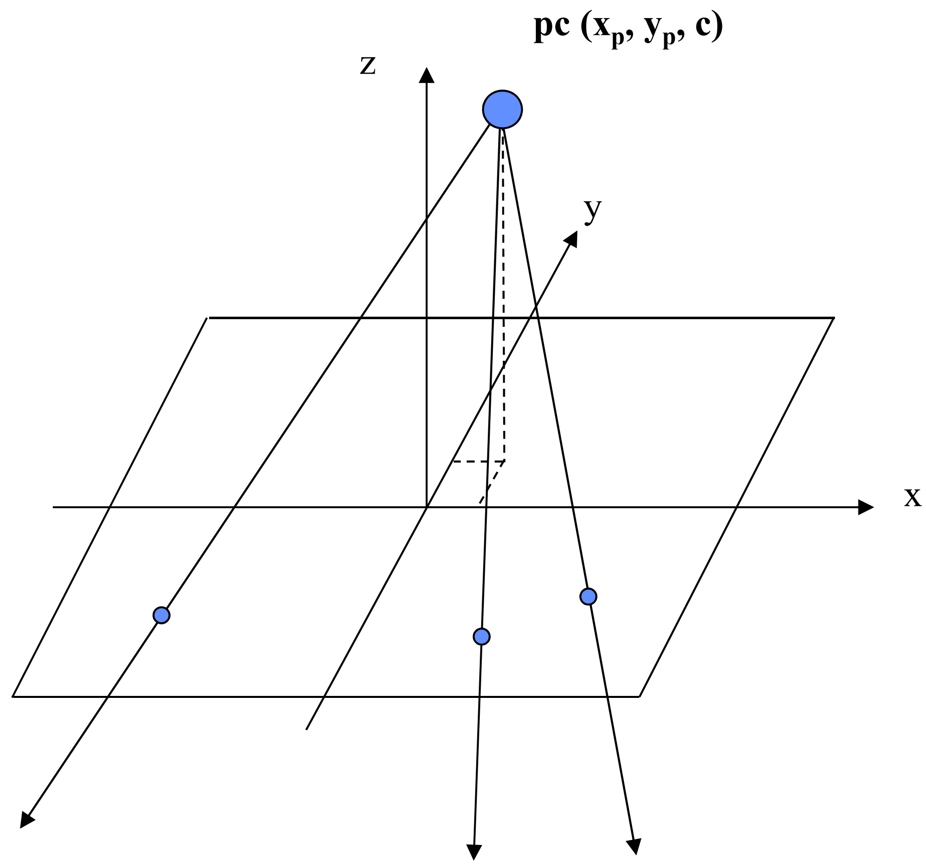
Interior Orientation



Interior Orientation

- Purpose: Reconstruct the bundle of light rays (as defined by the perspective center and the image points) in such a way that it is similar to the incident bundle onto the camera at the moment of exposure.
- Interior orientation is defined by the position of the perspective center w.r.t. the image coordinate system (x_p, y_p, c).
- Another component of the interior orientation is the distortion parameters.

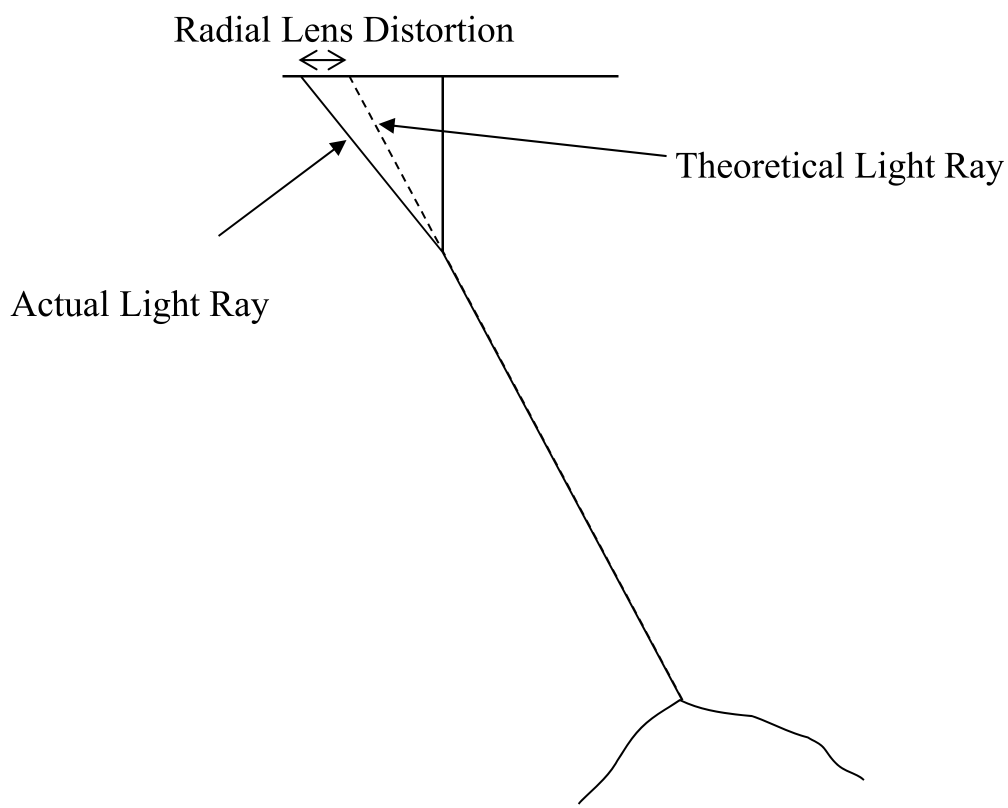
Interior Orientation



Distortion Parameters

- Assumed perspective geometry: The perspective center, the object point, and the corresponding image point are collinear.
- During camera calibration, we try to compensate for all deviations from the assumed perspective geometry:
  - Radial Lens Distortion
  - De-centering Lens Distortion
  - Affine Deformations
  - Out of Plane Deformations
  - Atmospheric Refraction

Radial Lens Distortion



Radial Lens Distortion

- The light ray changes its direction after passing through the perspective center.
- Radial lens distortion is caused by:
  - Large off-axial angle
  - Lens manufacturing flaws
- Radial lens distortion occurs along a radial direction from the principal point.
- Radial lens distortion increases as we move away from the principal point.

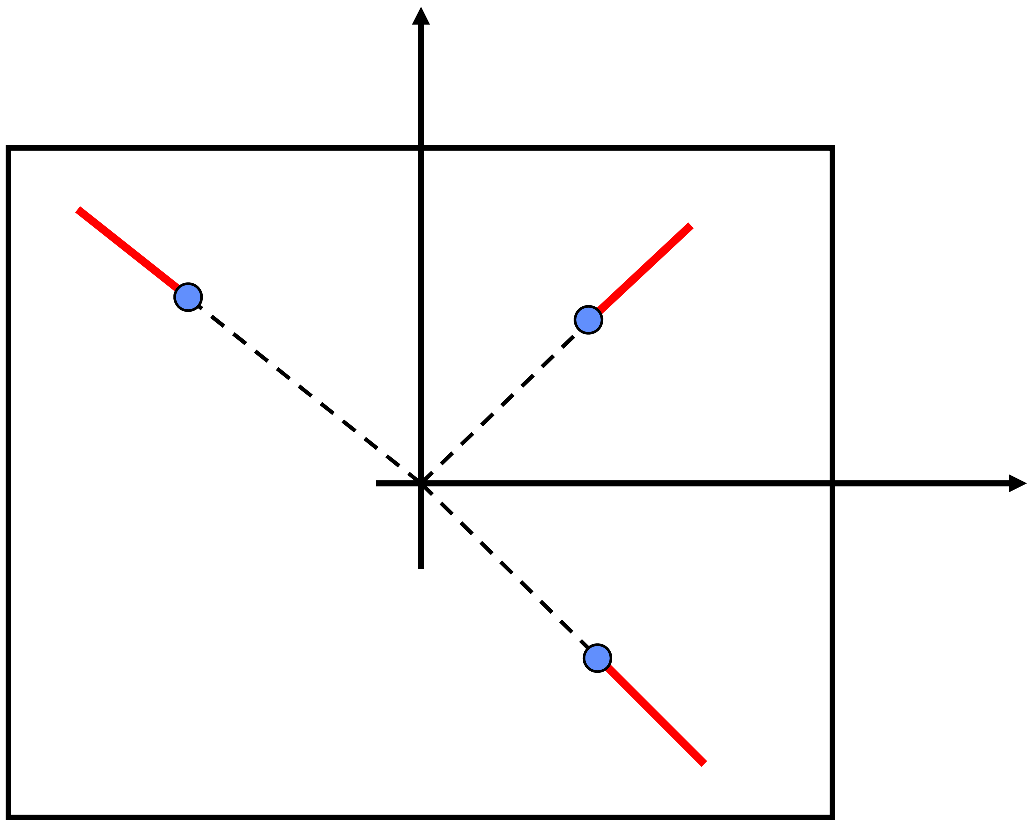
Radial Lens Distortion

$$\Delta x_{Radial\ \ Lens\ \ Distortion} = (x - x_p)\ \ (k_1 r^2 + k_2 r^4 + k_3 r^6 + ...)$$

$$\Delta y_{Radial\ \ Lens\ \ Distortion} = (y - y_p)\ \ (k_1 r^2 + k_2 r^4 + k_3 r^6 + ...)$$

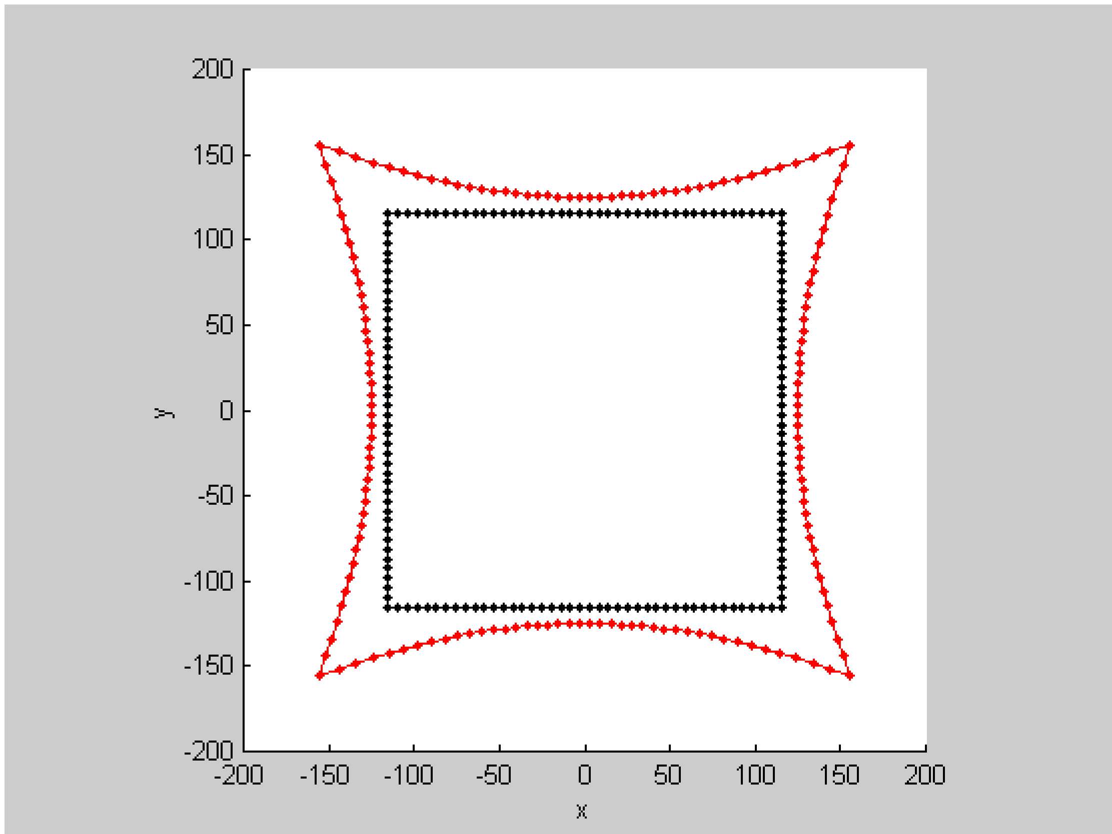
where: $r = \{(x - x_p)^2 + (y - y_p)^2\}^{0.5}$

