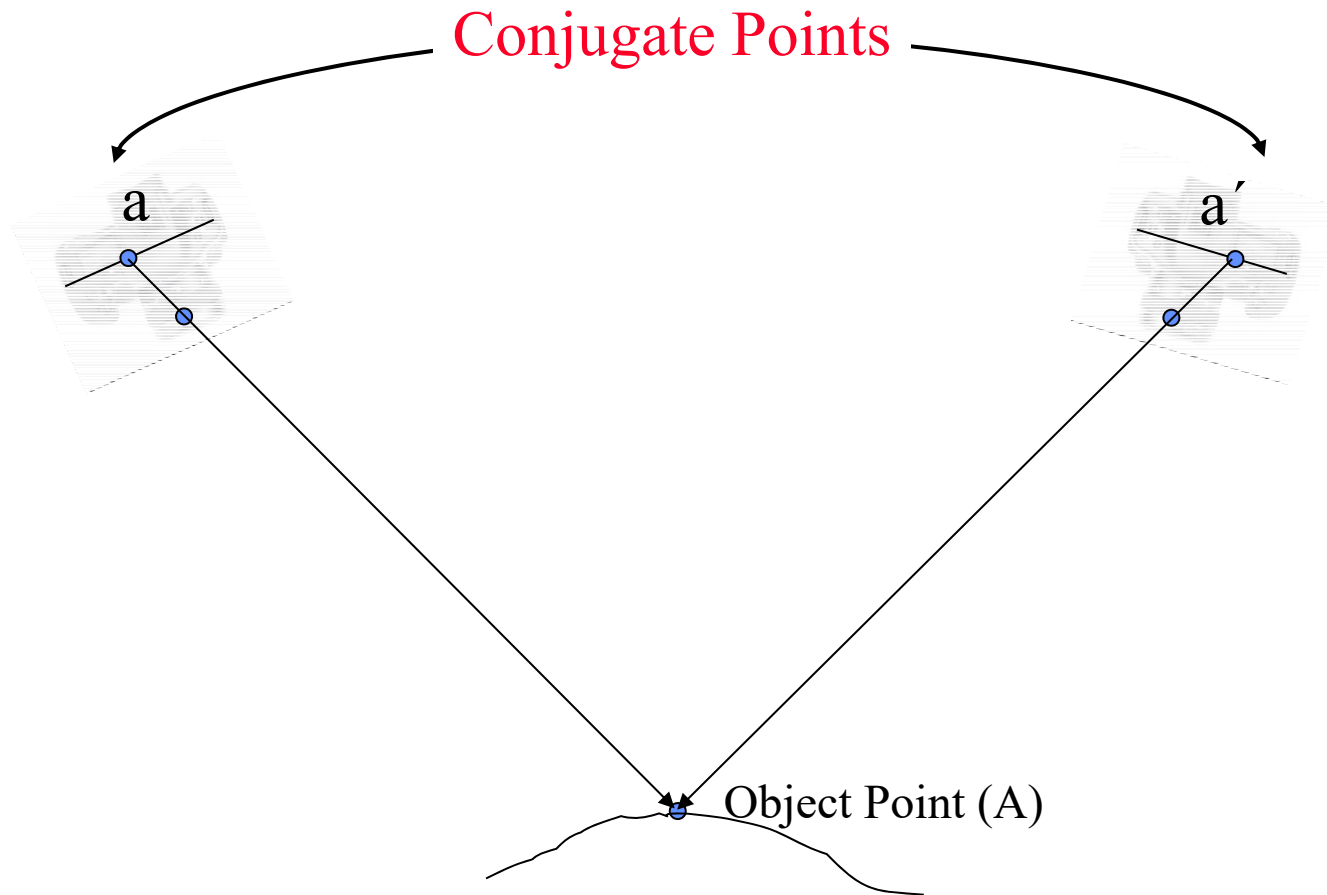


# Relative Orientation: Applications

Image Resampling According Epipolar  
Geometry

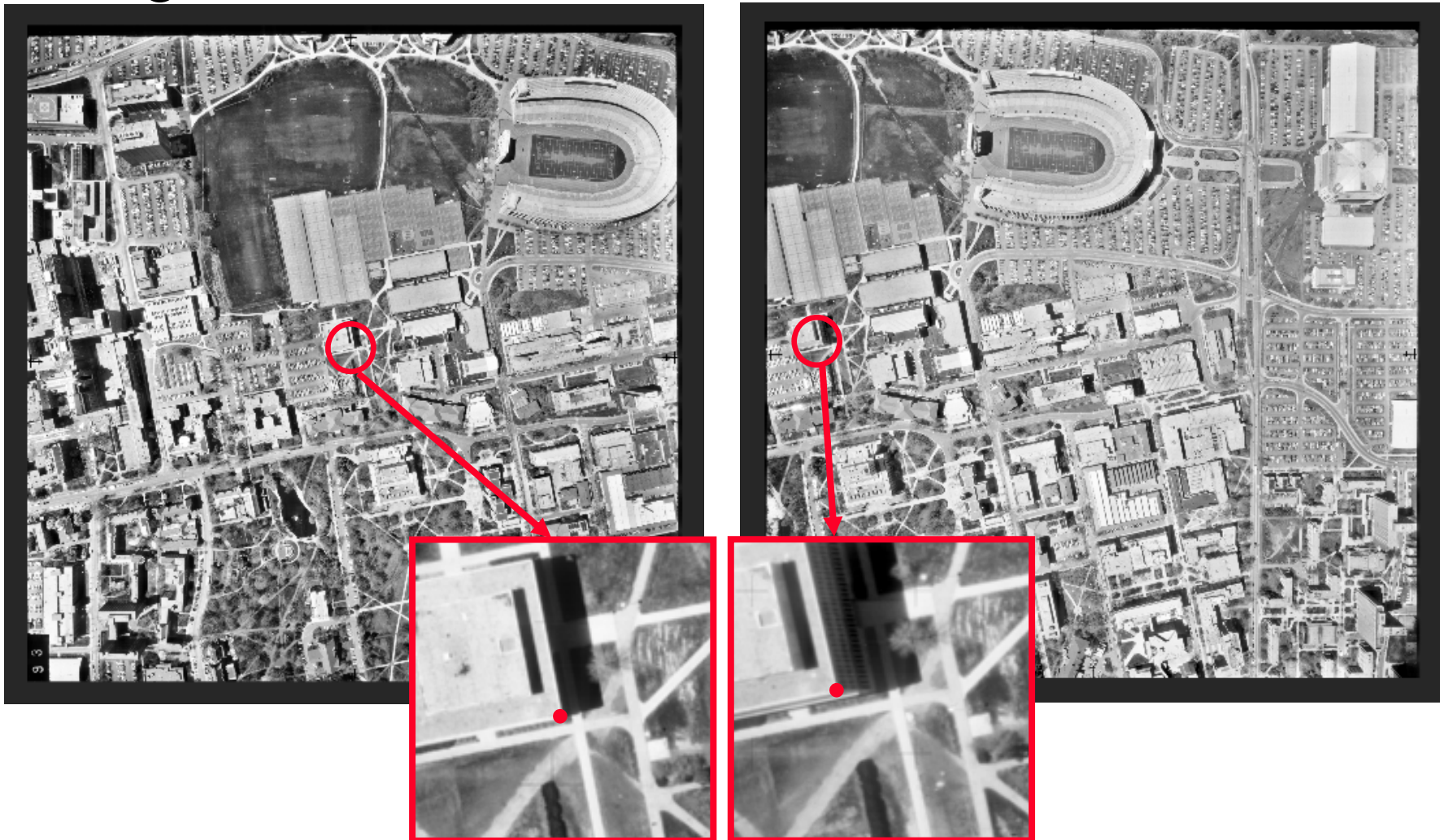
# 3D Reconstruction



- The position and orientation of each camera station have to be known.

# Image Matching

- How can we determine conjugate points in overlapping images?



# Terminology

- **Matching entity:**
  - The primitive that is being compared with similar primitives in overlapping images to find conjugate entities
- **Conjugate entities:**
  - Corresponding entities which are images of object space features such as points, lines, and areas
- **Similarity measure:**
  - A quantitative measure of how well the matching entities correspond to each other

# Terminology

- Matching method:
  - Evaluate the similarity measure between the matching entities.  
Possible options include:
    - Area-based matching,
    - Feature-based matching, and
    - Relational (symbolic) matching.
- Matching strategy:
  - Refers to the concept or overall scheme of the solution to the matching problem. This entails the analysis of the matching environment, the selection of the matching method, and the quality control of the matching results.

# Problem Statement

- The problem of image matching and surface reconstruction can be summarized as follows:
  - Select a matching entity in one image,
  - Find the conjugate entity in the overlapping image,
  - Assess the quality of the match, and
  - Compute the three-dimensional location of the matched entity in the object space.
- The second task is the most difficult to solve and is a major research area in the Photogrammetric and Computer Vision communities.

