Chapters 1 & 2: Overview

- Photogrammetry: Introduction & Applications
- Photogrammetric tools:
  - Rotation matrices
  - Photogrammetric point positioning
  - Photogrammetric bundle adjustment
- GNSS/INS for the direct georeferencing of photogrammetric and LiDAR mapping systems

- This chapter will introduce the principles of LiDAR mapping from static and mobile scanners onboard terrestrial and airborne platforms.
  - Point positioning equation, and
  - Error sources and their impact

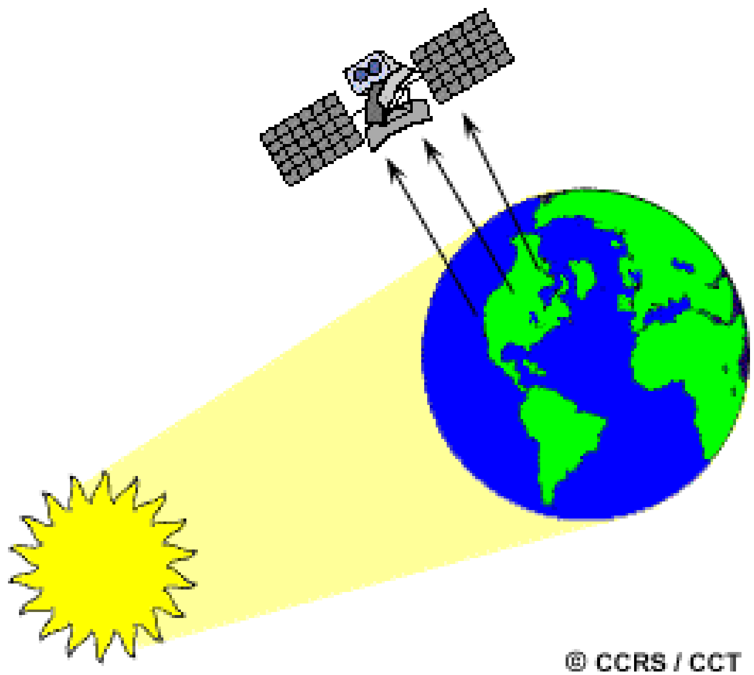
Chapter 3

LIDAR MAPPING PRINCIPLES

Overview

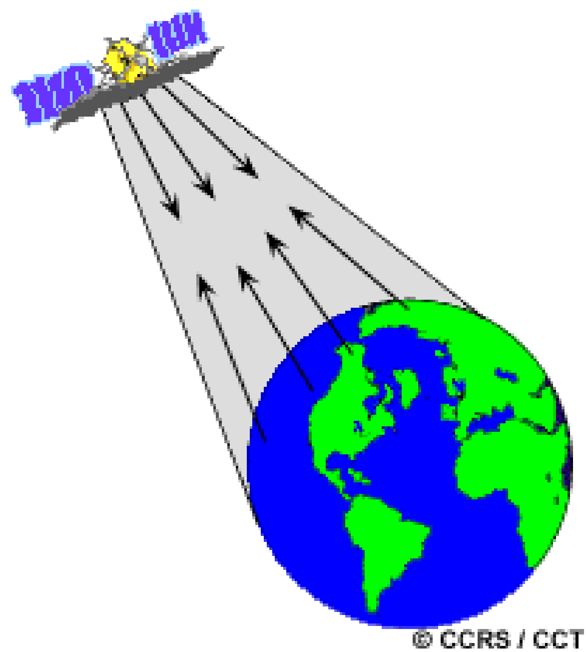
- Passive versus active sensors
- LASER principles
- LiDAR principles
- LiDAR equation
- Error sources (systematic and random errors) & their impact
- LiDAR vs. photogrammetric mapping

Passive Sensors



Source: http://www.nrcan.gc.ca/earth-sciences/geography-boundary/remote-sensing/fundamentals/1212

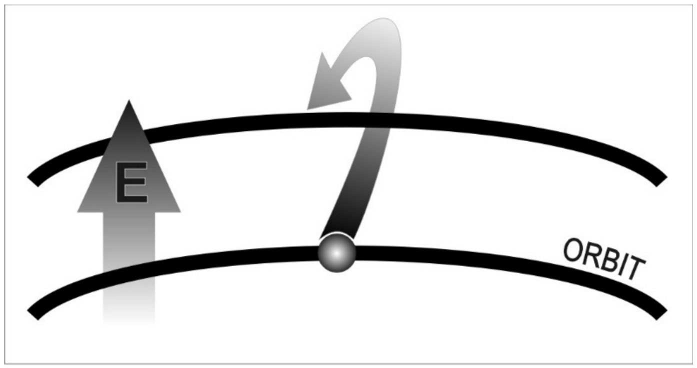
Active Sensors



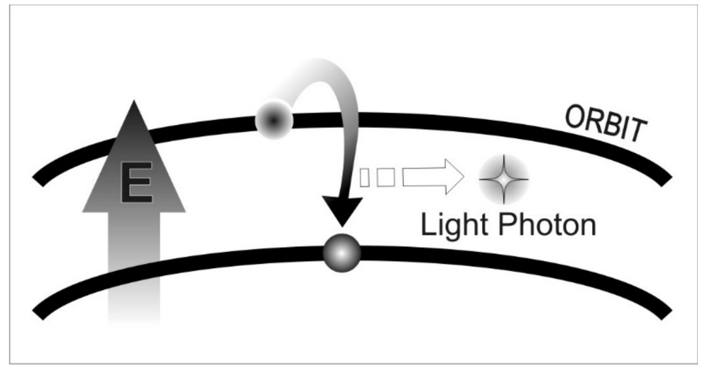
Source: http://www.nrcan.gc.ca/earth-sciences/geography-boundary/remote-sensing/fundamentals/1212

Laser Principles

LASER - **Light Amplification by Stimulated Emission of Radiation**



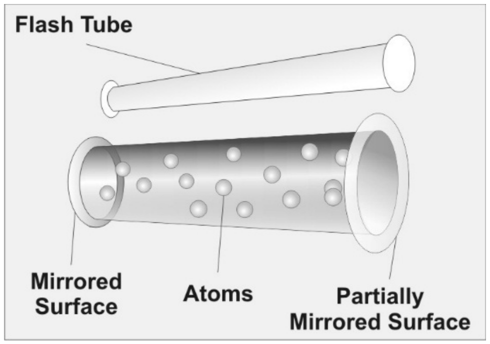
Energy given to an atom in ground state



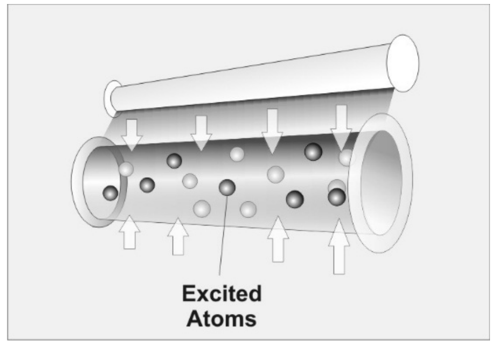
Excited atom returns to ground state by releasing energy

Source: Manual of Geospatial Science and Technology, Second Edition: Edited by J.D. Bossler

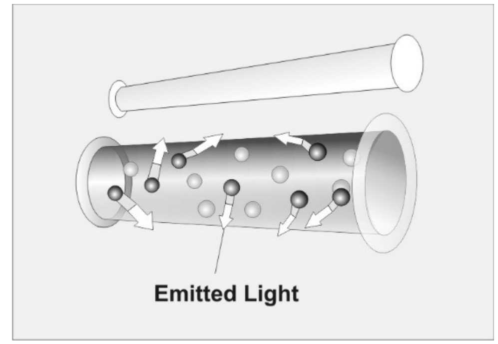
Laser Principles



1. The laser in its non-lasing state

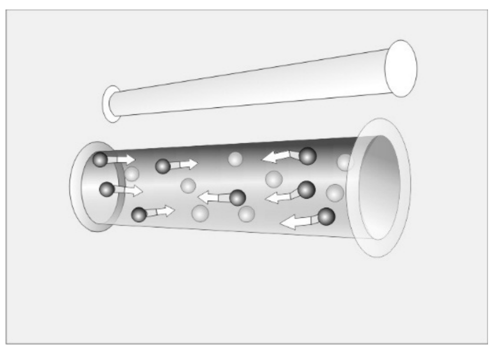


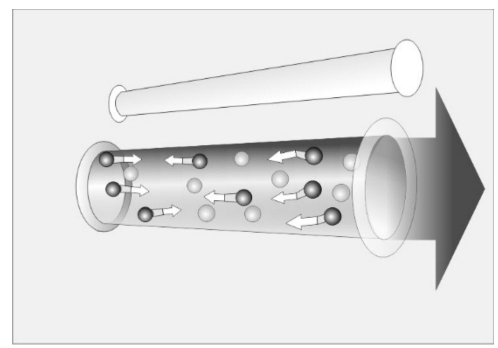
2. The flash tube fires and injects light into the rod. The light excites atoms in the tube.



3. Some of these atoms emit photons.

4. Some of these photons run in a direction parallel to the tube's axis, so they bounce back and forth off the mirror and they stimulate emission in other atoms.





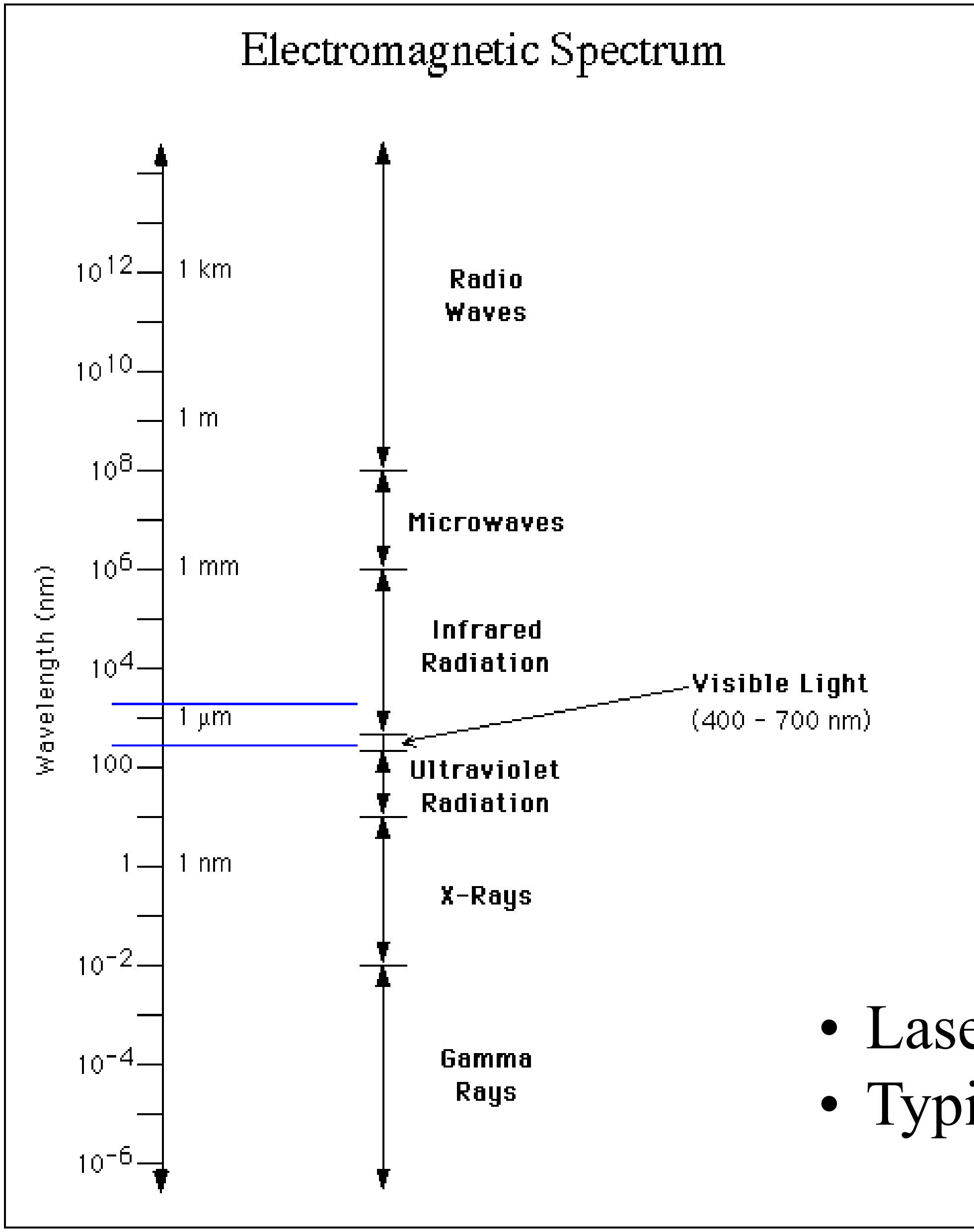
5. Monochromatic, single-phase, collimated light leaves the tube through the half-silvered mirror -- laser light!

Source: Manual of Geospatial Science and Technology, Second Edition: Edited by J.D. Bossler

Laser Principles

- Laser light is very different from normal light.
- Laser light has the following properties:
  - **Monochromatic:** It contains one specific wavelength of light (one specific color). The wavelength of light is determined by the amount of energy released when the electron drops to a lower orbit.
  - **Coherent:** It is "organized" -- each photon moves in step with the others.
  - **Directional:** A laser light has a very tight beam and is very strong and concentrated.
    - A flashlight, on the other hand, releases light in many directions, and the light is very weak and diffuse.

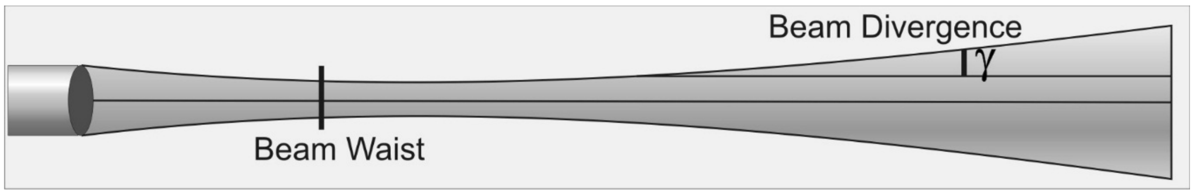
Laser Principles



- Laser wavelength 500-1500 nm
- Typical values 1040 – 1060 nm

Laser Principles

- Beam divergence from 0.2 - 1 mrad.

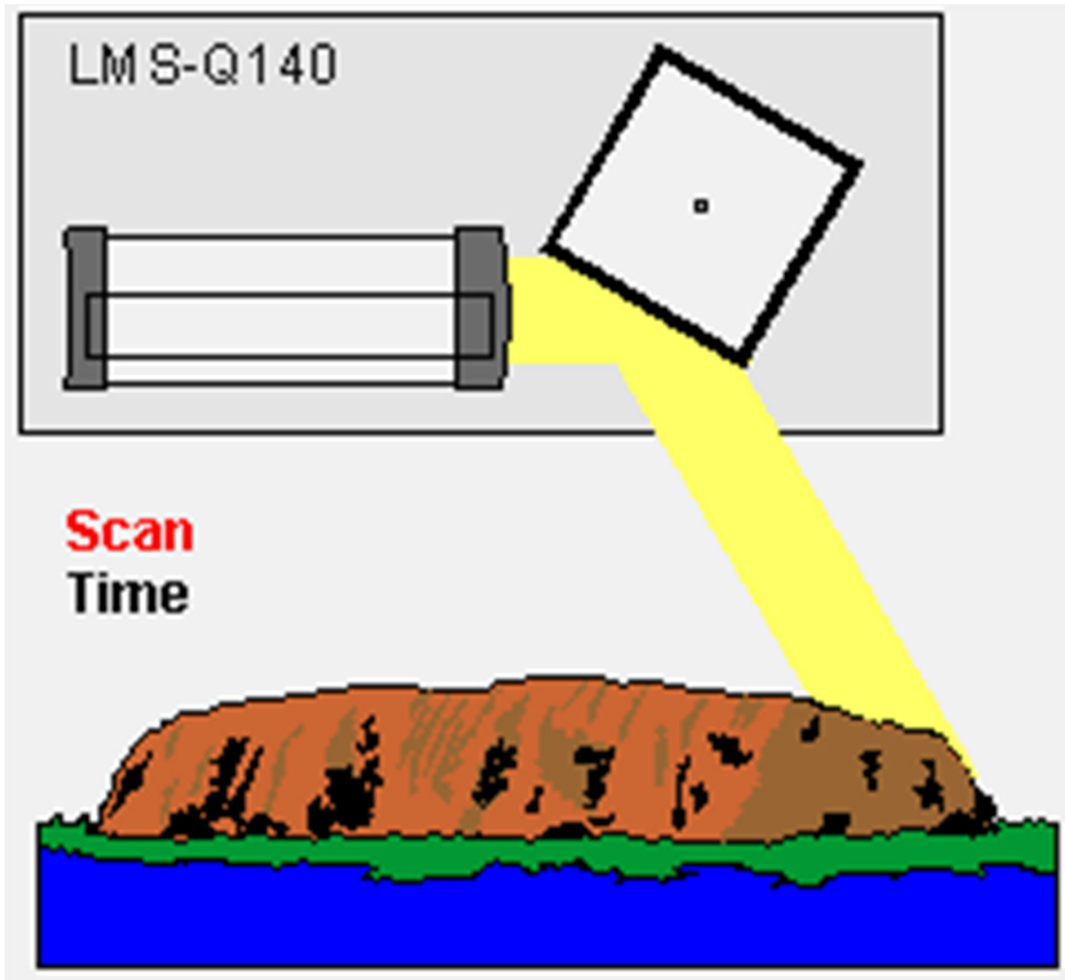


Source: Manual of Geospatial Science and Technology, Second Edition: Edited by J.D. Bossler

Laser Footprint

Wide Beam (0.8 mrad)	Narrow Beam (0.2 mrad)
• 0.8m diameter at 1000m • 2.4m diameter at 3000m	• 0.2m diameter at 1000m • 0.6m diameter at 3000m

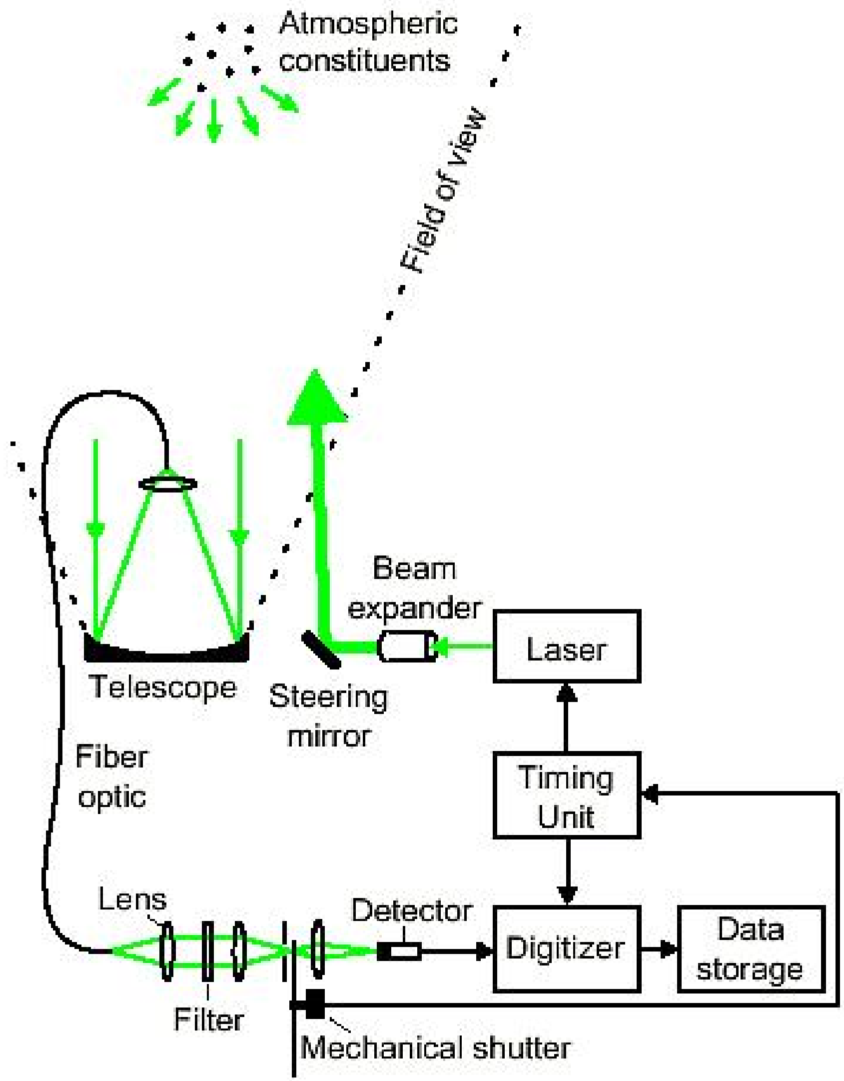
LiDAR Principles



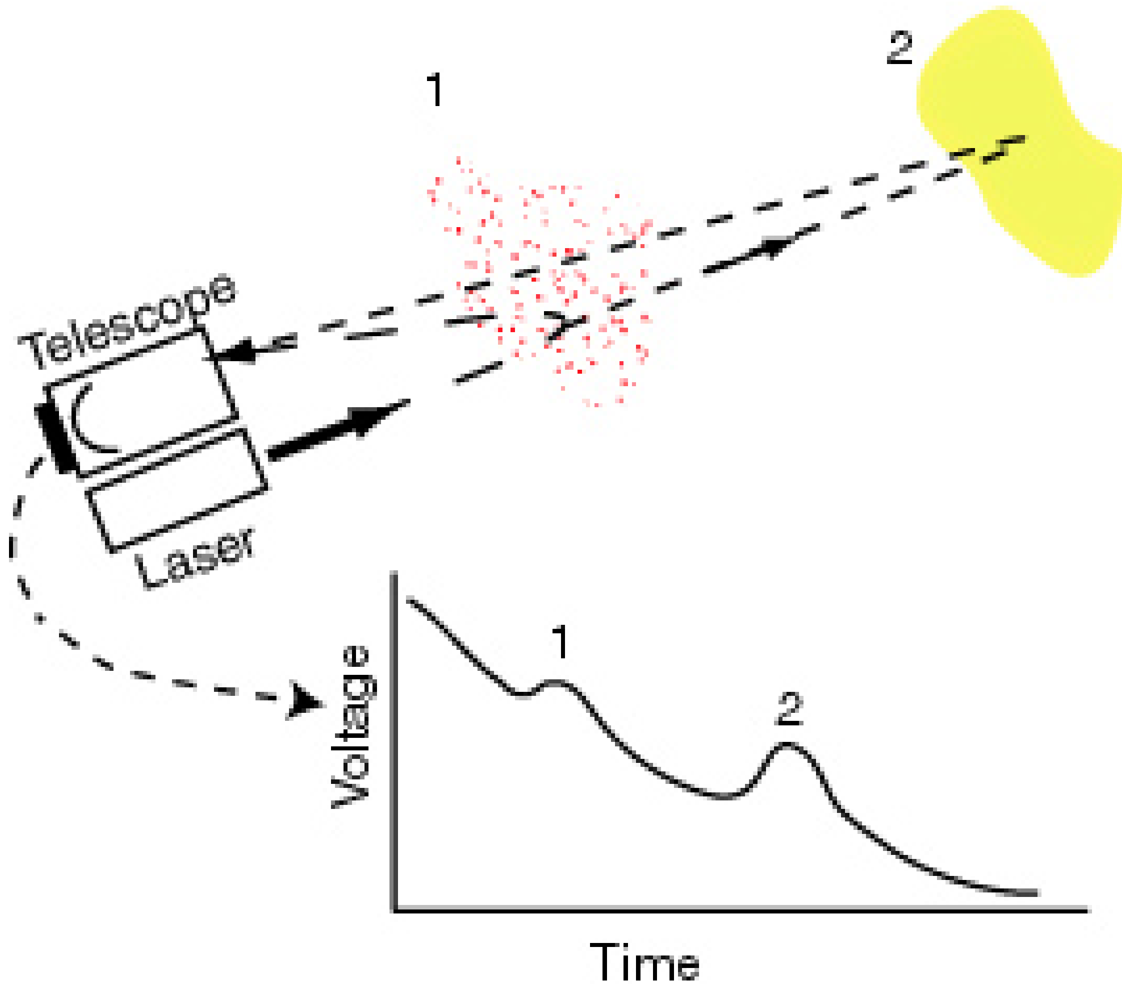
LiDAR Principles

- The LiDAR instrument transmits light out to a target.
- The transmitted light interacts with and is changed by the target.
- Some of this light is reflected/scattered back to the instrument where it is analyzed.
- The time for the light to travel out to the target and back to the LiDAR system is used to determine the range to the target.
- The change in the properties of the light enables some properties of the target to be determined.