Radial Lens Distortion



Fiducial Center ≈ Principal Point

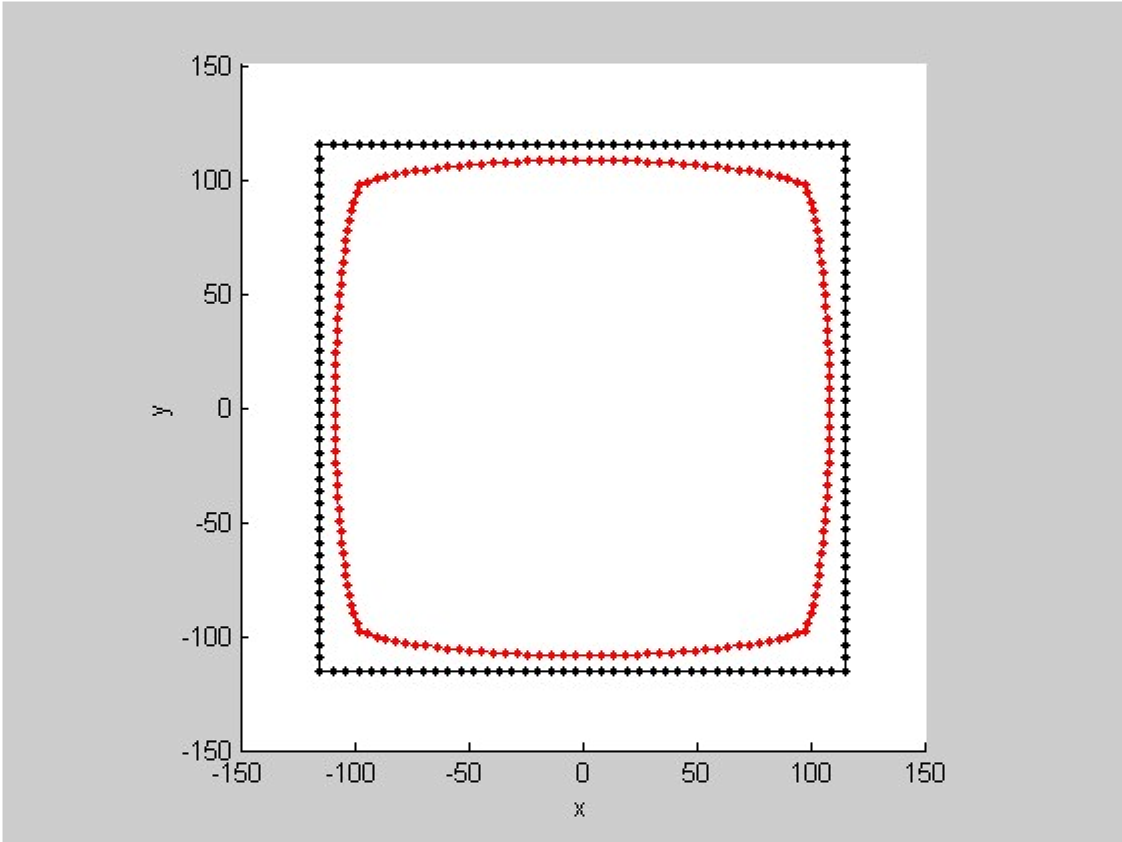
Radial Lens Distortion



Without distortions　　With distortions

Pin Cushion Type Radial Lens Distortion

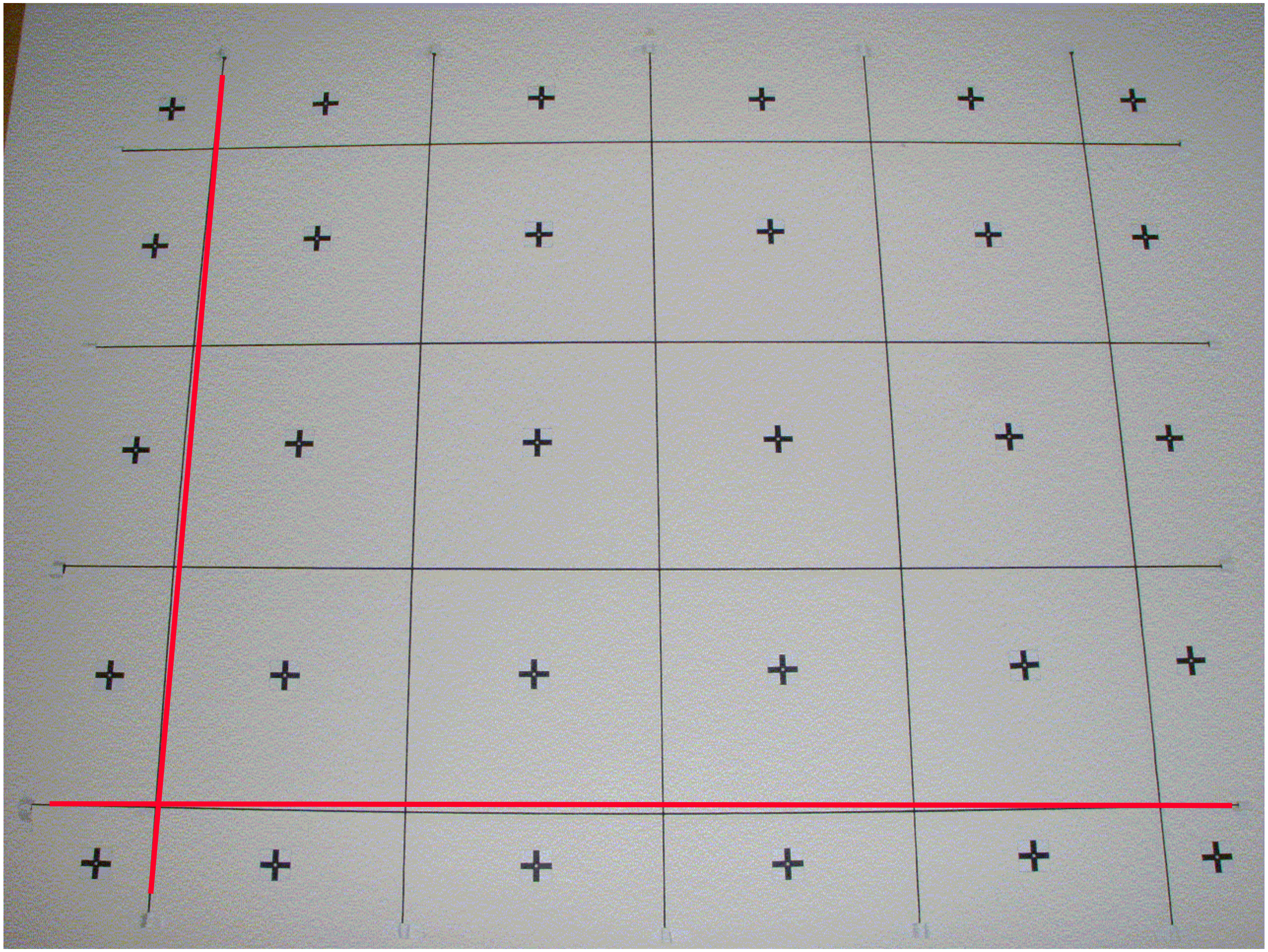
Radial Lens Distortion



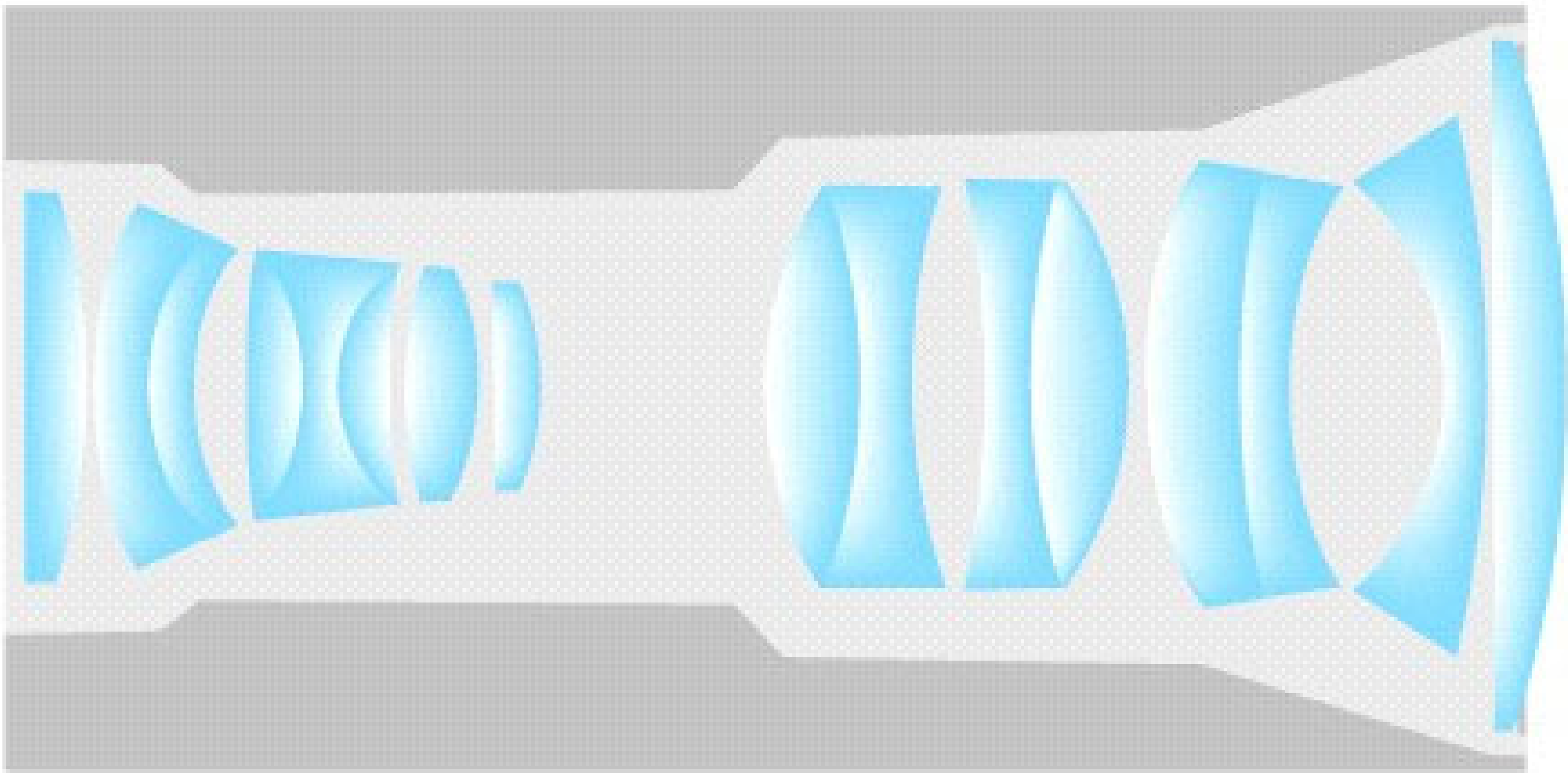
Without distortions — With distortions

Barrel Type Radial Lens Distortion

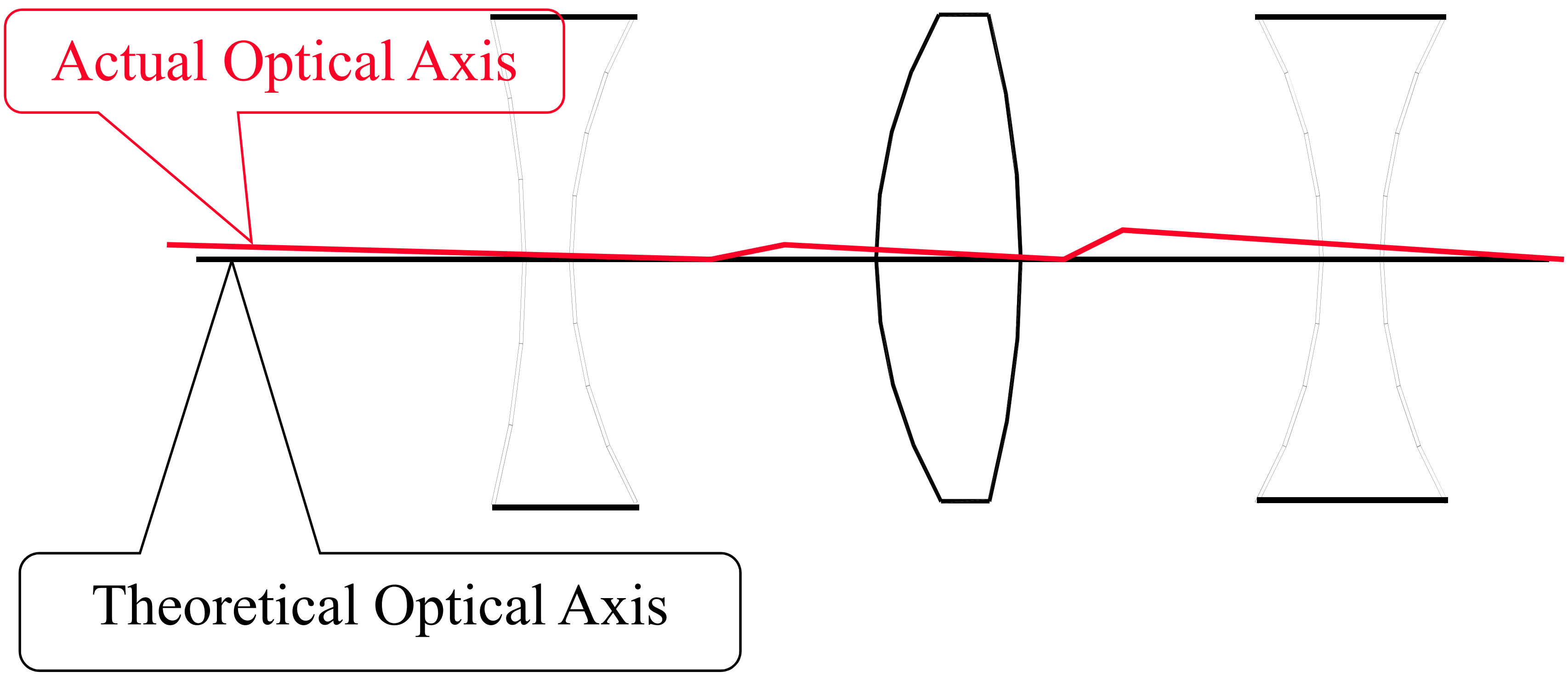
Radial Lens Distortion



Lens Cone Assembly



De-centering Lens Distortion



De-centering Lens Distortion

- De-centering lens distortion is caused by misalignment of the elements of the lens system.
- De-centering lens distortion has two components:
  - Radial component, and
  - Tangential component.

$$\Delta x_{Decentering\ Lens\ Distortion} = (1 + p_3 r^2)\{p_1(r^2 + 2\bar{x}^2) + 2 p_2 \bar{x}\ \bar{y}\}$$

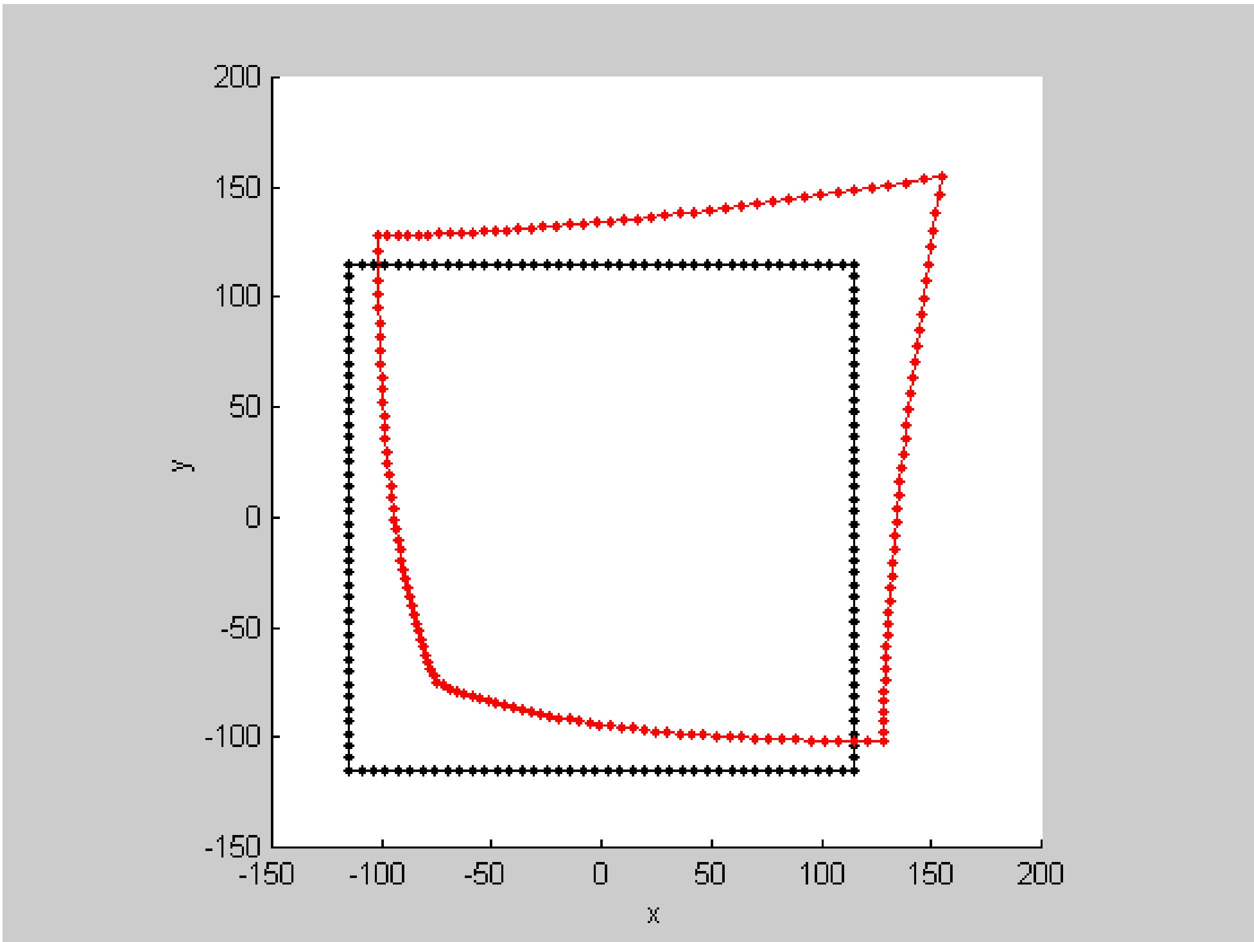
$$\Delta y_{Decentering\ Lens\ Distortion} = (1 + p_3 r^2)\{2 p_1 \bar{x}\ \bar{y} + p_2(r^2 + 2\bar{y}^2)\}$$

where: $r = \{(x - x_p)^2 + (y - y_p)^2\}^{0.5}$

$$\bar{x} = x - x_p$$

$$\bar{y} = y - y_p$$

De-centering Lens Distortion



Without distortions

With distortions

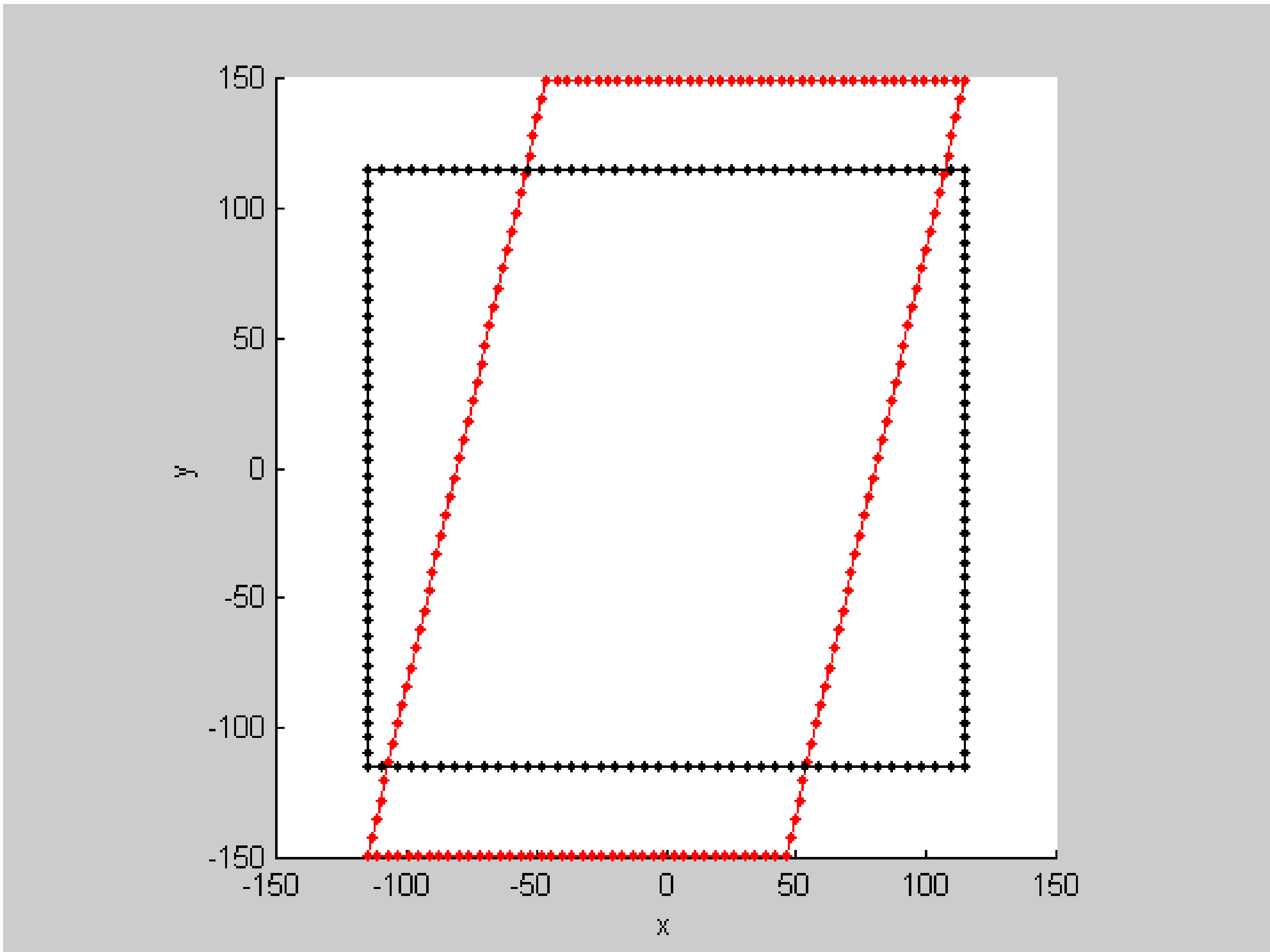
Affine Deformations

- Affine deformations in the focal plane will be manifested as:
  - Non-uniform scale along the x and y directions, and
  - Non orthogonality between the xy-axes.

$$\Delta x_{AD} = -A_1 \ x + A_2 \ y$$

$$\Delta y_{AD} = A_1 \ y$$

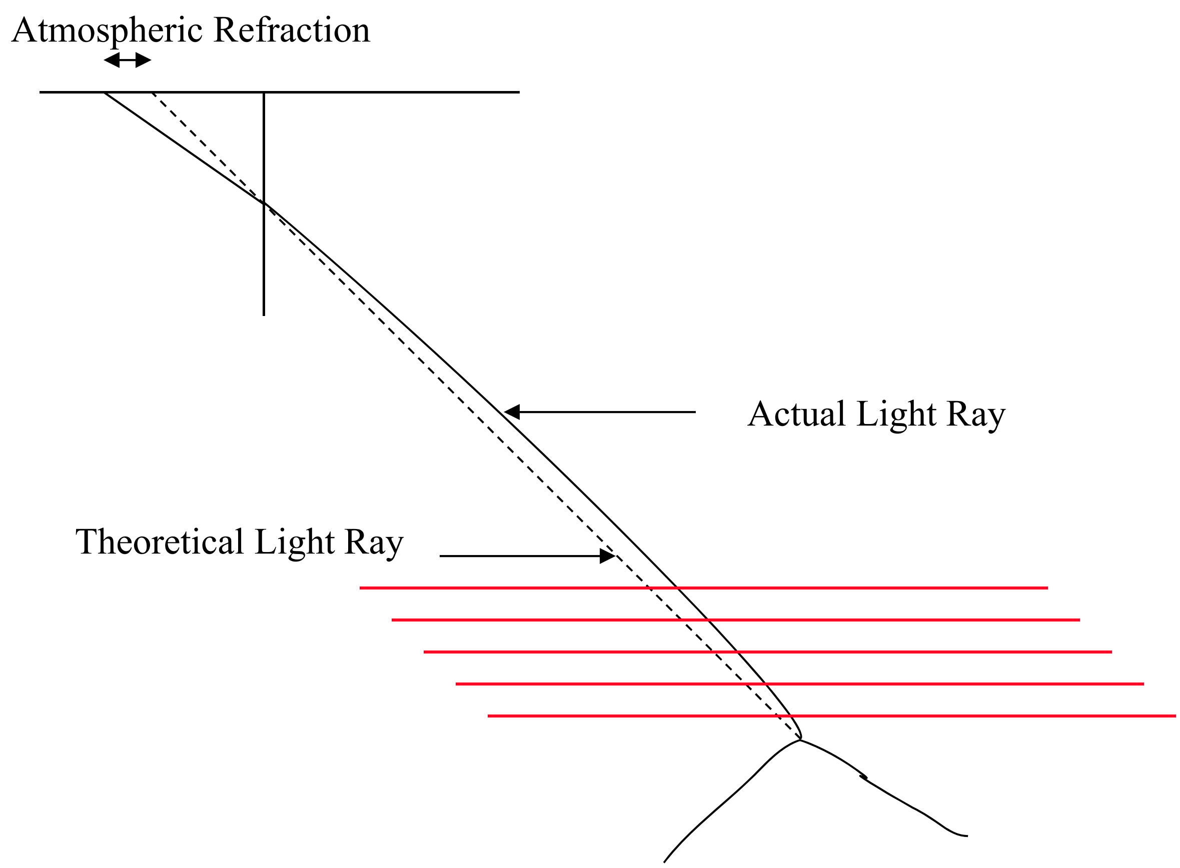
Affine Deformations



Without distortions

With distortions

Atmospheric Refraction



Atmospheric Refraction

- The light ray from the object point to the perspective center passes through layers with different temperature, pressure, and humidity.
- Each layer has its own refractive index.
- Consequently, the light ray will follow a curved not a straight path.
- The distortion occurs along the radial direction from the nadir point.
- It increases as the radial distance increases.

Atmospheric Refraction

- $\Delta r = k\, r\, \{1 + r^2 / c^2\}$
- "k" is the atmospheric refraction coefficient.
- Image points are always displaced outwardly along the radial direction.
- The above equation is only valid for almost vertical photography.

Atmospheric Refraction

$$k = 0.00241 \left\{ \frac{Z_o}{Z_o^2 - 6\,Z_o + 250} - \frac{Z^2}{Z_o(Z^2 - 6\,Z + 250)} \right\}$$

Where:

Z & Z_o *are in Km above the sea level*

$$\Delta x = k\;x\;(\frac{r^2}{c^2} + 1)$$

$$\Delta y = k\;y\;(\frac{r^2}{c^2} + 1)$$

Extended Collinearity Equations

$$M = R_m^c$$

$$x_a = x_p - c\frac{m_{11}(X_A - X_o) + m_{12}(Y_A - Y_o) + m_{13}(Z_A - Z_o)}{m_{31}(X_A - X_o) + m_{32}(Y_A - Y_o) + m_{33}(Z_A - Z_o)} + dist_x$$

$$y_a = y_p - c\frac{m_{21}(X_A - X_o) + m_{22}(Y_A - Y_o) + m_{23}(Z_A - Z_o)}{m_{31}(X_A - X_o) + m_{32}(Y_A - Y_o) + m_{33}(Z_A - Z_o)} + dist_y$$

$$R = R_c^m$$

$$x_a = x_p - c\frac{r_{11}(X_A - X_o) + r_{21}(Y_A - Y_o) + r_{31}(Z_A - Z_o)}{r_{13}(X_A - X_o) + r_{23}(Y_A - Y_o) + r_{33}(Z_A - Z_o)} + dist_x$$

$$y_a = y_p - c\frac{r_{12}(X_A - X_o) + r_{22}(Y_A - Y_o) + r_{32}(Z_A - Z_o)}{r_{13}(X_A - X_o) + r_{23}(Y_A - Y_o) + r_{33}(Z_A - Z_o)} + dist_y$$

Extended Collinearity Equations

- $dist_x = \Delta x_{\text{Radial Lens Distortion}} + \Delta x_{\text{De-centering Lens Distortion}} + \Delta x_{\text{Atmospheric Refraction}} + \Delta x_{\text{Affine Deformation}} +$ etc….

- $dist_y = \Delta y_{\text{Radial Lens Distortion}} + \Delta y_{\text{De-centering Lens Distortion}} + \Delta y_{\text{Atmospheric Refraction}} + \Delta y_{\text{Affine Deformations}} +$ etc….

Camera Calibration

Camera Calibration

http://www.dpreview.com/reviews/sonydscf828/3

Camera Calibration

- Objective: Recover the camera's internal characteristics
  - Coordinates of the principal point (x_p, y_p)
  - Principal distance (c)
  - Radial lens distortion
  - De-centering lens distortion
  - Affine deformations
- Calibration alternatives:
  - Laboratory calibration using multi-collimators
  - Analytical camera – in situ – calibration (bundle adjustment with self calibration)

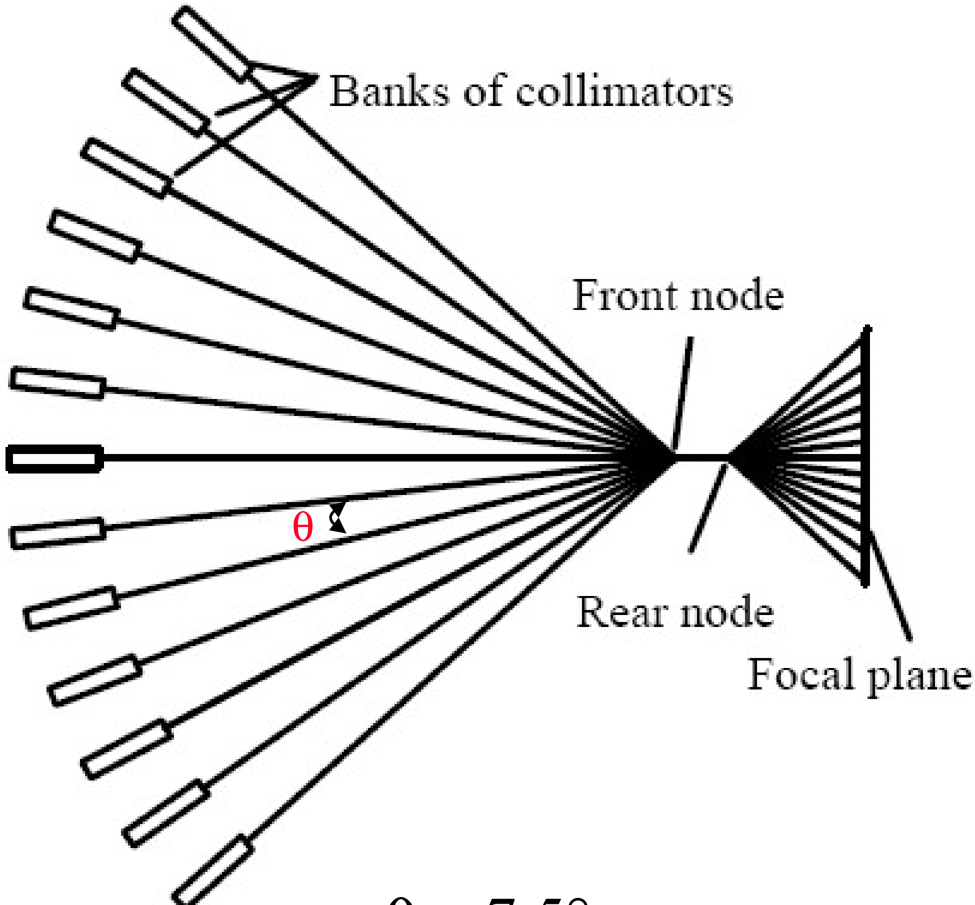
Laboratory Camera Calibration

- This method uses a device known as multi-collimator.
  - Each collimator consists of a lens with a cross fixed at its plane of infinite focus.
  - The collimators are fixed at precisely known angular positions.
  - Collimators are arranged in two perpendicular planes with their axes intersecting at the front nodal point of the lens of the calibrated camera.
  - The camera is placed with its focal plane perpendicular to the axis of the central collimator.
    - The central collimator is aligned along the optical axis of the lens system.