# Automatic Determination of Conjugate Points

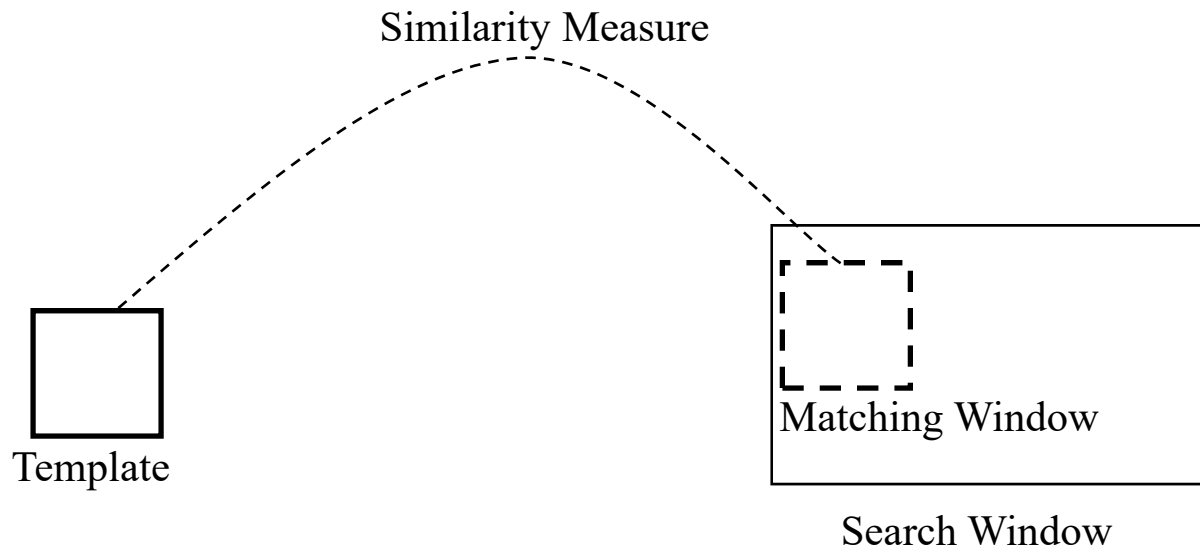
Left Image



Right Image

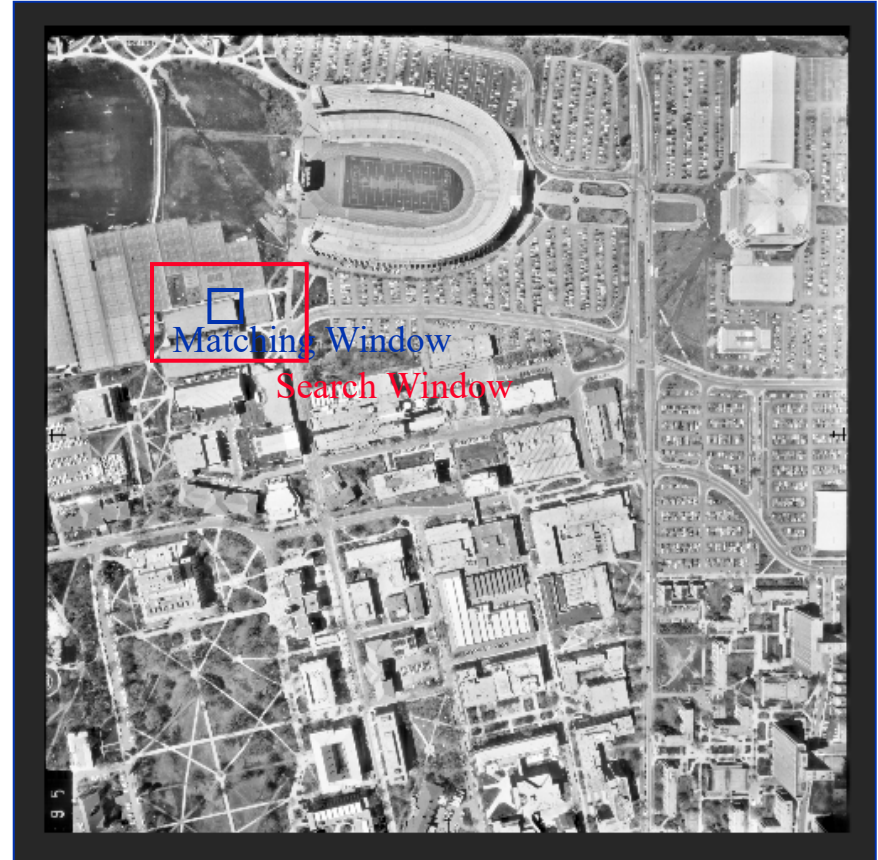
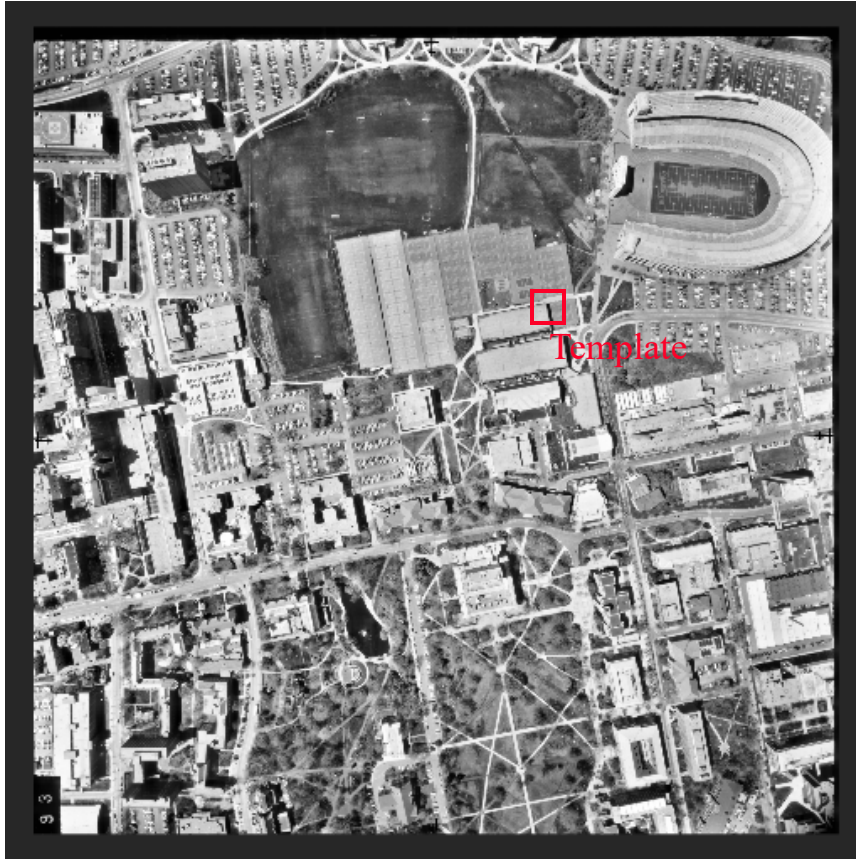


# Image Matching





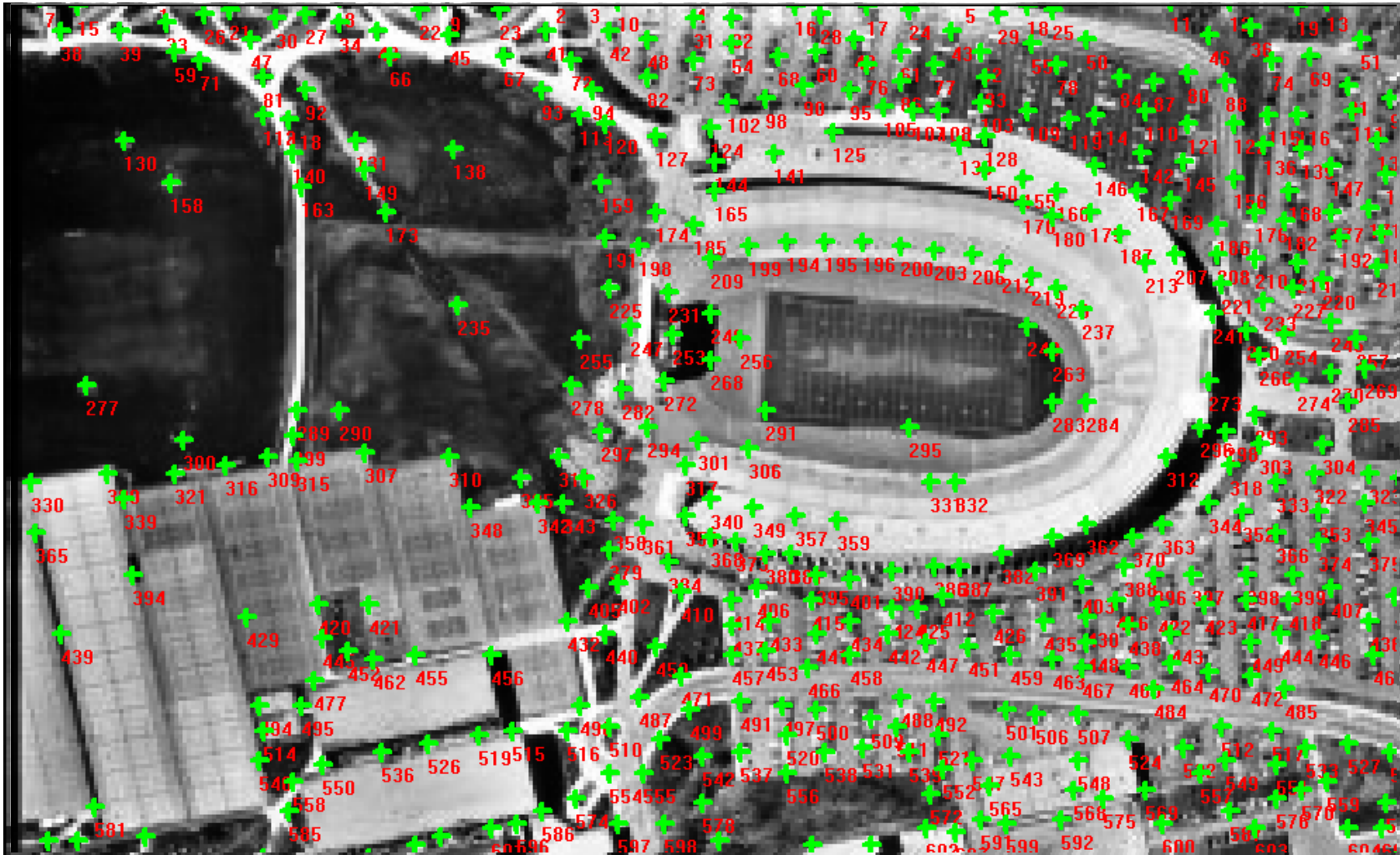
# Image Matching



# Fundamental Issues in Image Matching

- Search Space:
  - How can we determine the location (the center) and the size of the search space?
  - One should reduce the size of the search space to avoid combinatorial explosion, which will be the case if the search space covers the whole image.
- The uniqueness of the Matching Entity:
  - The matching entity should be unique to avoid ambiguities in the matching process.
  - Interest points are used to avoid this problem.