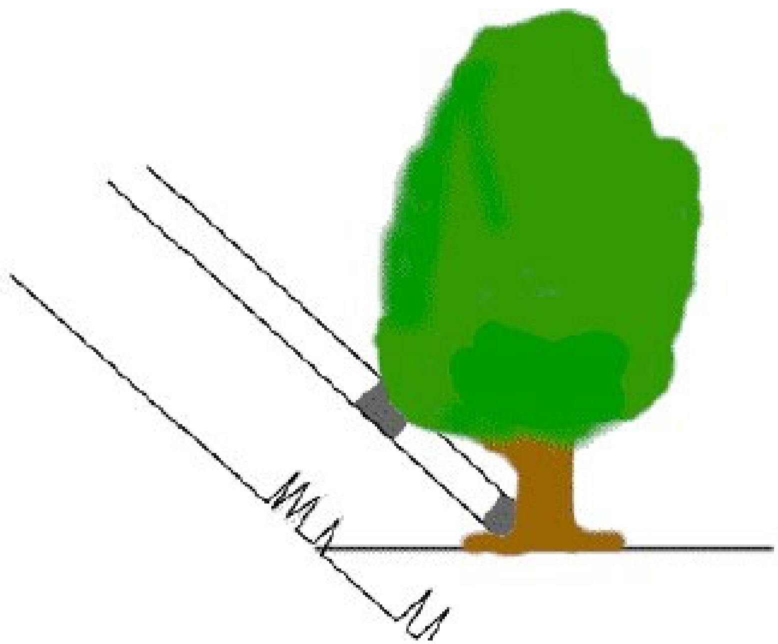
LiDAR Principles



LiDAR Principles

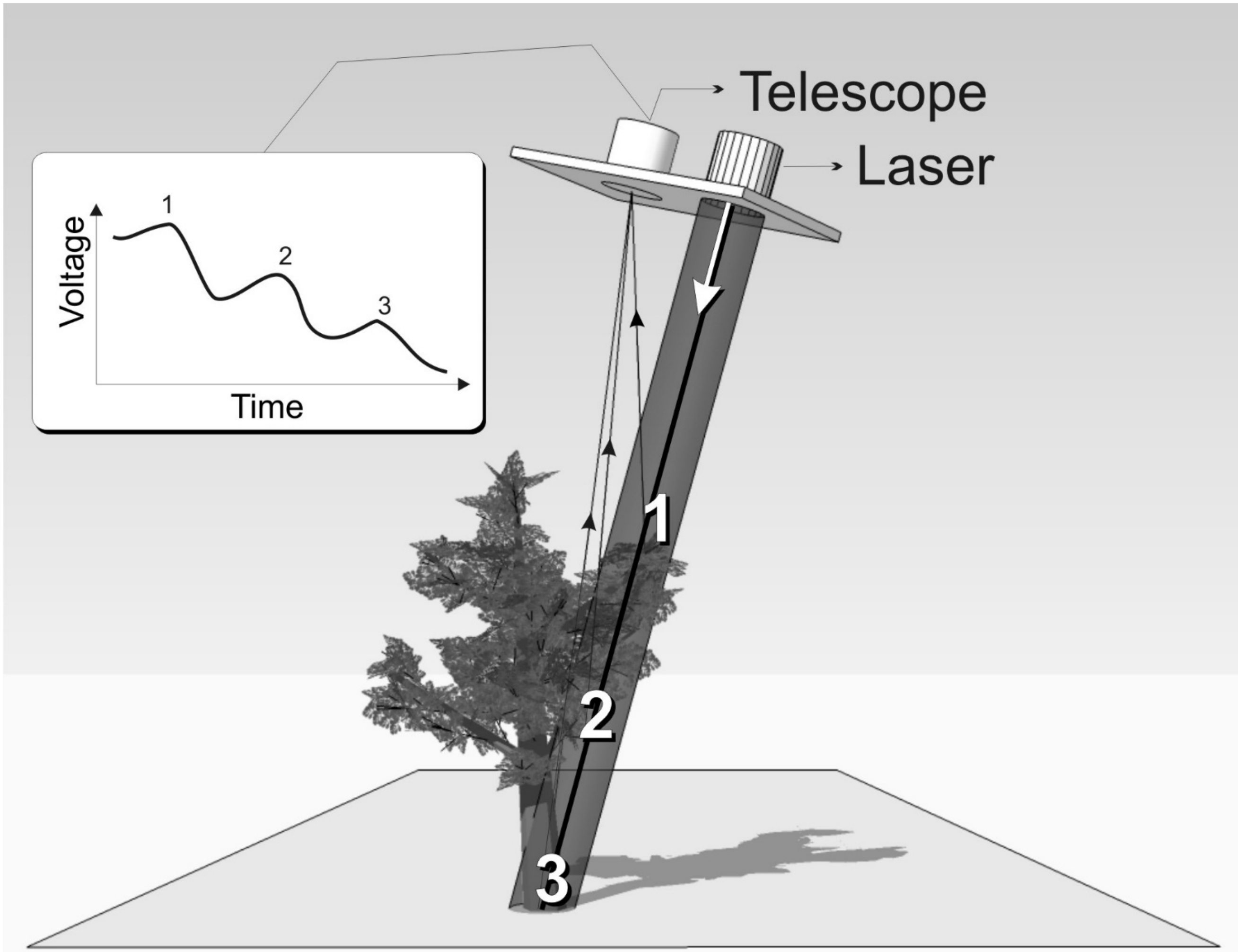


LiDAR Principles



- Range = (travel time * speed of light) / 2.0
- Range + pointing direction ⇒ XYZ (relative to the scanner coordinate system)

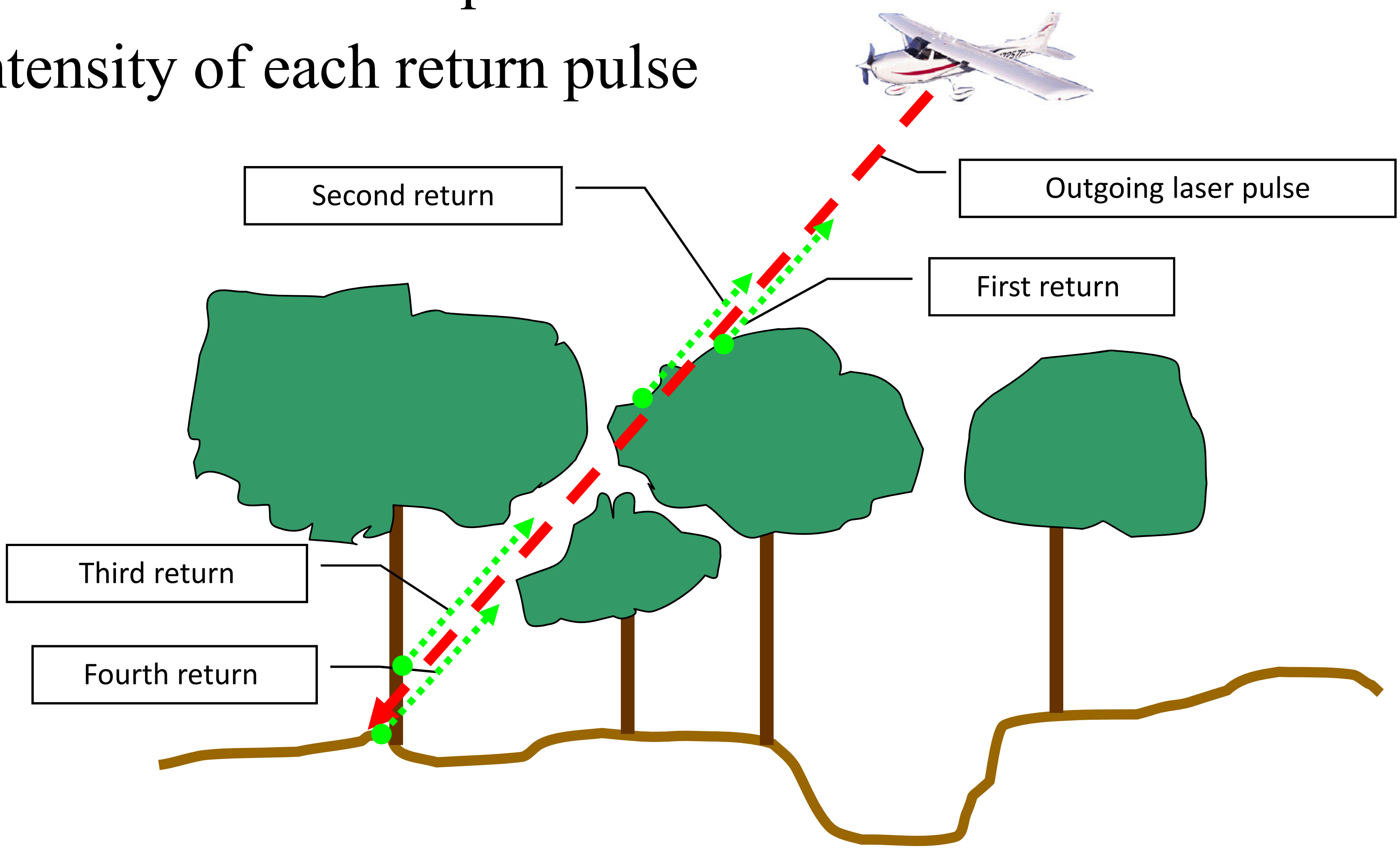
LiDAR Principles



Source: Manual of Geospatial Science and Technology, Second Edition: Edited by J.D. Bossler

LiDAR Principles

- Outgoing laser pulse
- One or more return pulses
- Intensity of each return pulse

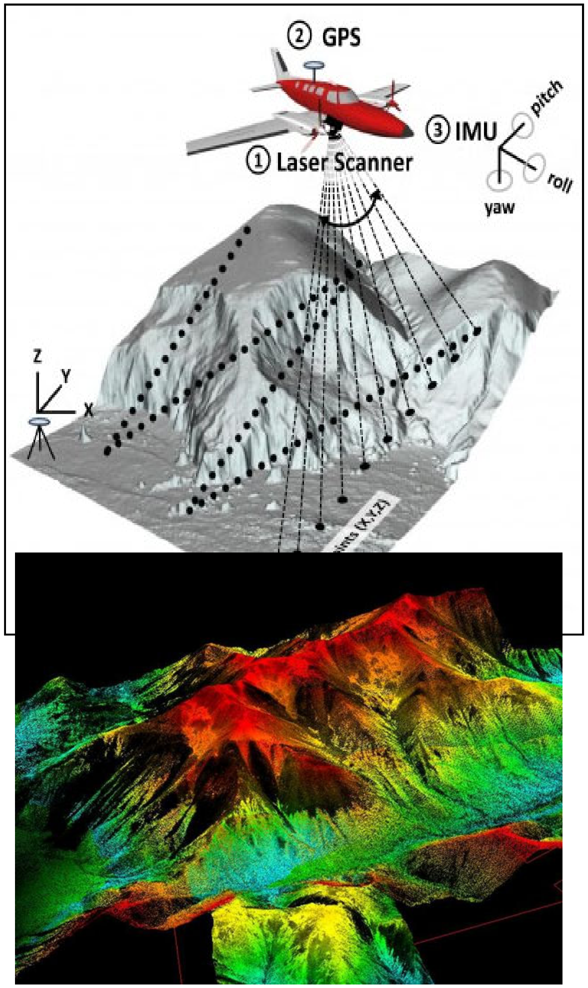


LiDAR Principles

- Depending on the targets to be measured, LiDAR equipment can be mounted on:
  - Tripod,
  - Ground vehicle,
  - Airplane, or
  - Satellite.
- For urban remote sensing, airborne topographic LiDAR is the most popular one.

LiDAR Principles

Airborne Laser Scanning



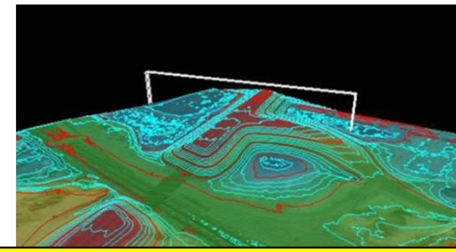
Static Terrestrial Laser Scanning

Kinematic Terrestrial Laser Scanning

LiDAR Principles

Heritage Documentation

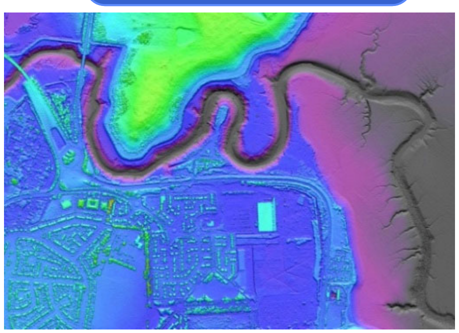
Transportation Planning



LiDAR Data

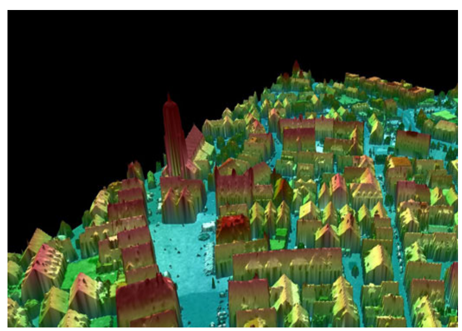
We need to understand the underlying principles of LiDAR point positioning and the factors that affect the quality of derived points.

Flood Plain Mapping



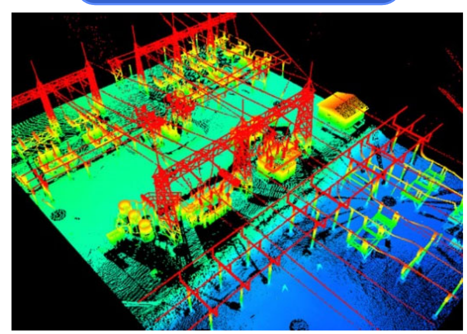
Source: www.maritimejournal.com

3D City Modeling



Source: www.trimble.com

Power Line Mapping



Source: www.merrick.com

Static Terrestrial Laser Scanning

Static Terrestrial Laser Scanning

Pulse Based

Hybrid Type

Trimble, http://www.trimble.com/trimblegx.shtml, (accessed March 16, 2010)

Phase Based

Panoramic Type

Leica Geosystems, http://hds.leica-geosystems.com/en/index.htm, (accessed October 7, 2009)

Triangulation Based

Camera Type

Konica Minolta, http://www.konicaminolta.com/instruments/products/3d/index.html, (accessed October 7, 2009)

Accuracy

1mm → 0.01mm

Range

100m ← 10m ← 1m

Static Terrestrial Laser Scanning

- A static terrestrial laser scanner (pulse/phase-based) is an automatically driven total station/EDM.
- It measures distances to objects at uniform increments in the horizontal and vertical directions.
- These measurements are then converted into a Cartesian coordinate system.
- Most terrestrial laser scanners would even provide intensity and RGB values, although this is not always the case.

Static Terrestrial Laser Scanning

Examples of Operational Systems:
Mensi GS200, Leica (Cyrax) HDS3000, Riegl LMS Z210

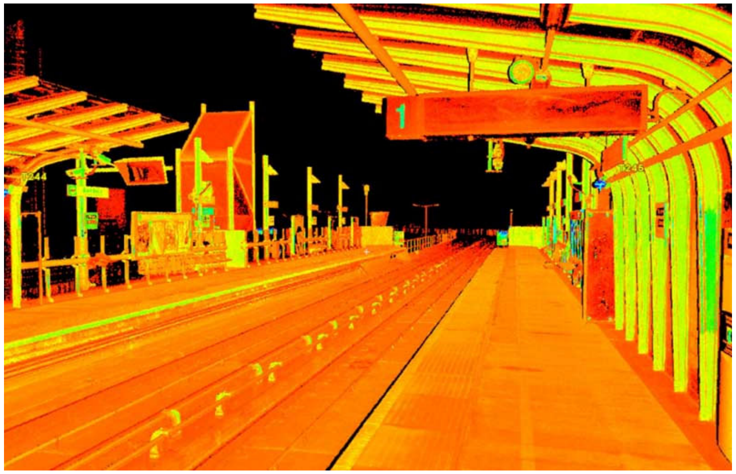
Static Terrestrial Laser Scanning

FARO Focus3D X 330
976,000 points/second
330 m range
±2 mm range error
*http://faro.com

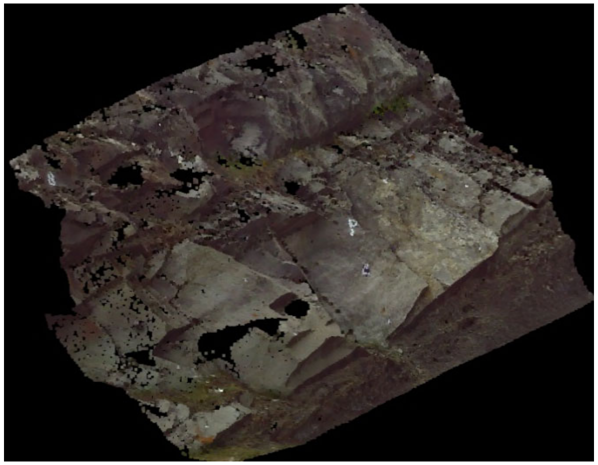
Leica Scanner P20
1 million points/second
120 m range
±6mm at100 M position error
*http://leica-geosystems.com

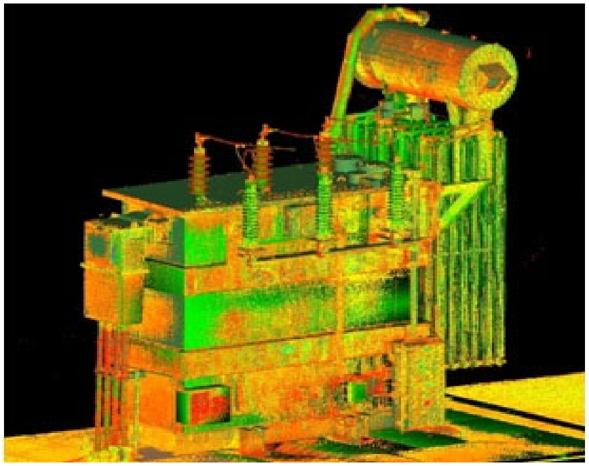
Static Terrestrial Laser Scanning

- 3D city modeling
- Cultural heritage documentation
- Industrial sites modeling
- Land slide monitoring
- Many other civilian and military applications



http://lidarusa.com



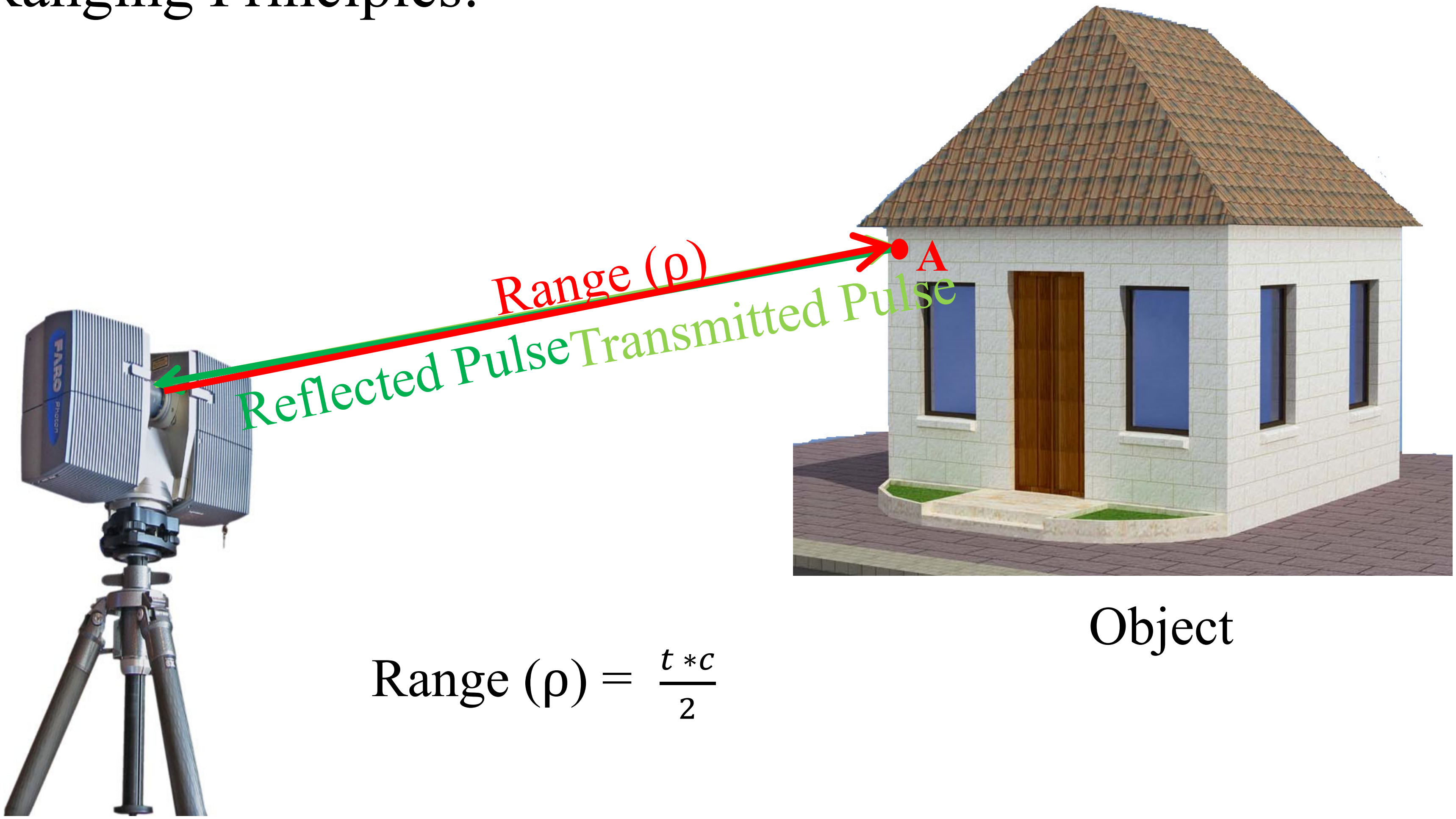


http://lidarusa.com

http://www.3deling.com

Static Terrestrial Laser Scanning

- Ranging Principles:



Object

$$\text{Range } (\rho) = \frac{t * c}{2}$$

Terrestrial Laser Scanner*

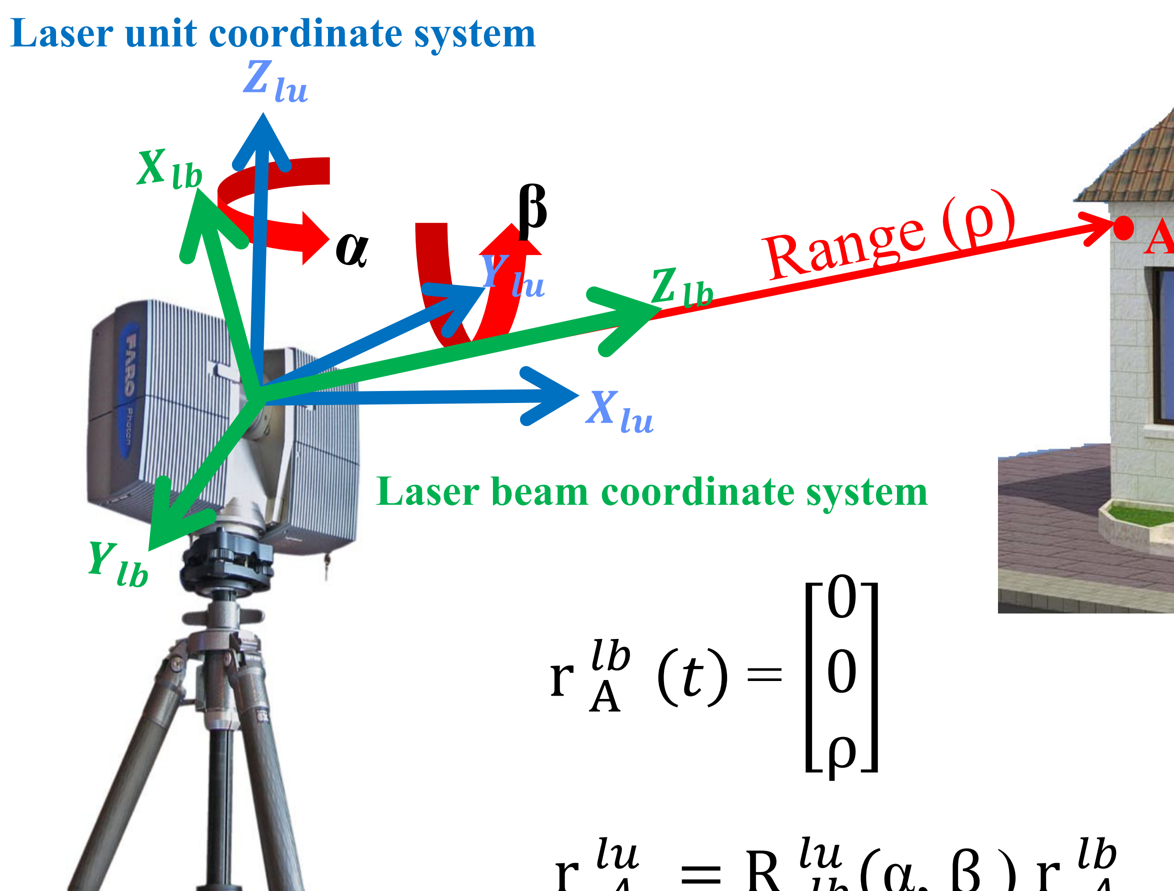
*http://faro.com

t: Time between transmited and recived pulse

c: Speed of laser light

Static Terrestrial Laser Scanning

- Coordinate Systems



$$r_A^{lb}(t) = \begin{bmatrix} 0 \\ 0 \\ \rho \end{bmatrix}$$

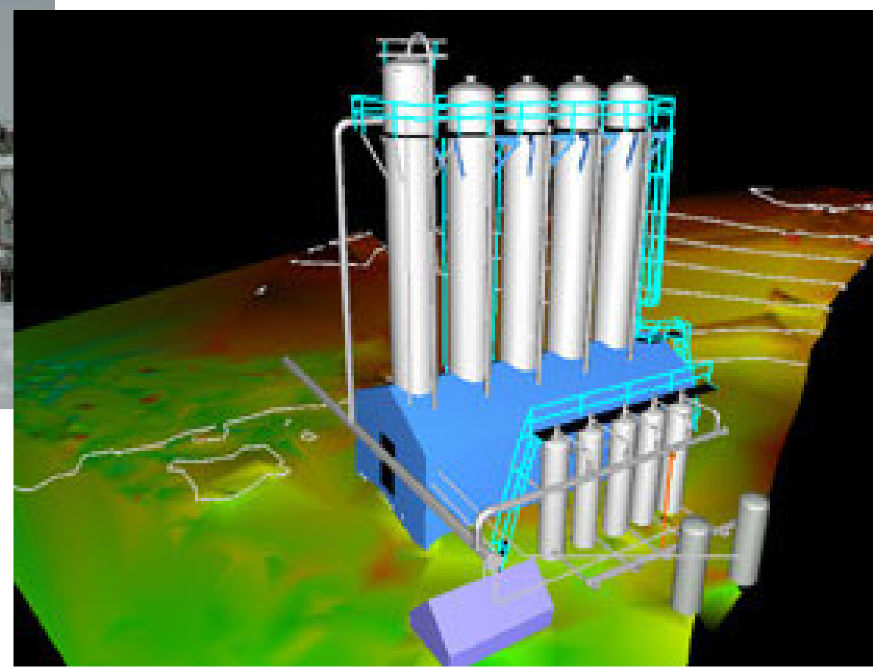
$$r_A^{lu} = R_{lb}^{lu}(\alpha, \beta)\, r_A^{lb}$$