Laboratory Calibration: Multi-Collimators

http://calval.cr.usgs.gov/

Laboratory Calibration: Multi-Collimators

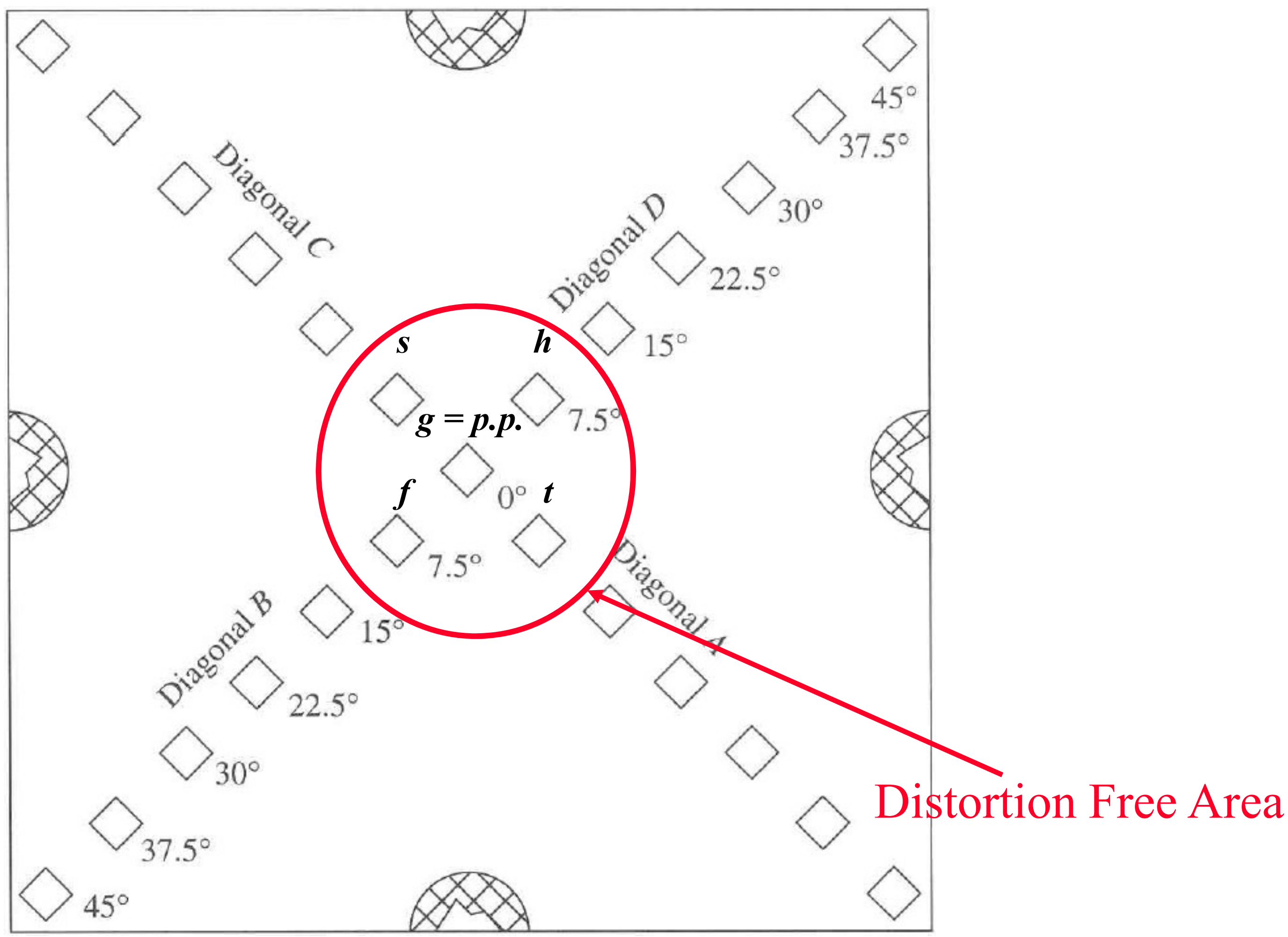


$\theta = 7.5°$

Principal Point Coordinates

- Light rays from the telescopes are directed towards the lens of an aerial camera.
  - The crosses are imaged on the camera's focal plane.
- The image of the central cross will be located at the principal point.
  - The coordinates of the central cross with respect to the image coordinate system as defined by the Fiducial marks are used as the principal point coordinates.
    - $x_p = x_g$
    - $y_p = y_g$

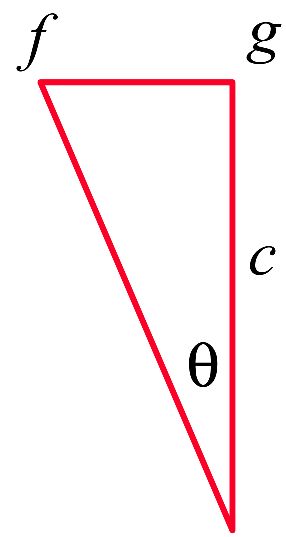
Image of Photographed Collimator Targets



Principal Distance

- Assuming that the central part of the image is a distortion-free area, one can compute the principal distance as follows:

$$c = \frac{gf + gh + gs + gt}{4 \tan \theta}$$



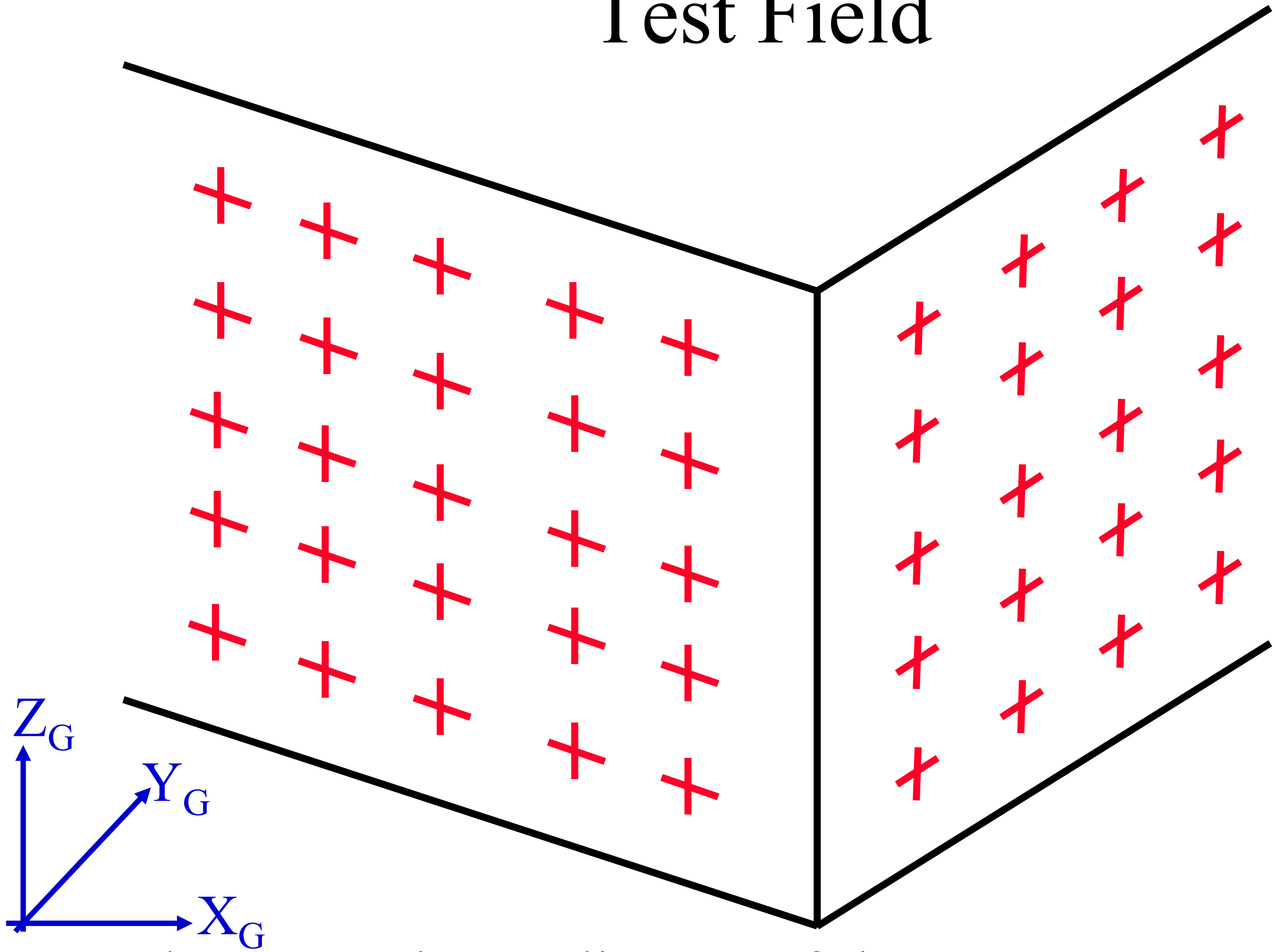
Radial Lens Distortion

- Using the estimated principal distance, one can compute non-distorted distances from the central collimator image to all other collimator images.
- Lens distortion, at a certain radial distance, can be computed as the difference between:
  - The measured distance, and
  - The calculated distance.
- The radial lens distortion curve can be drawn by plotting the lens distortion versus the radial distance.

Analytical Camera Calibration

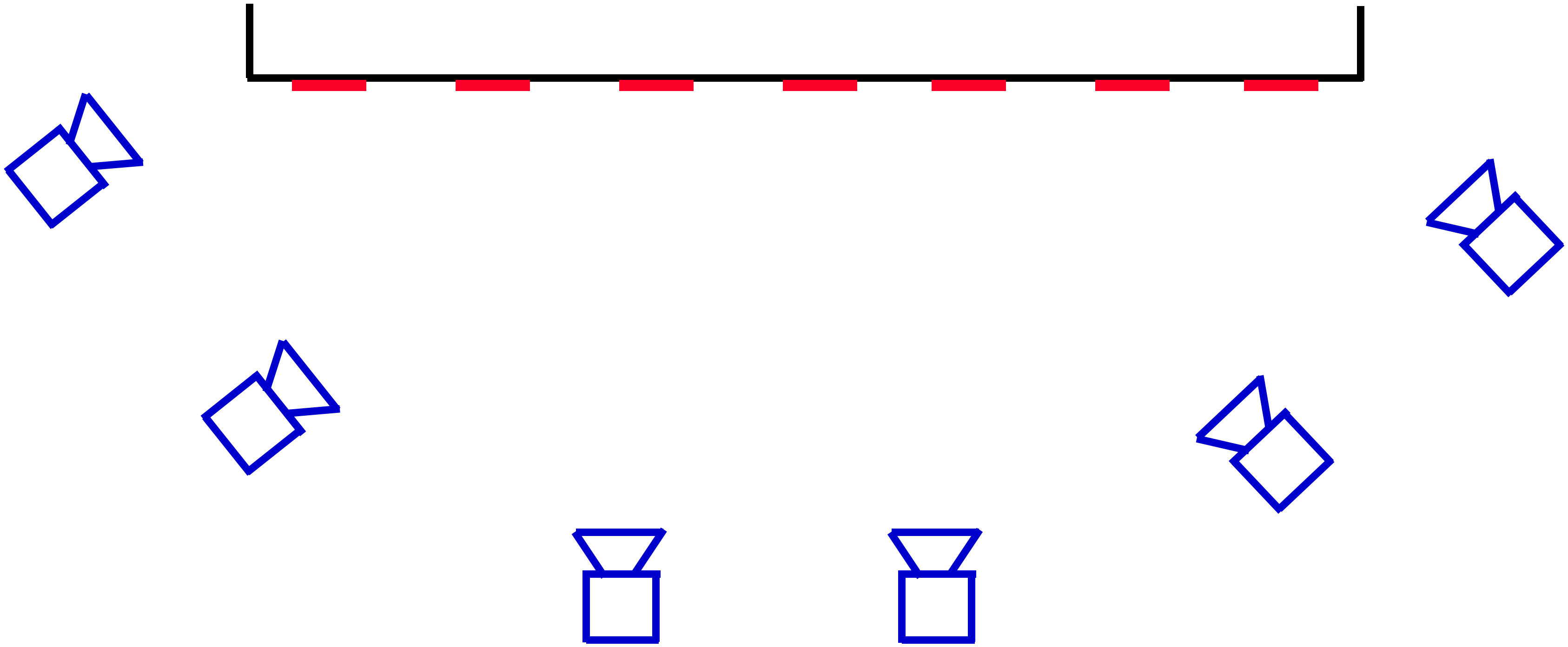
- Capture imagery over a test-field (sufficient number of GCPs should be available).
- Measure image coordinates (GCPs and tie points).
- Perform bundle adjustment and solve for:
  - The ground coordinates of tie points,
  - The EOPs of involved imagery, and
  - The IOPs of involved camera/cameras.
- Caution:
  - Make sure that we have enough number of GCPs.

Test Field

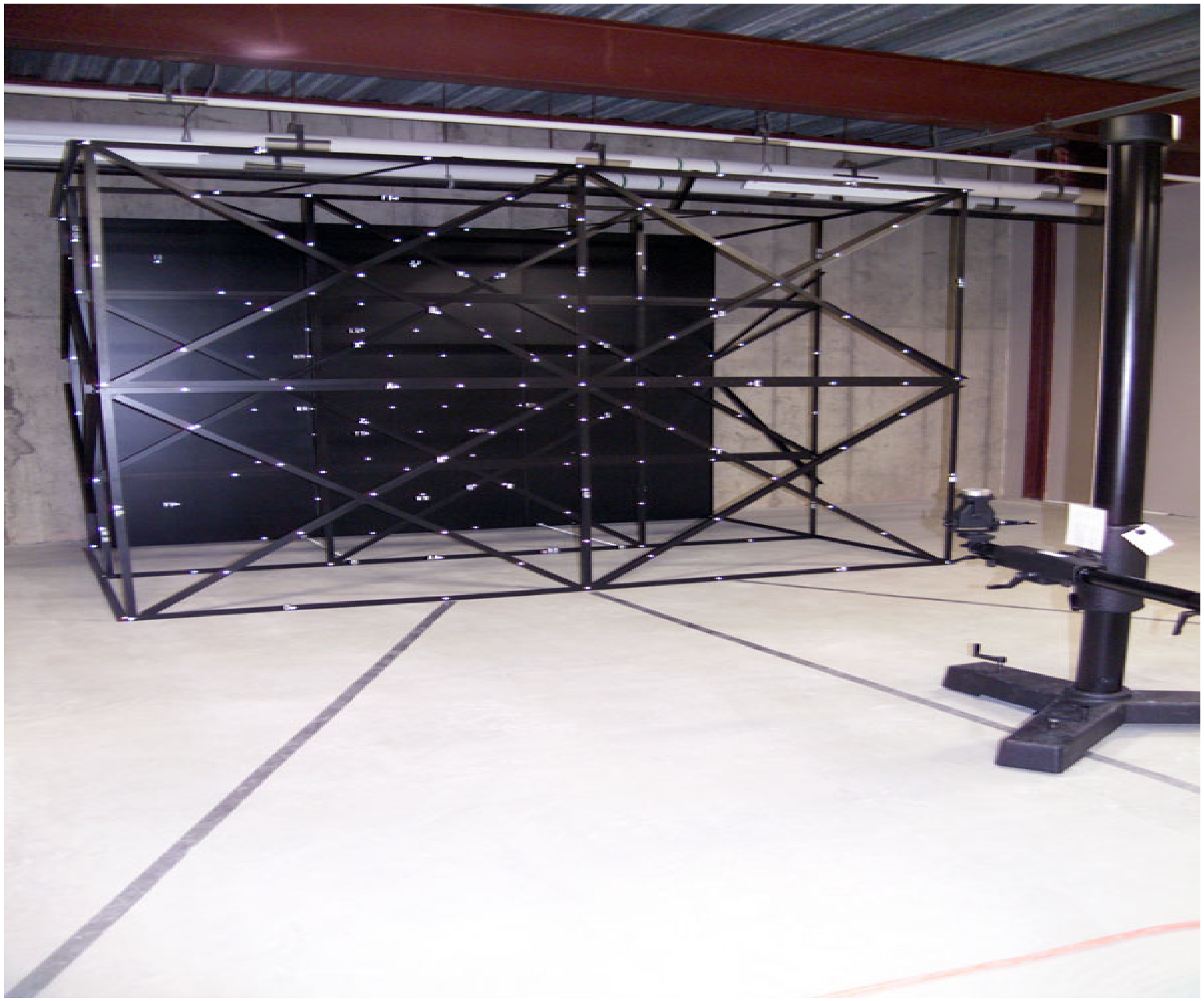


- The ground coordinates of those targets are accurately surveyed.