# Interest Points



# Automatic Determination of Conjugate Points

Left Image



Right Image



# Automatic Determination of Conjugate Points

Left Image



# Automatic Determination of Conjugate Points

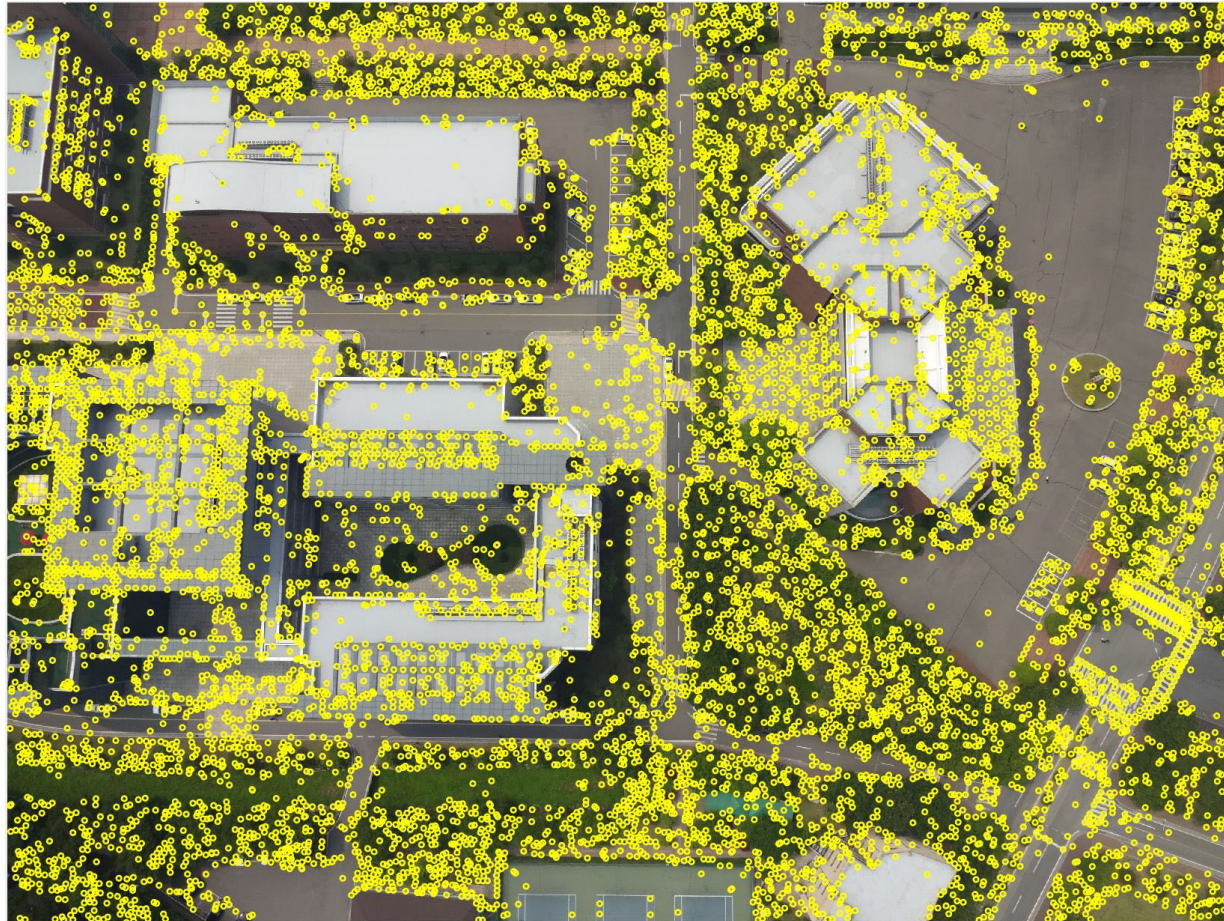
Right Image



# Automatic Determination of Conjugate Points

Left Image

Interest  
Points (Key  
Points)



# Automatic Determination of Conjugate Points

Right Image

Interest  
Points (Key  
Points)

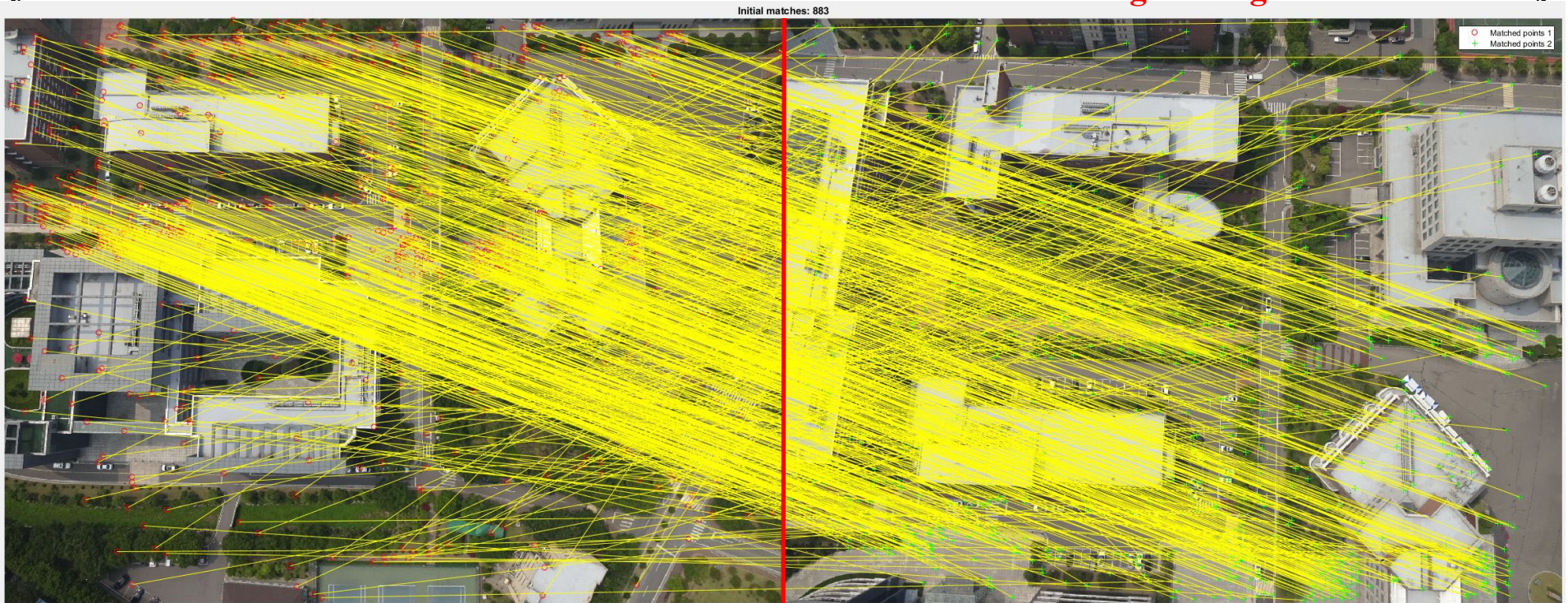




# Automatic Determination of Conjugate Points

**Left Image**

**Right Image**

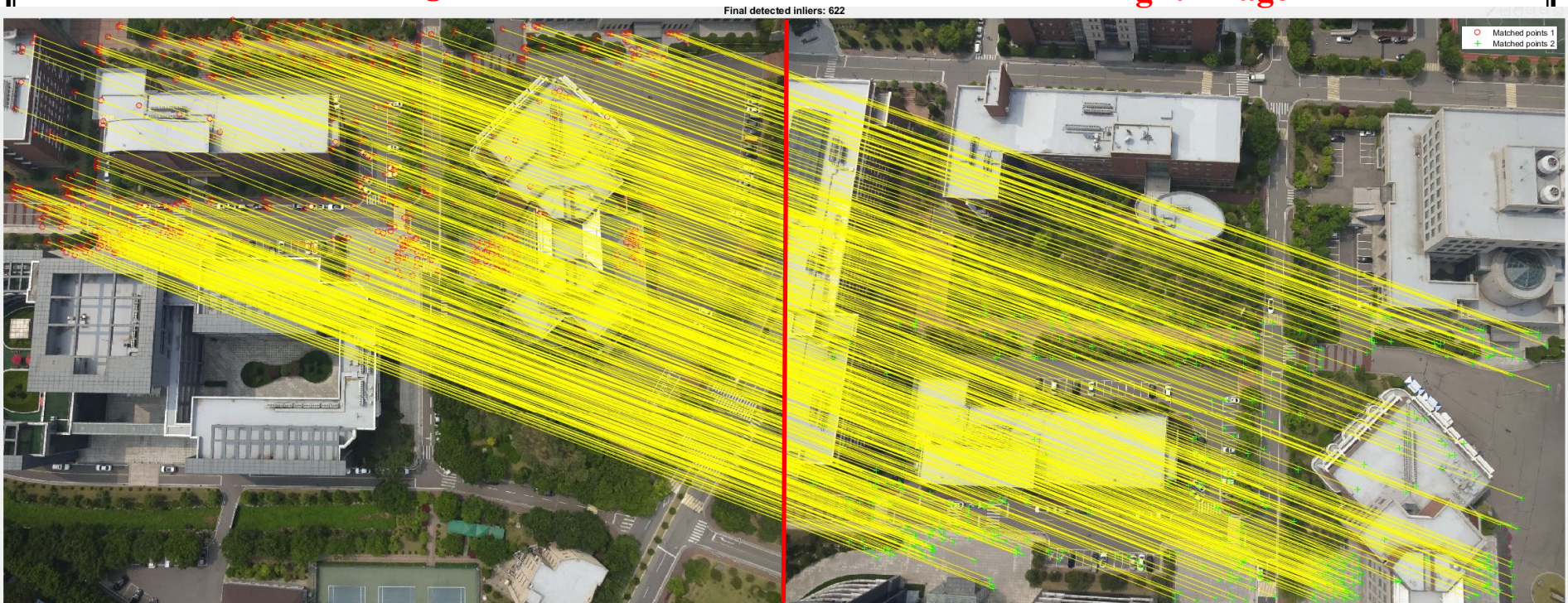


**Initial Matches**

# Automatic Determination of Conjugate Points

Left Image

Right Image



Final Matches

# Automatic Determination of Conjugate Points

**Left Image**

**Right Image**



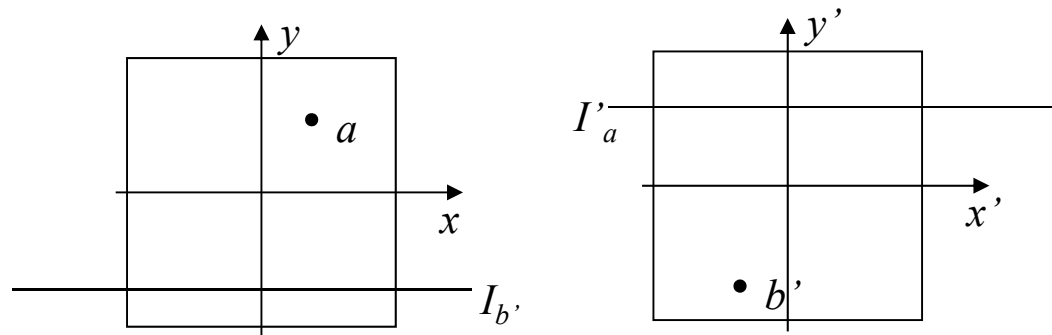
**Final Matches (Examples)**

# Normalized Image Generation

## Image Resampling According to Epipolar Geometry

# Normalized Image Generation

- Ensure that conjugate points in overlapping images have the same  $y$ -coordinates.
  - Reducing matching ambiguities
  - Reducing search space/time