Object

r : range vector
lb: laser beam C.S.
lu:Laser unit C.S.
β : rotation around Y_{lu}
α: rotation around Z_{lu}

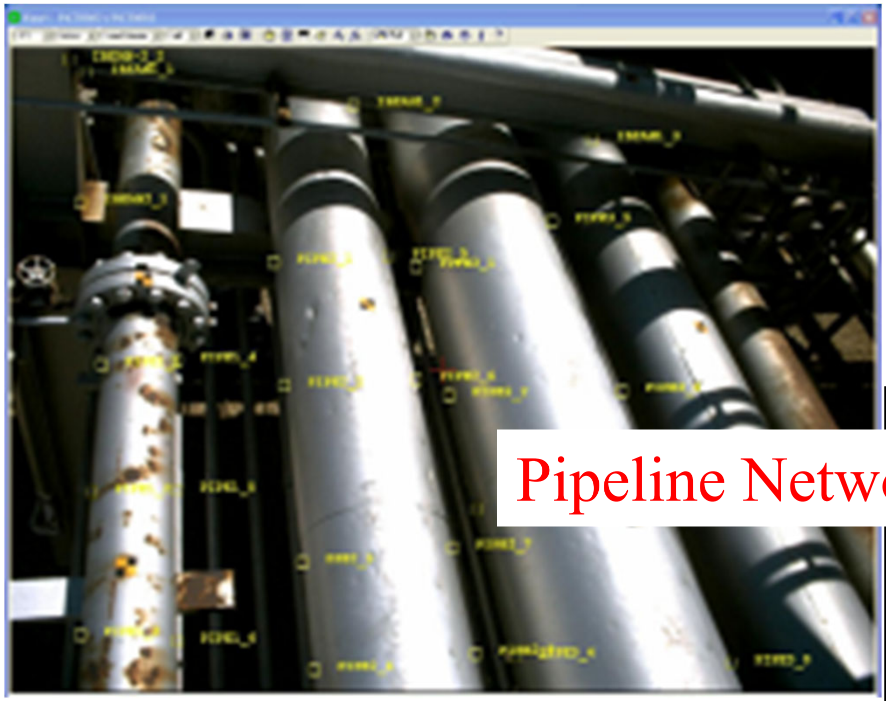
Terrestrial Laser Scanner*
*http://faro.com

Static Laser Scanning

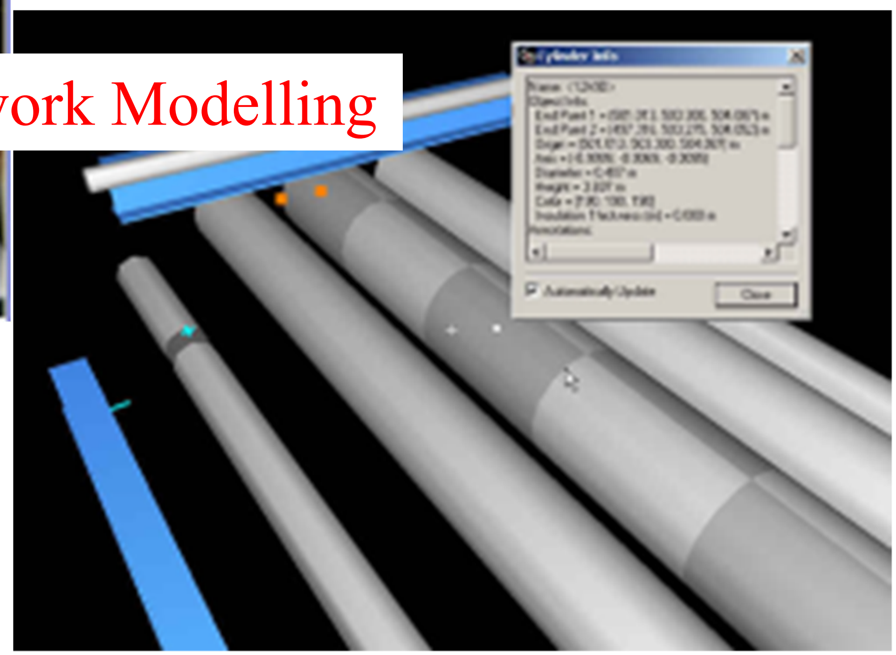


Generation of as-built Plans/Models

Static Laser Scanning



Pipeline Network Modelling



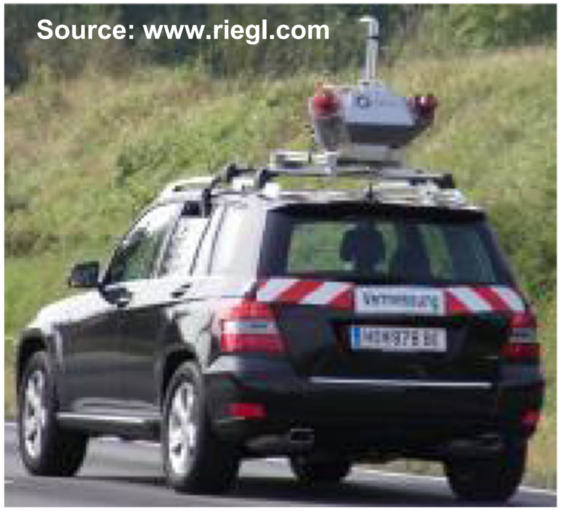
Generation of as-built Plans/Models

Kinematic Terrestrial Laser Scanning

Kinematic Terrestrial Laser Scanning

- Three Measurement Systems
  - GNSS
  - IMU
  - Laser scanner emits laser beams with high frequency and collects the reflections.

Kinematic Terrestrial Laser Scanning

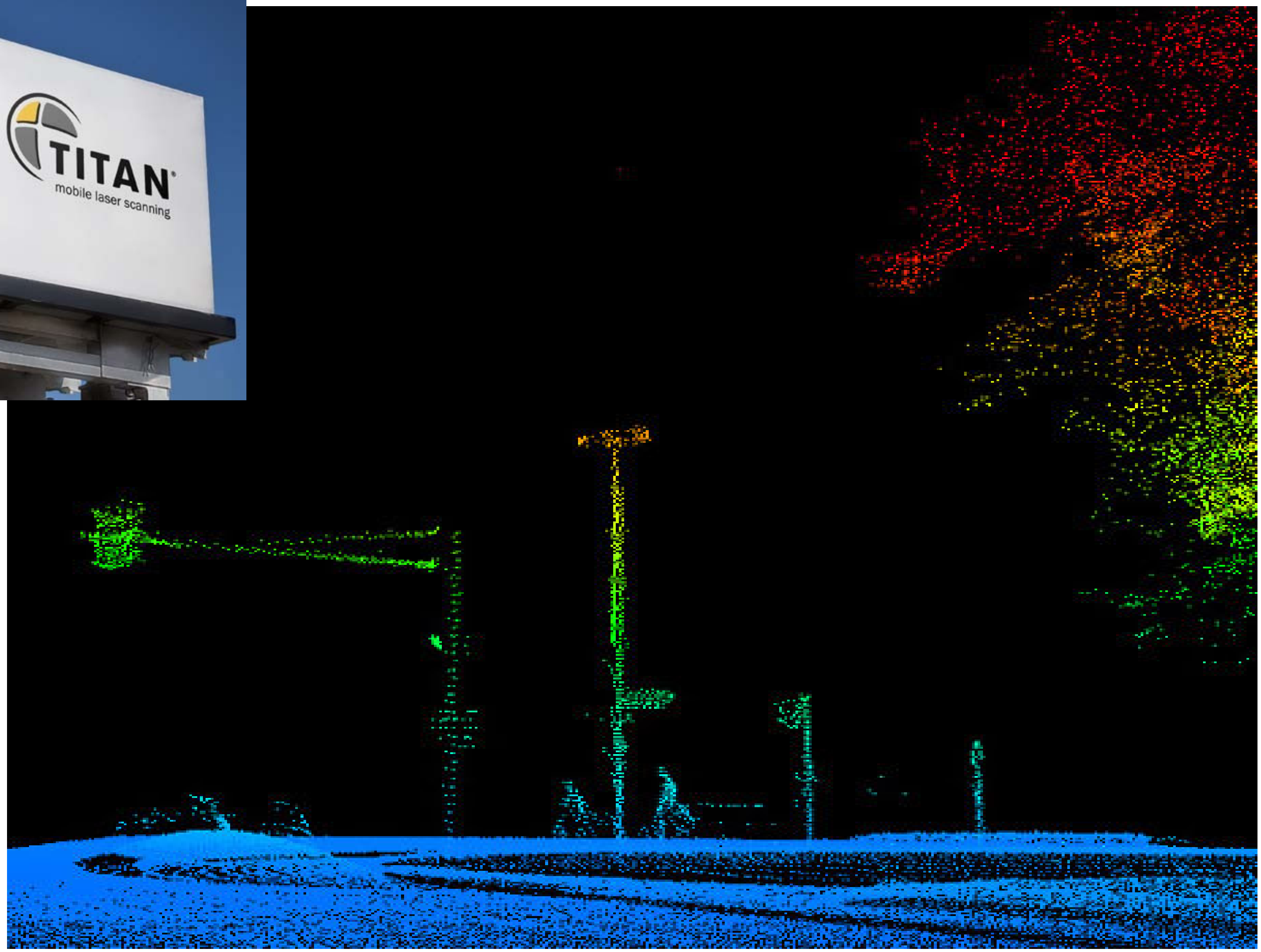

Source: www.riegl.com

Source: http://www.ambercore.com/titan.php

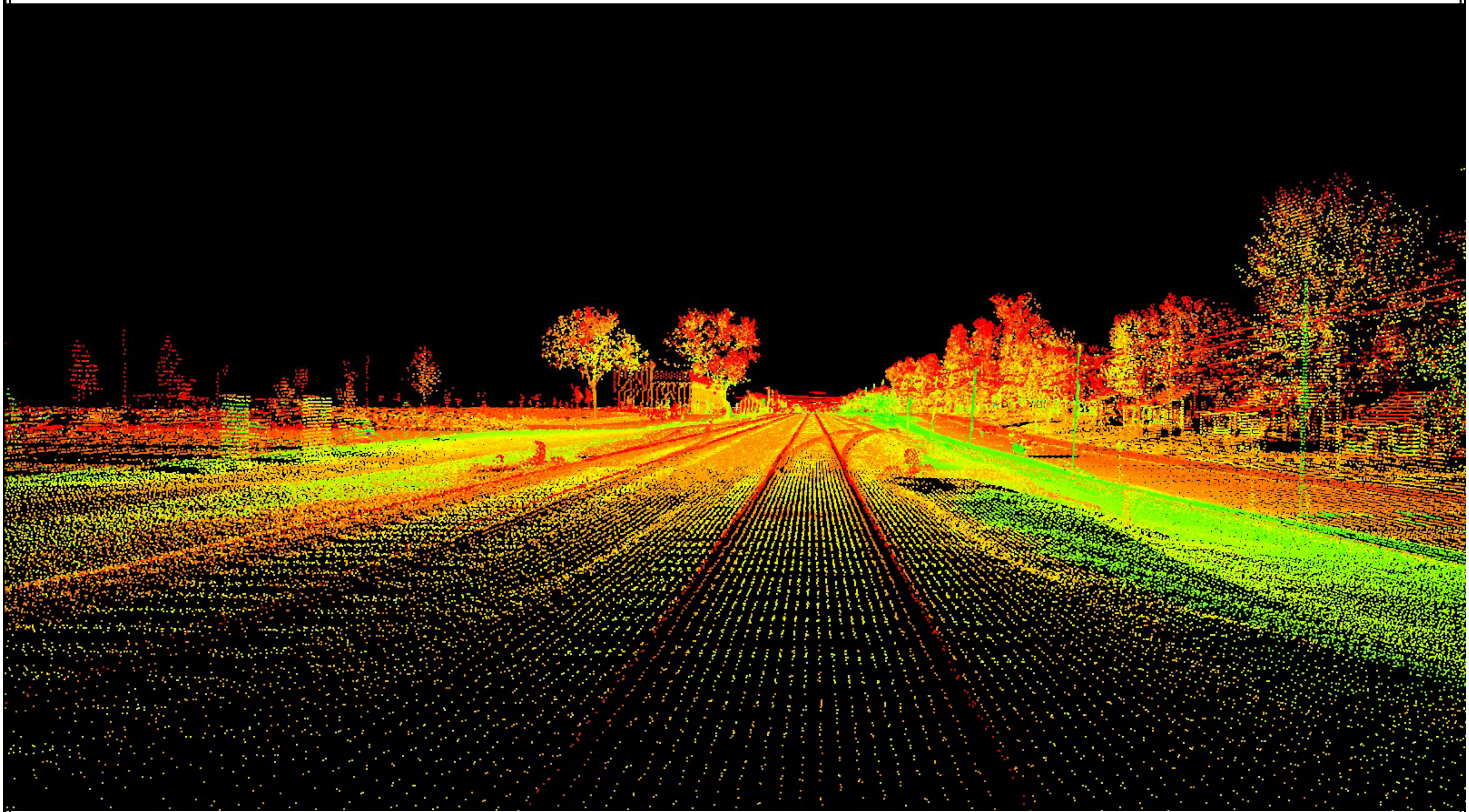
Source: http://www.streetmapper.net/

Source: http://www.riegl.com/uploads/tx_pxpriegldownloads/10_DataSheet_RIEGL_VMX-250_08-04-2010_PRELIMINARY_pdf.pdf

Kinematic Terrestrial Laser Scanning



Kinematic Terrestrial Laser Scanning

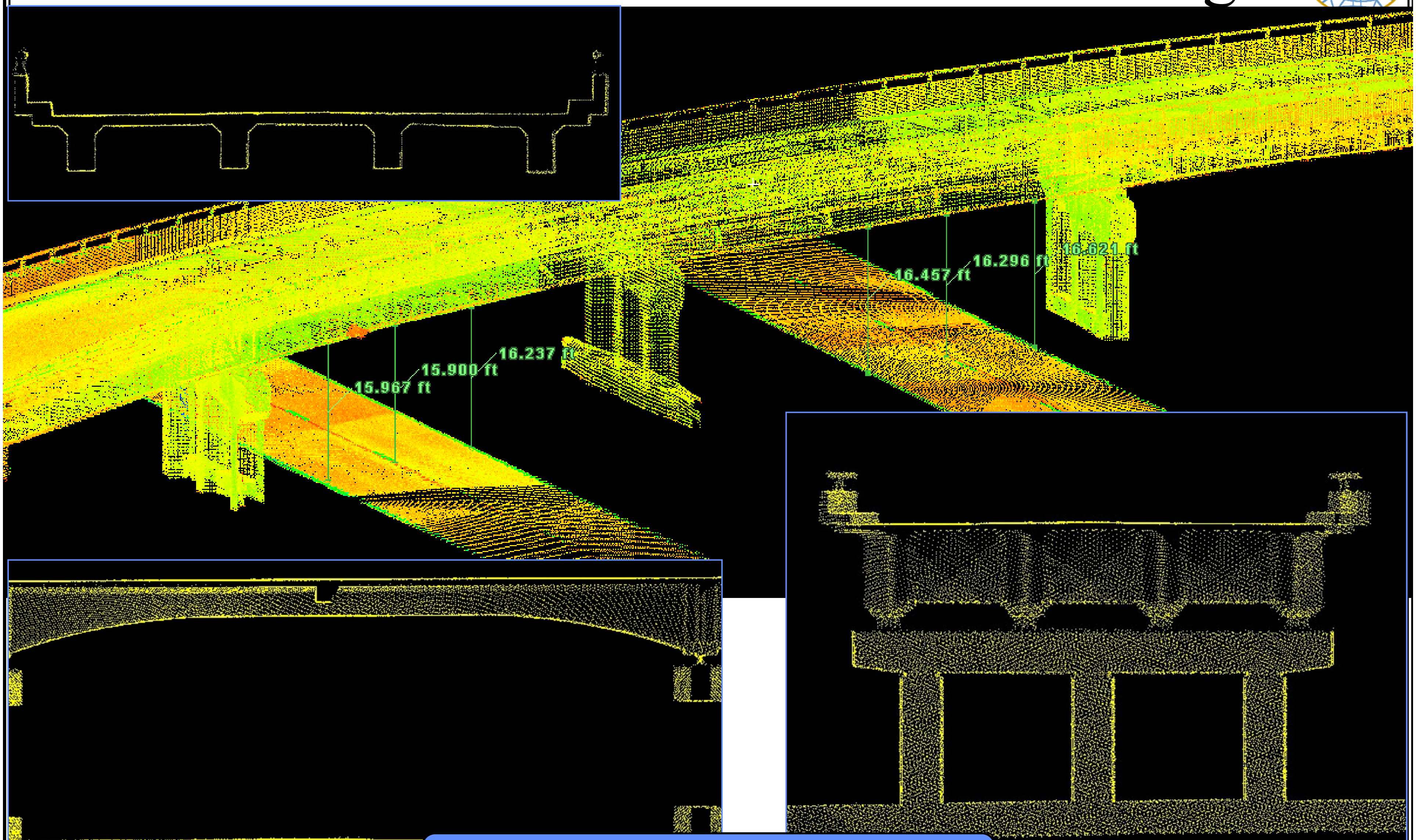


Mapping of Railroad Transportation Network

Kinematic Terrestrial Laser Scanning

Power line and communication network management

Kinematic Terrestrial Laser Scanning

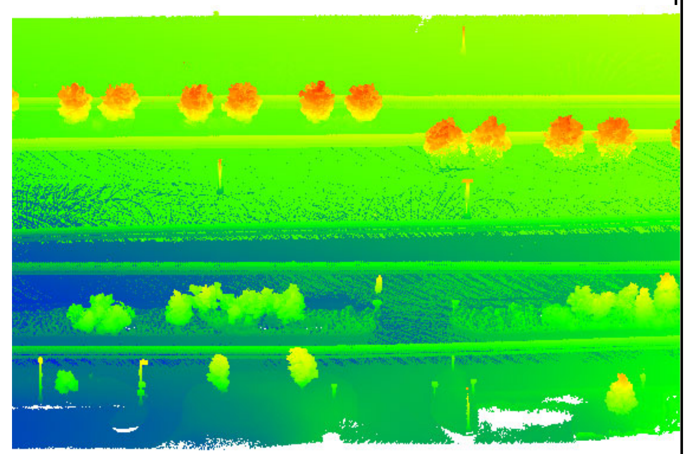


Structural health monitoring

Kinematic Terrestrial Laser Scanning

Platform: Truck

Test Area: Stadium

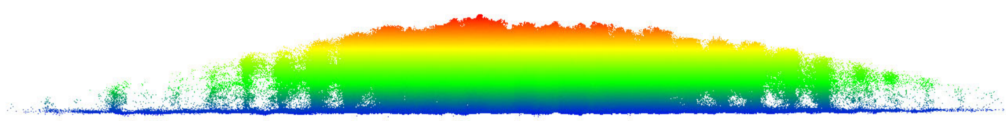


Collected point cloud
(Colored by height)