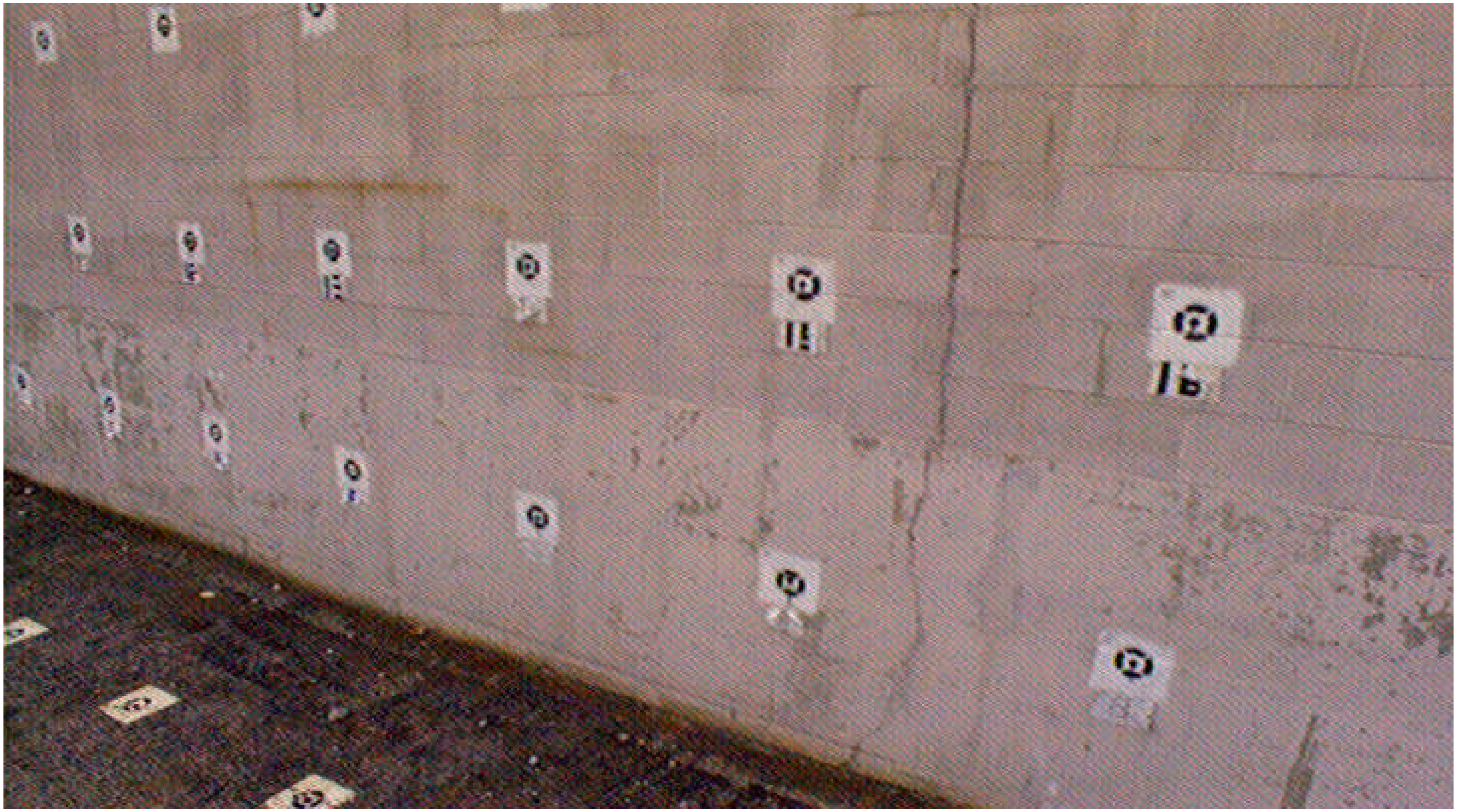
Analytical Camera Calibration



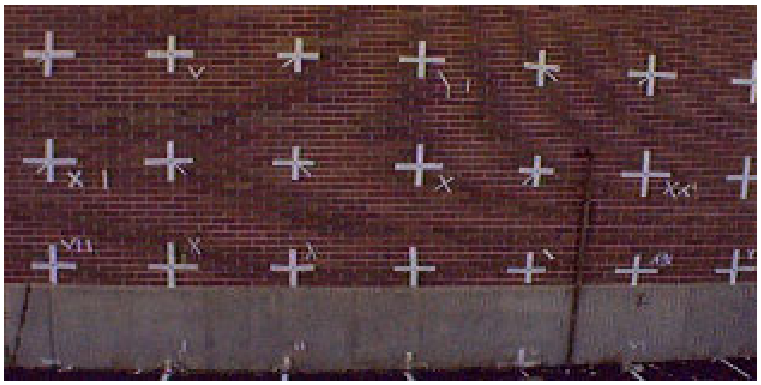
3-D Calibration Test Field

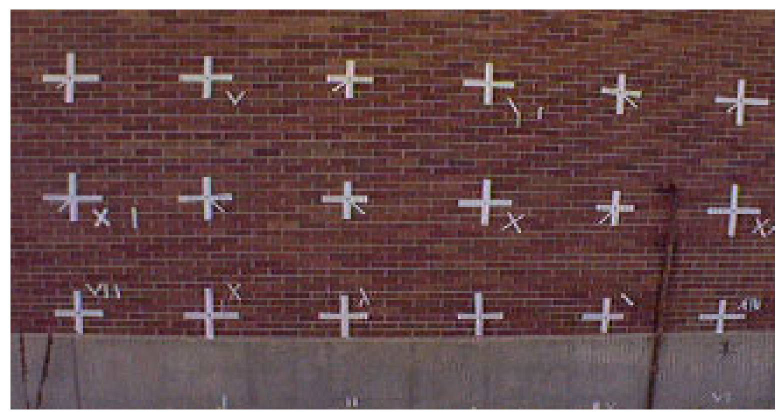


2-D Calibration Test Field



Sample Calibration Images





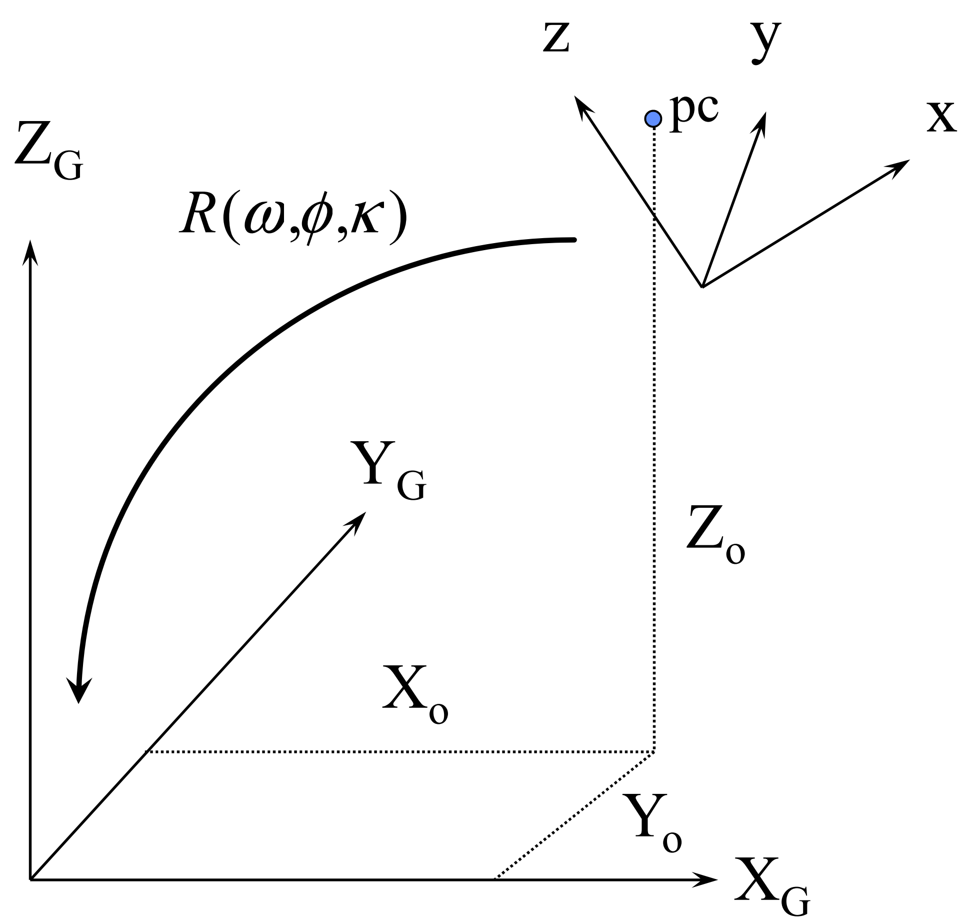
Sample Calibration Image

Exterior Orientation

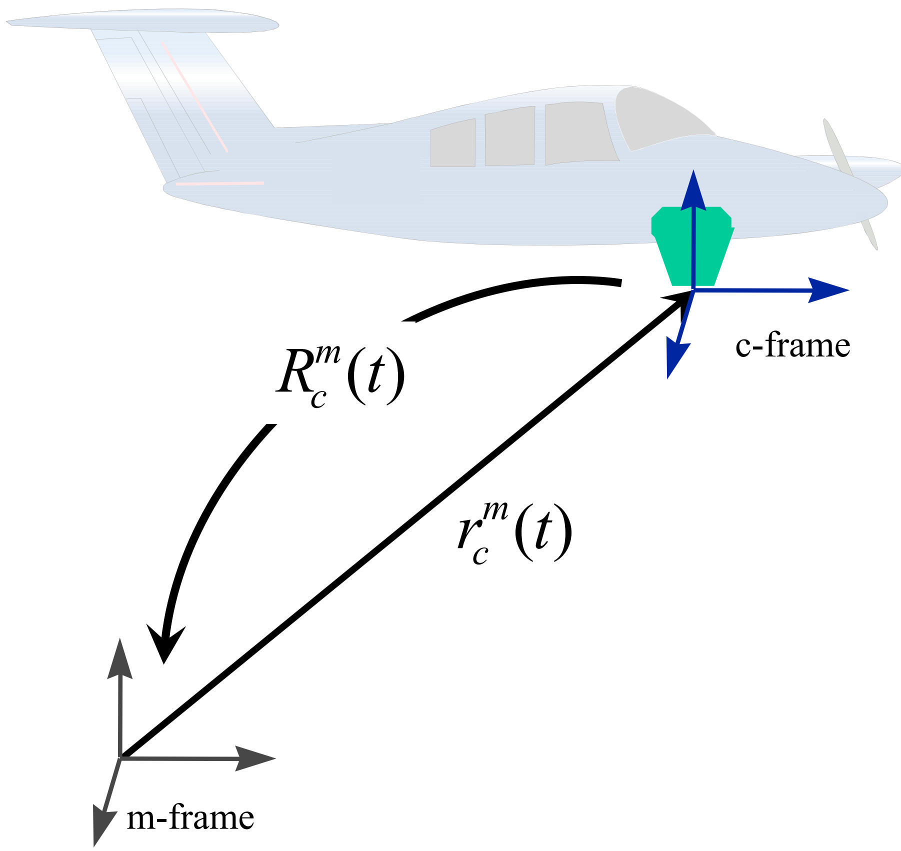
Exterior Orientation

- Exterior Orientation has two components:
  - The position of the perspective center w.r.t. the ground coordinate system (X_o, Y_o, Z_o), and
  - The rotational relationship between the image and the ground coordinate systems (ω, ϕ, κ).
    - These are the rotation angles we need to apply to the ground coordinate system to make it parallel to the image coordinate system.

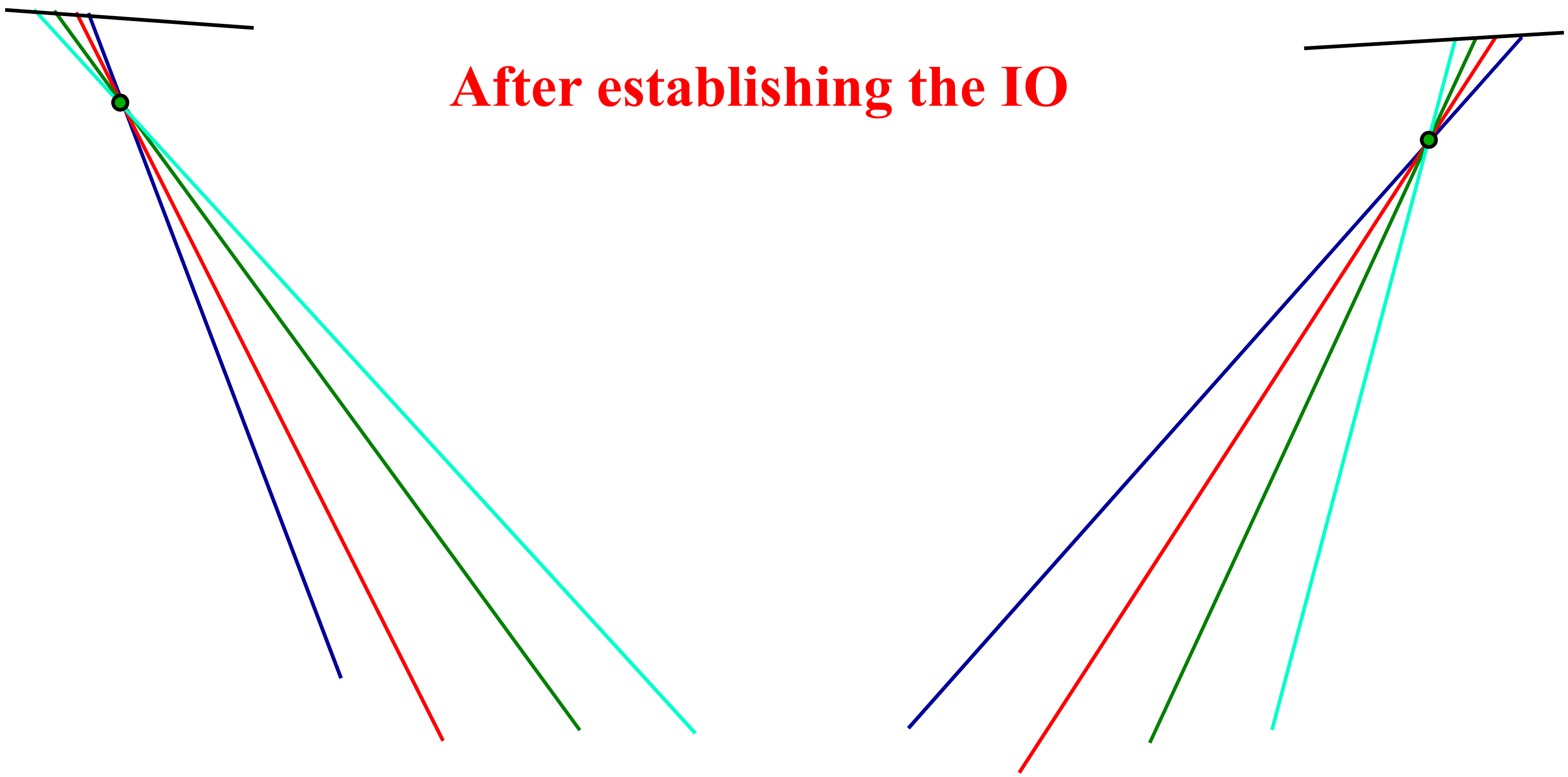
Exterior Orientation



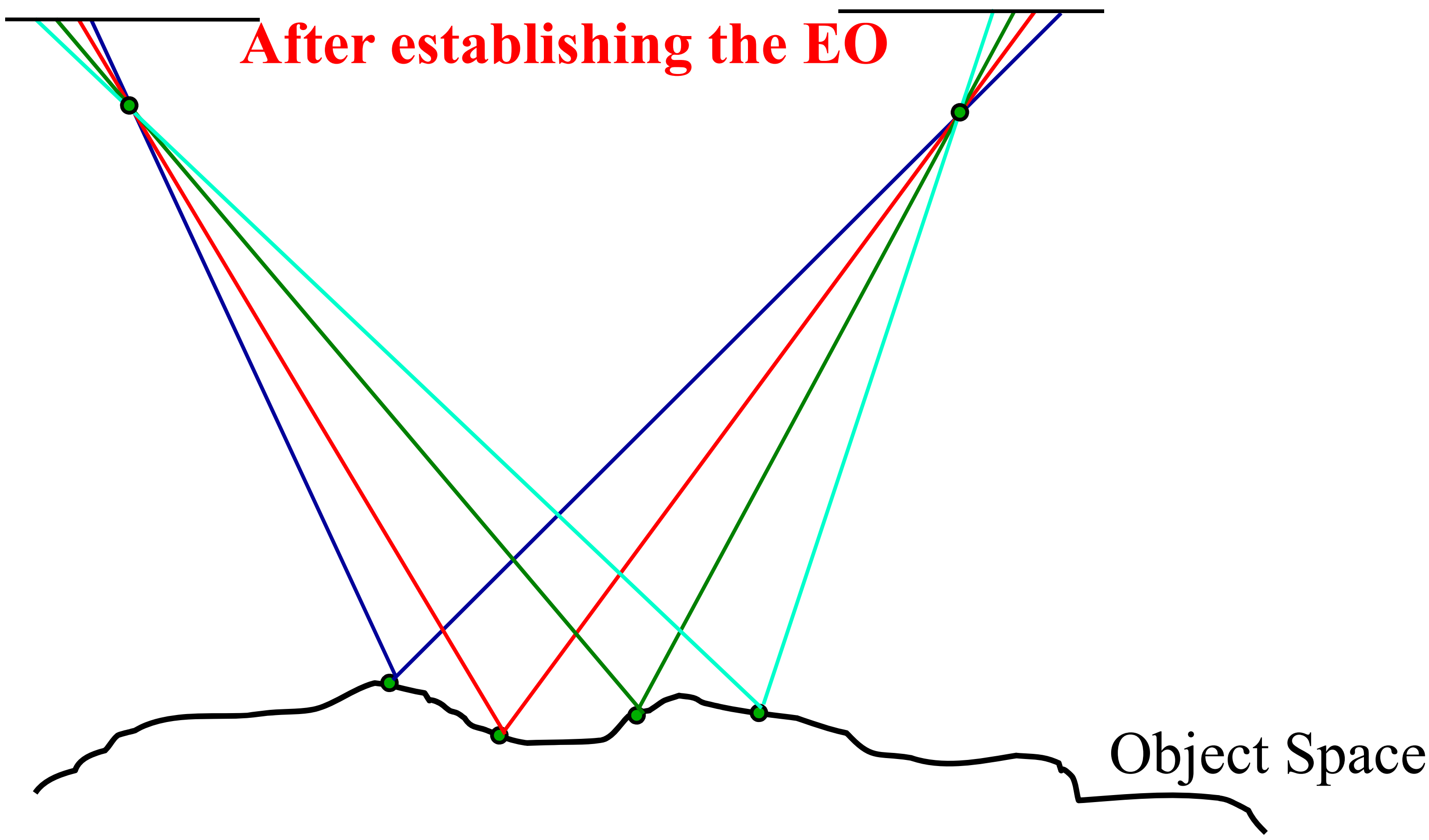
Exterior Orientation



Exterior Orientation of a Stereo-Pair



Exterior Orientation of a Stereo-Pair



Exterior Orientation of a Stereo-Pair

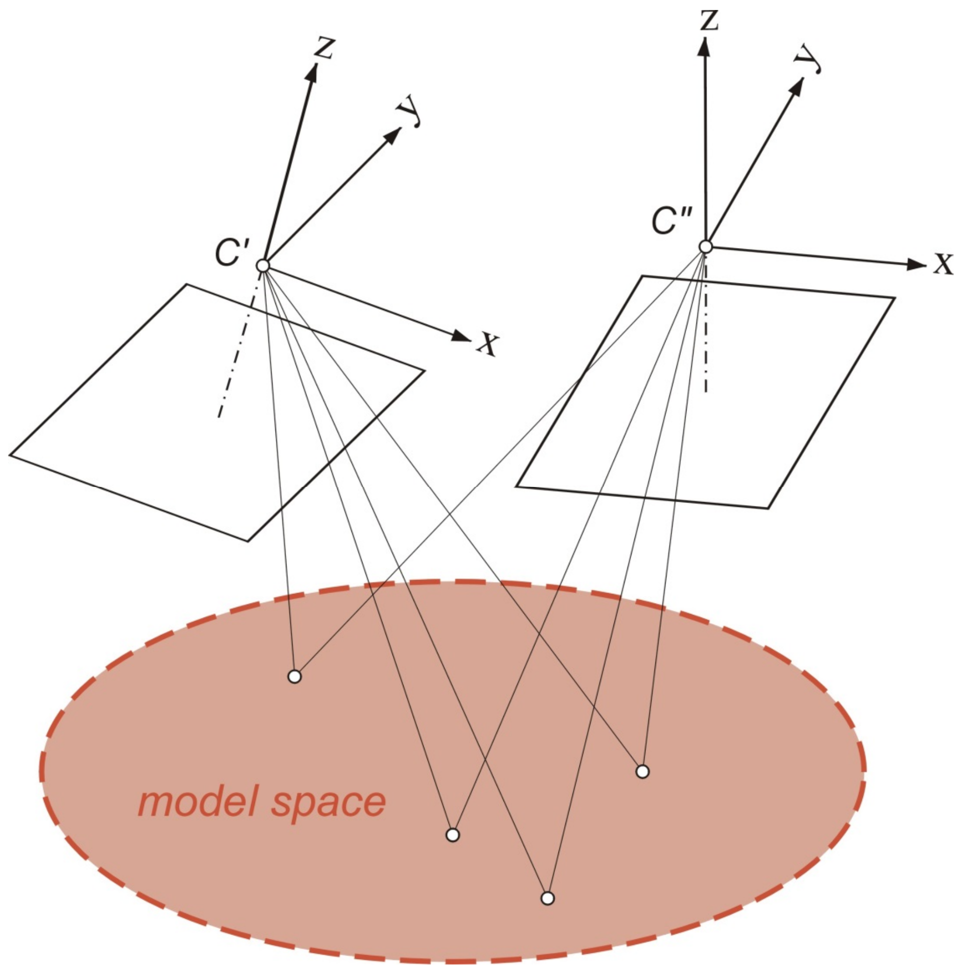
- Exterior Orientation of a stereo-pair is defined by twelve parameters.
  - $X_{o_l}, Y_{o_l}, Z_{o_l}, \omega_l, \phi_l, \kappa_l$
  - $X_{o_r}, Y_{o_r}, Z_{o_r}, \omega_r, \phi_r, \kappa_r$
- Exterior Orientation of a stereo-pair can be decomposed into:
  - Relative Orientation – RO (five parameters), and
  - Absolute Orientation – AO (seven parameters).

Relative Orientation

Relative Orientation – RO

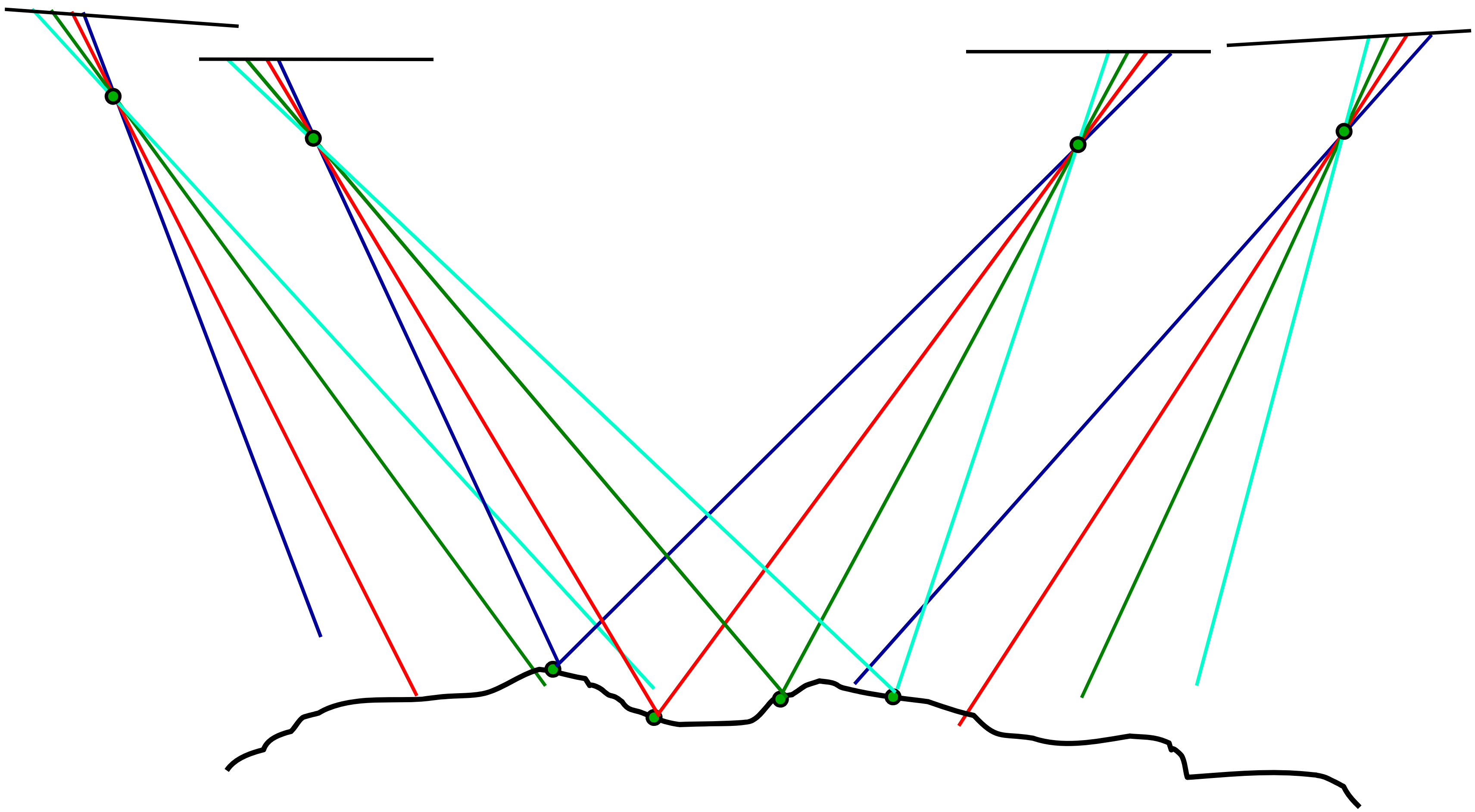
- Objective: Orient the two bundles of a stereo-pair relative to each other in such a way that all conjugate light rays intersect.
- Result: A stereo-model, which is a 3-D representation of the object space w.r.t. an arbitrary local coordinate system.
- If we make at least five conjugate light rays intersect, all the remaining light rays will intersect at the surface of the stereo-model.