- Important Applications:
  - Automatic matching
  - Automatic aerial triangulation
  - Orthophoto generation
  - Automatic DEM generation
  - Dense matching
  - Stereo viewing

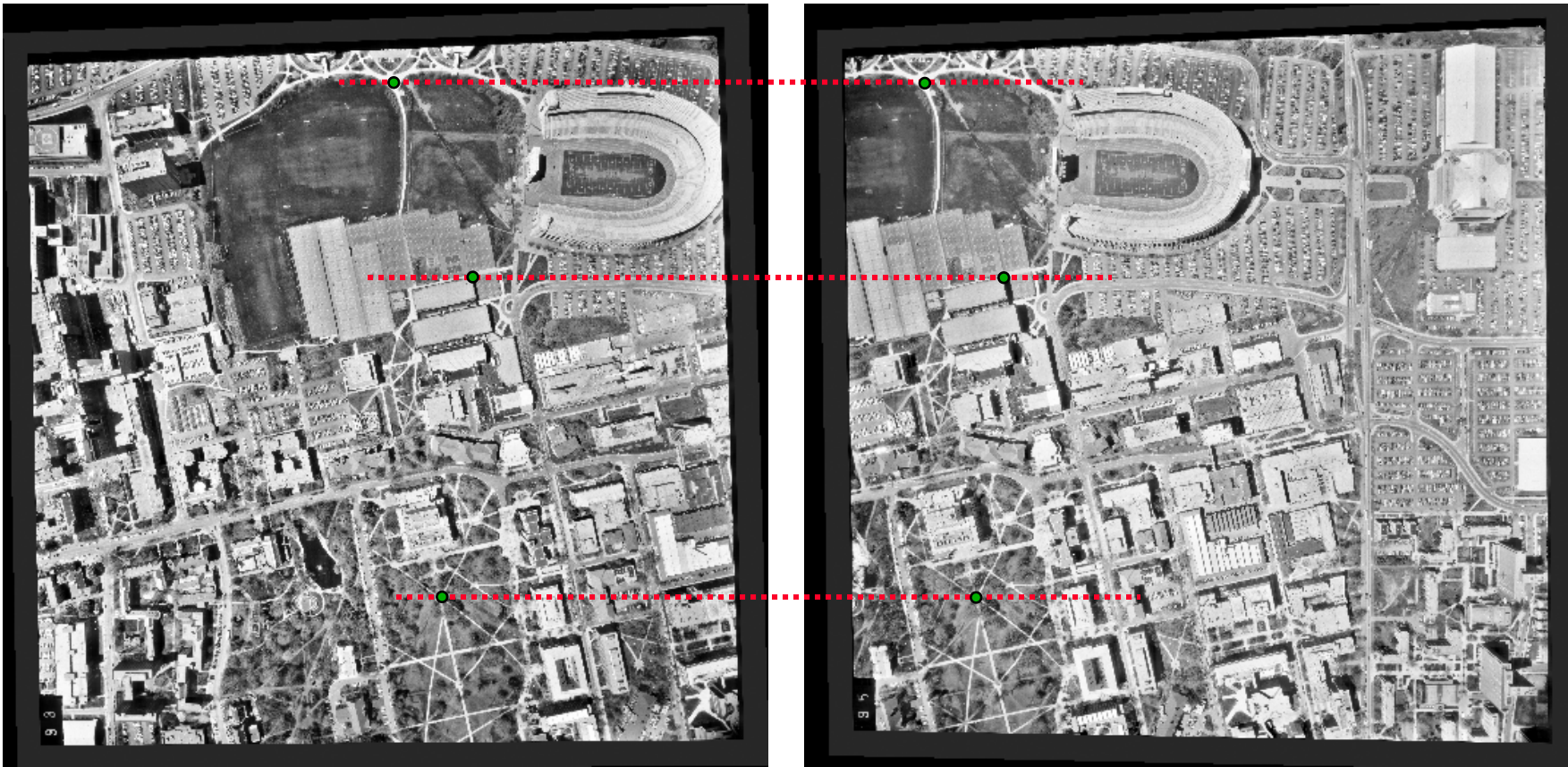
# Original Images

Conjugate points do not have the same y-coordinates.

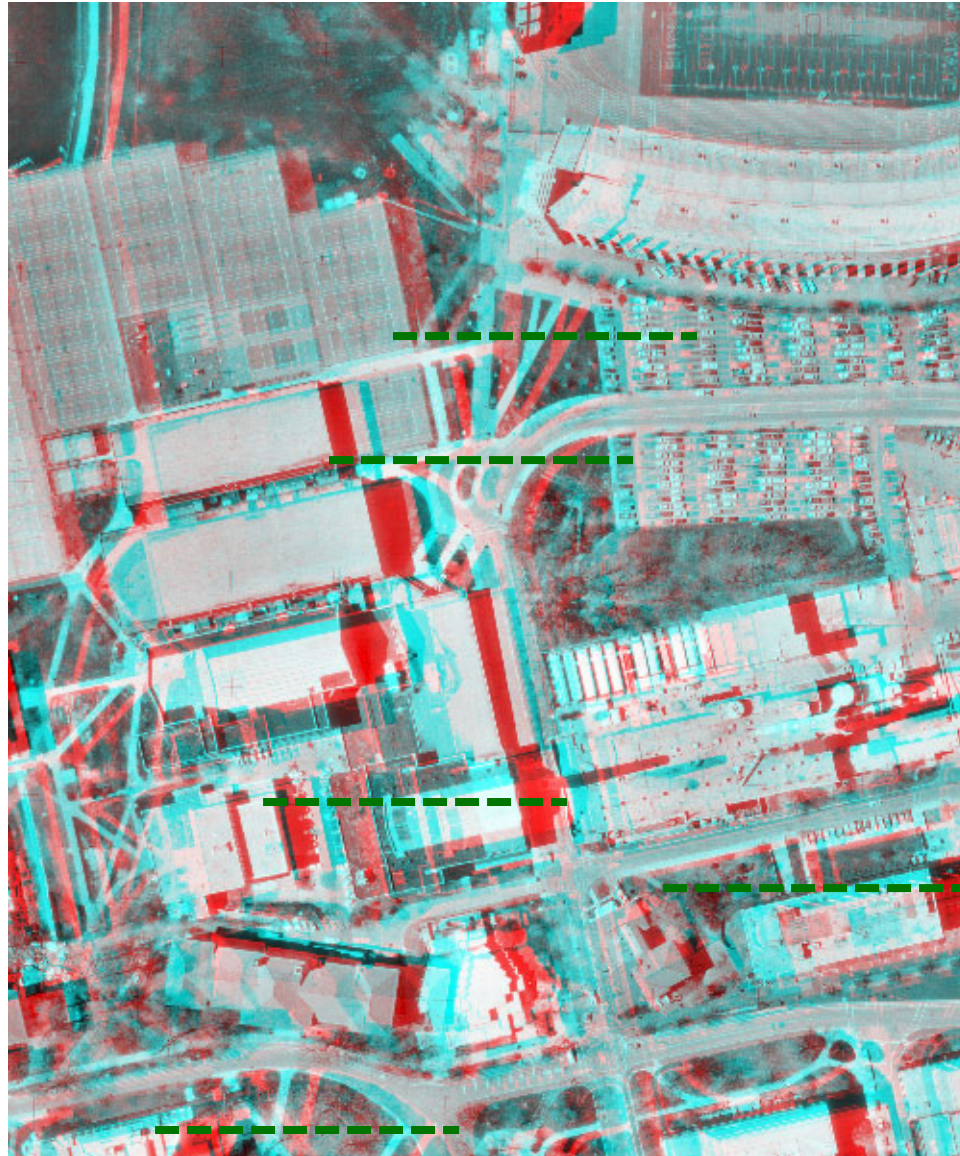


# Normalized Images

Conjugate points have the same y-coordinates.



# Normalized Images

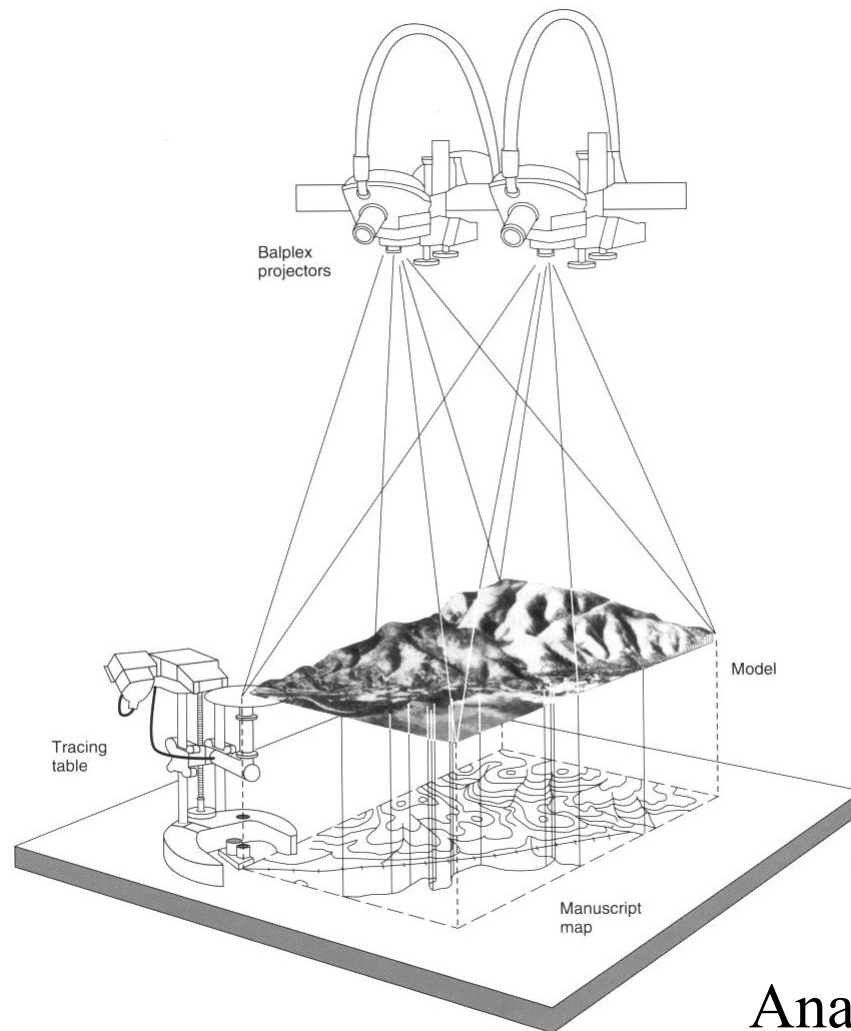




# Normalized Image Generation: Why?

- Normalized image generation is important for the following reasons:
  - 3-D viewing of 2-D images can be established without the need for photogrammetric plotters.
    - Photogrammetric plotters are expensive and need an experienced operator to establish the orientation procedure.
  - In automatic matching, the search space for conjugate points will be reduced from 2-D to 1-D.
    - Save time and avoid matching ambiguities whenever dealing with repetitive patterns.