Purdue University System

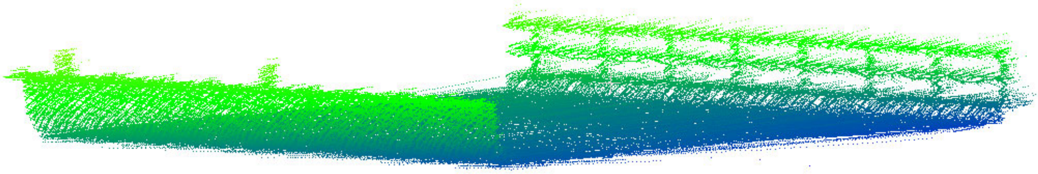
Kinematic Terrestrial Laser Scanning

Phenomobile: RGB, Hyperspectral, and LiDAR



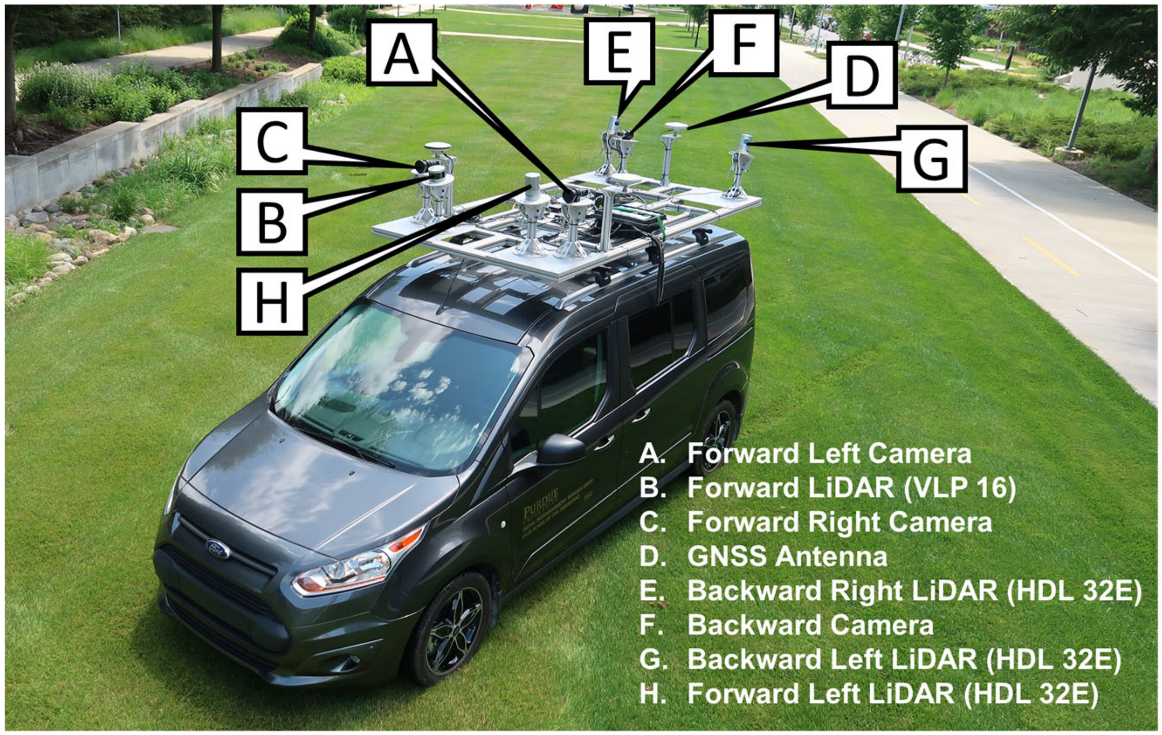
Purdue University System

Kinematic Terrestrial Laser Scanning



Purdue University System

Kinematic Terrestrial Laser Scanning

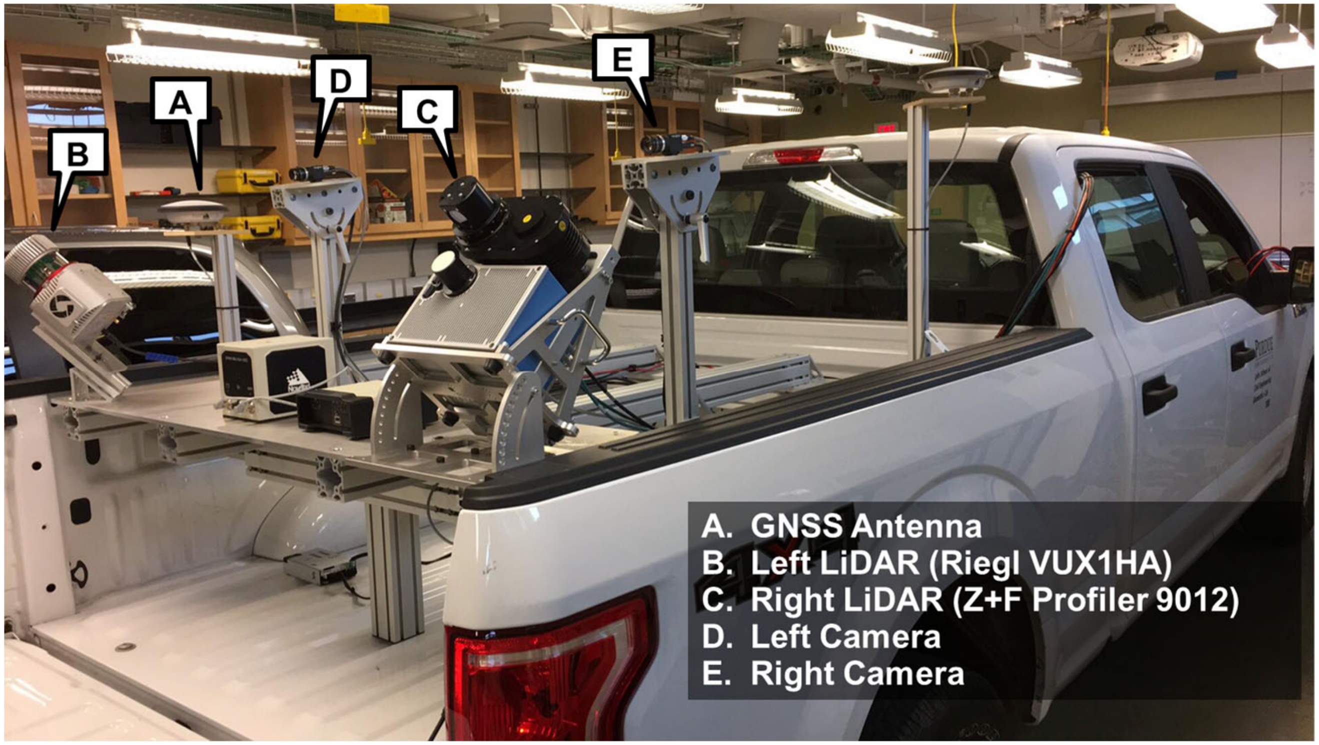


PWMMS1-HA

Purdue Wheel-based Mobile Mapping Systems (PWMMS): High Accuracy

Purdue University System

Kinematic Terrestrial Laser Scanning

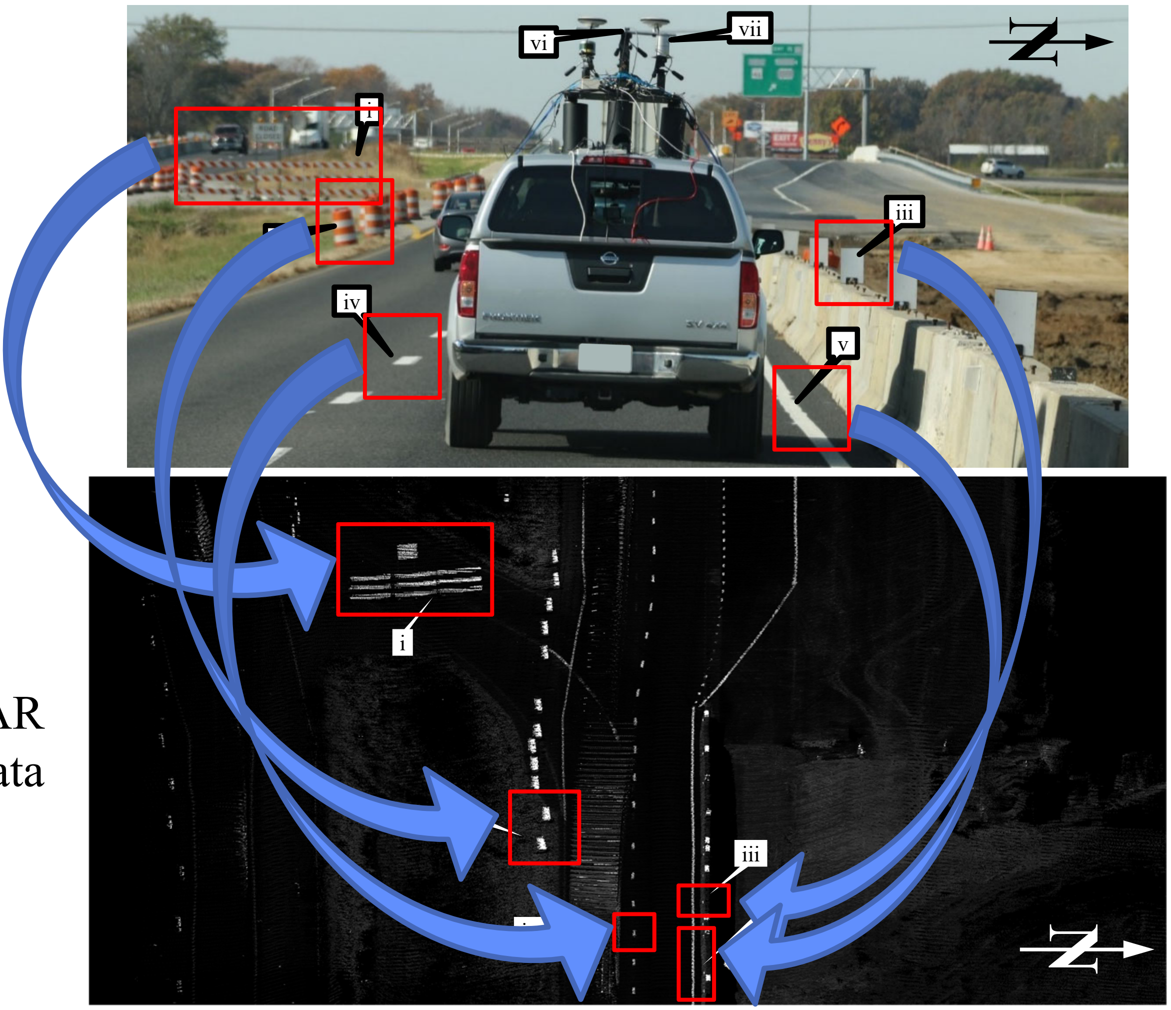


PWMMS2-UHA

Purdue Wheel-based Mobile Mapping Systems (PWMMS): Ultra High Accuracy

Purdue University System

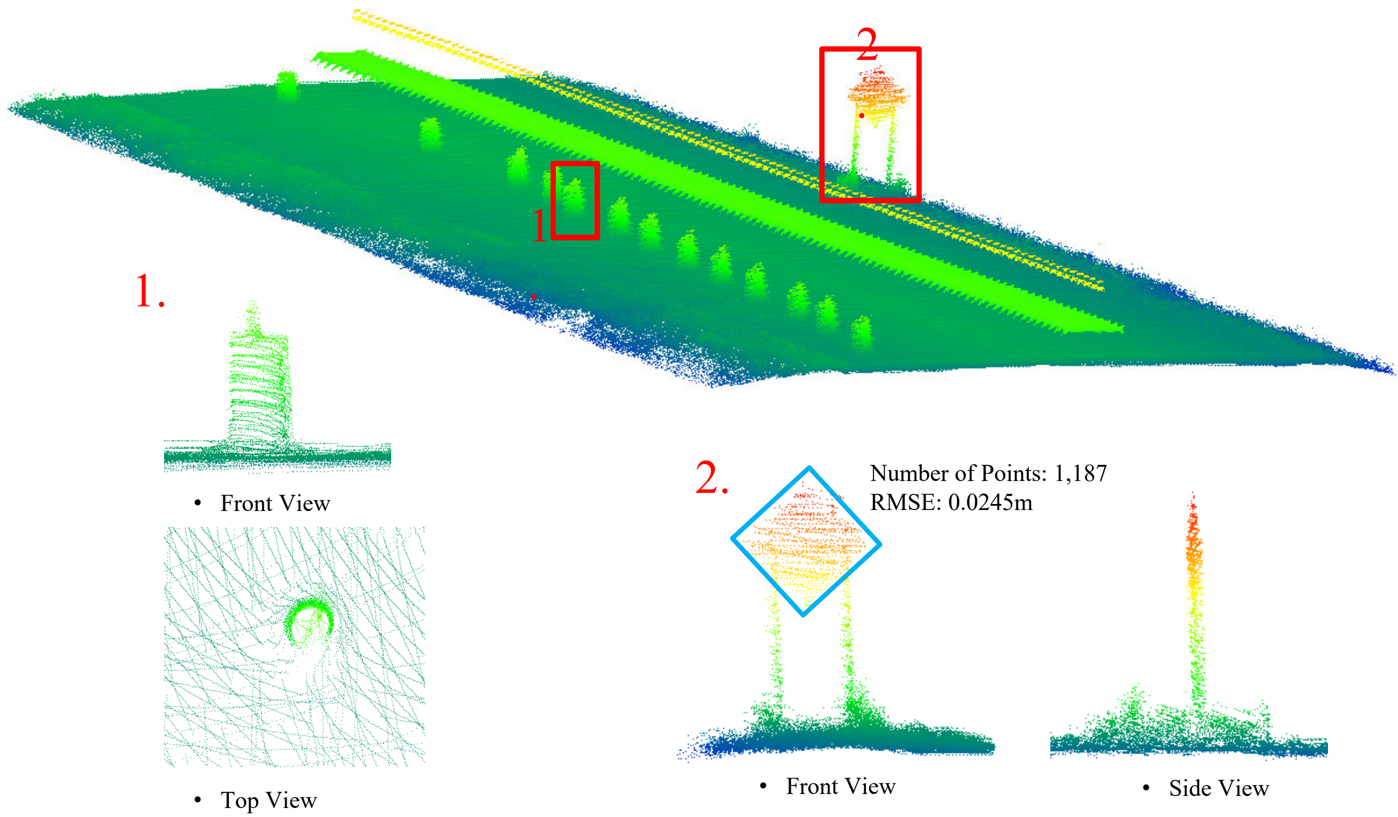
Kinematic Terrestrial Laser Scanning



LiDAR
Intensity Data

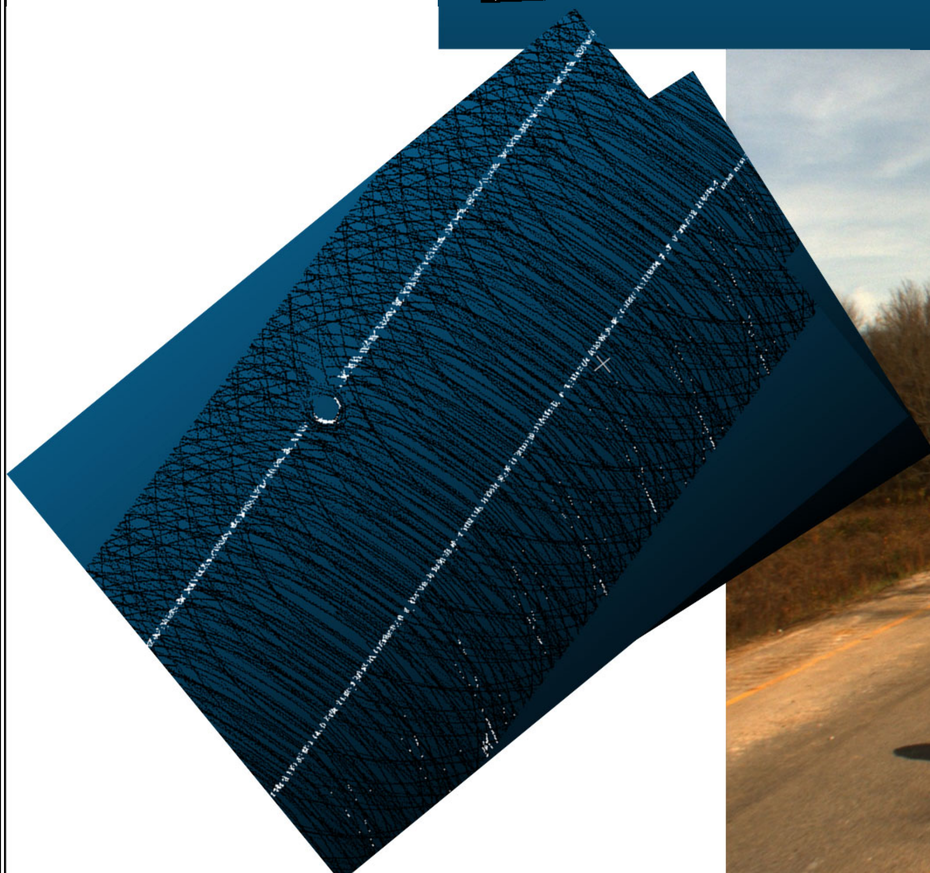
Purdue University System

Kinematic Terrestrial Laser Scanning



Purdue University System

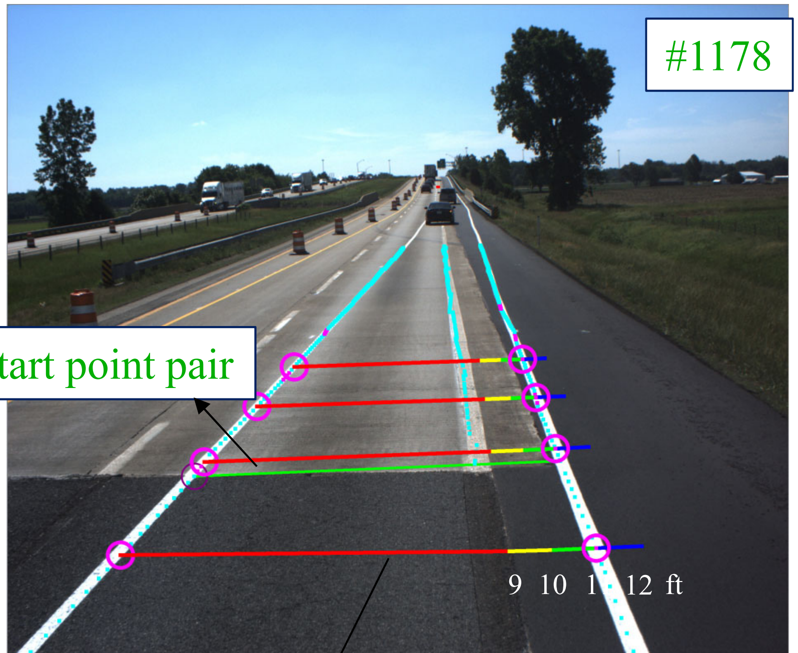
Kinematic Terrestrial Laser Scanning



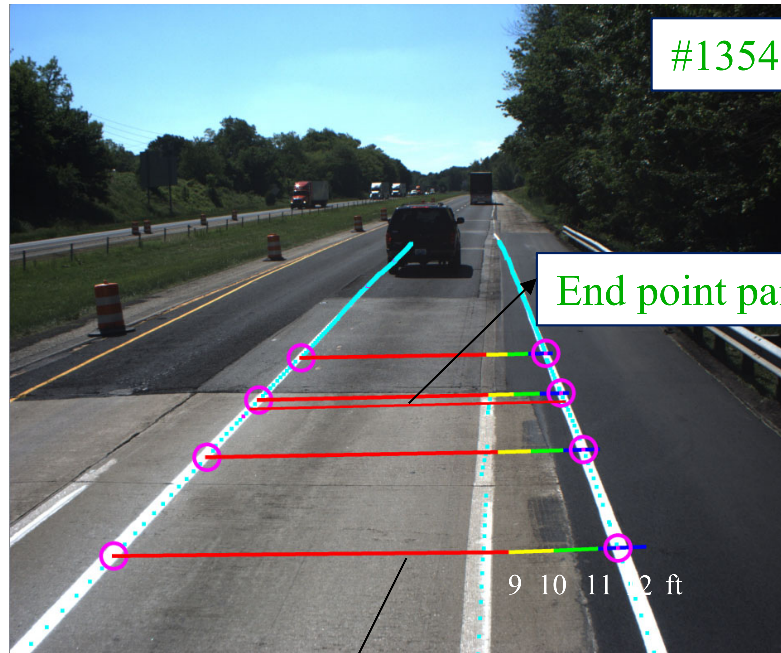
Purdue University System

Kinematic Terrestrial Laser Scanning

Type: Confusing Lane Marker



Standard lane width



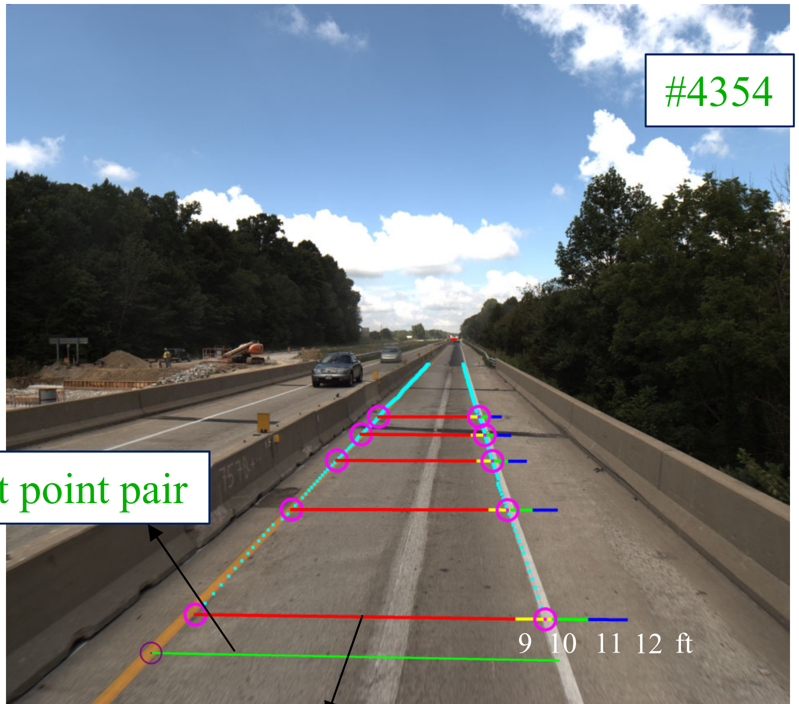
Standard lane width

● Lane Maker point
● Interpolation point

Lane Marking Detection for Work Zone Monitoring

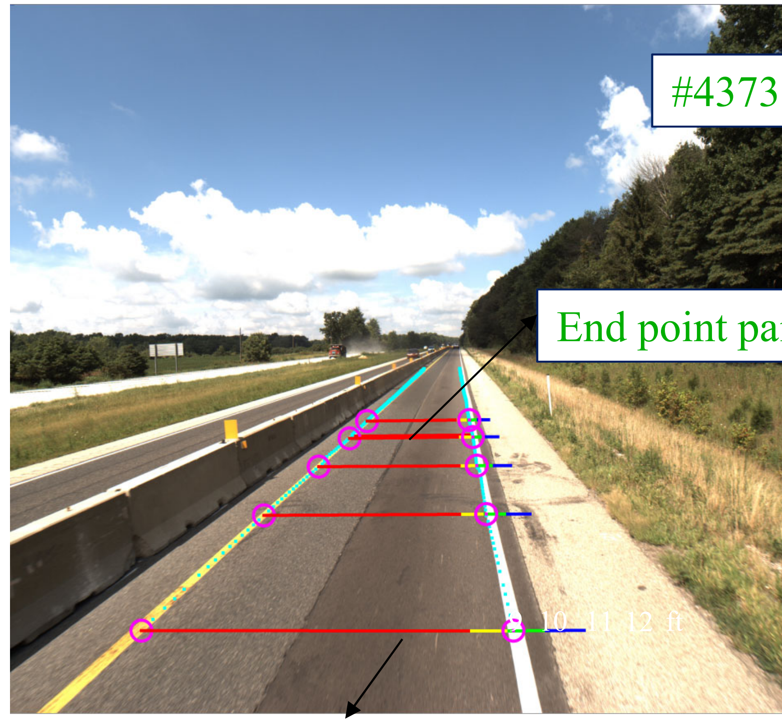
Kinematic Terrestrial Laser Scanning

Type: Narrow Lane



Start point pair

Standard lane width

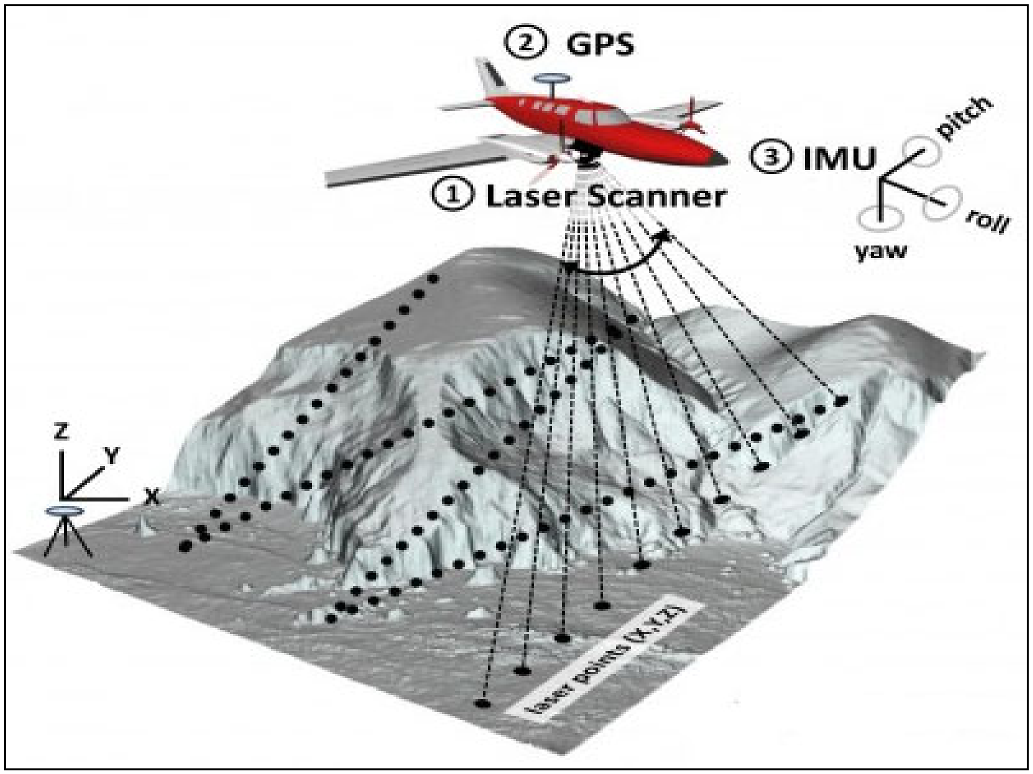


Standard lane width

- Lane Maker point
- Interpolation point

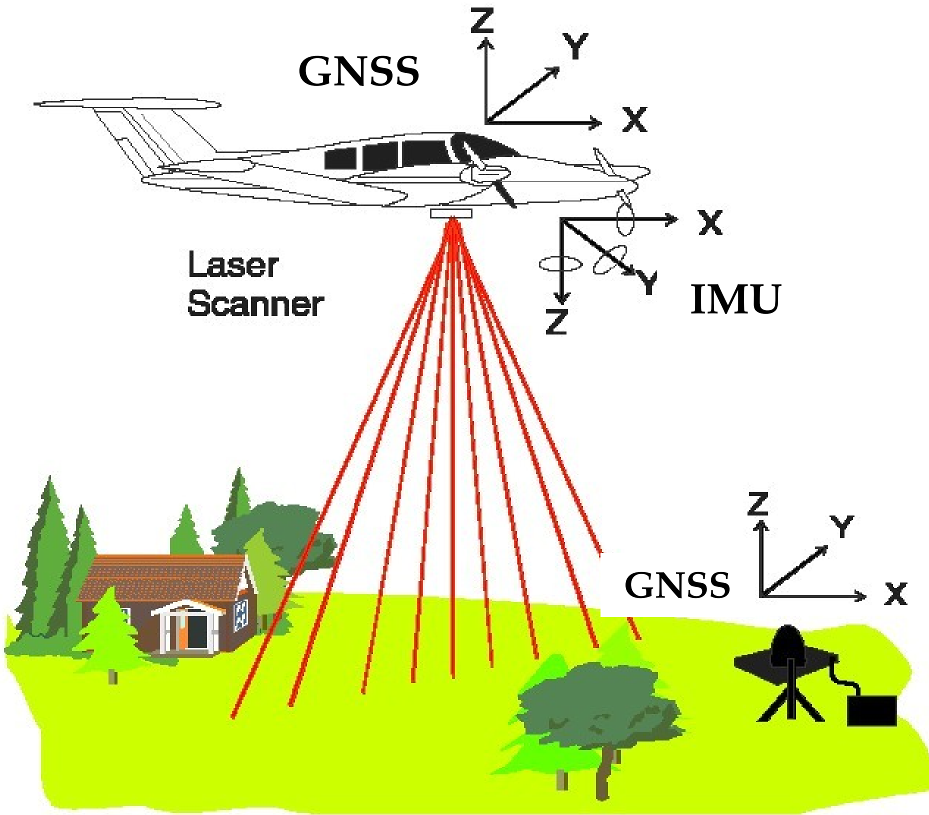
Lane Marking Detection for Work Zone Monitoring

Airborne LiDAR Mapping



Airborne LiDAR Mapping

- Three Measurement Systems
  - GNSS
  - IMU
  - Laser scanner emits laser beams with high frequency and collects the reflections.