Relative Orientation – RO

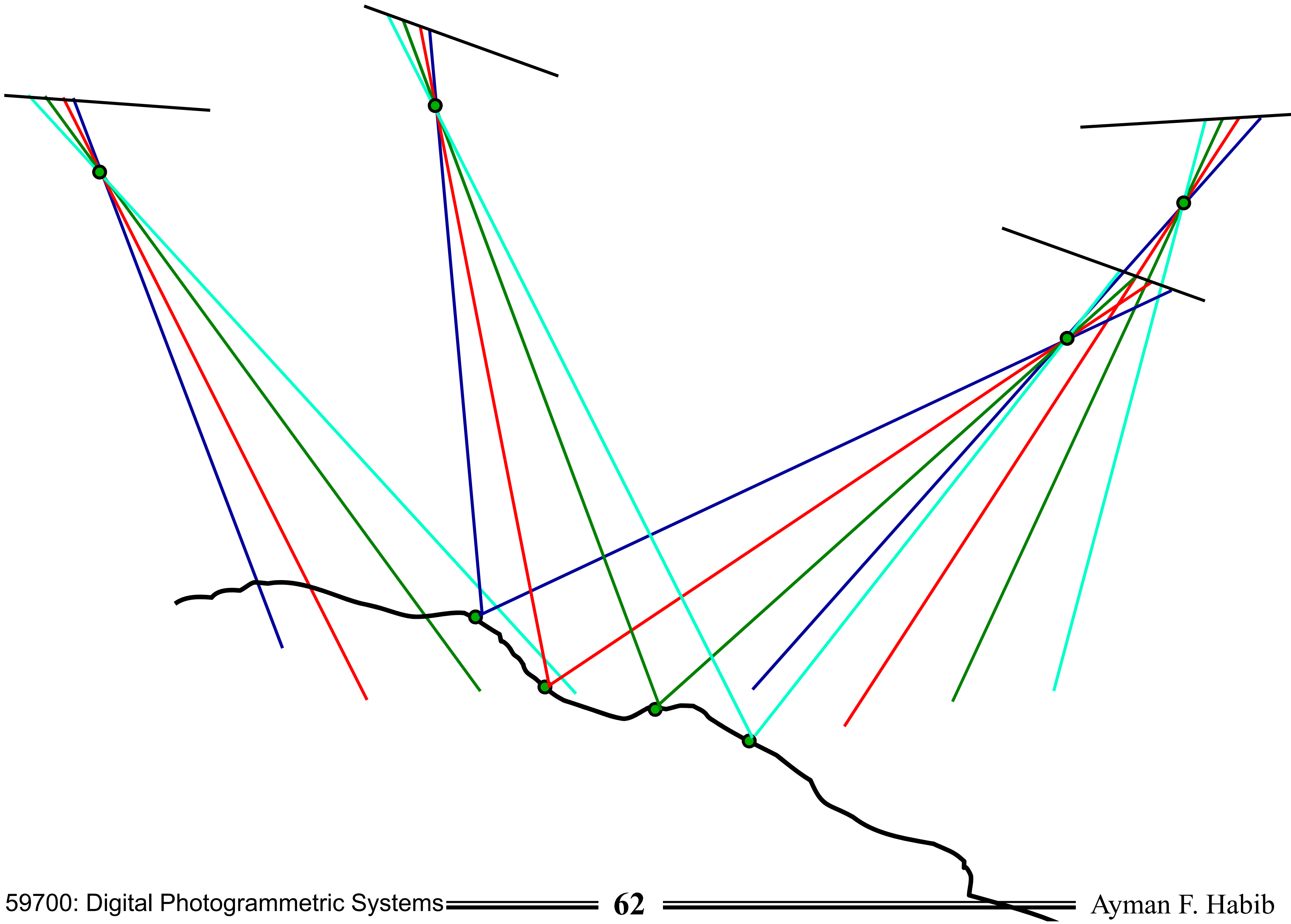


- Concept: Force conjugate light rays to intersect

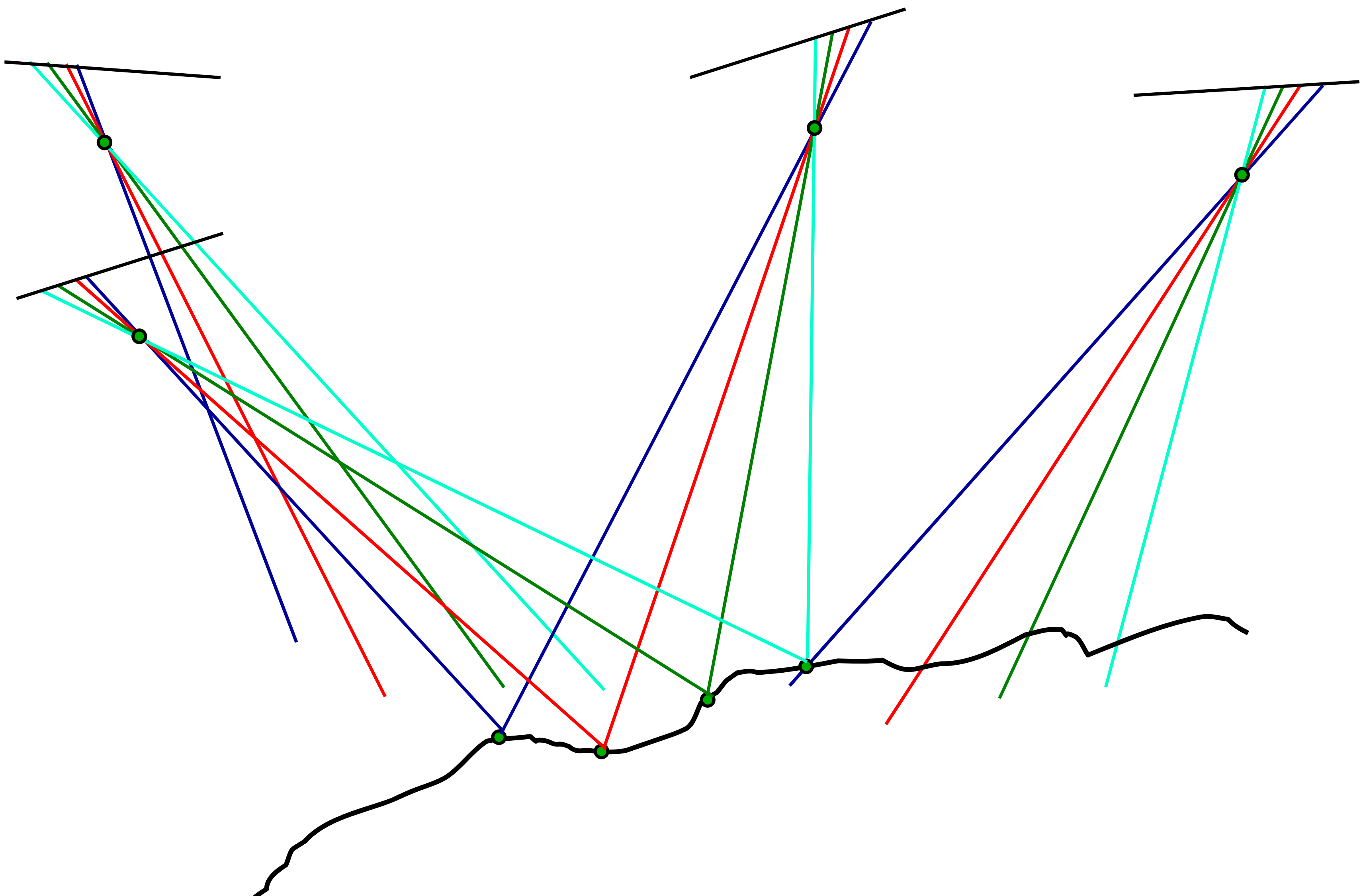
Relative Orientation – RO



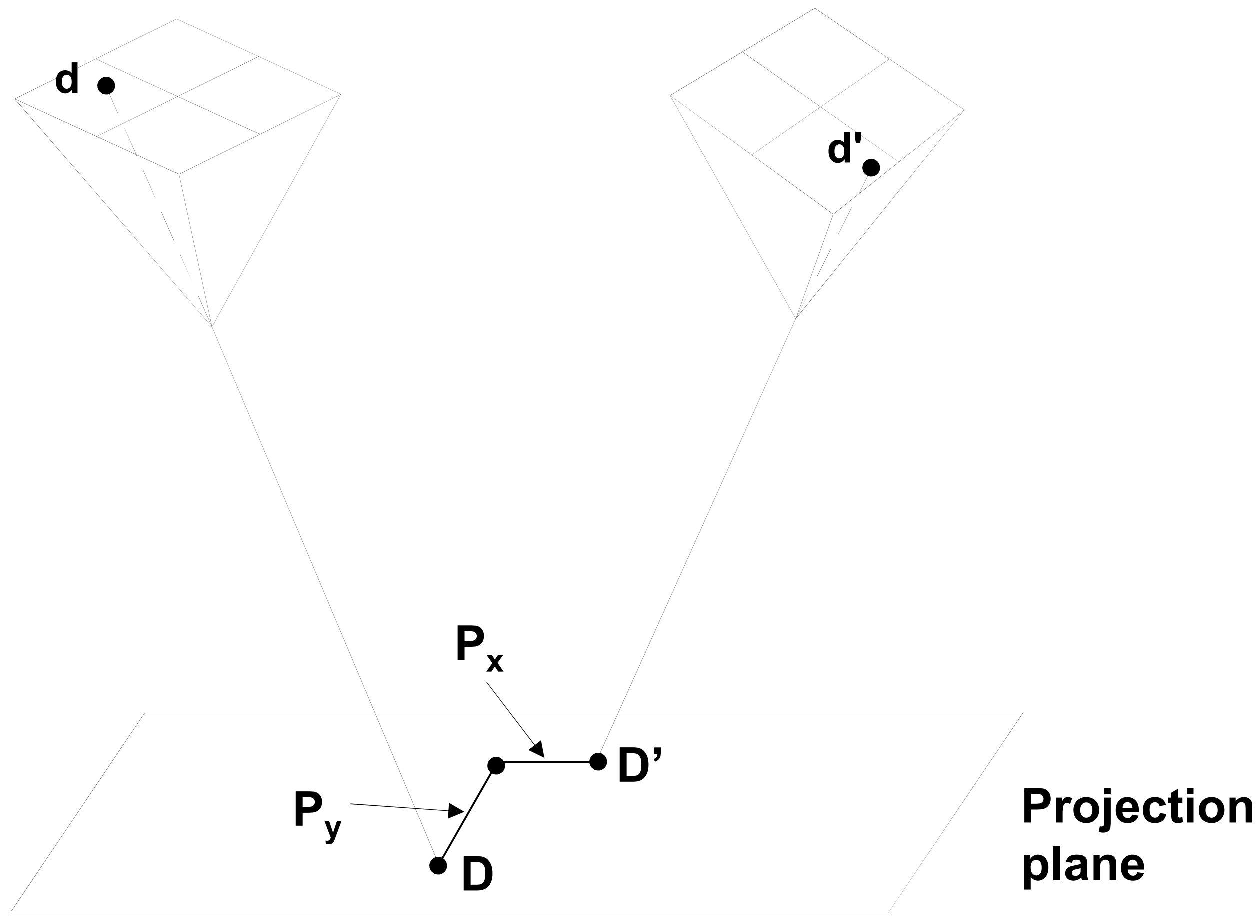
Relative Orientation – RO



Relative Orientation – RO



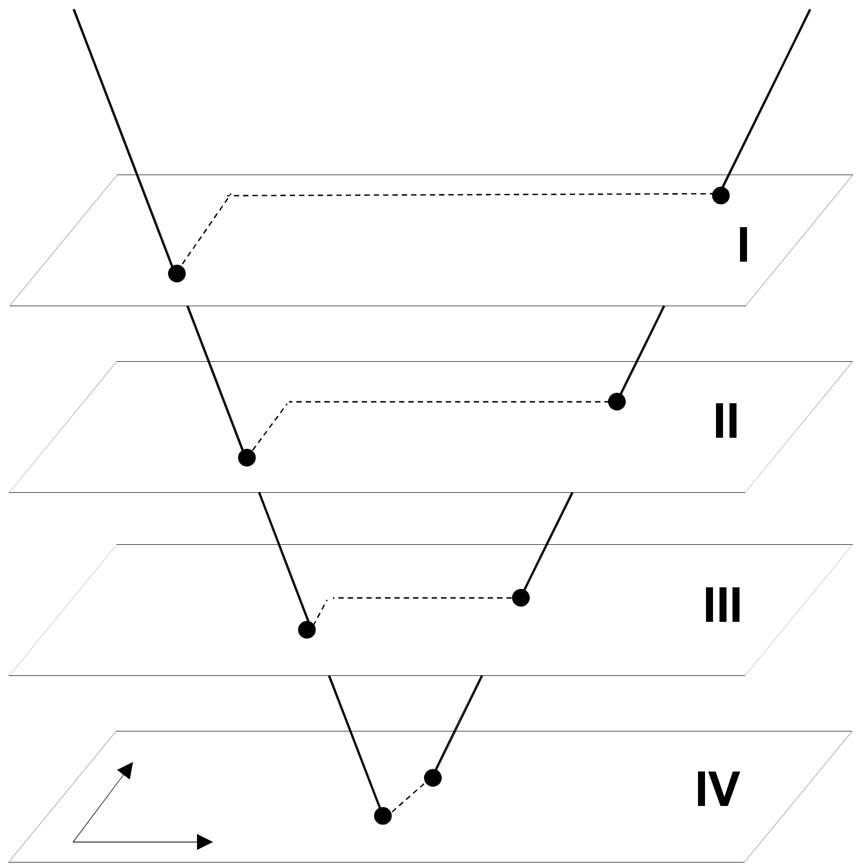
x-parallax & y-parallax



x-parallax & y-parallax

- Two conjugate light rays will be separated at any projection plane (DD`).
- This separation can be decomposed into:
  - x-parallax (along the base connecting the two perspective centers), and
  - y-parallax (along the perpendicular direction).
- x-parallax is responsible for depth perception.
- During relative orientation, we clear the y-parallax.

x-Parallax & Height



Relative Orientation of a Stereo-Pair

- Question:
  - Do we need ground control points to establish the relative orientation?
- Answer:
  - No
- Question:
  - What do we need to establish the relative orientation?
- Answer:
  - Measure the image coordinates of at least five tie points (why)

Relative Orientation of a Stereo-Pair

- For a stereo-pair, we have twelve degrees of freedom to position and orient the two bundles of a stereo-pair in space.
  - X_{o_l}, Y_{o_l}, Z_{o_l}, ω_l, ϕ_l, κ_l
  - X_{o_r}, Y_{o_r}, Z_{o_r}, ω_r, ϕ_r, κ_r
- These twelve parameters establish the relative and absolute orientation of that stereo-pair.
- The absolute orientation establishes the ground coordinate system (datum).

Relative Orientation of a Stereo-Pair

- The ground coordinate system is defined by:
  - Origin (three parameters),
  - Orientation in space (three parameters), and
  - Scale (how long is one unit along the axes of this coordinate system – one parameter).
- For relative orientation, we can define any arbitrary coordinate system (model coordinate system).

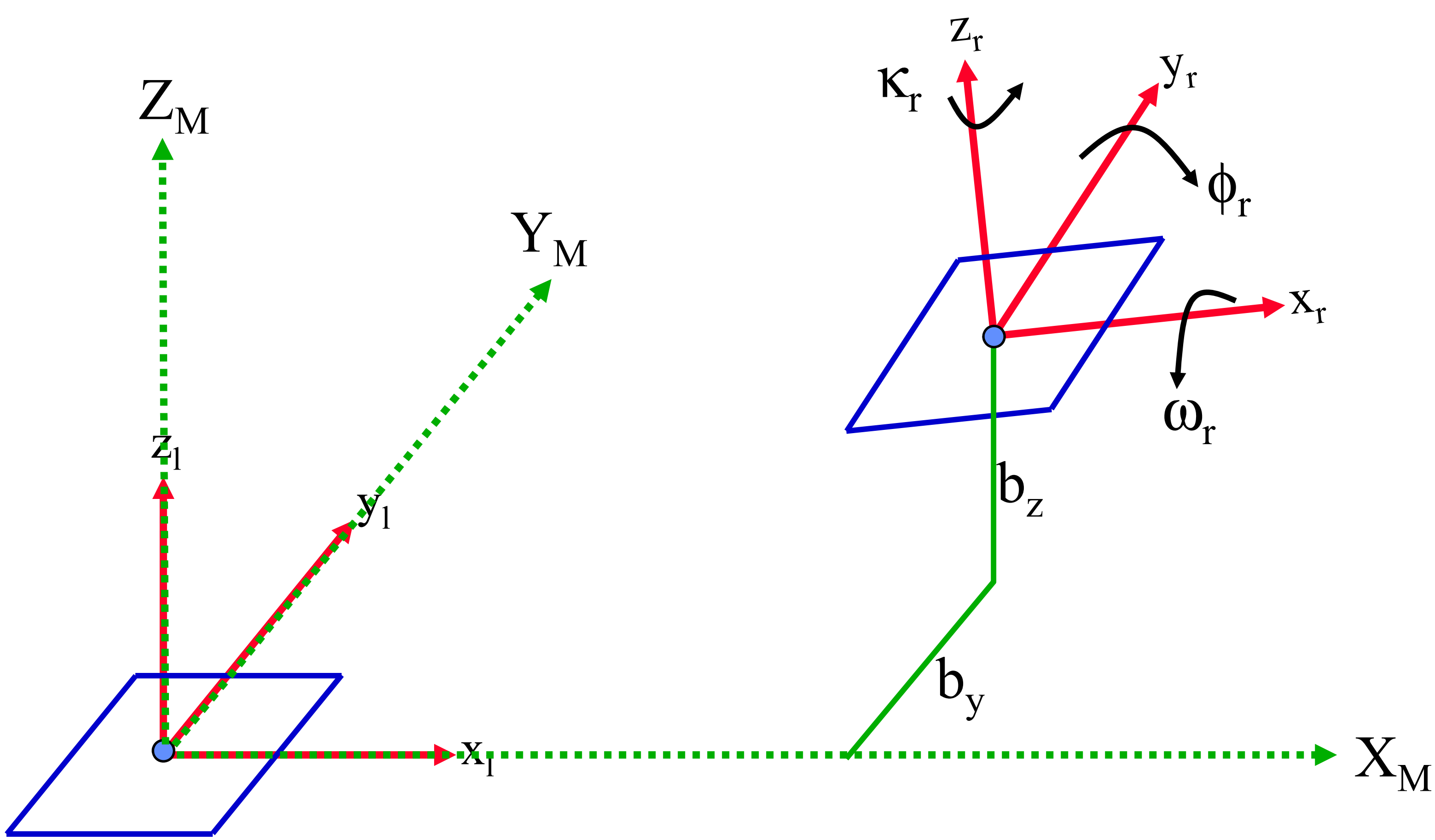
Relative Orientation of a Stereo-Pair

- Therefore, we need to fix seven parameters (out of the twelve) to any arbitrary values.
- During relative orientation, we solve for the remaining five parameters.
- Alternatives for Relative Orientation (RO) include:
  – Dependent Relative Orientation, and
  – Independent Relative Orientation.