# 3-D Viewing of 2-D imagery



Analog Photogrammetric Plotter

# 3-D Viewing of 2-D imagery



Analytical Photogrammetric Plotter  
Leica Geosystems: SD2000/3000

# 3-D Viewing of 2-D imagery



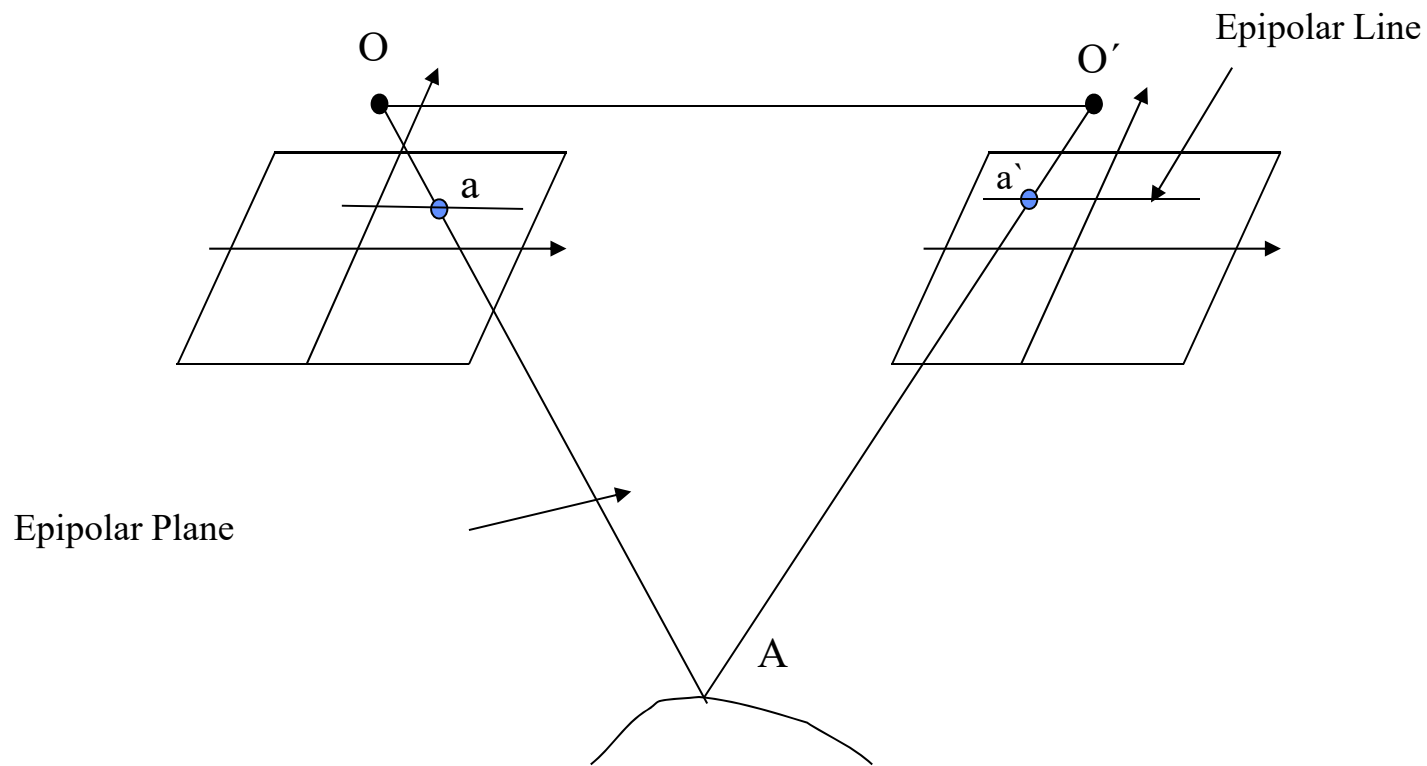
Digital Photogrammetric Plotter  
Z/I Image Station

## 3-D Viewing of 2-D imagery

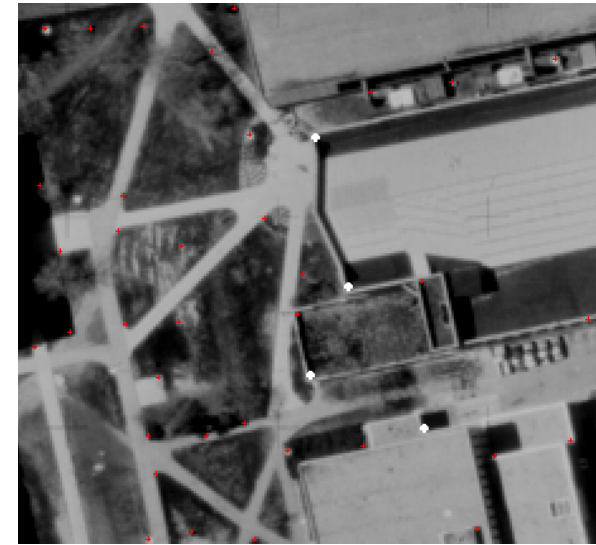
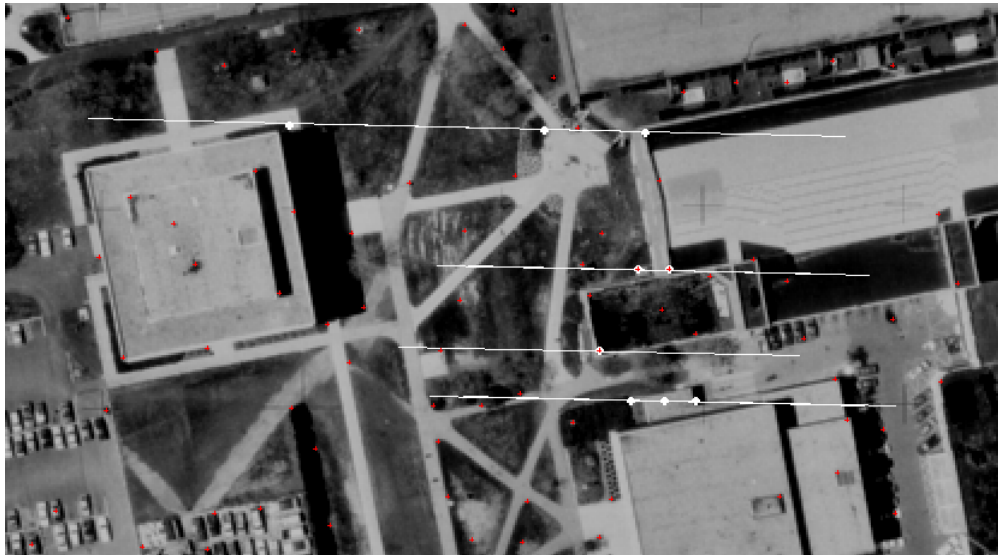


Digital Photogrammetric Plotter  
Leica Geosystems

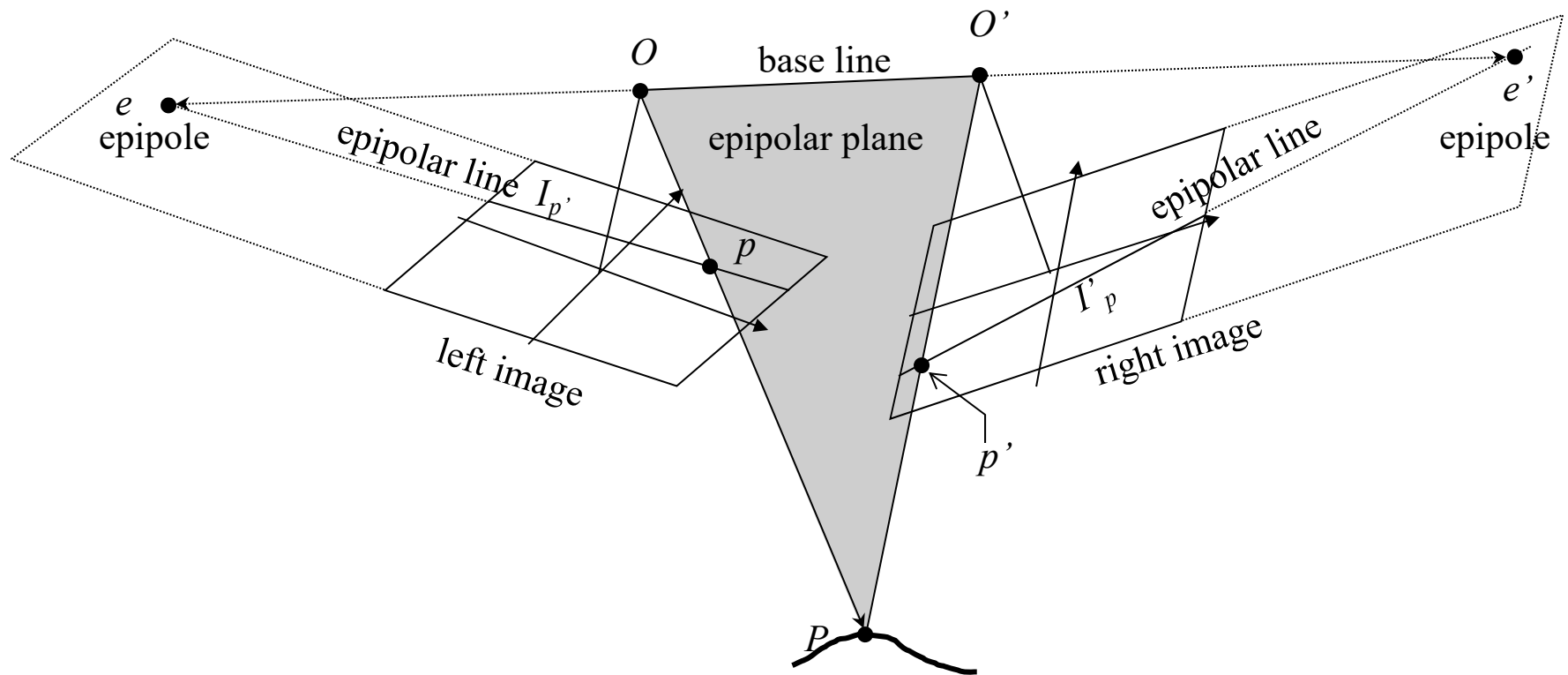
# Epipolar Geometry



# Epipolar Geometry



# Epipolar Geometry: Definitions





# Epipolar Geometry

- Epipolar plane for a given **object** point is the plane containing the perspective centers of a stereo-pair and the object point under consideration.
- Epipolar plane for a given **image** point is the plane containing the perspective centers of a stereo-pair and the image point under consideration.
- Epipolar line is the intersection of the epipolar plane with the image plane.