Airborne LiDAR Mapping

Airborne LiDAR Mapping

ALS 40 (Leica Geosystems)

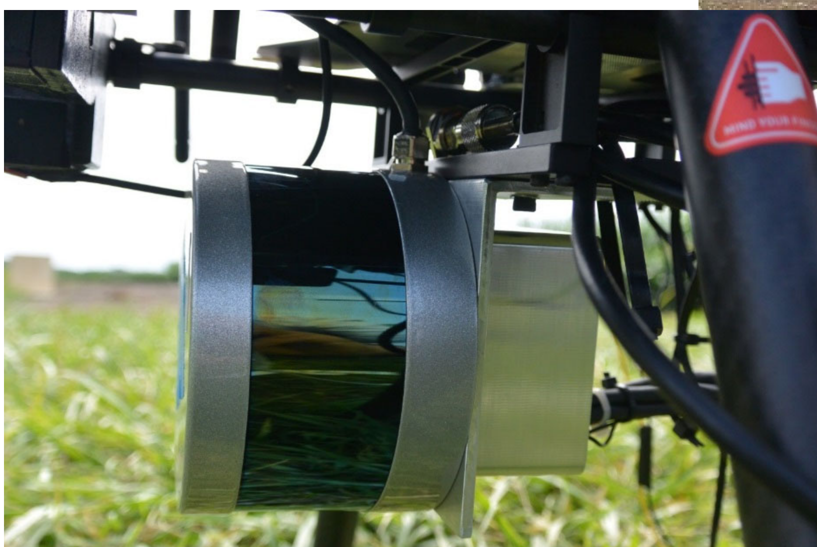
Airborne LiDAR Mapping

OPTECH ALTM 3100

Airborne LiDAR Mapping

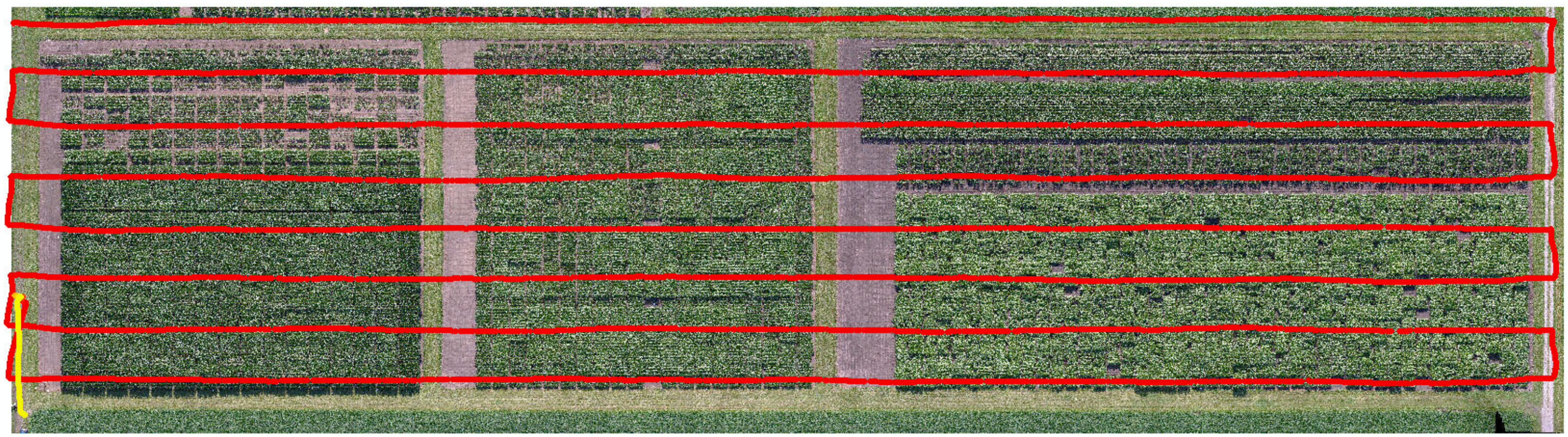
Reigl 680i LiDAR

UAV-Based LiDAR Mapping



Purdue University System

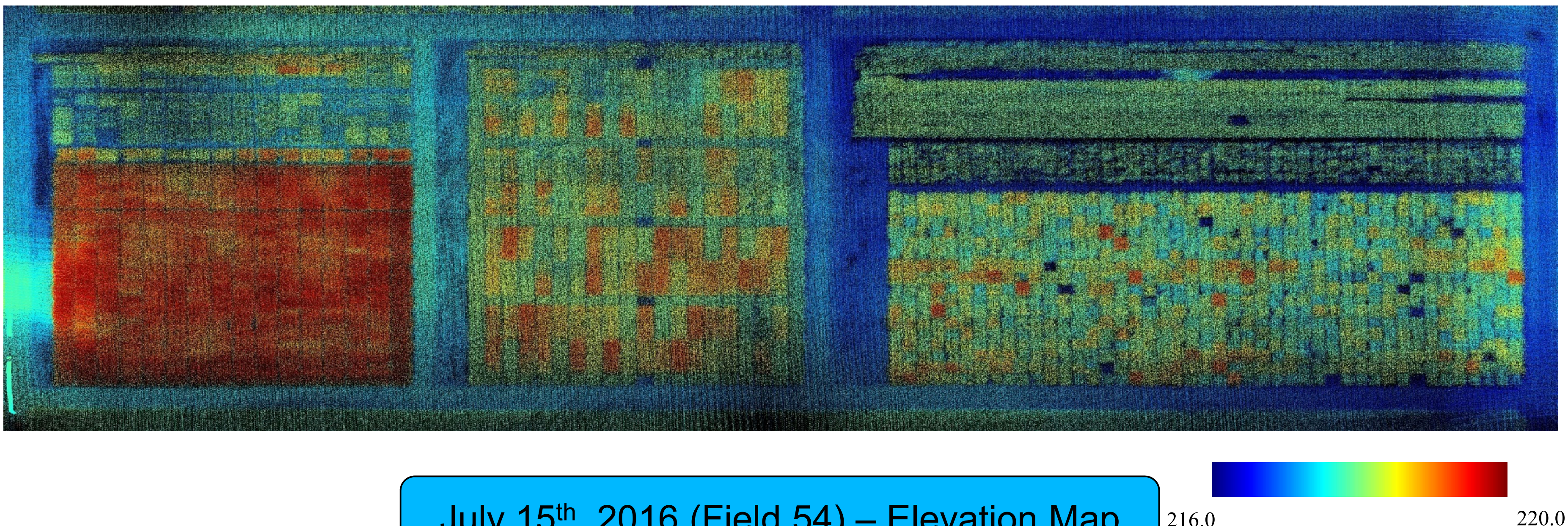
UAV-Based LiDAR Mapping



July 15th, 2016 (Field 54) – Trajectory

Purdue University System

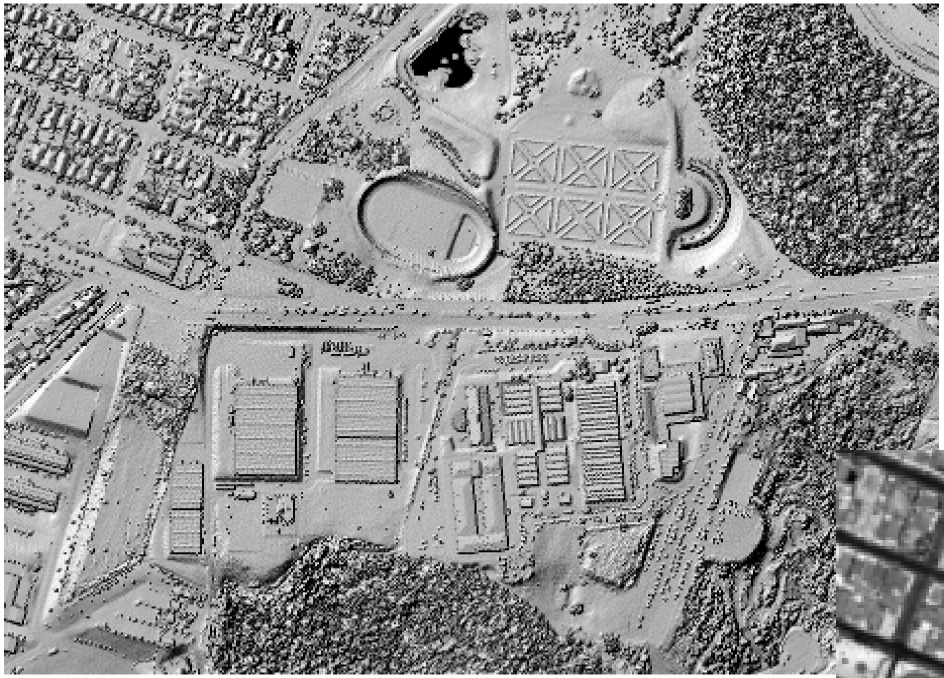
UAV-Based LiDAR Mapping



July 15th, 2016 (Field 54) – Elevation Map

Purdue University System

Airborne LiDAR Mapping



Range Data (Shaded Relief)
Interpolated

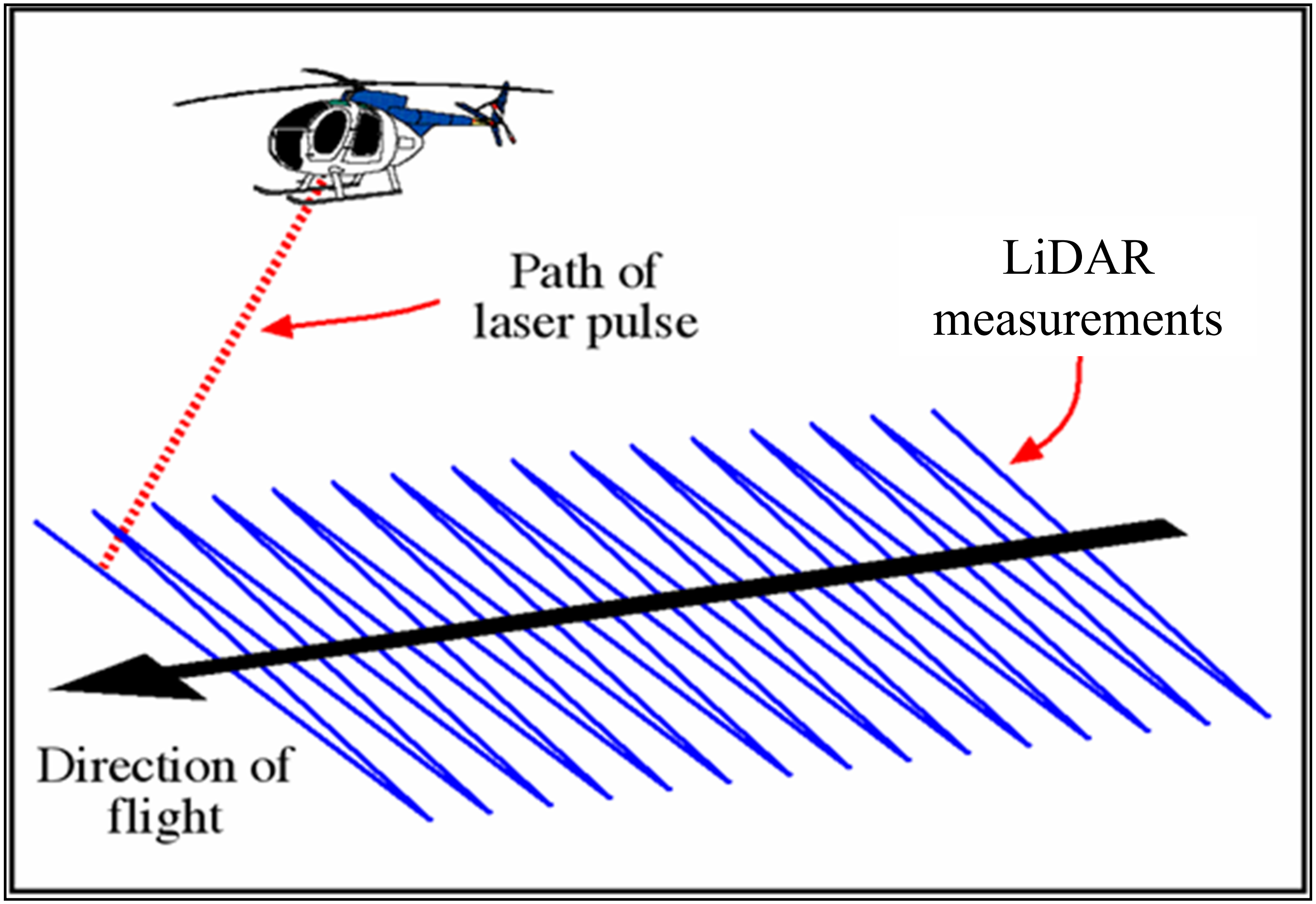
Intensity Data
Interpolated

LiDAR Principles

Sensor Specifications and Data Characteristics

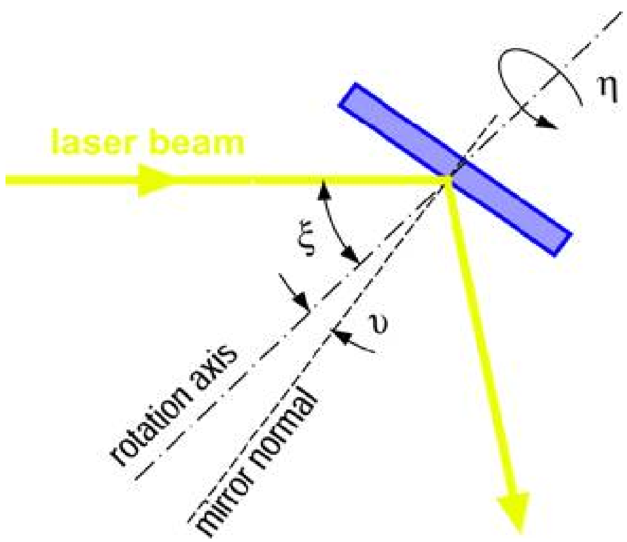
LiDAR Principles

Linear LiDAR Scanners

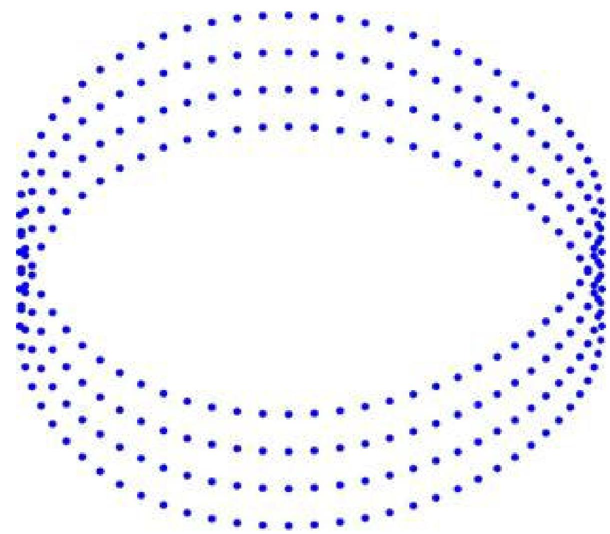


LiDAR Principles

Conical/Elliptical LiDAR Scanners

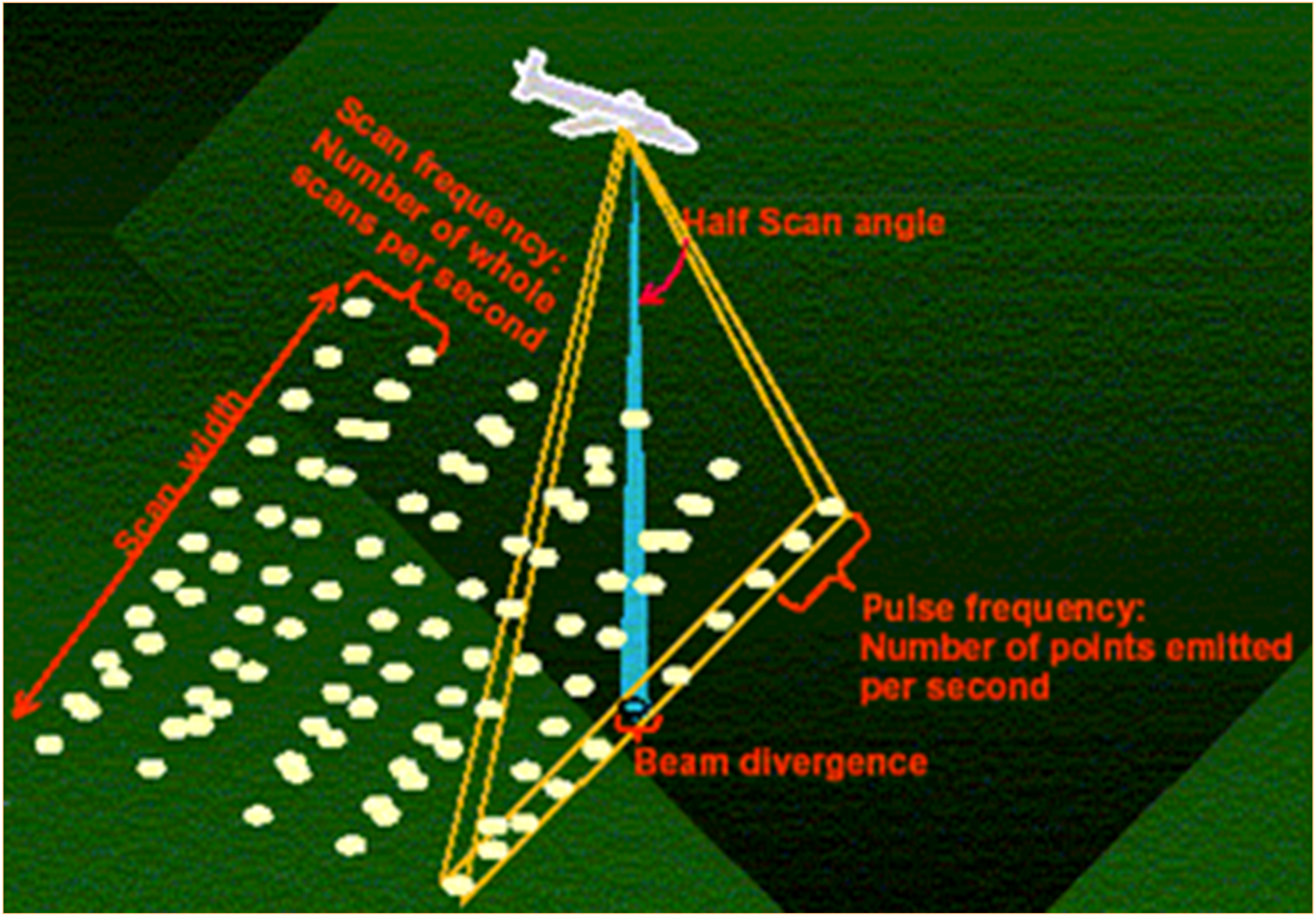


Nutating Mirror



Scanning Pattern

LiDAR Principles



LiDAR Principles

- Scan Rate/Frequency:
  - Number of scanned swaths per second
- Pulse Rate/Frequency:
  - Number of transmitted pulses per second
- Ground Spacing:
  - The distance between the footprints of two adjacent laser pulses
- Other specifications include:
  - Wavelength, scan angle, scan pattern, beam divergence, operating altitude,….

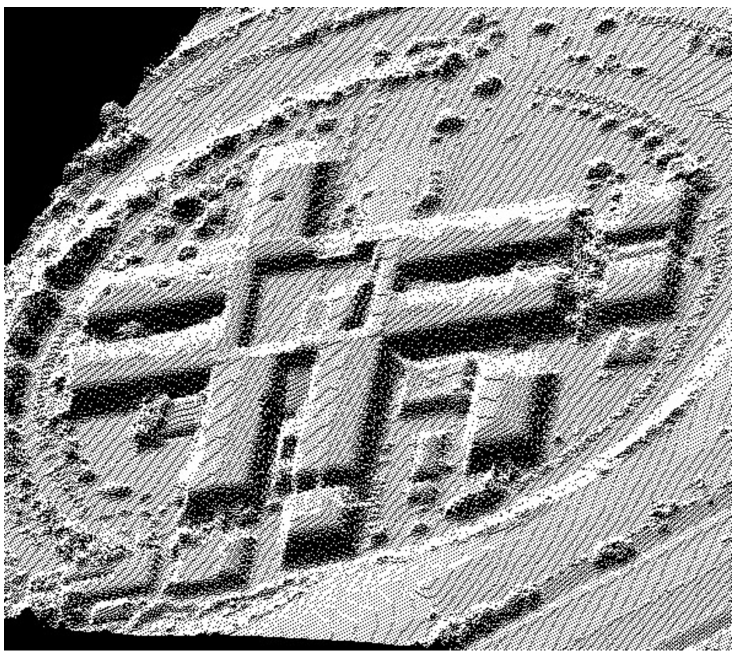
LiDAR Principles

Specification	Typical values
Laser wavelength	1.064 µm
Pulse repetition rate	50 – 500 kHz
Pulse energy	100s µJ
Pulse width	< 10 ns
Beam divergence	0.25 – 2 mrad
Scan angle (full angle)	40º – 75º
Scan rate	25 – 90Hz
Scan pattern	Zig-zag; parallel; elliptical; sinusoidal
GNSS frequency	1 – 10 Hz
INS frequency	200 – 300 Hz
Operating altitude	80 – 3,500 m (6,000m max)
Footprint	0.25 – 2m @ 1,000m AGL
Multiple elevation capture	1 – 4 (Full waveform)
Ground spacing	0.5 – 2m
Vertical accuracy	< 5 – 30 cm (1,000 –3,000 m altitude AGL); 1 σ
Horizontal accuracy	1/5,500 – 1/2000 x altitude (m AGL); 1 σ

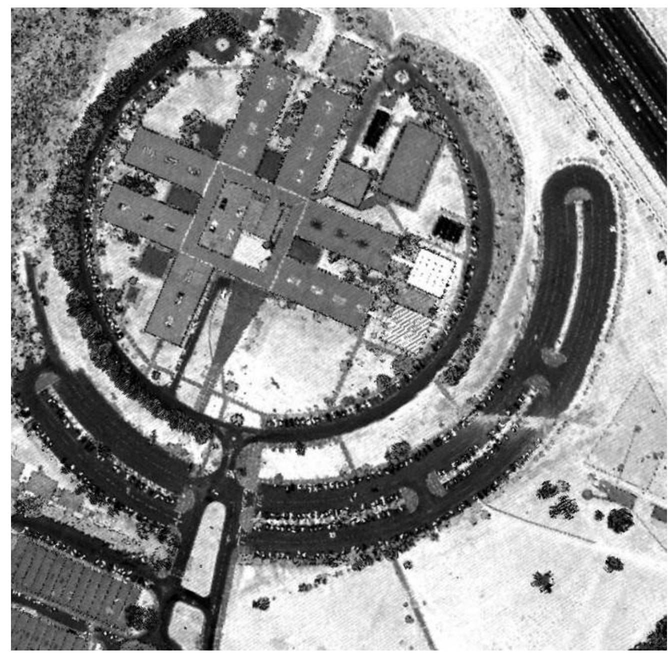
Specifications of Typical LiDAR Systems

LiDAR Principles

- LiDAR produces accurate point cloud measurements of surfaces in addition to intensity images.

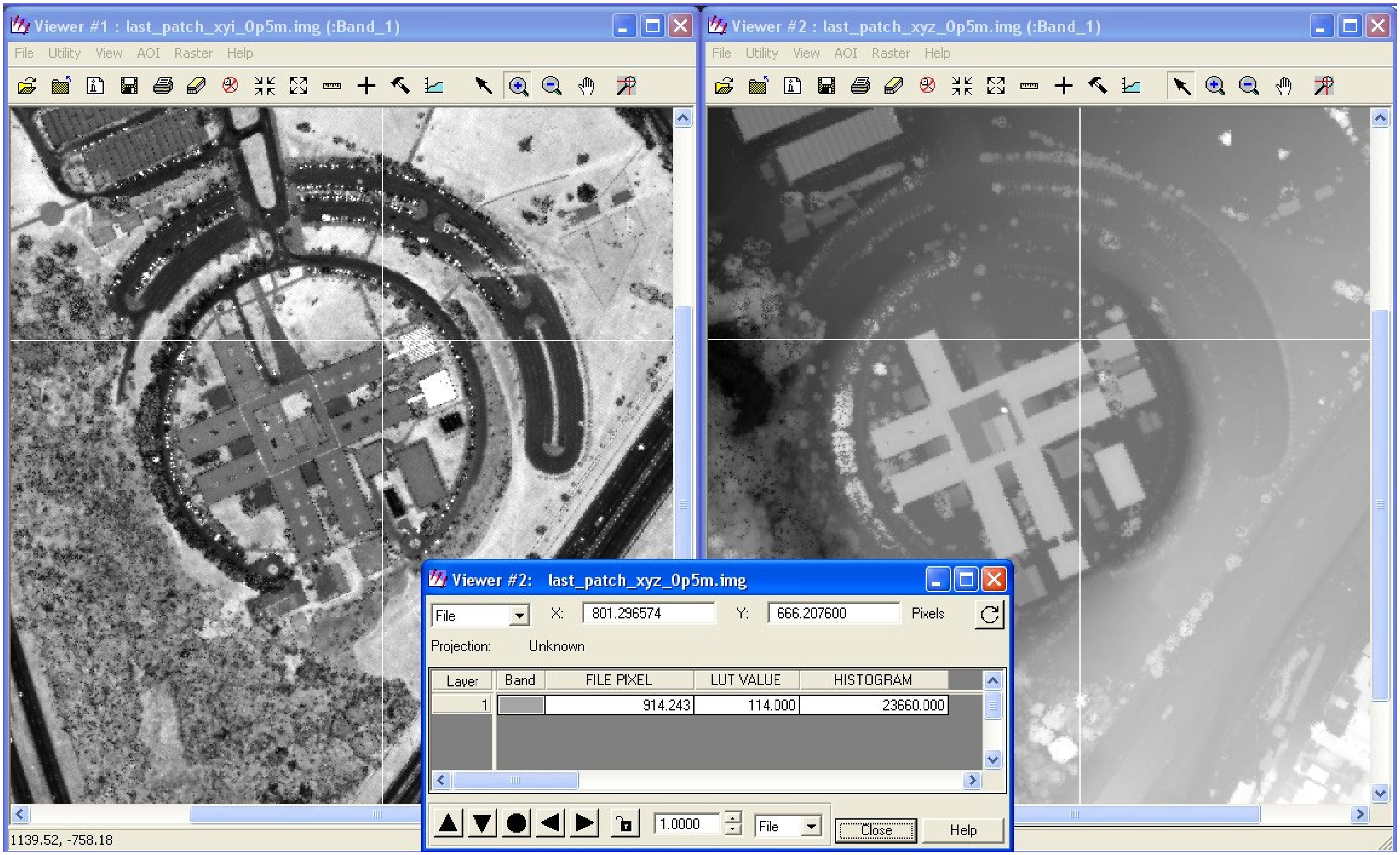


Elevation Data



Intensity Image

LiDAR Principles



LiDAR Principles

- Some restrictions
  - Restricted platforms: with few space-borne systems
    - Ground-based and airborne systems are more common.
  - Flying height:
    - Restricted by laser power, sensitivity of sensor, unambiguous maximum pulse rate
    - 2,000 m or less, it recently reached 6,000 m with unknown accuracies.
    - Minimum height restricted by safety
  - Flying speed restricted by scan rate and point density requirements as well as GNSS/INS

LiDAR Principles

- Compared to photogrammetric systems, LiDAR systems are **<u>not transparent</u>**.
  - Still a provided service
    - Raw measurements are not always accessible.
  - No single system to process the data
  - No interoperability between available systems
  - No standards for calibration, strip adjustment, number and distribution of control points …
  - High initial cost

LiDAR Principles

- LiDAR output:
  - 232802.510 319978.600 44.300 41.0 9 First
  - 232802.510 319978.600 44.300 41.0 9 Last
  - 232802.360 319979.590 44.460 38.0 9 First
  - 232802.360 319979.590 44.460 38.0 9 Last
  - 232802.250 319980.340 44.550 41.0 9 First
  - 232802.250 319980.340 44.550 41.0 9 Last
  - 232802.100 319981.420 44.470 37.0 9 First
  - 232802.100 319981.420 44.470 37.0 9 Last
  - ………….
  - …………….