Dependent Relative Orientation

- Fix the following parameters to any arbitrary values:
  - $X_{o_l}, Y_{o_l}, Z_{o_l}, \omega_l, \phi_l, \kappa_l, X_{o_r}$
- Solve for:
  - $Y_{o_r}, Z_{o_r}, \omega_r, \phi_r, \kappa_r \equiv b_y, b_z, \omega_r, \phi_r, \kappa_r$

Dependent Relative Orientation

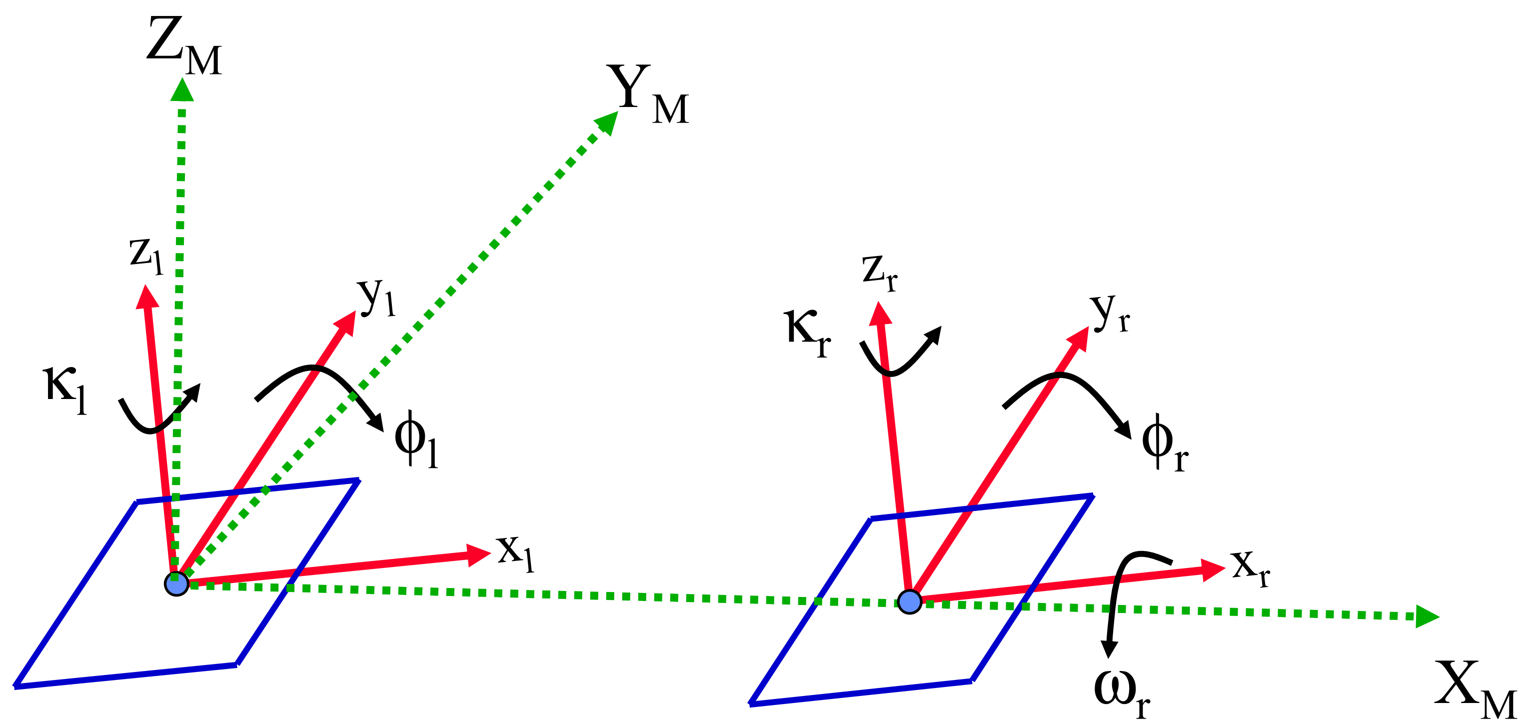


- Determine b_y, b_z, ω_r, ϕ_r, κ_r

Independent Relative Orientation

- Fix the following parameters to any arbitrary values:
  - X_{o_l}, Y_{o_l}, Z_{o_l}, ω_l, X_{o_r}, Y_{o_r}, Z_{o_r}
- Solve for:
  - ϕ_l, κ_l, ω_r, ϕ_r, κ_r

Independent Relative Orientation



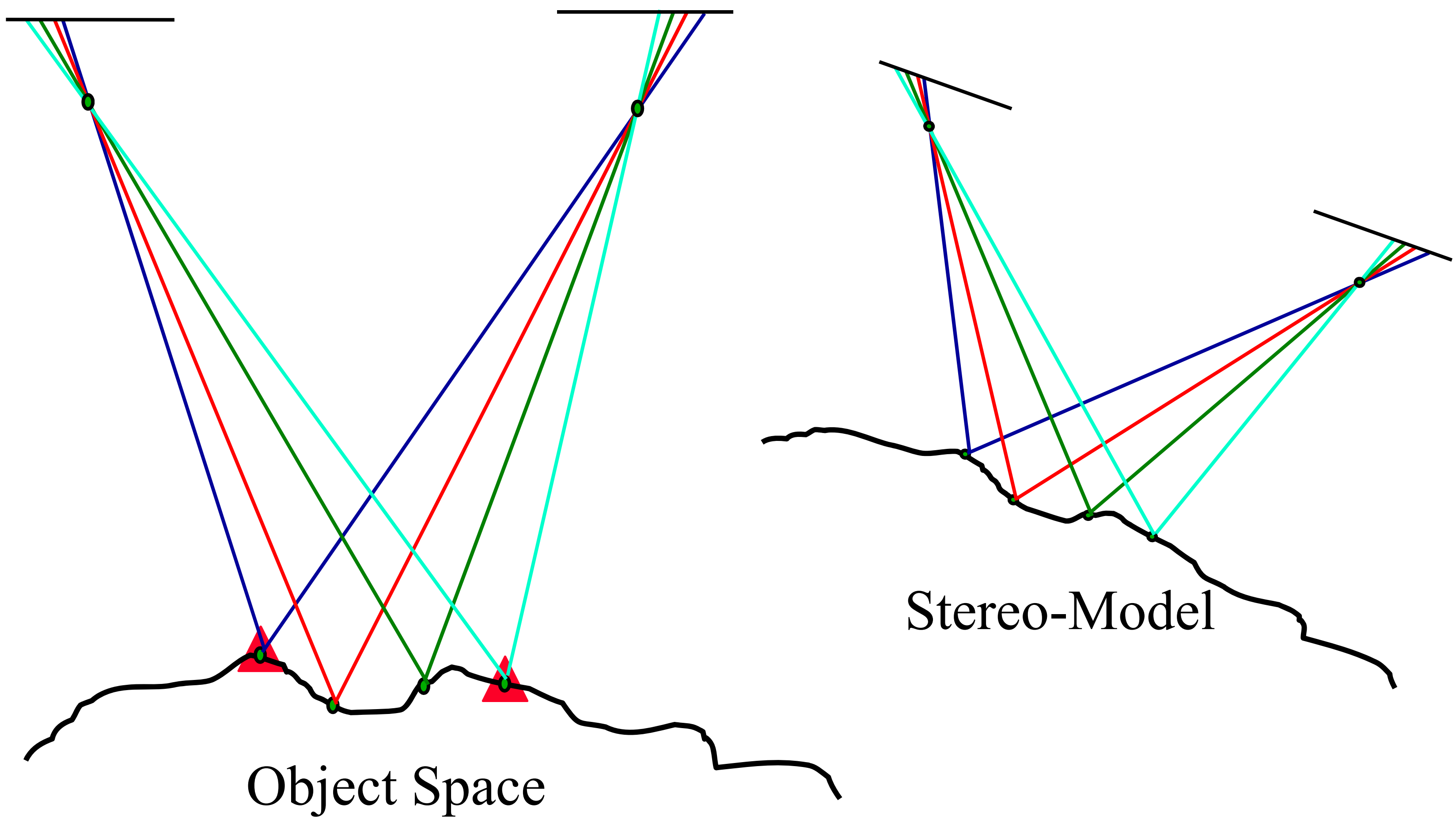
- Determine ϕ_l, κ_l, ω_r, ϕ_r, κ_r

Relative Orientation: Coplanarity Model

- Another alternative for estimating the relative orientation among the images of a stereo-pair is the Coplanarity model.
- Refer to Chapter 8 – Addendum 1 for:
  - The mathematical model for the Coplanarity Equation
  - The Computer Vision (CV) equivalent of the Coplanarity Equation
    - Fundamental Matrix
    - Essential Matrix

Absolute Orientation

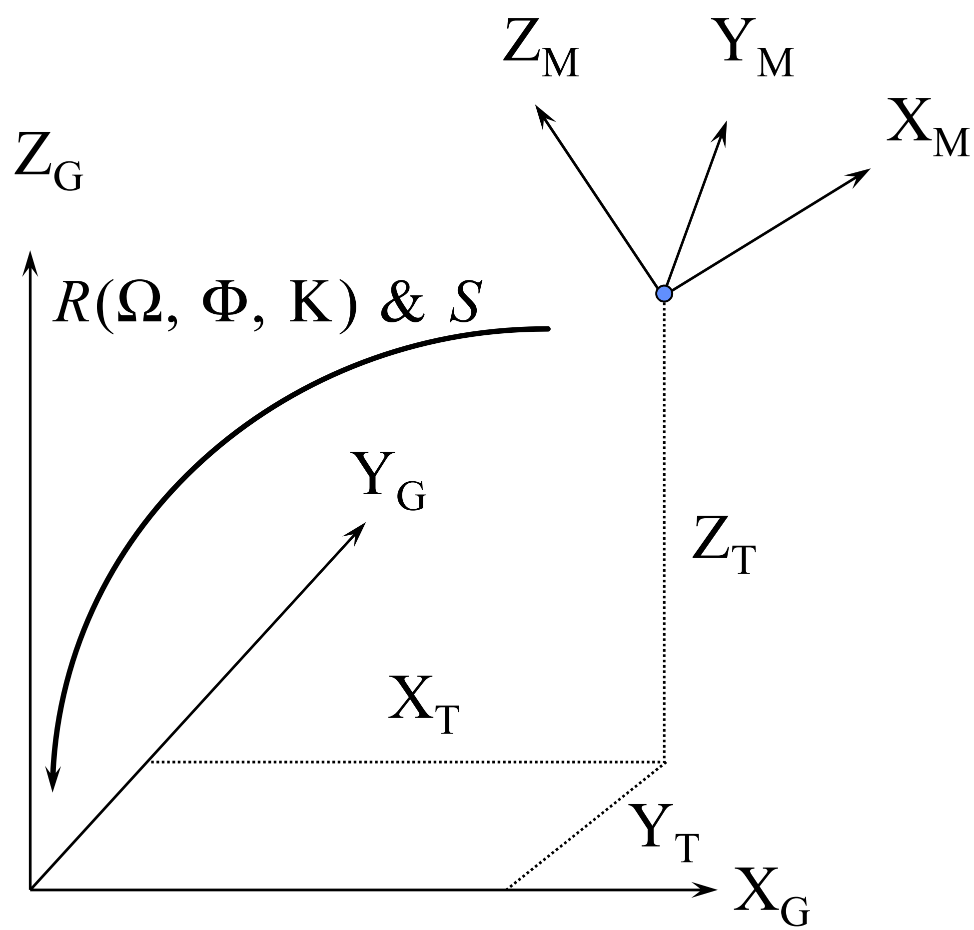
Absolute Orientation



Absolute Orientation

- Purpose: Rotate, scale, and shift the stereo-model (resulting from relative orientation) until it fits at the location of control points
- Absolute orientation is defined by:
  - Three rotations,
  - One scale factor, and
  - Three shifts.
- The absolute orientation is described mathematically by a 3-D similarity transformation.

Absolute Orientation



Absolute Orientation: Mathematical Model

$$\begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} = \begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} + S \; R(\Omega, \Phi, \mathrm{K}) \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix}$$

- A control point, with known ground and model coordinates, yields three equations.
- A minimum of three control points (two full and one vertical) should be available to solve for the seven parameters of the absolute orientation.
- These points should not be collinear.
  - In this case, the roll angle across the line defined by the control points cannot be determined.

Photogrammetric Triangulation

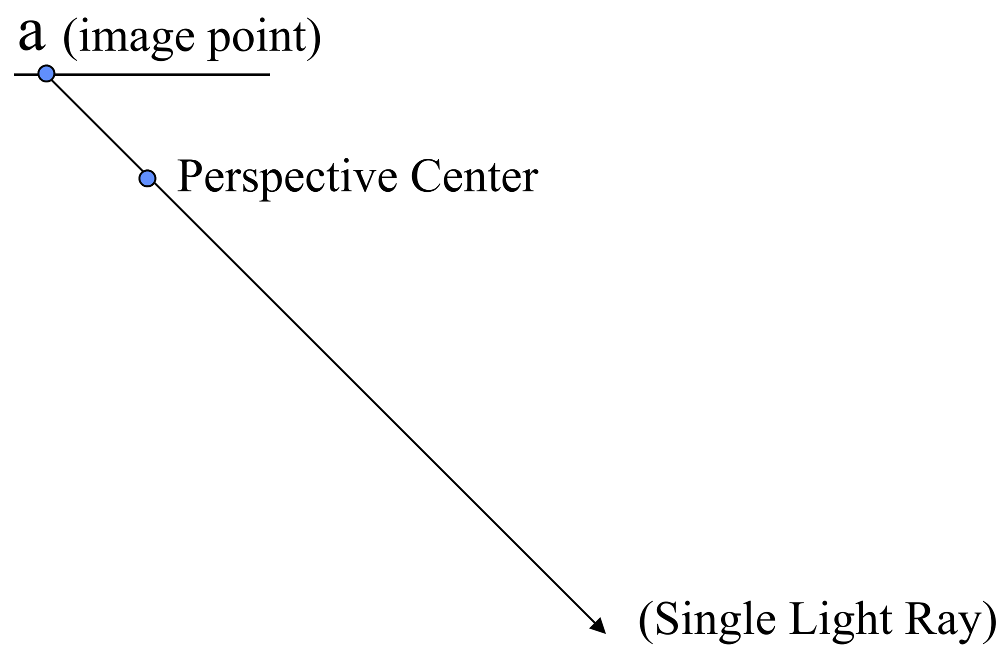
Photogrammetry: The Reconstruction Process

- The main objective of photogrammetry is to derive ground coordinates of object points from imagery.
- We would like to investigate the feasibility of performing this task using single image, stereo-pair, or more.
- The mathematical model we are going to use is the collinearity equations.

Single Photograph

- An image point is defined by its coordinates relative to the image coordinate system.
- We have two equations (collinearity equations) in three unknowns (ground coordinates of the corresponding object point – Best Case Scenario).
  - IOPs & EOPs are available.
- Consequently, this problem is under determined.
- Conceptually: An image point will define a single (infinite) light ray.
  - The object point can be anywhere along this light ray.

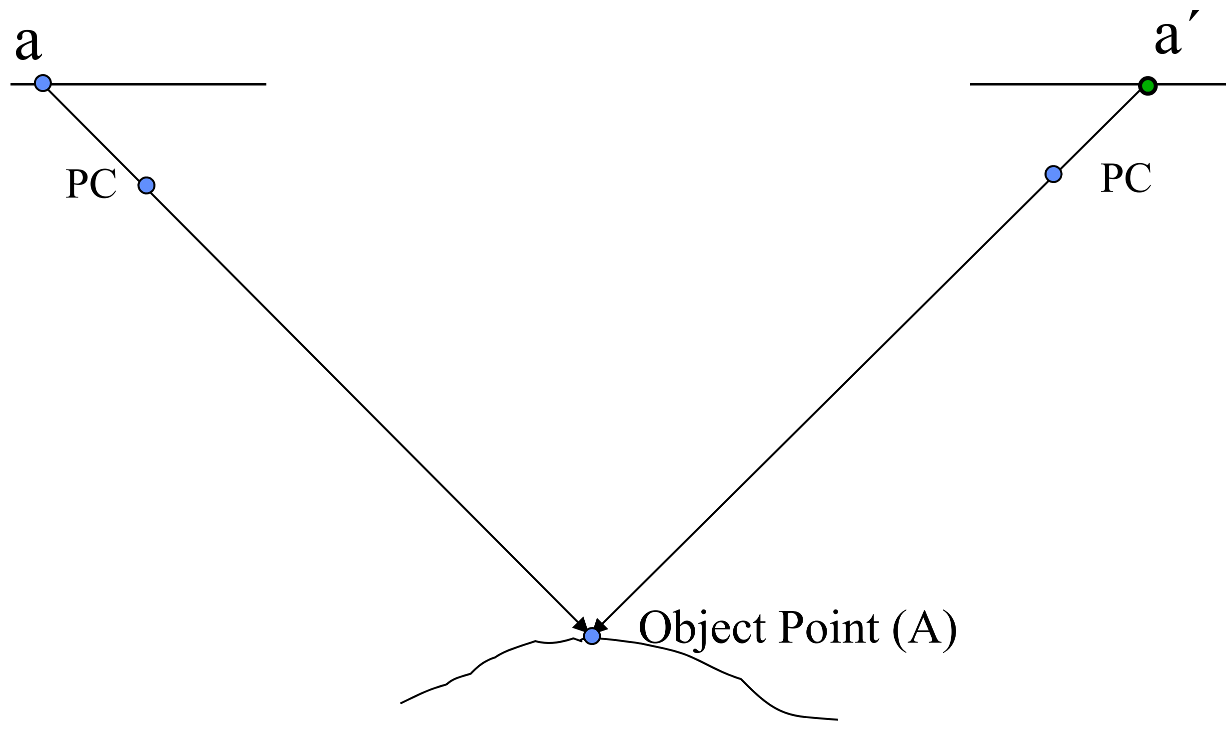
Single Photograph



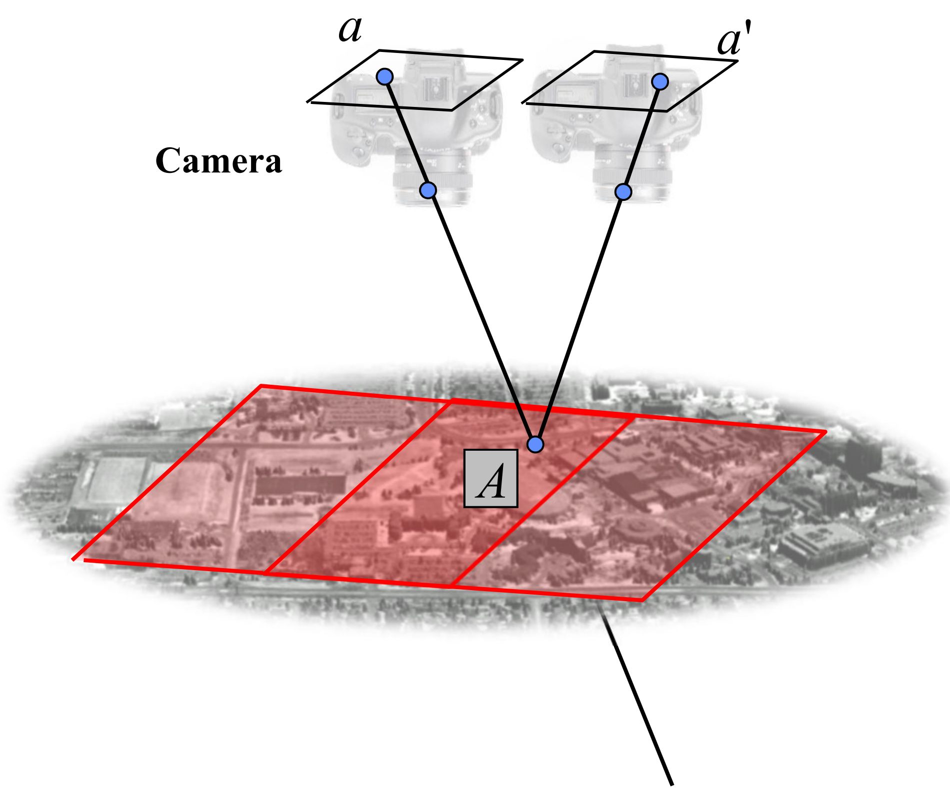
Stereo Photogrammetry

- We have the same point appearing in two images.
- We have four equations (two collinearity equations in each image).
- We have three unknowns (ground coordinates of the corresponding object points – Best Case Scenario).
  - IOPs & EOPs are available.
- Thus, we have a redundancy of one (which will contribute towards the computation of the ROP).
- Conceptually: The two image points define two light rays.
  - The object point is the intersection of these light rays.