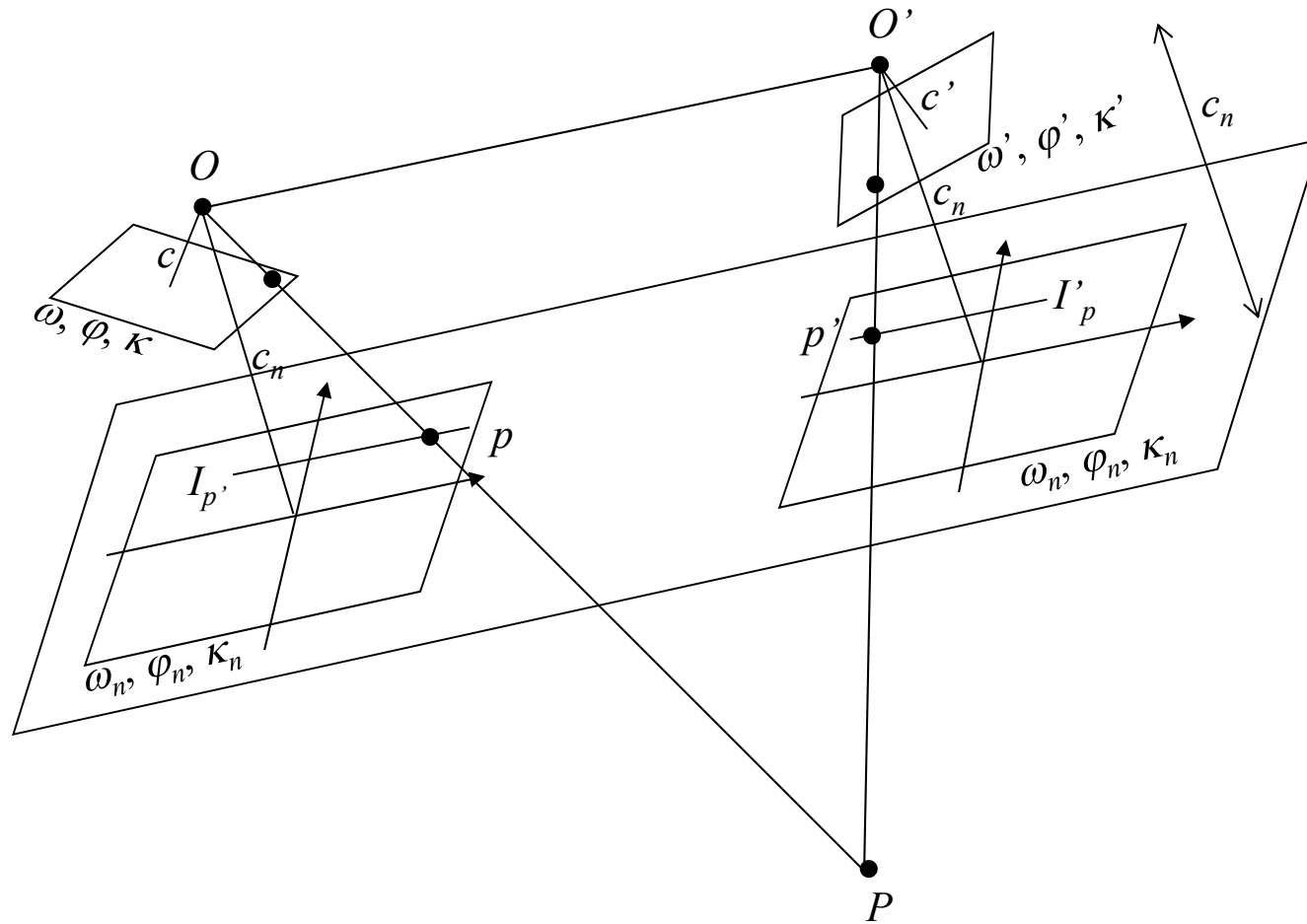
# Epipolar Geometry

- The epipolar plane can be defined once we have:
  - **Relative Orientation Parameters (ROP)** relating the two images of a stereo-pair, and
  - Image coordinate measurements in either the left or right image.
- Epipole: The intersection of the base line with the image plane
  - All the epipolar lines pass through the epipole.
- Epipolar lines associated with different object points are not parallel except when:
  - **The two images of a stereo-pair are parallel to the base line (the line connecting the two perspective centers).**

# Normalized Images

- Concept:
  - Create a new image at the same exposure station
  - This image is parallel to the base line.
  - The rows of the new image should be parallel to the base line.
  - The rows of the new image are the epipolar lines.
- This is commonly known as “**Image Resampling according to Epipolar Geometry**”.
- EOPs of the normalized image:
  - $(X_o, Y_o, Z_o)$  are the same as the original image.
  - $(\omega_n, \phi_n, \kappa_n)$  are chosen in such a way that:
    - The image plane is parallel to the base line, and
    - The rows are parallel to the base line.

# Normalized Image Generation



# Original Image

$$\vec{v}_i = \lambda R^T(\omega, \phi, \kappa) \vec{V}_O$$

$$\begin{bmatrix} x_a \\ y_a \\ -c \end{bmatrix} = \lambda \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X_A - X_O \\ Y_A - Y_O \\ Z_A - Z_O \end{bmatrix}$$

Where:  $\lambda$  is a scale factor

$$\vec{V}_O = 1/\lambda R(\omega, \phi, \kappa) \vec{v}_i$$

# Normalized Image

$$\vec{v}_{i_n} = \lambda_n R_n^T(\omega_n, \phi_n, \kappa_n) \vec{V}_O$$

$$\begin{bmatrix} x_{a_n} \\ y_{a_n} \\ -c \end{bmatrix} = \lambda_n \begin{bmatrix} r_{11_n} & r_{21_n} & r_{31_n} \\ r_{12_n} & r_{22_n} & r_{32_n} \\ r_{13_n} & r_{23_n} & r_{33_n} \end{bmatrix} \begin{bmatrix} X_A - X_O \\ Y_A - Y_O \\ Z_A - Z_O \end{bmatrix}$$

Where:  $\lambda_n$  is a scale factor

$$\vec{V}_O = 1 / \lambda_n R_n(\omega_n, \phi_n, \kappa_n) \vec{v}_{i_n}$$

# Normalized Image

$$\vec{V}_o = 1/\lambda R(\omega, \phi, \kappa) \vec{v}_i$$

$$\vec{V}_o = 1/\lambda_n R_n(\omega_n, \phi_n, \kappa_n) \vec{v}_{i_n}$$

$$1/\lambda R(\omega, \phi, \kappa) \vec{v}_i = 1/\lambda_n R_n(\omega_n, \phi_n, \kappa_n) \vec{v}_{i_n}$$

$$\vec{v}_{i_n} = \lambda_n / \lambda R_n^T(\omega_n, \phi_n, \kappa_n) R(\omega, \phi, \kappa) \vec{v}_i$$

$$\vec{v}_{i_n} = \lambda_n / \lambda M(\omega_n, \phi_n, \kappa_n, \omega, \phi, \kappa) \vec{v}_i$$

# Normalized Image

$$x_{a_n} = -c \frac{m_{11} x_a + m_{12} y_a - m_{13} c}{m_{31} x_a + m_{32} y_a - m_{33} c}$$

$$y_{a_n} = -c \frac{m_{21} x_a + m_{22} y_a - m_{23} c}{m_{31} x_a + m_{32} y_a - m_{33} c}$$



# Normalized Image

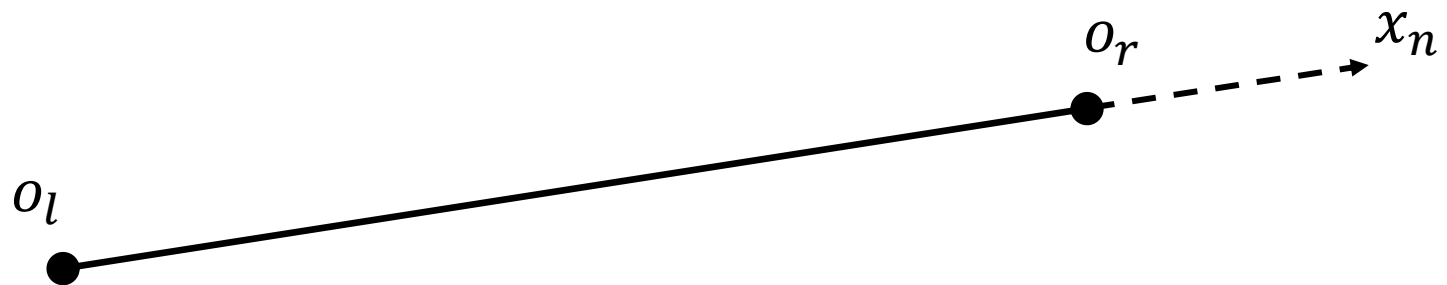
$$\vec{v}_i = \lambda / \lambda_n M^T (\omega_n, \phi_n, \kappa_n, \omega, \phi, \kappa) \vec{v}_{i_n}$$

$$x_a = -c \frac{m_{11} x_{a_n} + m_{21} y_{a_n} - m_{31} c}{m_{13} x_{a_n} + m_{23} y_{a_n} - m_{33} c}$$

$$y_a = -c \frac{m_{12} x_{a_n} + m_{22} y_{a_n} - m_{32} c}{m_{13} x_{a_n} + m_{23} y_{a_n} - m_{33} c}$$