Black Box (non-transparent model)

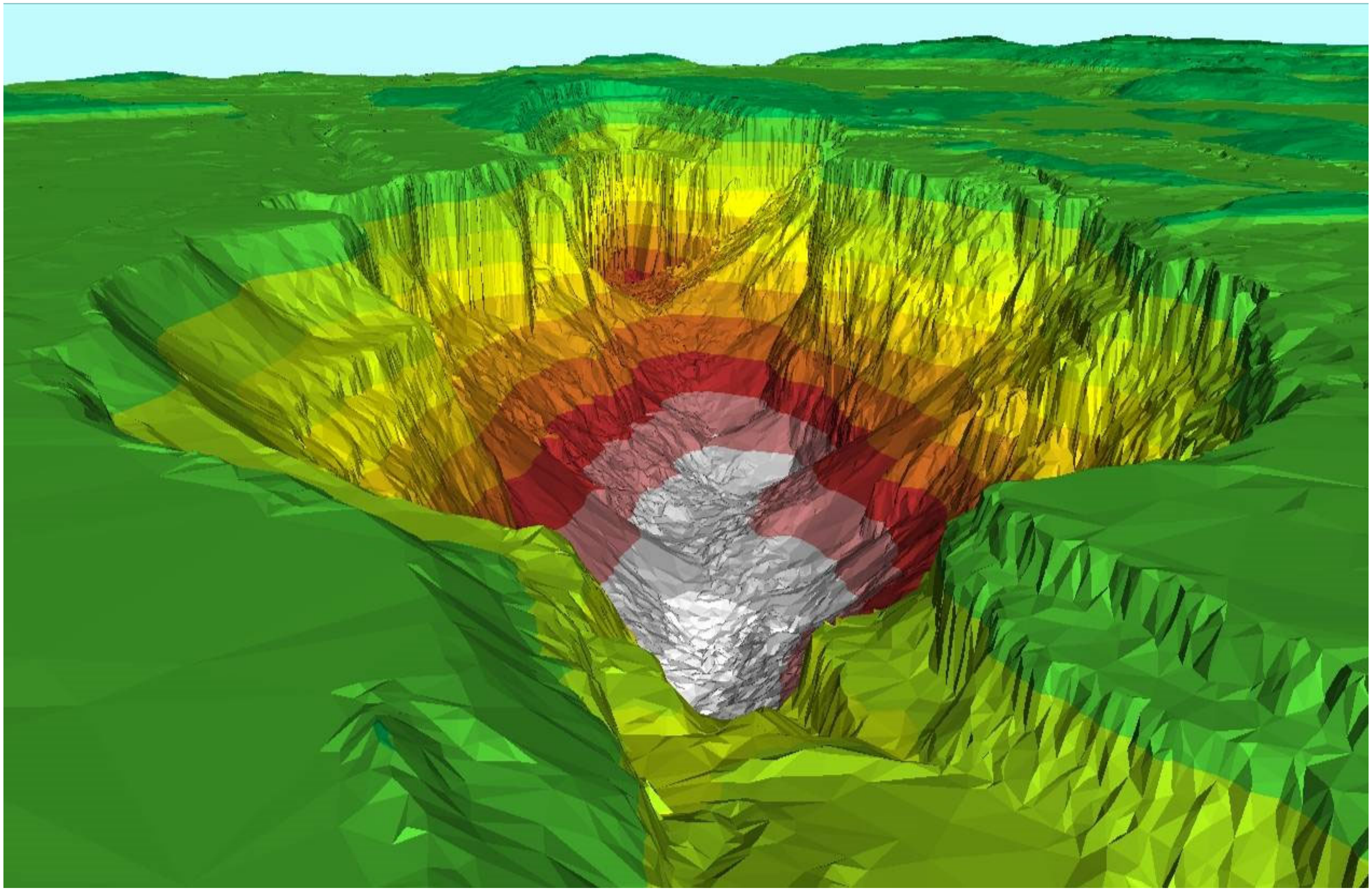
Why Use LiDAR?

- Fast and Accurate
  - 10's – 100's km^2/hour; 5 cm $RMSE_z$ on hard surfaces possible
- Flexible Collection:
  - Maps through canopy
    - Ground measurement is possible.
  - Independent of sun angle
  - Day or night
  - Light rain is tolerated.
  - Mapping of surfaces with very little/no texture or poor definition; ice/snow surfaces, sand, wetlands, ...

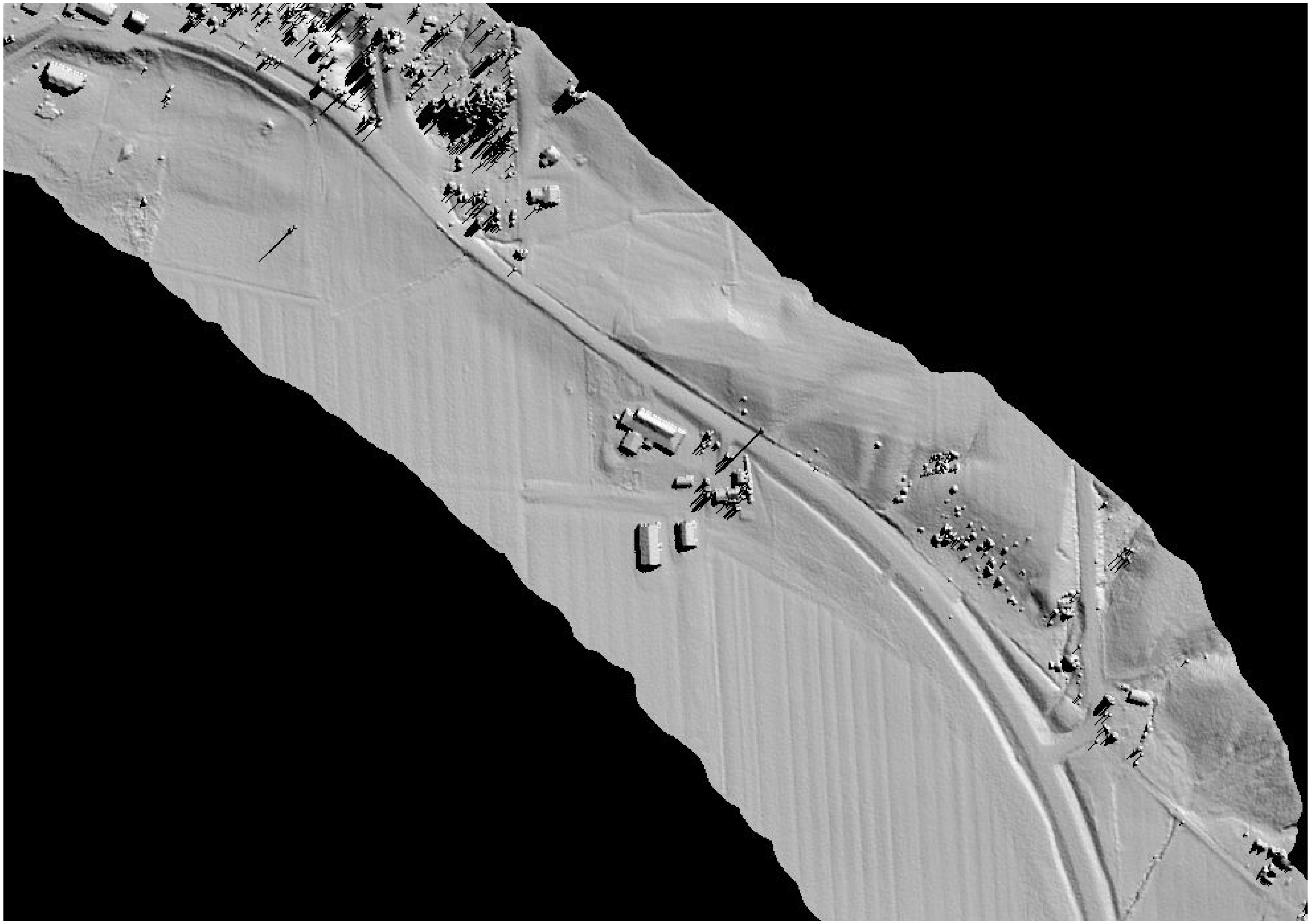
Why Use LiDAR?

- High Resolution 3D Surface:
  - Dense point clouds; Millions of points/km^2
- Diverse Data Products:
  - Full-feature,
  - Bare Earth,
  - Contours,
  - Building Footprints,
  - Land Usage,
  - Transportation/Utility Corridors, and
  - Many more …

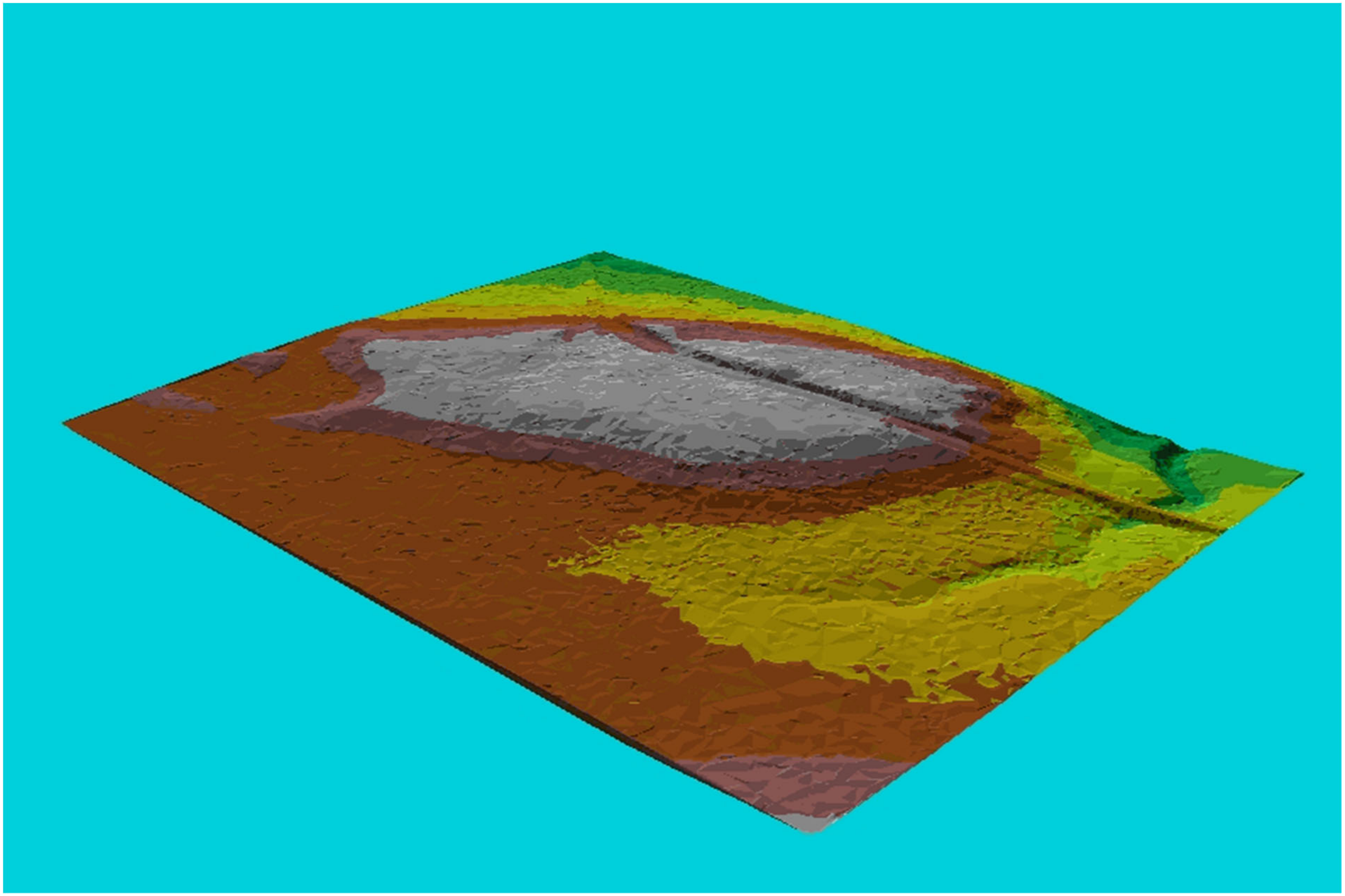
Volumetric Calculations



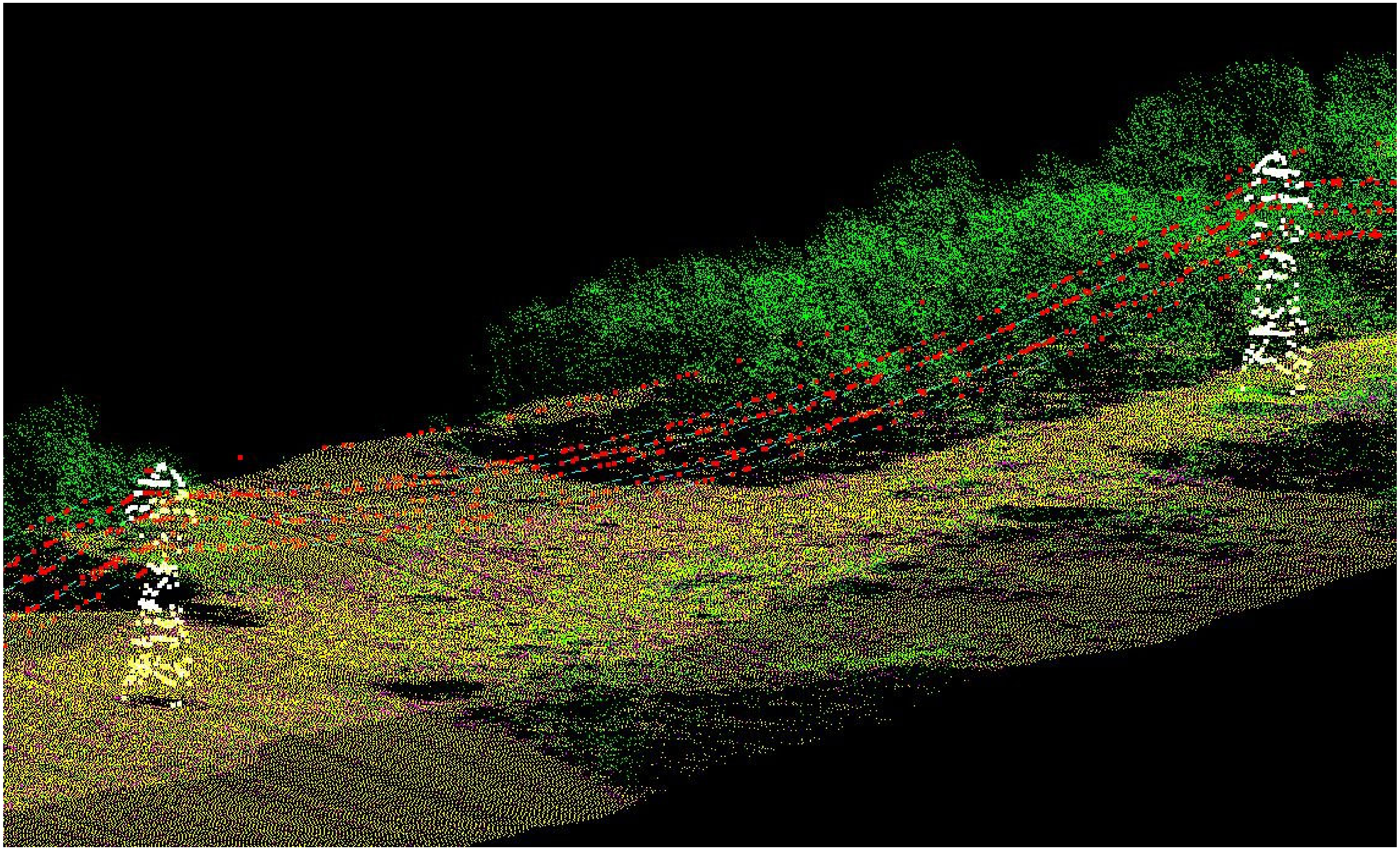
Transportation - Highway Expansion



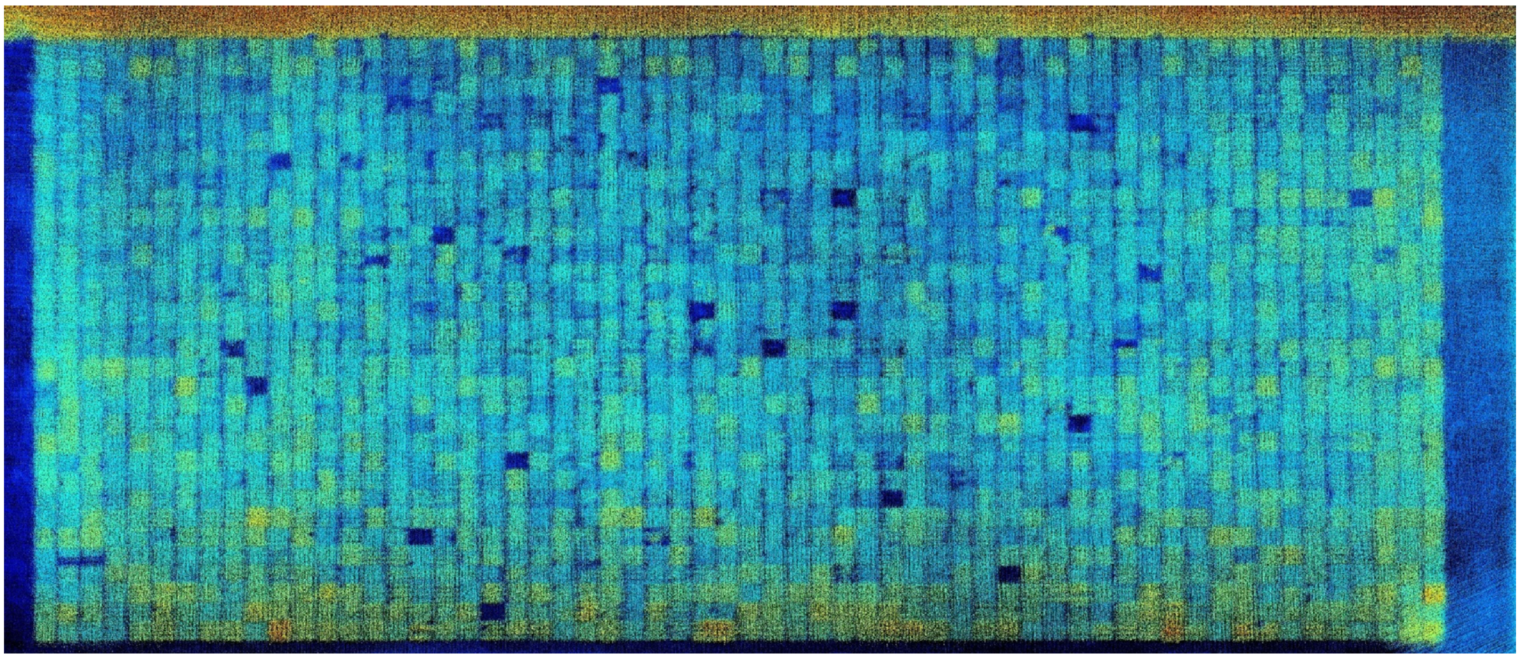
Cut and Fill Applications



Power Line Mapping

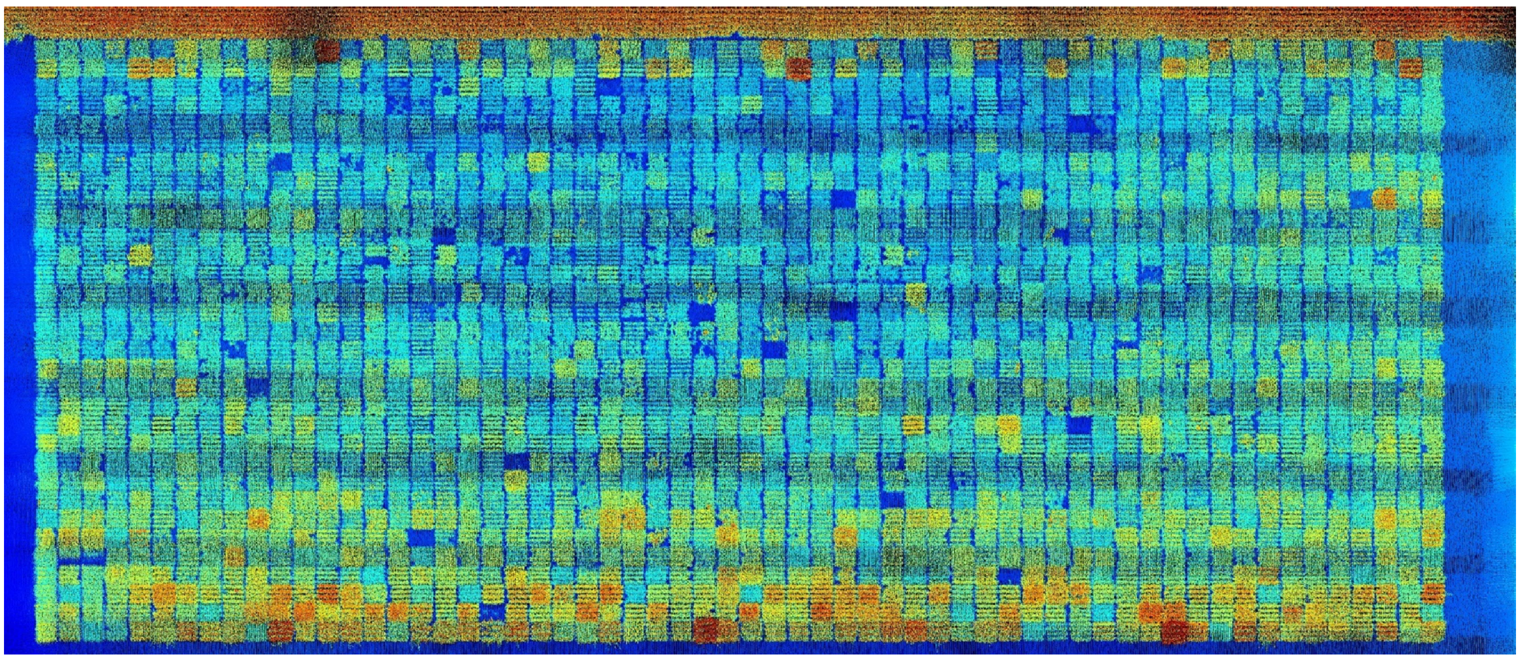


Precision Agriculture



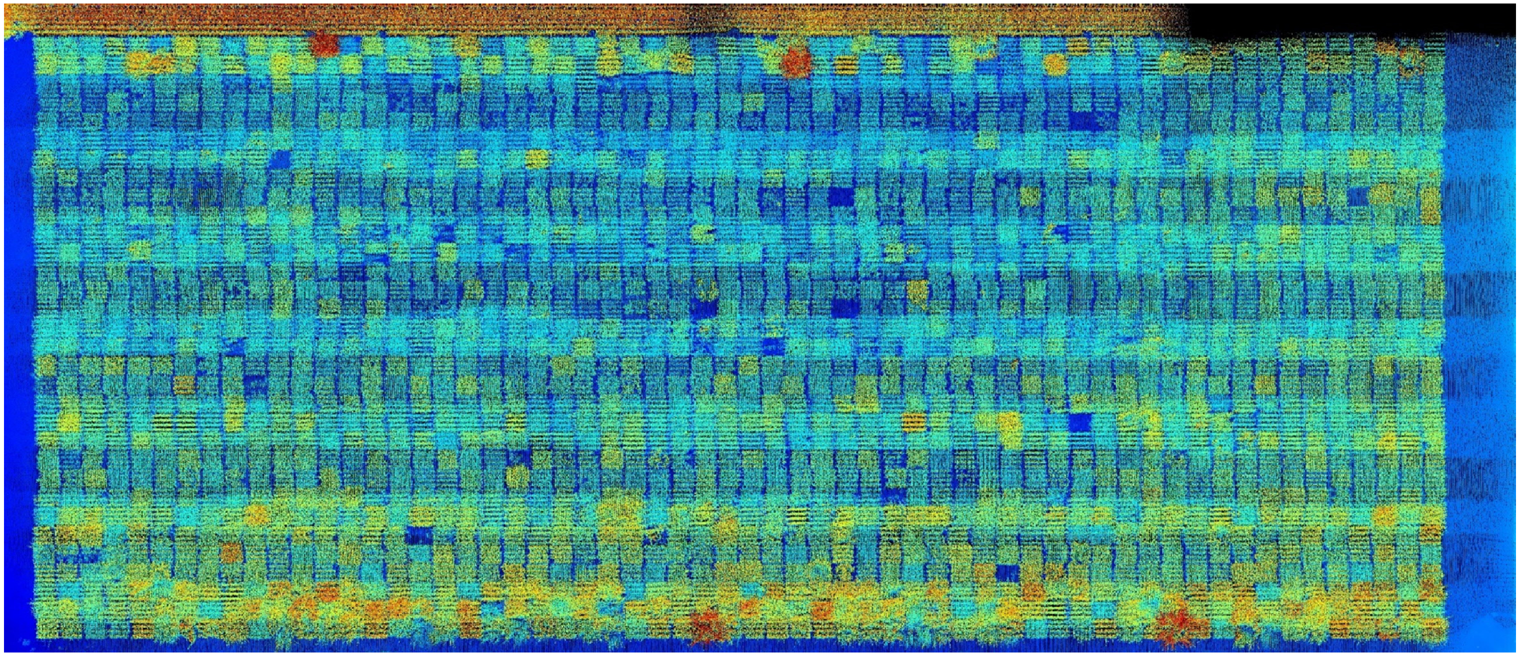
July 19th, 2016 (Field 57 E) – Elevation Map