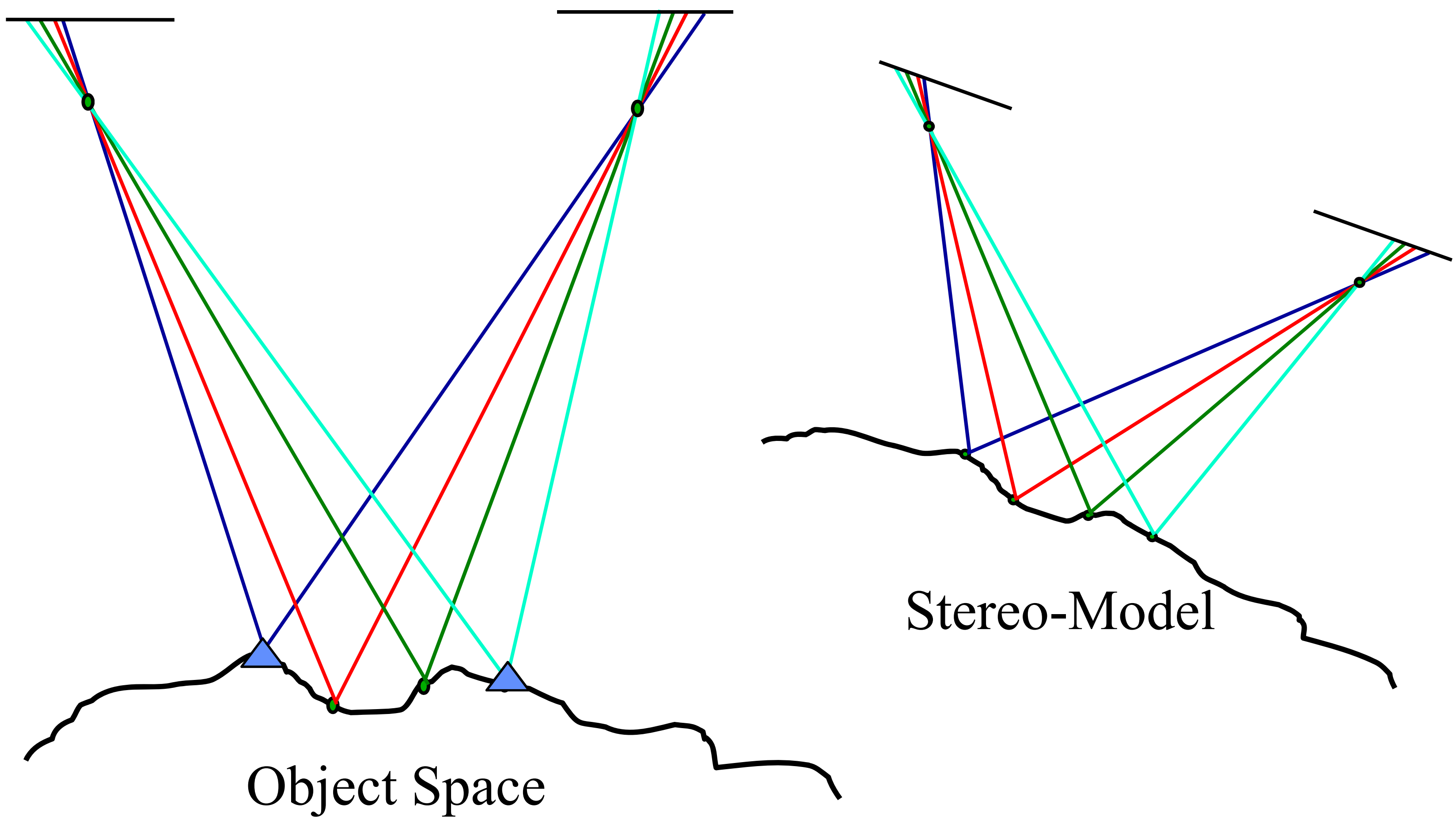
Stereo Photogrammetry



Stereo Photogrammetry



From Images to Object Space



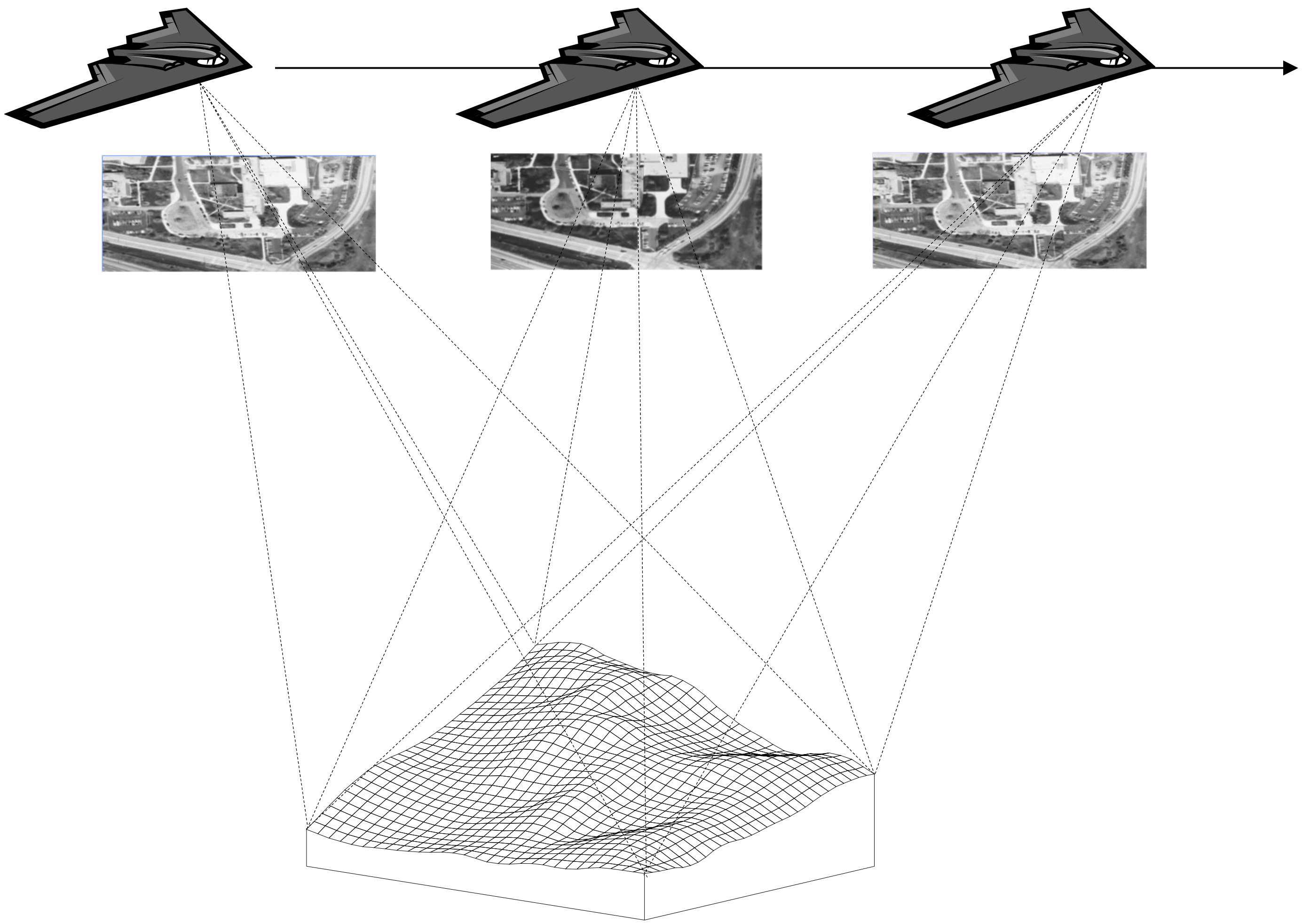
From Images to Object Space

- For a stereo-pair, we can derive a 3-D representation of the object space covered by the overlap area as follows:
  - Perform RO of the stereo-pair under consideration (using at least five conjugate/tie points) → stereo-model
  - Using some GCPs, we can rotate, scale, and shift the stereo-model (AO) until it fits at the location of the ground control points (i.e., the residuals at the GCPs are as small as possible).
  - For the absolute orientation, we need at least three GCPs (those points should not be collinear).

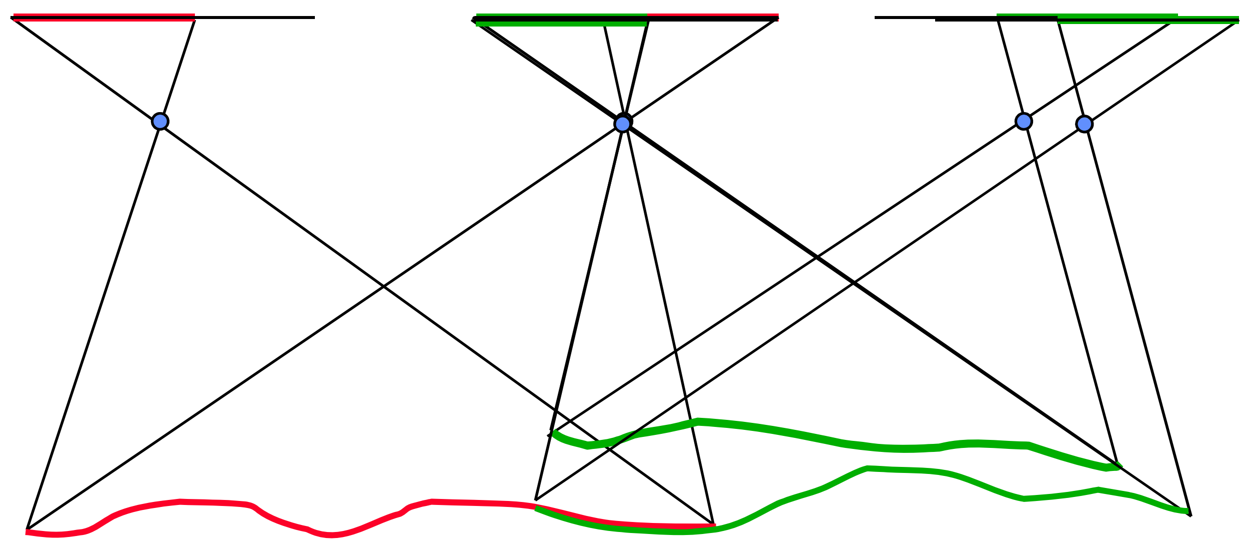
Aerial Triangulation

- Objective:
  - How can we reconstruct the object space from imagery without the need for three ground control points in each stereo-model?
- Alternatives:
  - Strip Triangulation,
  - Block Adjustment of Independent Models (BAIM), and
  - Bundle Block Adjustment (Covered in Chapter 7)

Strip Triangulation



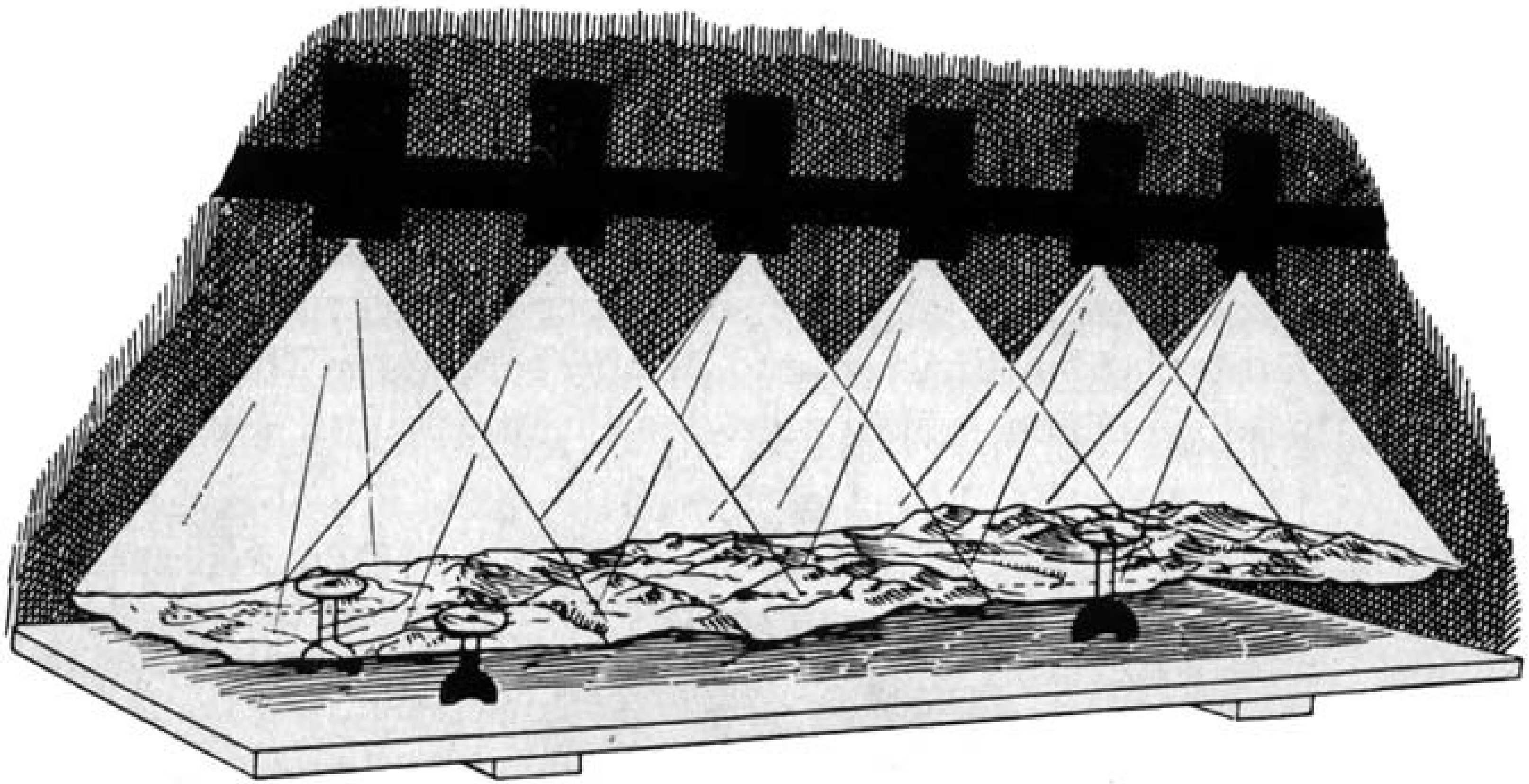
Strip Triangulation



Strip Triangulation

- Procedure:
  - Perform RO of the first stereo-model (DRO or IRO)
  - Perform AO of the first stereo-model using 3GCPs
  - Perform RO for the second-model using only DRO
    - We do not want to disturb the orientation parameters for the second image.
  - Only scale is required to establish the AO for the second model.
  - Adjust the base until a point in the overlap area between the first and second models have the same elevation
  - For the remaining models, repeat the above three steps.

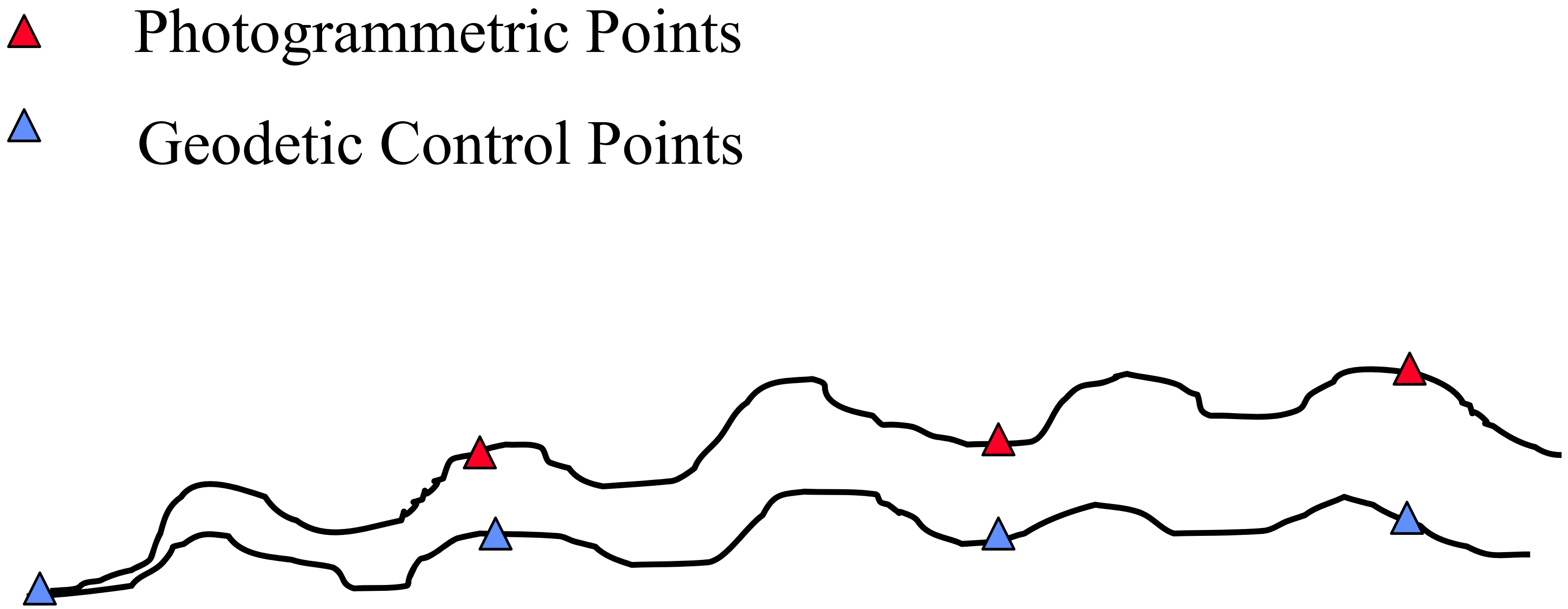
Strip Triangulation



Strip Triangulation

- Error Propagation:
  - Similar to an open traverse, errors will increase as the length of the strip increases.
- Solution:
  - Implement GCPs every three or four models, and
  - Apply corrections polynomials
- Correction Polynomials:
  - They are used to reduce the difference between the photogrammetric and geodetic coordinates of the control points.

Correction Polynomials



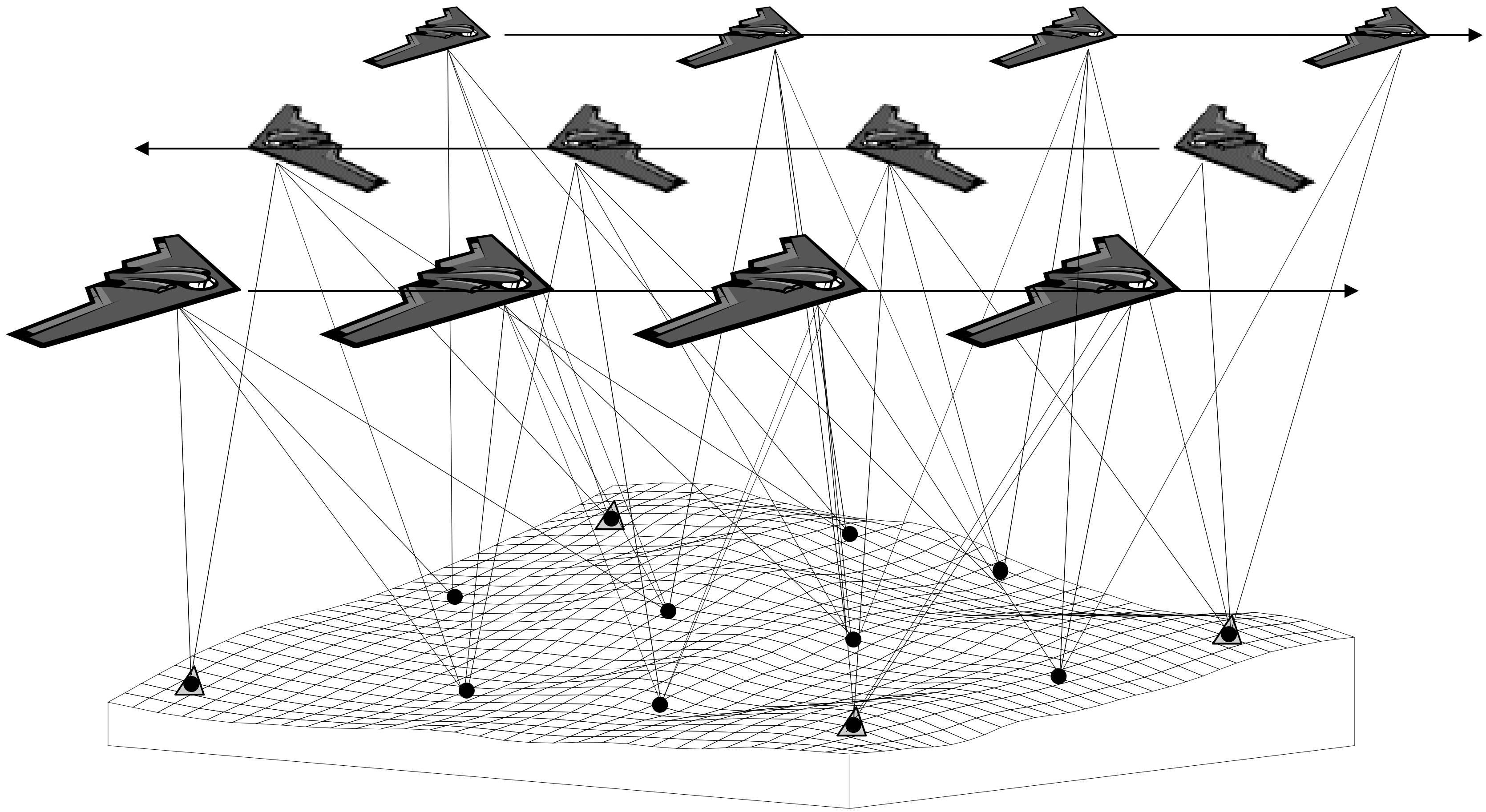
Correction Polynomials

$$Z_g - Z_p = a_o + a_1 \; X + a_2 \; Y + a_3 \; XY$$

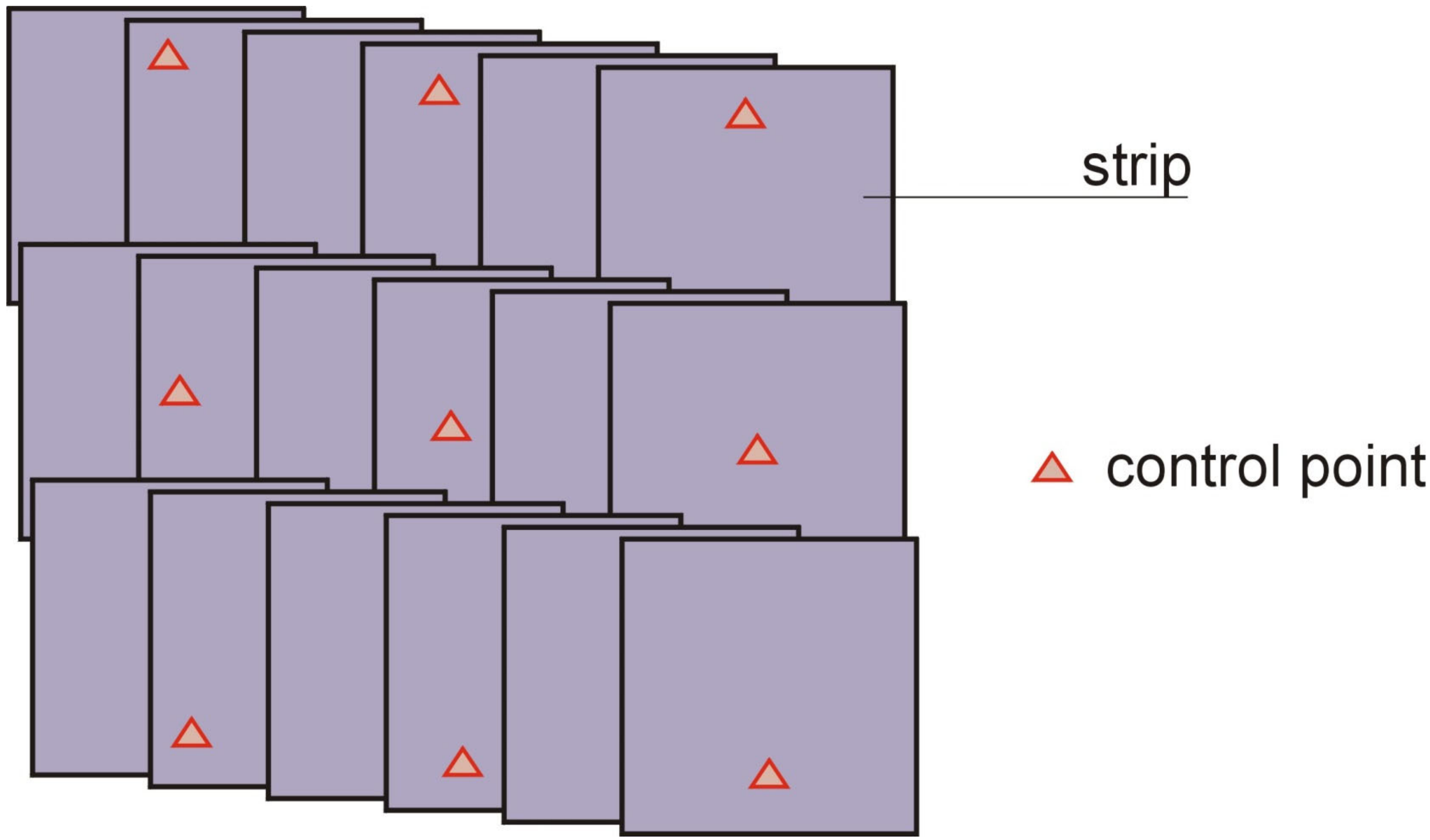
- Zg $\equiv$ Geodetic Ground Coordinates
- Zp $\equiv$ Photogrammetric Ground Coordinates

- Using ground control points, we can solve for the polynomial coefficients.