# Rotation Matrix for the Normalized Images

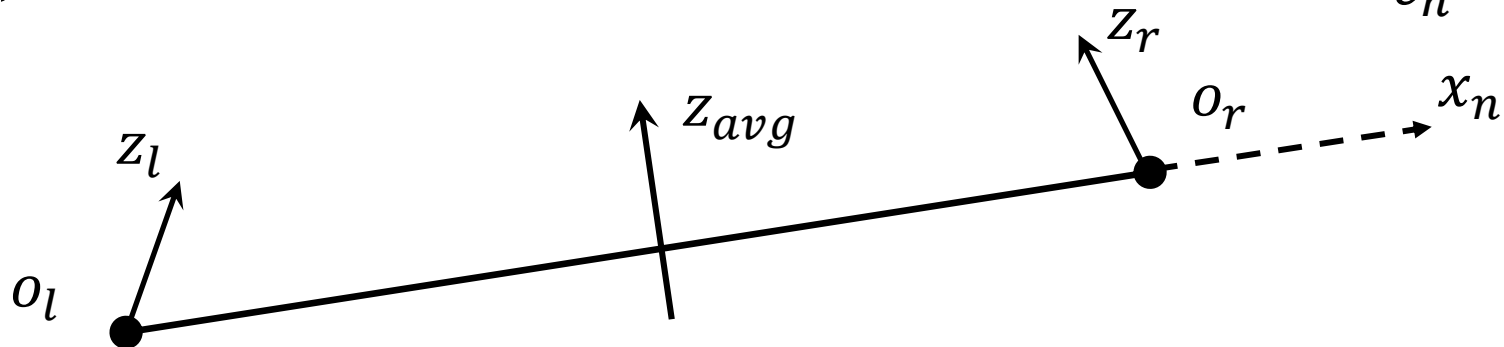
- Now we need to determine the rotation matrix  $R_{c_n}^m = R_n$ .
- There are different alternatives for deriving  $R_{c_n}^m$ .
- In the following discussion, we will present an approach for directly deriving  $R_{c_n}^m$  without the need to derive  $\omega_n, \varphi_n, \kappa_n$ .



- Using the **coordinates of the left and perspective centers**, can we derive one of the columns of  $R_{c_n}^m$ ?
- Yes, we can derive the **first column** of  $R_{c_n}^m$  ( $r_{1_n}$ ).

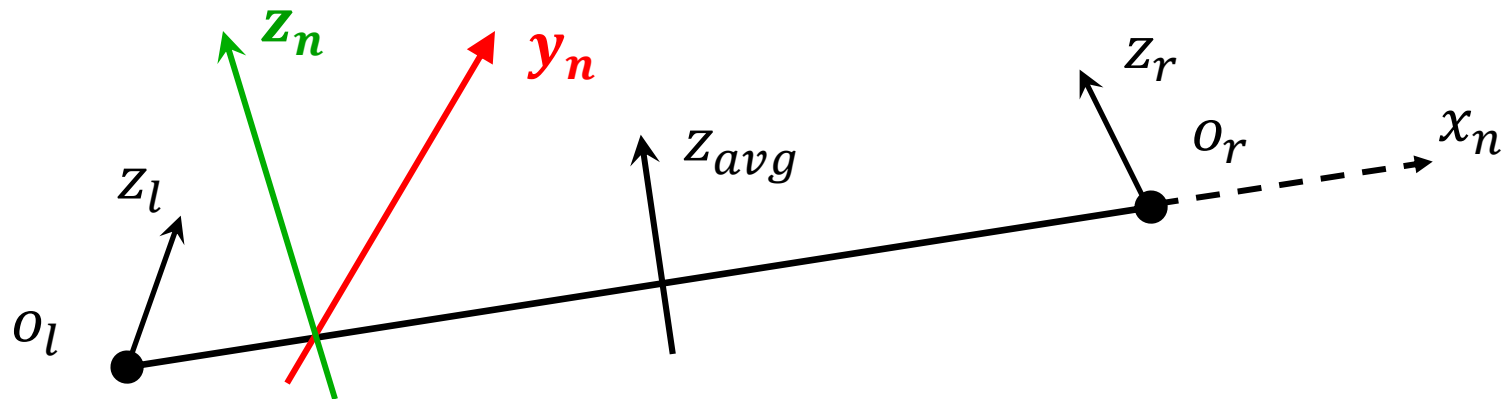
# Rotation Matrix for the Normalized Images

- Now, we need to derive the other two columns of  $R_{C_n}^m$ .



- Note: The optical axis of the left image ( $z_l$ ) is represented by the third column of  $R_{C_l}^m$ .
- Note: The optical axis of the right image ( $z_r$ ) is represented by the third column of  $R_{C_r}^m$ .
- $z_{avg}$  is derived as the average of the third columns of  $R_{C_l}^m$  and  $R_{C_r}^m$ .

# Rotation Matrix for the Normalized Images



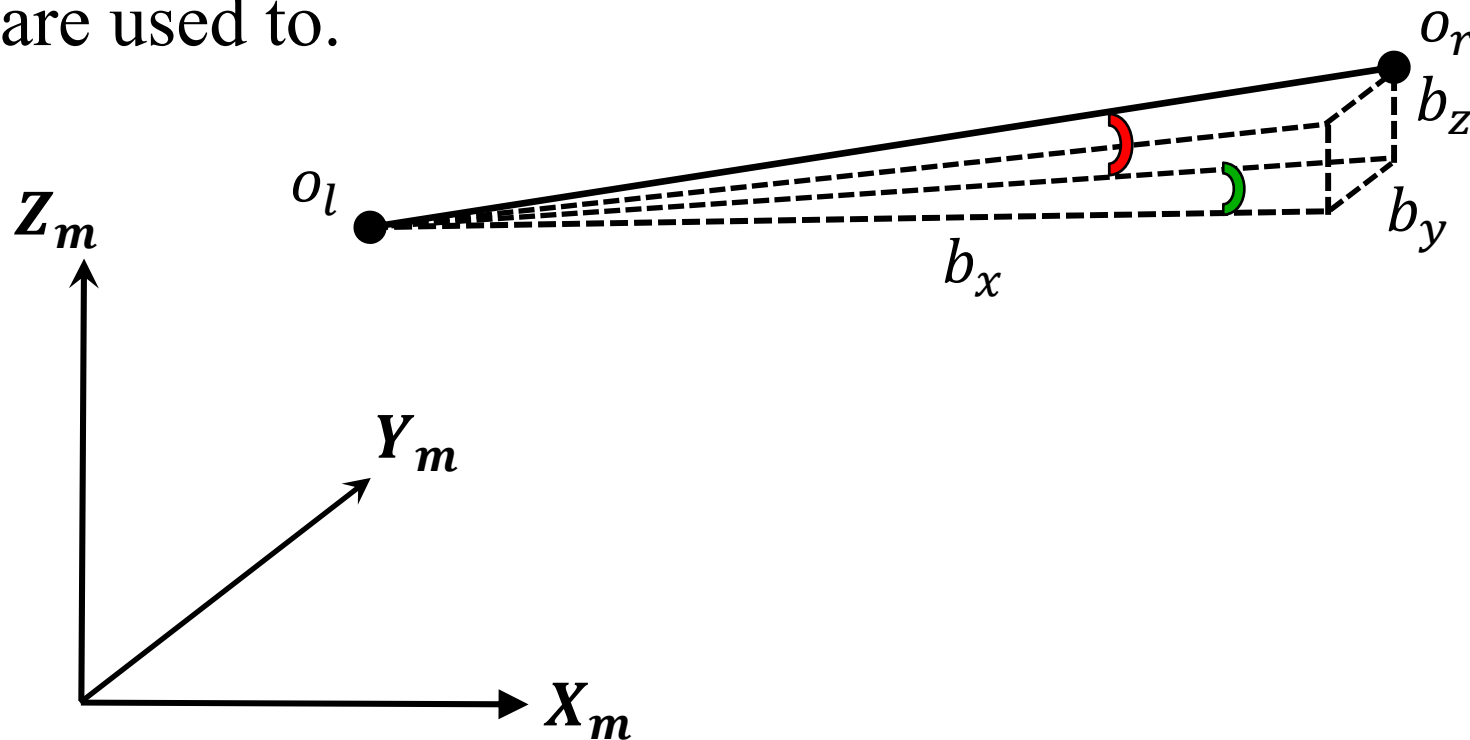
- The **second column** of  $R_{c_n}^m$  ( $r_{2_n}$ ) can be derived through the cross product of  $z_{avg}$  and  $r_{1_n}$ .
- The **third column** of  $R_{c_n}^m$  ( $r_{3_n}$ ) can be derived as the cross product of  $r_{1_n}$  and  $r_{2_n}$ .
- NOTE: Make sure to normalize the cross products (i.e., convert them to unit vectors).