Precision Agriculture

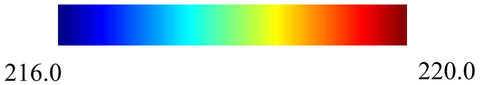


August 10th, 2016 (Field 57 E) – Elevation Map

Precision Agriculture

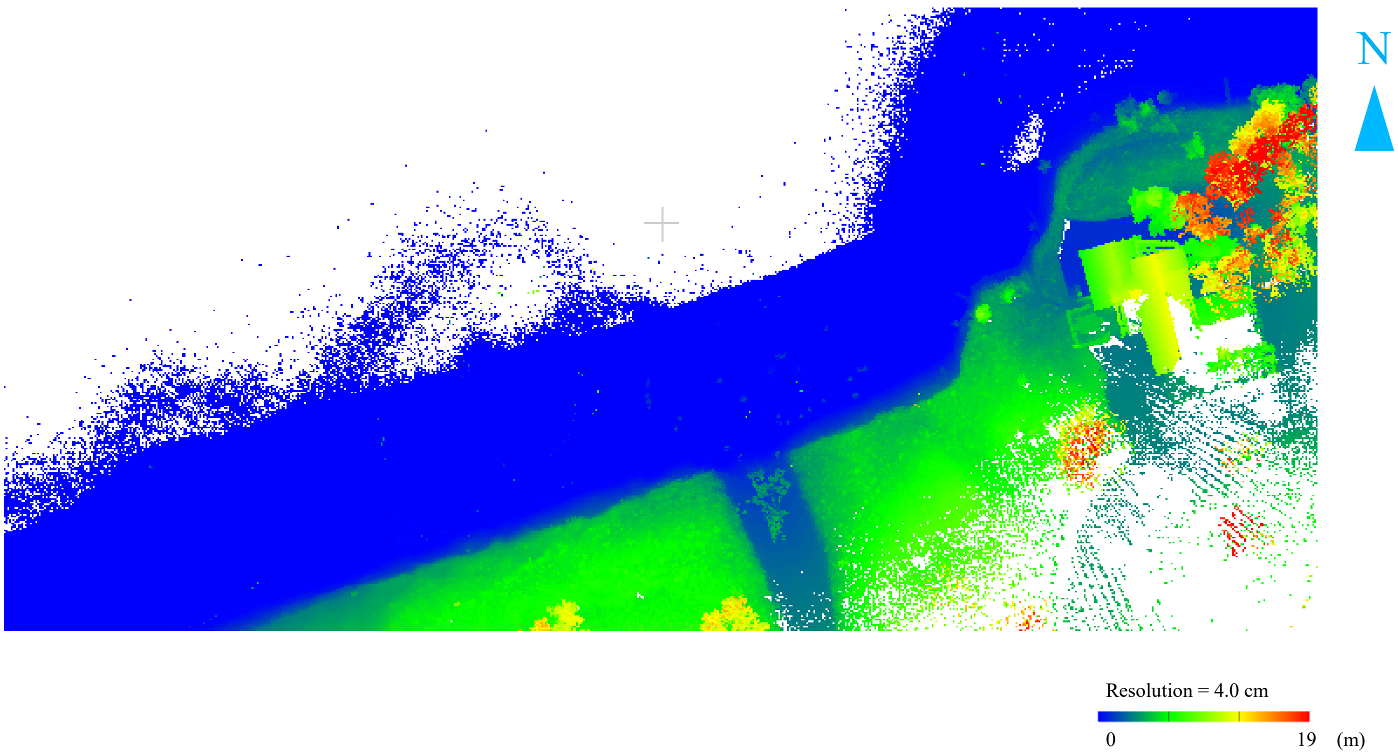


August 30th, 2016 (Field 57 E) – Elevation Map



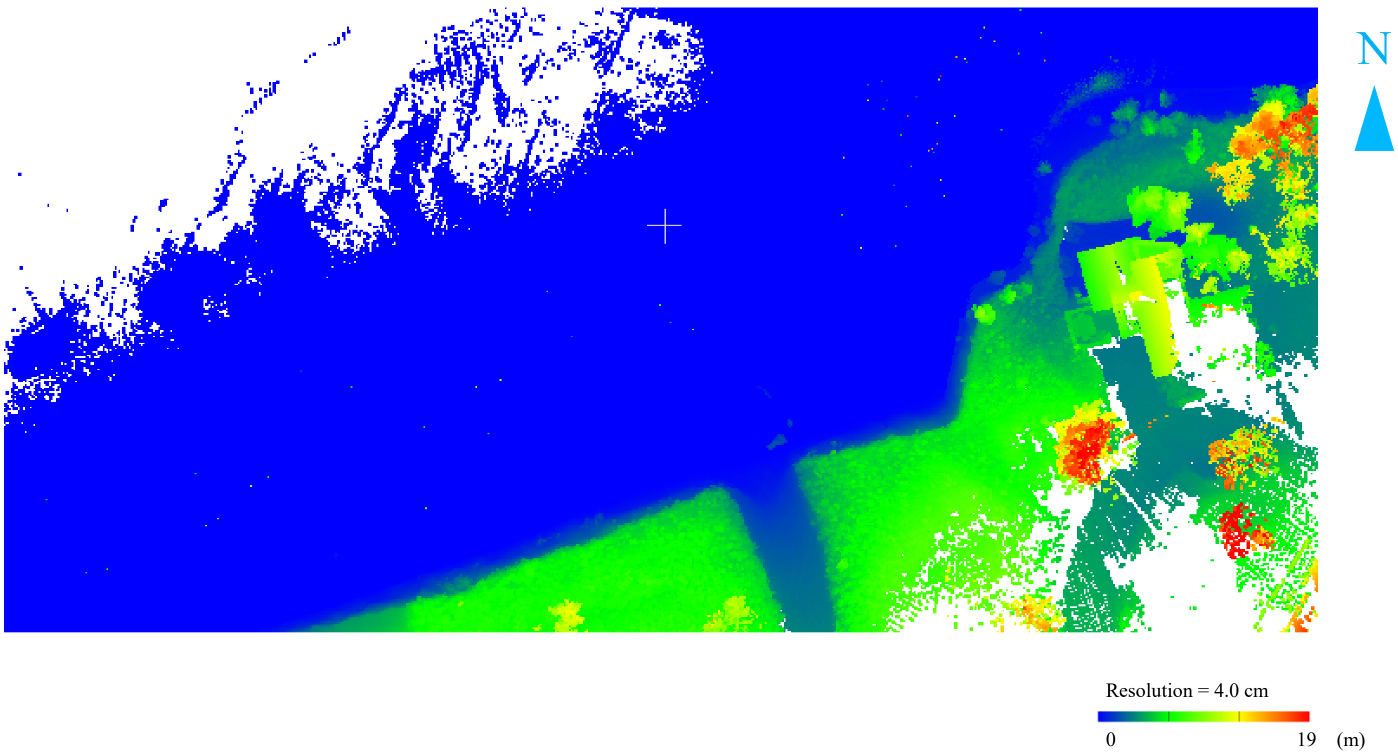
Shoreline Monitoring

Shoreline Monitoring



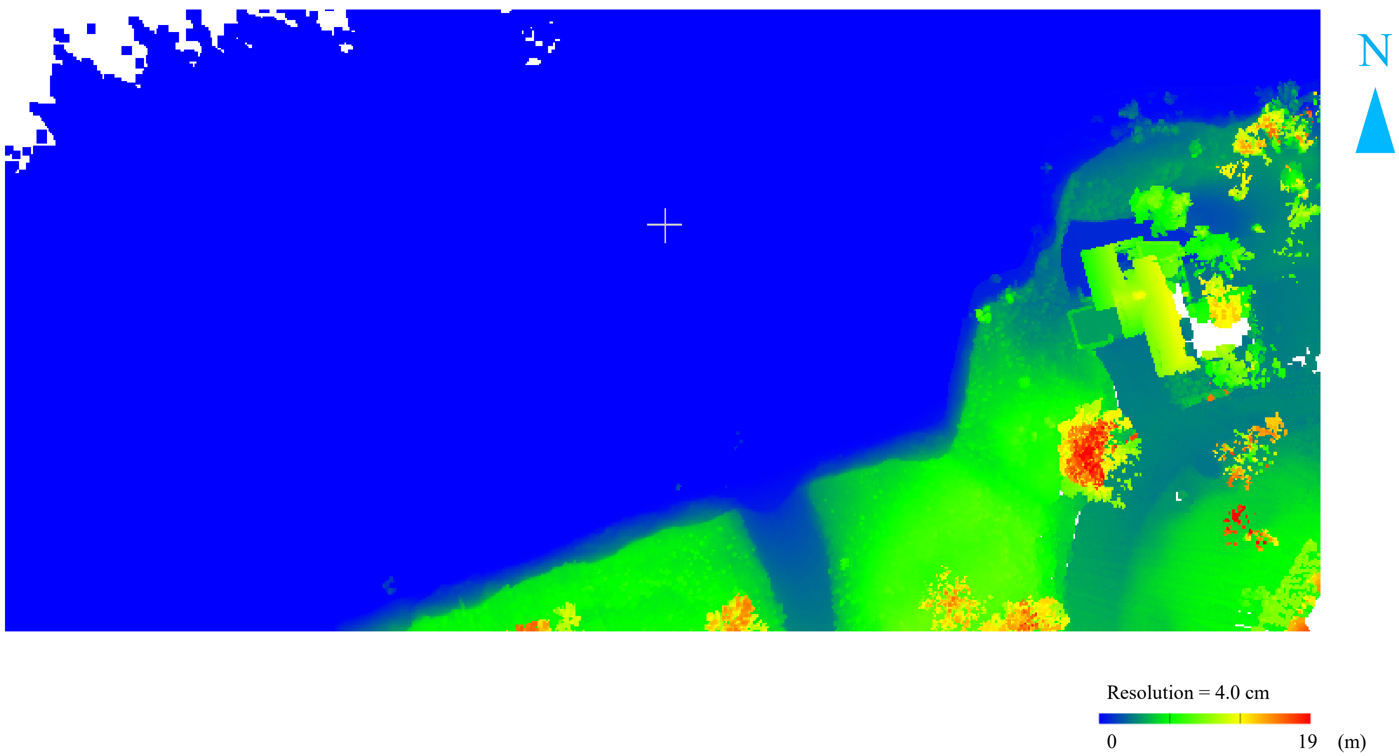
Digital Surface Model: May 17th

Shoreline Monitoring



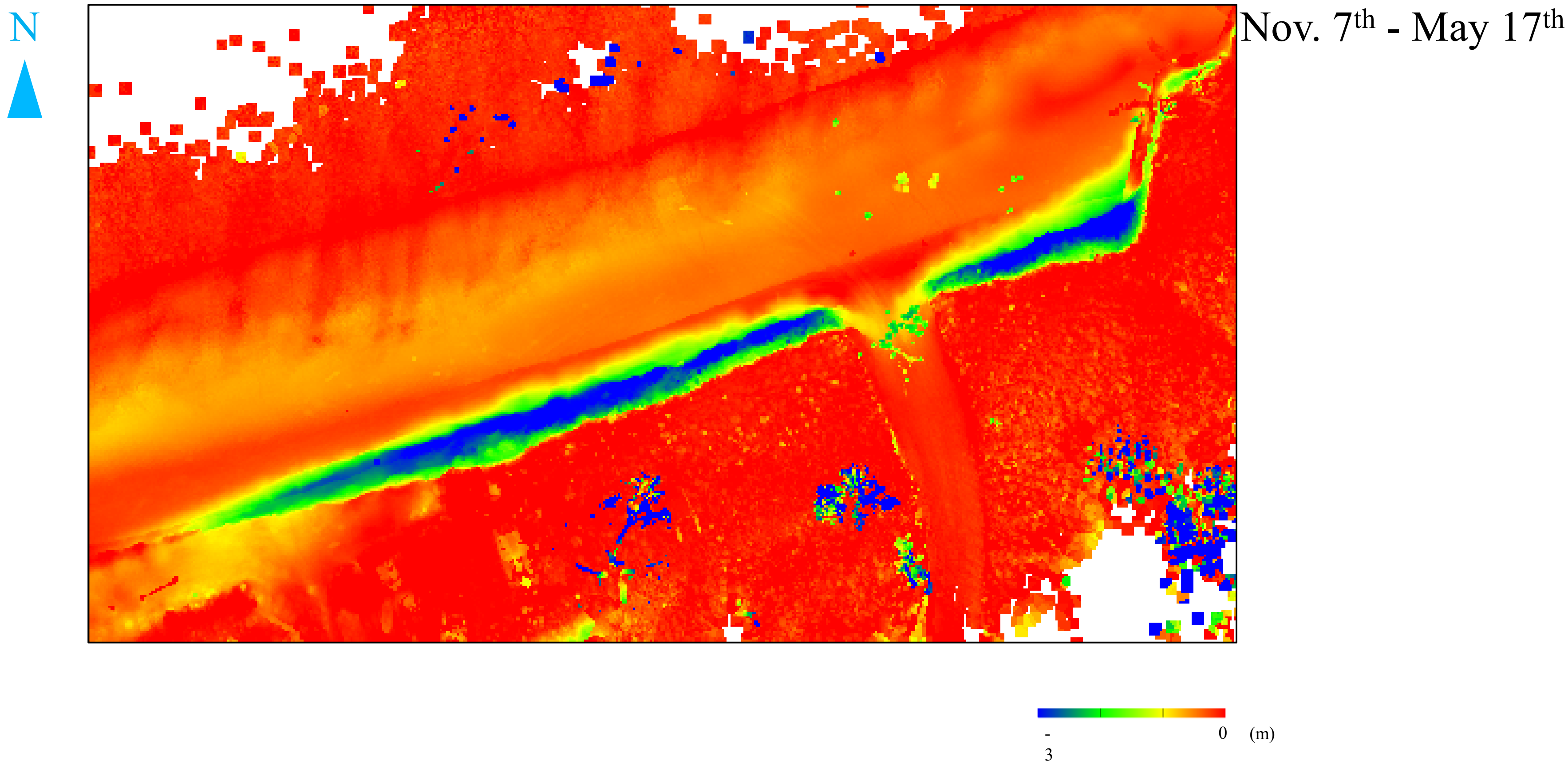
Digital Surface Model: Nov. 7th

Shoreline Monitoring



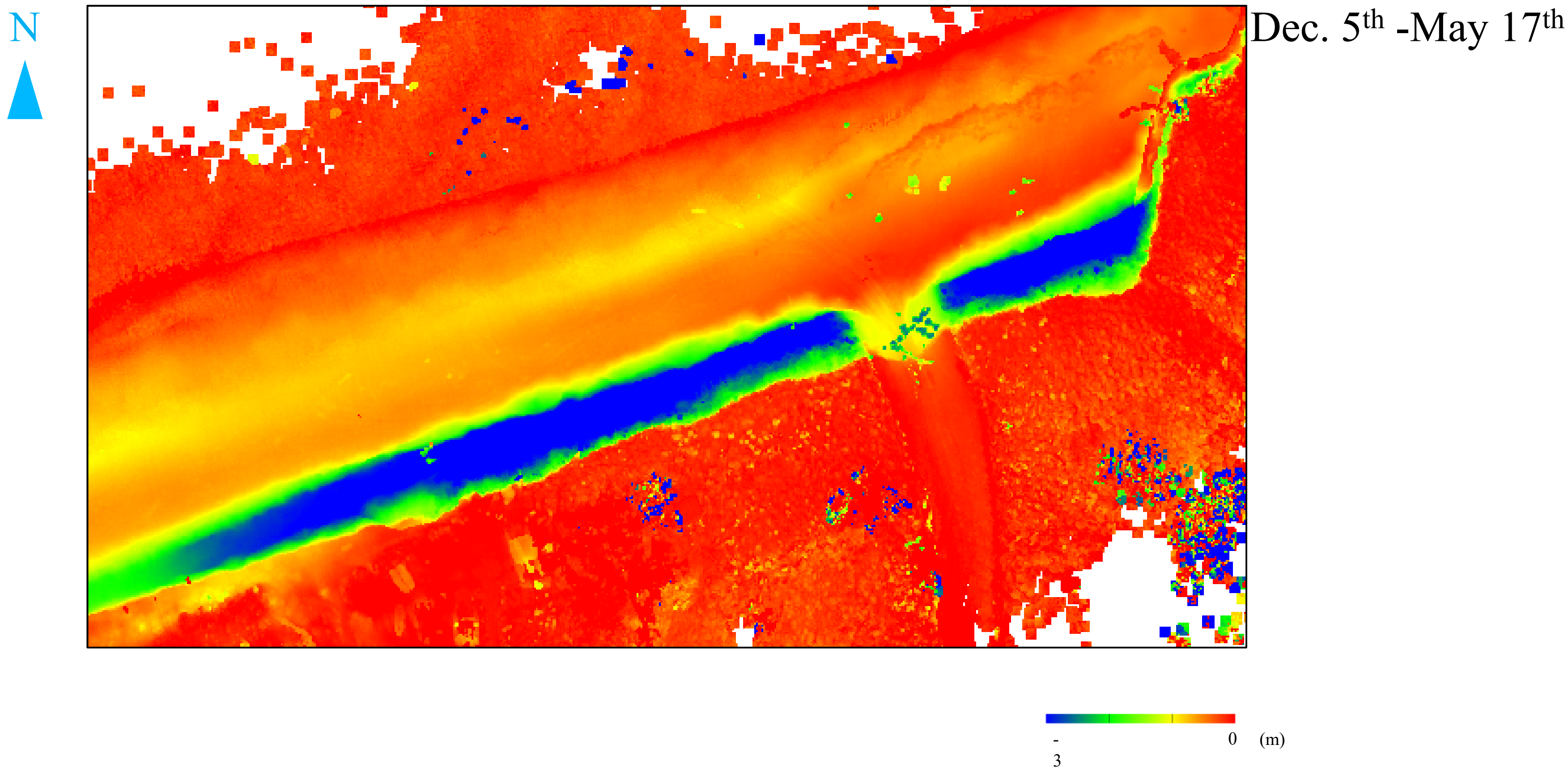
Digital Surface Model: Dec. 5th

Shoreline Monitoring



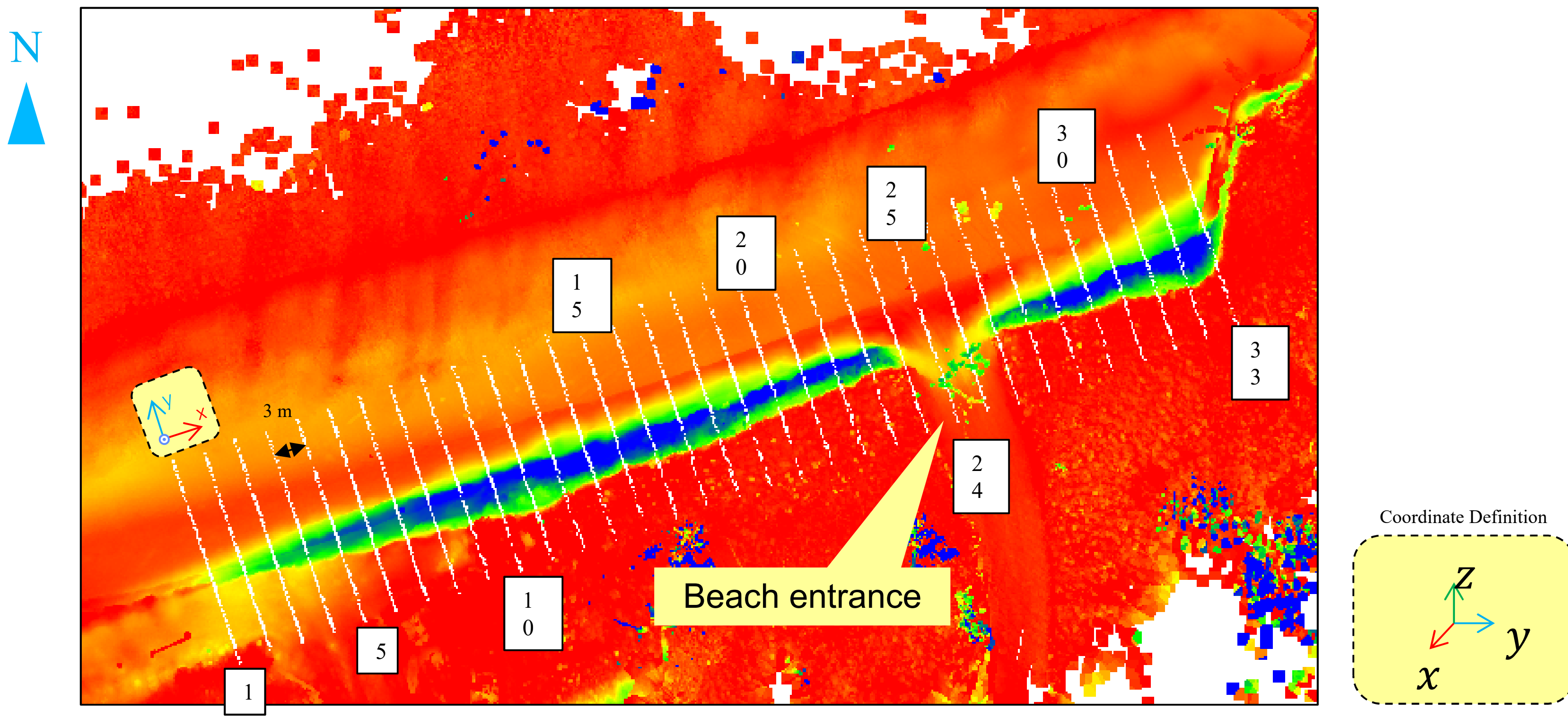
Z-difference: Nov. 7th – May 17th

Shoreline Monitoring



Z-difference: Dec. 5th – May 17th

Shoreline Monitoring



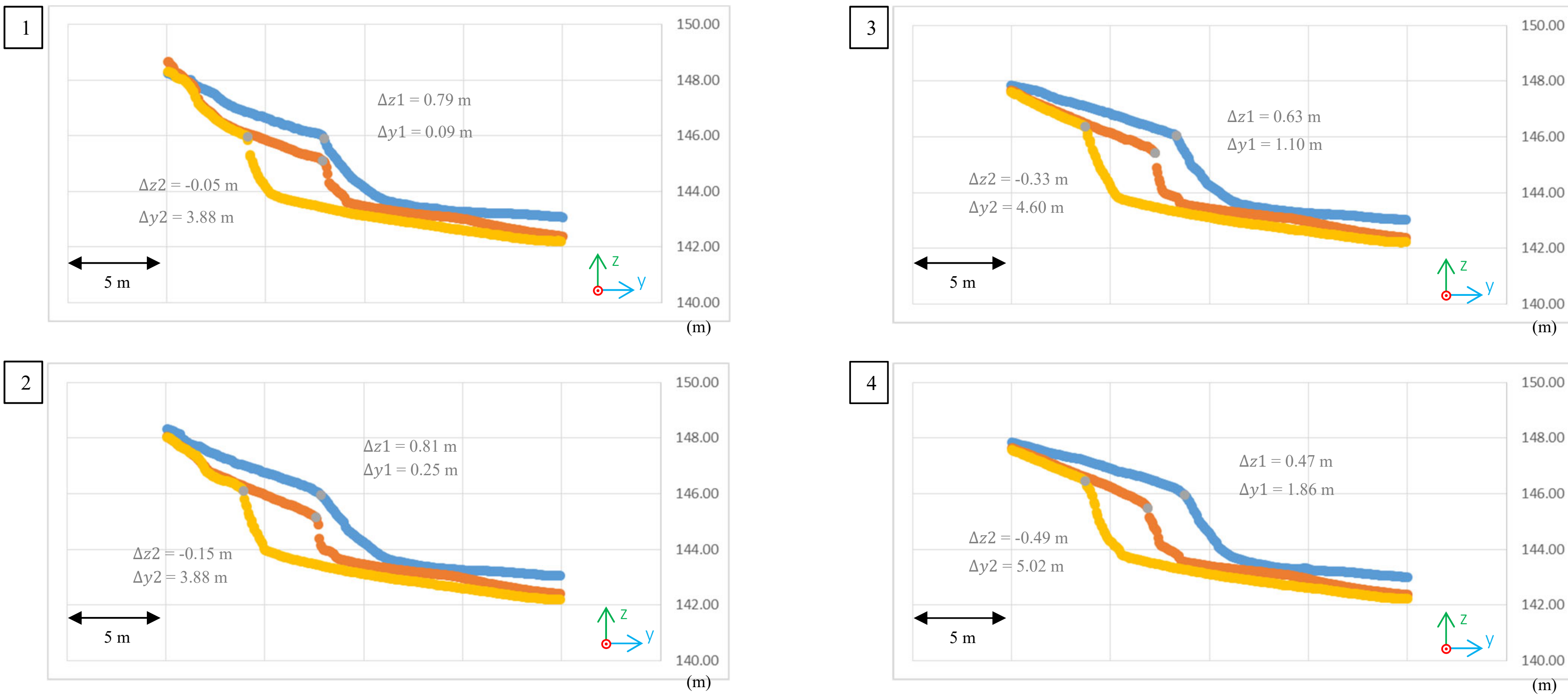
Base Map: z-difference map (Nov. 7th - May 17th)

Profile width = 2.5 cm

-3 0 (m)

Cross-sections: Profile Locations

Shoreline Monitoring



$\Delta z1 = z_{May} - z_{Nov.}$ $\Delta y1 = y_{May} - y_{Nov.}$ $\Delta z2 = z_{May} - z_{Dec.}$ $\Delta y2 = y_{May} - y_{Dec.}$

5th Dec. · 17th May · Slope change point (manually) · 7th Nov.