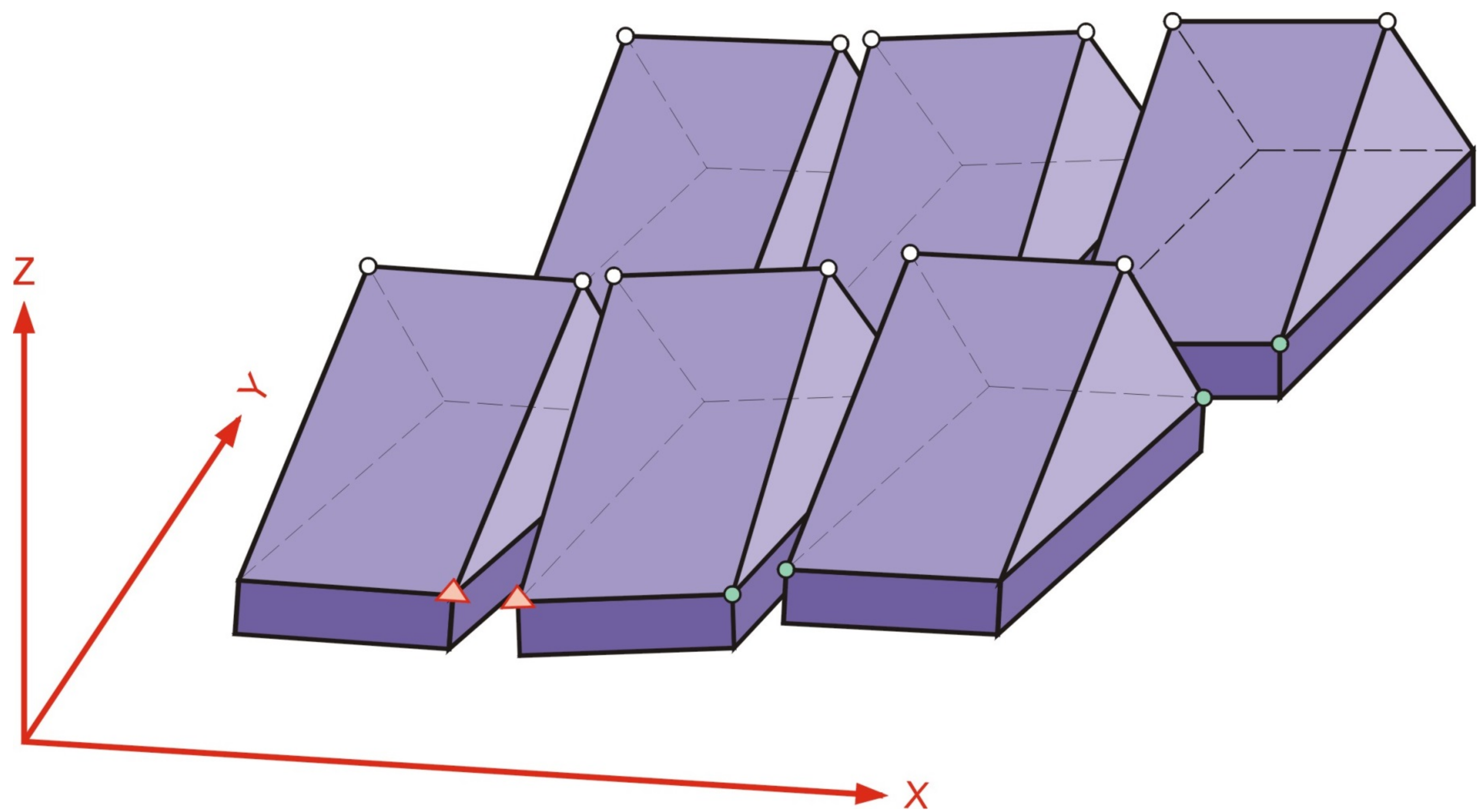
Block Adjustment



Block Adjustment



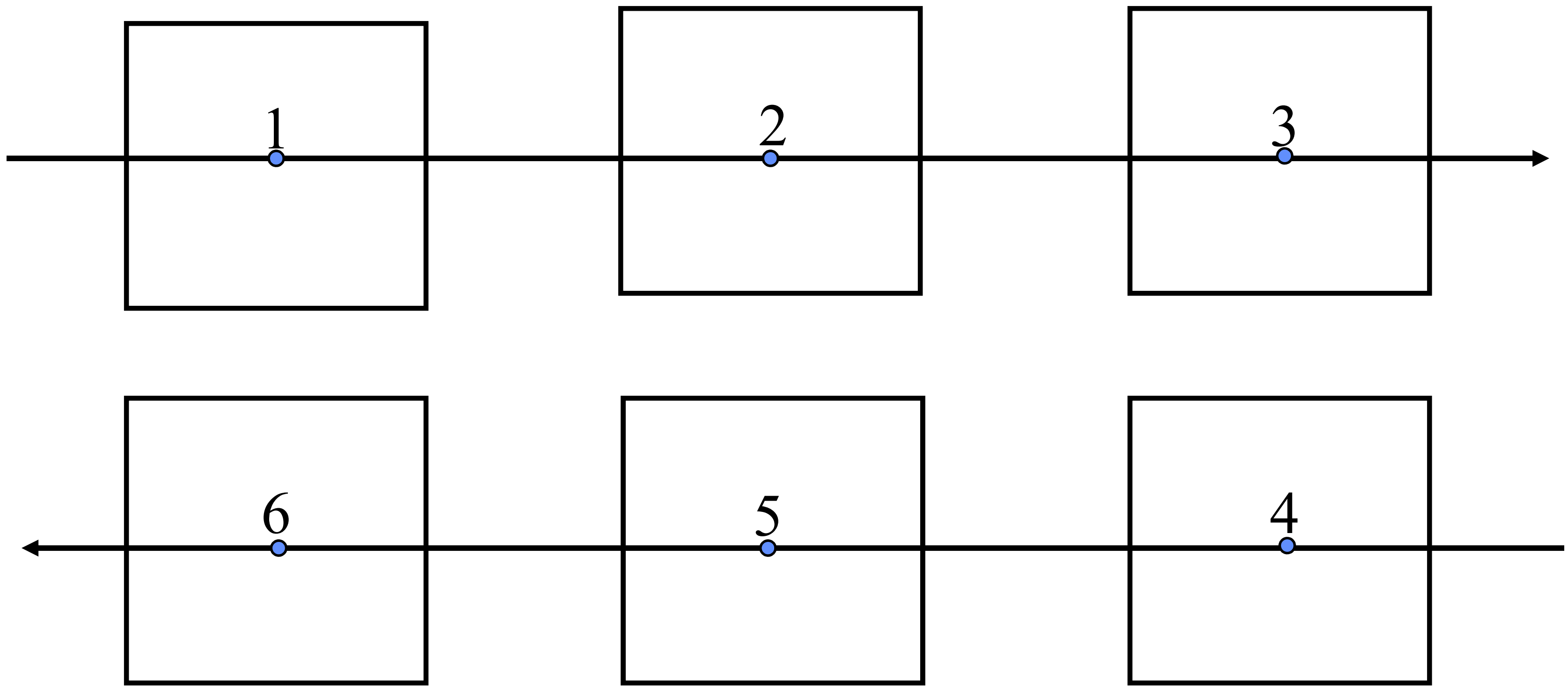
Block Adjustment of Independent Models



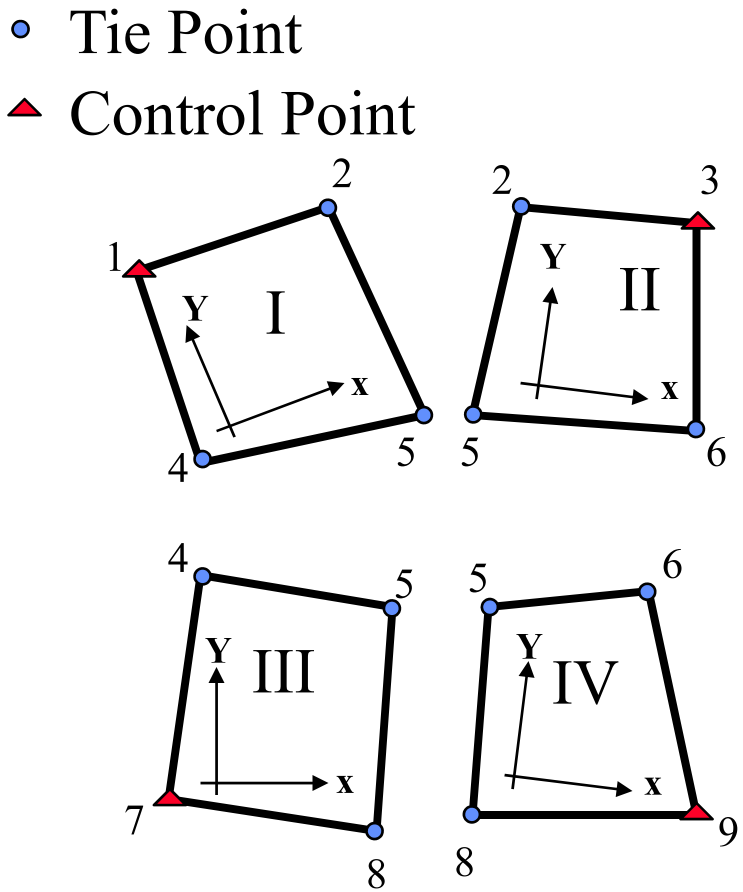
Block Adjustment of Independent Models

- Procedure:
  - Perform RO (DRO or IRO) for all the stereo-models in the project
  - The models are simultaneously rotated, scaled, and shifted until:
    - The tie points among adjacent models fit together as well as possible, and
    - The residuals at the locations of the ground control points are as small as possible.

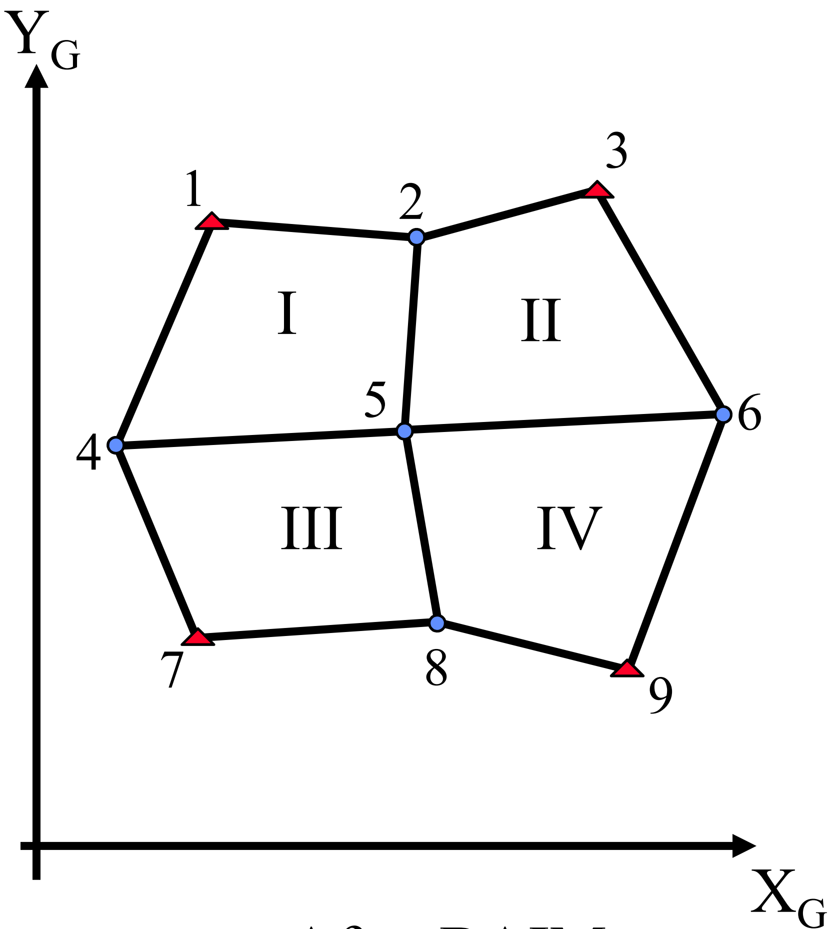
BAIM: Concept



BAIM: Concept

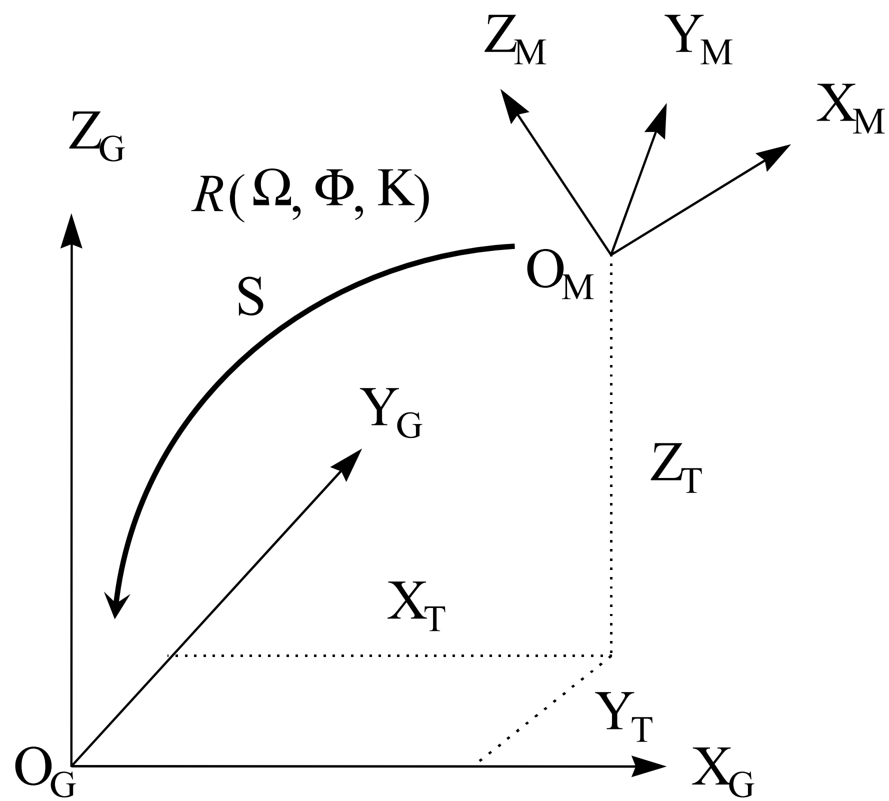


Before BAIM



After BAIM

Mathematical Model (BAIM)



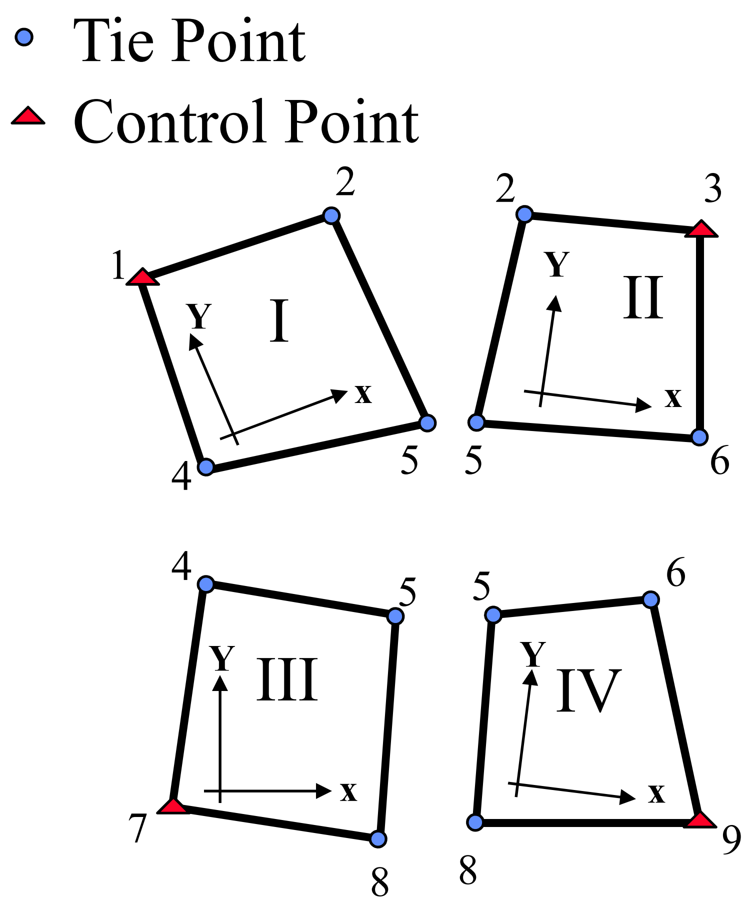
Mathematical Model (BAIM)

$$\begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} = \begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} + S \; R(\Omega,\Phi,\mathrm{K}) \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix}$$

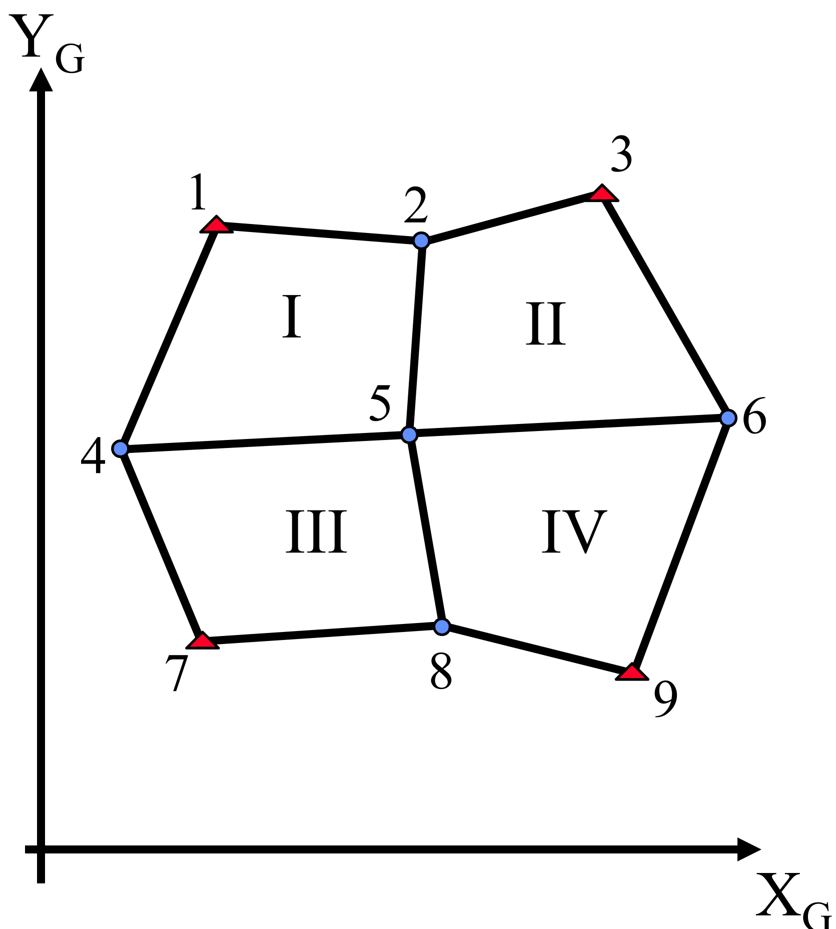
Mathematical Model (BAIM)

$$\begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = 1/S \;\; R^T(\Omega,\Phi,K) \begin{bmatrix} X_G - X_T \\ Y_G - Y_T \\ Z_G - Z_T \end{bmatrix}$$

Example (BAIM)



Before BAIM



After BAIM

Example (BAIM)

- What is the balance between the observations and unknowns?
  - Unknowns:
    - 4 * 7 AO parameters
    - 5 * 3 GC of tie points
    - Total of 43
  - Equations:
    - 4 (models) x 4 (points/model) x 3 (equations/point) = 48
  - Redundancy:
    - 5
- This is a non-linear system: Approximations and partial derivatives are required.

BAIM: Adjustment Model

- $Y = a(X) + e \qquad e \sim (0, \sigma^2 P^{-1})$
- Using approximate values for the unknown parameters (X^o) and partial derivatives, the above equations can be linearized leading to the following equations:
- $y_{48x1} = A_{48x43}\, x_{43x1} + e_{48x1} \qquad e \sim (0, \sigma^2 P^{-1})$
- Assignment: Illustrate the structure of
  - The design matrix and the observation vector, and
  - The normal equation matrix (N) and normal equation vector (c)

Bundle Block Adjustment

- Direct relationship between image and ground coordinates
- We measure the image coordinates of selected features in the images of the block.
- Using the collinearity equations, we can relate the image coordinates, the corresponding ground coordinates, the IOPs, and the EOPs.
- Using simultaneous least squares adjustment, we can solve for the:
  - The ground coordinates of tie points,
  - The EOPs, and
  - The IOPs – if needed (Bundle Adjustment with Self Calibration – BASC).

Bundle Block Adjustment

- The image coordinate measurements and the IOPs define a bundle of light rays.
- The EOPs define the position and the attitude of the bundles in space.
- During the adjustment: The bundles are individually rotated (ω, ϕ, κ) and shifted (X_o, Y_o, Z_o) until:
  - Conjugate light rays intersect as well as possible at the locations of object space tie points, and
  - Light rays corresponding to ground control points pass through the object points as close as possible to the measured ground coordinates.

Bundle Block Adjustment