# Rotation Matrix for the Normalized Images

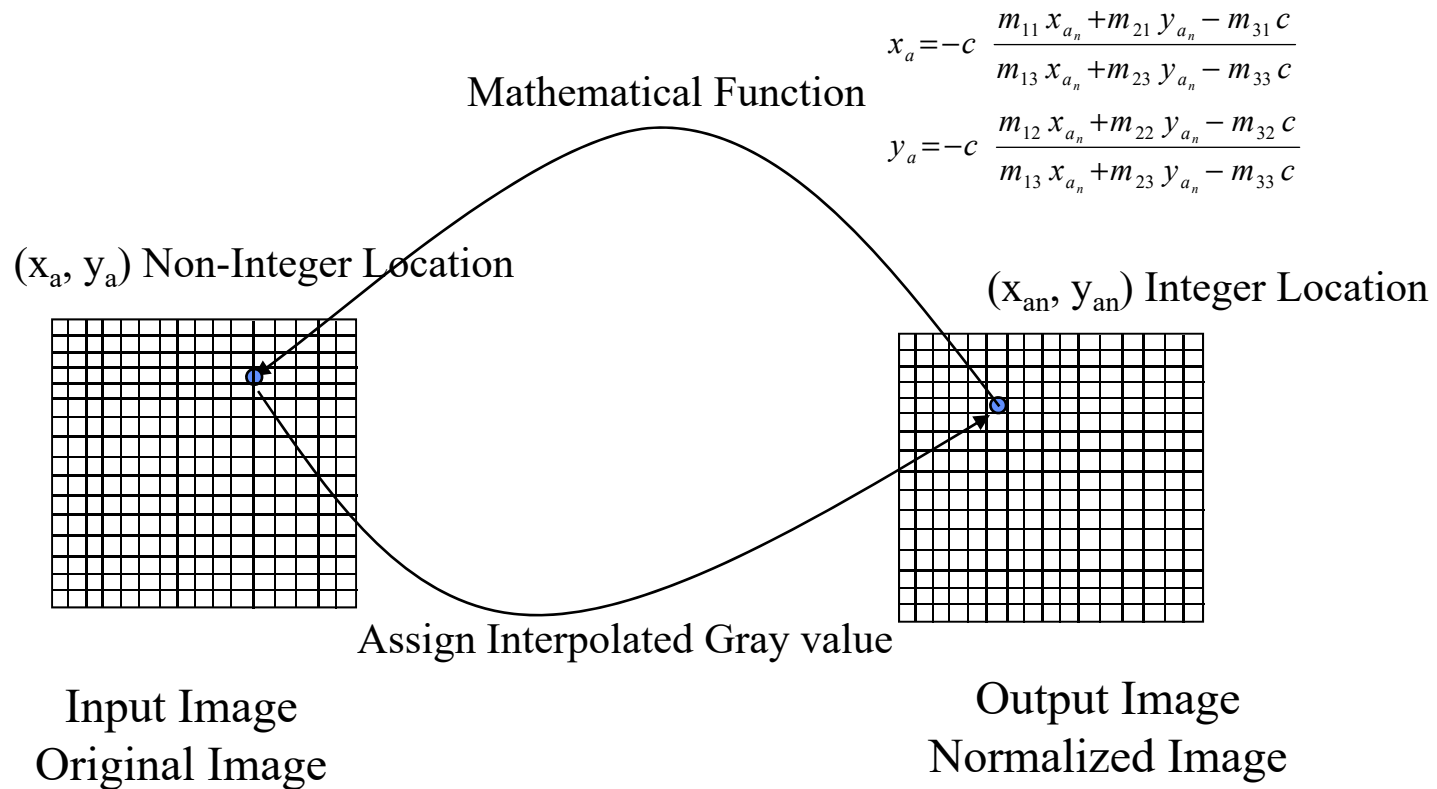
- Alternatively,  $R_{c_n}^m$  can be derived as follows.

$$- R_z\{\tan^{-1}(b_y/b_x)\}R_y\left\{-\tan^{-1}\left(b_z/\sqrt{b_x^2 + b_y^2}\right)\right\}R_x\{0.5(\omega_l + \omega_r)\}$$

- NOTE: this is a different order of rotation than what we are used to.



# Normalized Image Generation

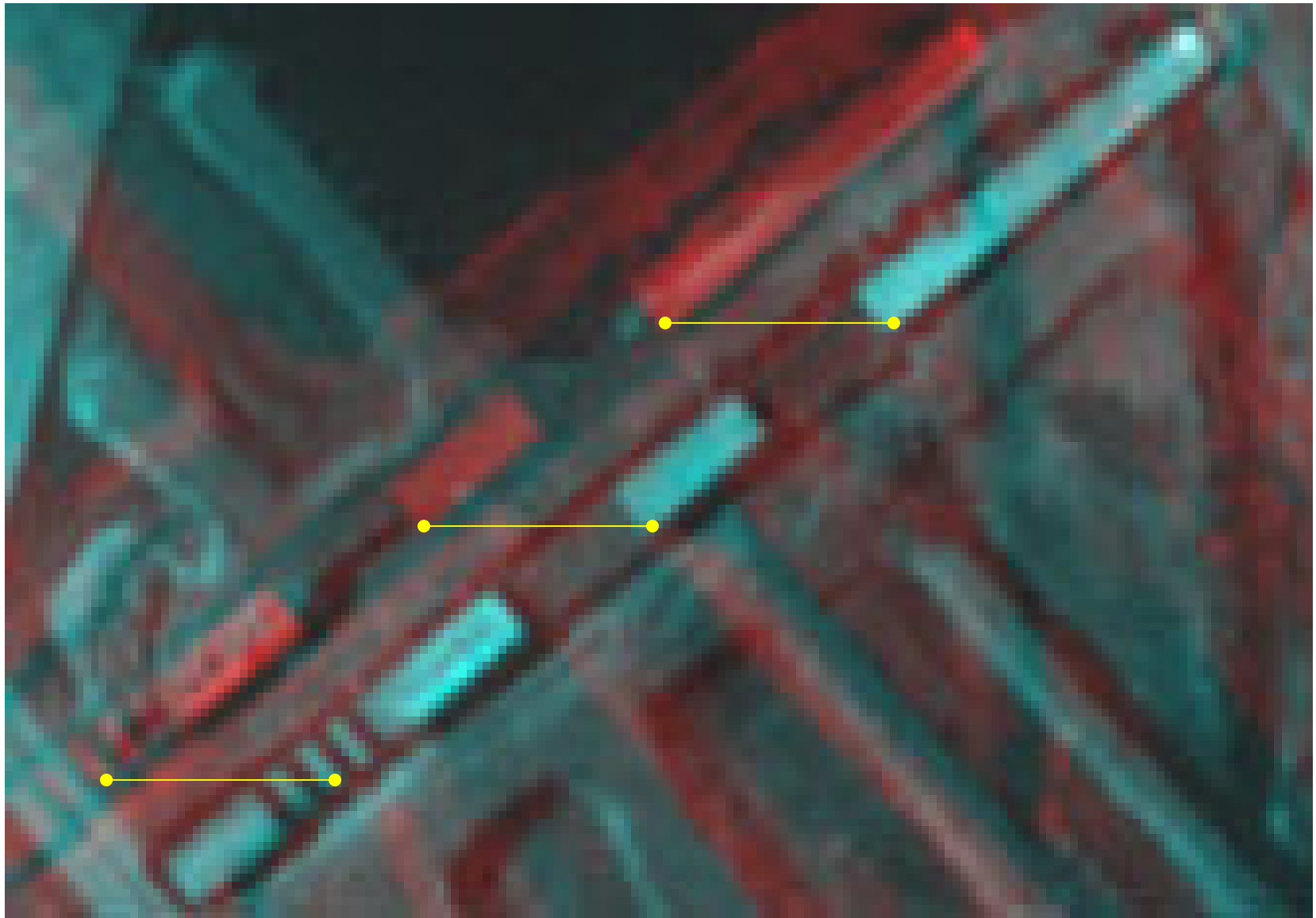


Interpolate for the gray value at  $(x_a, y_a)$

# Normalized Image Generation

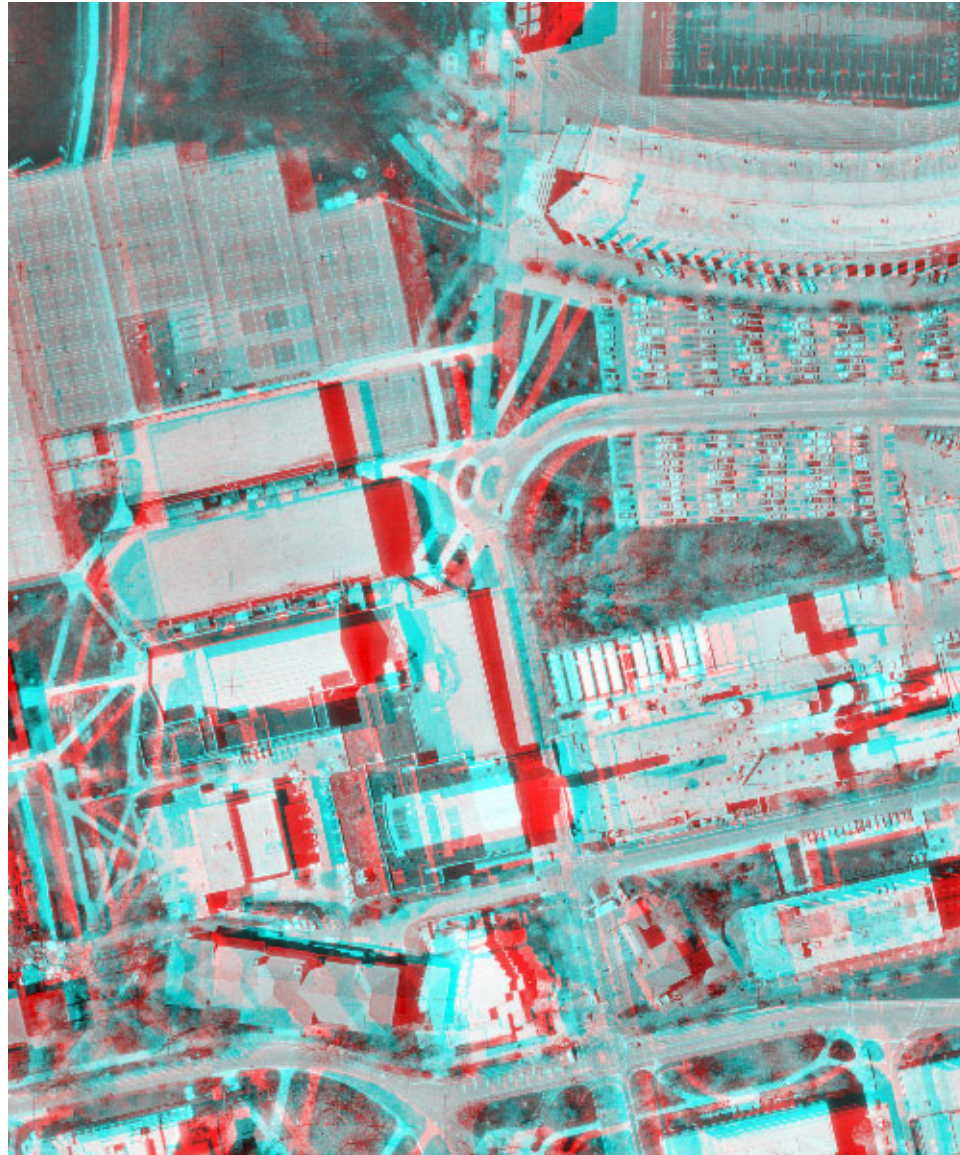
- Procedure:
  - Start from a certain pixel  $(x_{a_n}, y_{a_n})$  in the normalized image,
  - Compute the corresponding  $(x_a, y_a)$  using the previous equations,
  - Compute  $g(x_a, y_a)$  using image resampling,
  - $g(x_{a_n}, y_{a_n}) = g(x_a, y_a)$ , and
  - Repeat the above mentioned steps for every pixel in the normalized image.

# 3-D Viewing of 2-D Imagery





# 3-D Viewing of 2-D Imagery



# 3-D Viewing of 2-D Imagery



# 3-D Viewing of 2-D Imagery



# 3-D Viewing of 2-D Imagery