Profile width = 2.5 cm

Cross-sections: Ridge Point Locations

Stockpile Volume Estimation

UAS photos

INDOT Lafayette Maintenance Facility

Stockpile Volume Estimation

UAV-based Mobile Mapping System

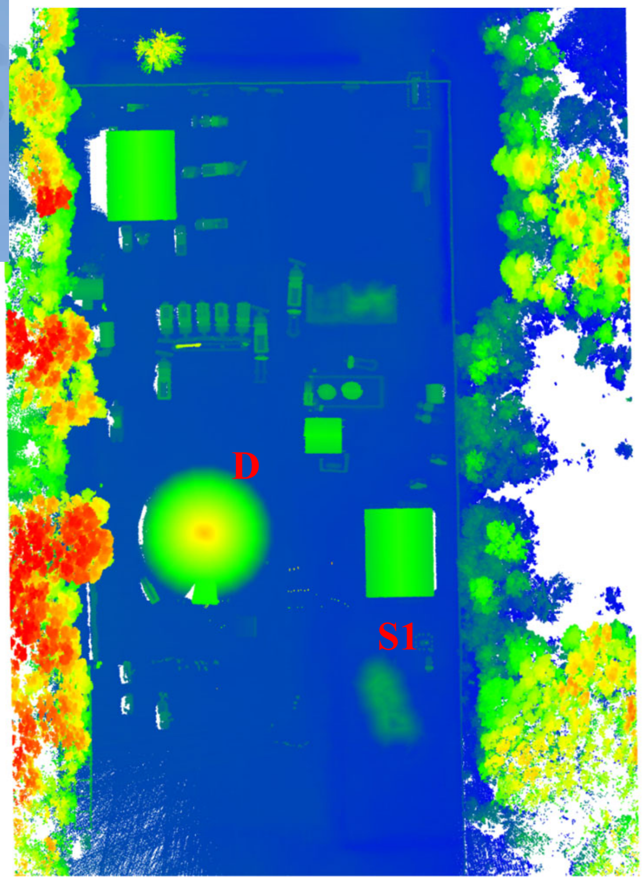
- Platform: DJI M600
- Camera: Sony Alpha 7R
- LiDAR: Velodyne VLP-32C
- GNSS/INS: APX-15 V2

Terrestrial LiDA

- FARO Focus 3D

INDOT Lafayette Maintenance Facility

Stockpile Volume Estimation

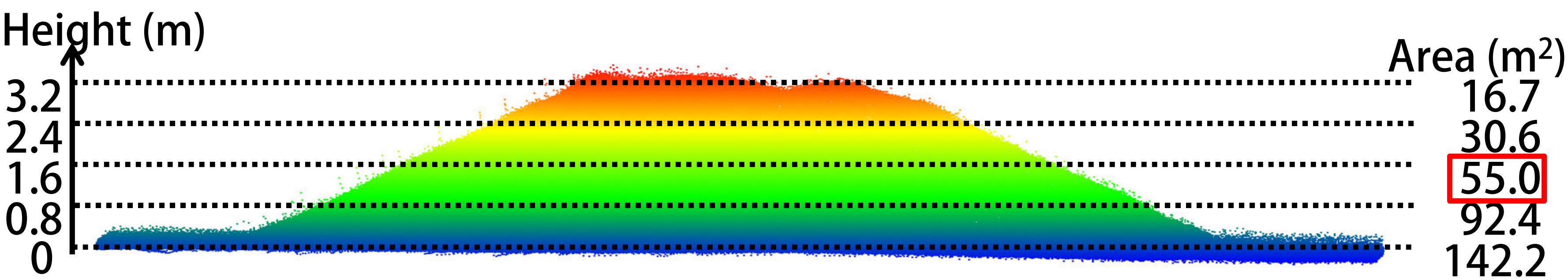


Point cloud colored by height

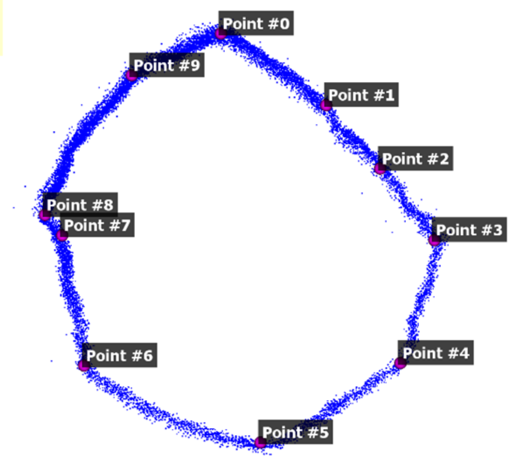
Color-coded LiDAR DSM

VLP-32C LiDAR Point Cloud

Stockpile Volume Estimation

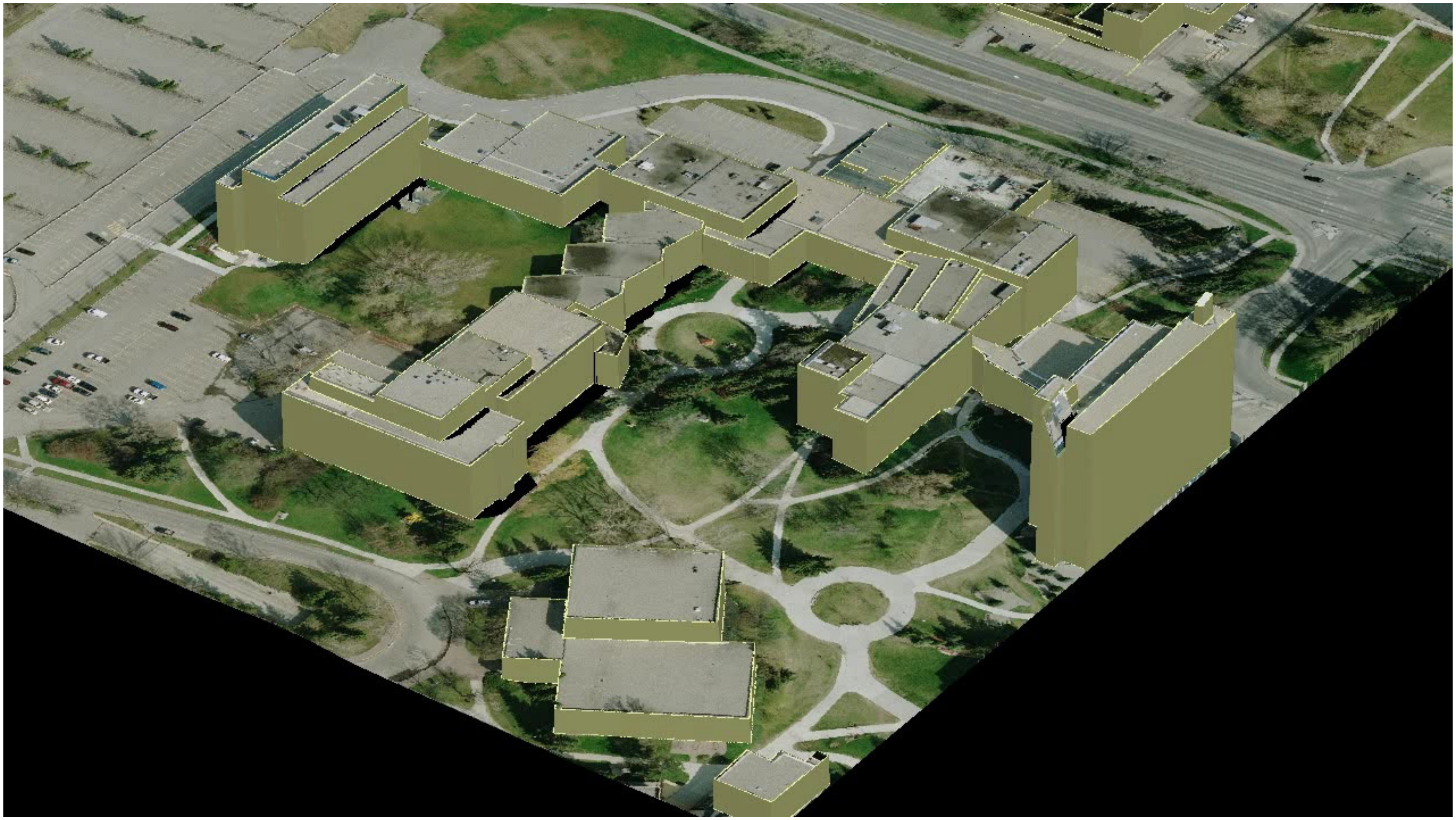


Volume = 269 yard^3 (206 m^3)



Stockpile S2

3D Realistic View Generation



LiDAR Mathematics

LiDAR Equation

LiDAR Mathematics

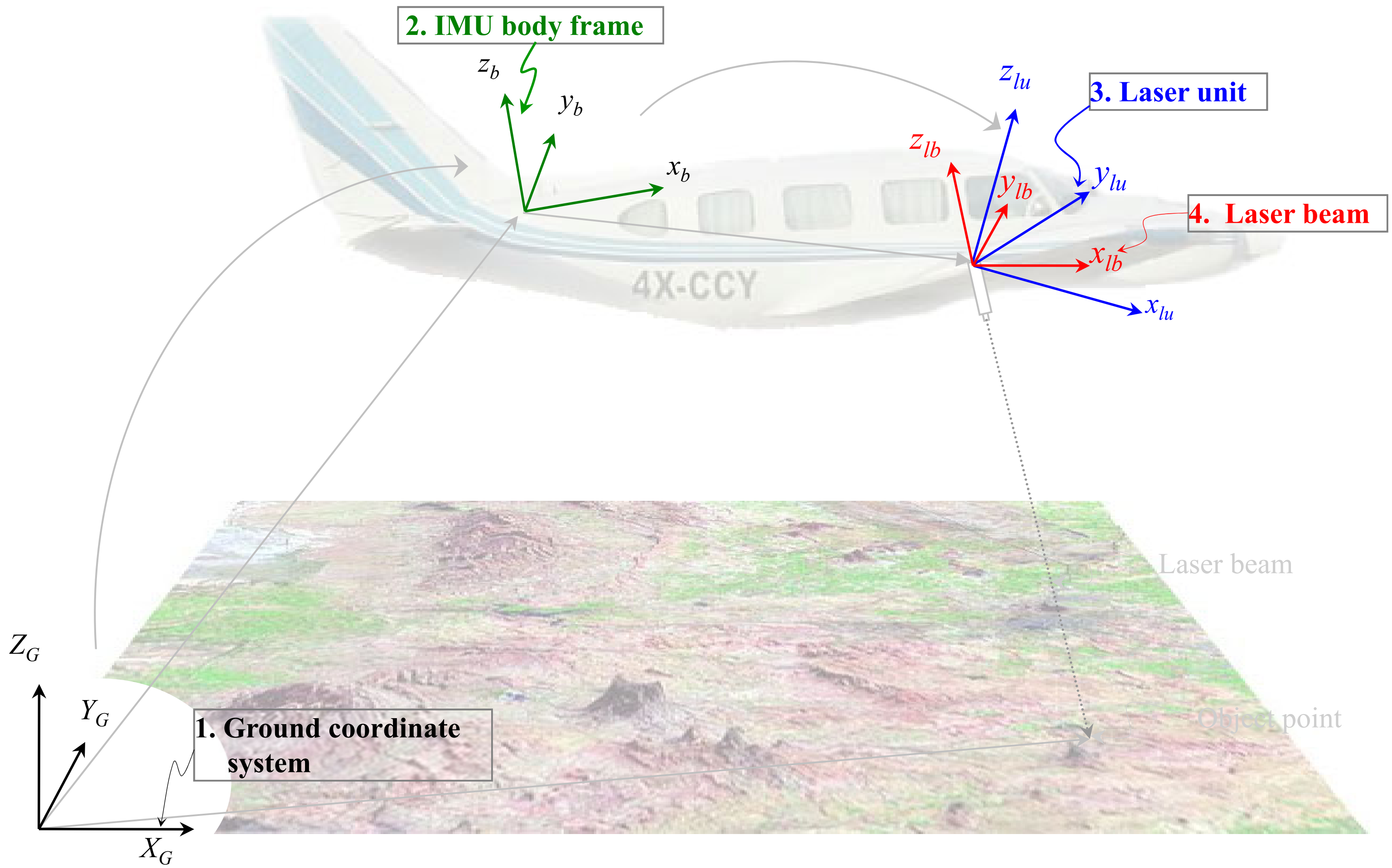
- Objective:
  - How are the LiDAR measurements used to generate the ground coordinates of the laser footprints?
- We will be focusing on Mobile LiDAR Systems since they are the most general ones.
  - The model for static LiDAR can be derived as a special case.
- Procedure:
  - Involved coordinate systems
  - Relationship between these coordinate systems (mounting parameters)
  - LiDAR equation
  - Error sources
  - Impact of these error sources

Notation

- r_a^b Stands for the coordinates of point ***a*** relative to point ***b*** – this vector is defined relative to the coordinate system associated with point ***b***.

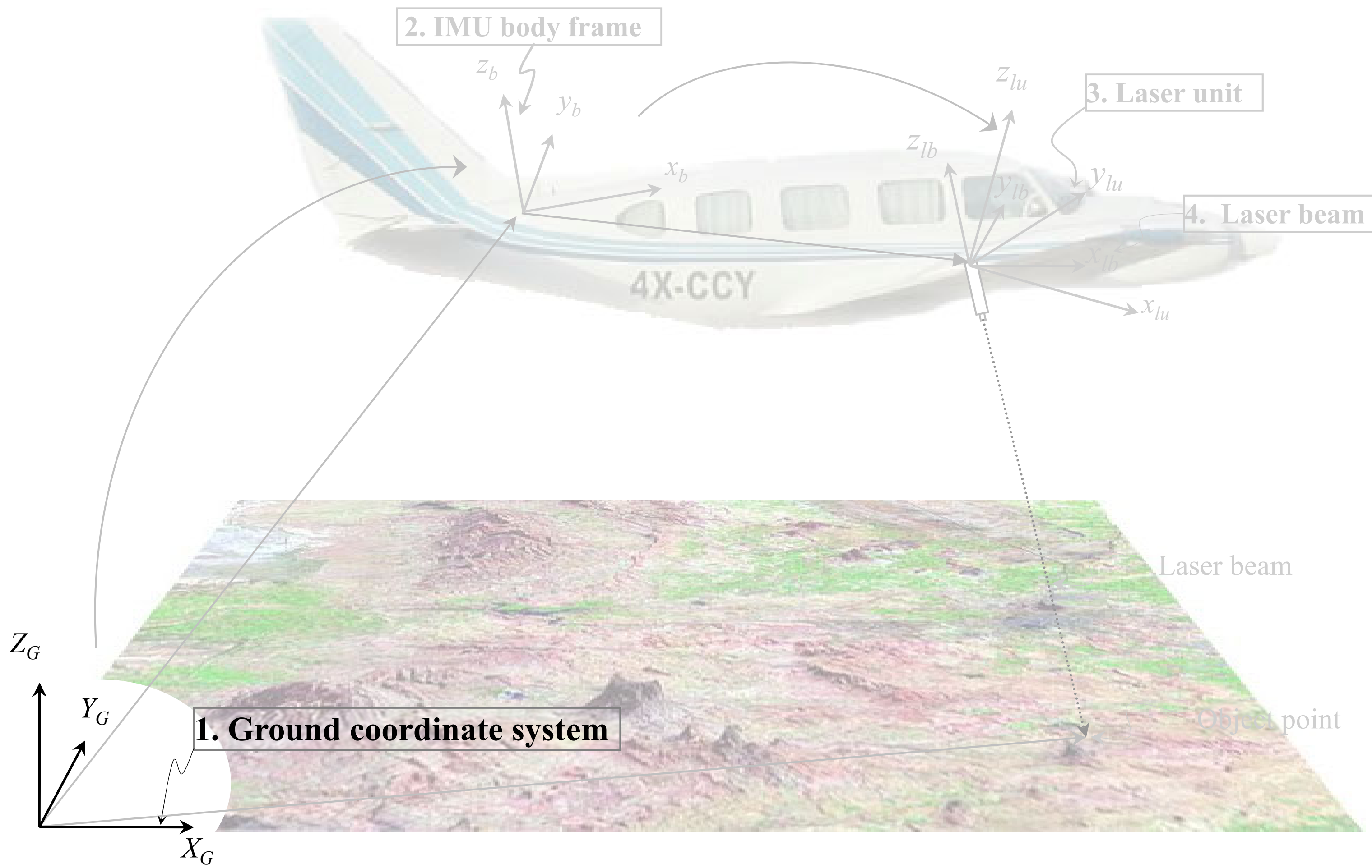
- R_a^b Stands for the rotation matrix that transforms a vector defined relative to the coordinate system denoted by ***a*** into a vector defined relative to the coordinate system denoted by ***b***.

LiDAR Equation: Coordinate Systems



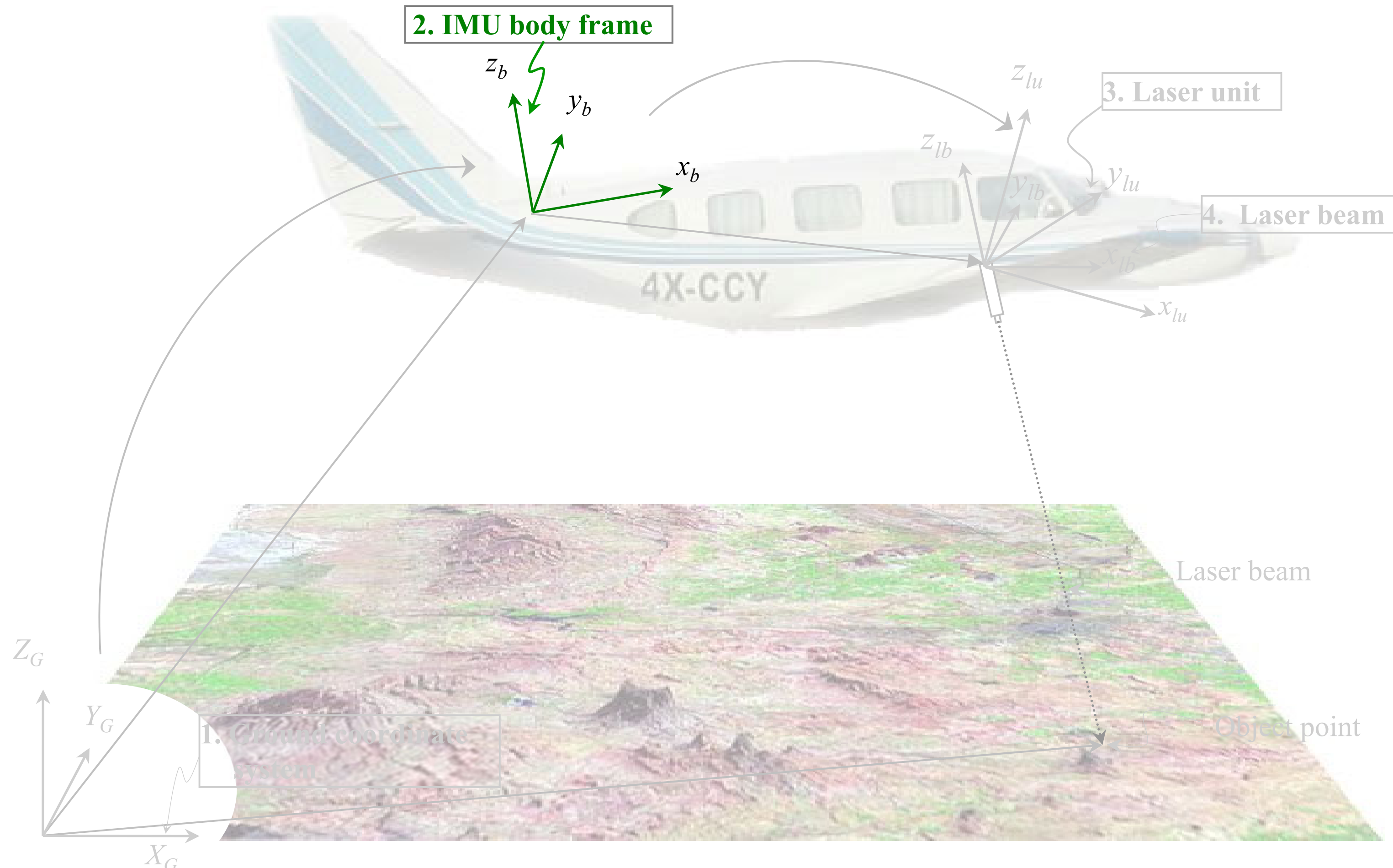
Four coordinate systems involved

LiDAR Equation: Coordinate Systems



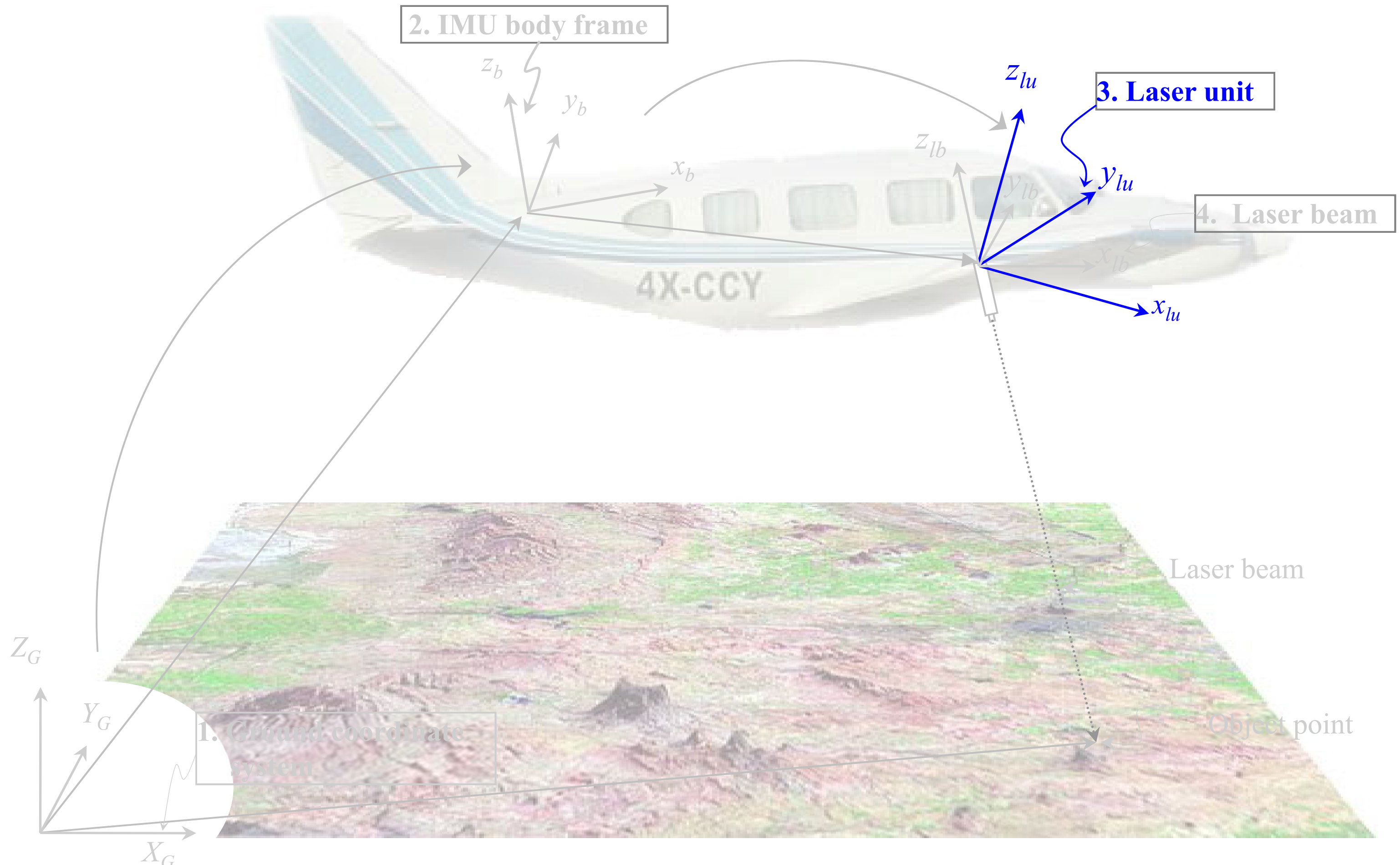
Ground coordinate system (X_G, Y_G, Z_G)

LiDAR Equation: Coordinate Systems



IMU body frame coordinate system (x_b, y_b, z_b)

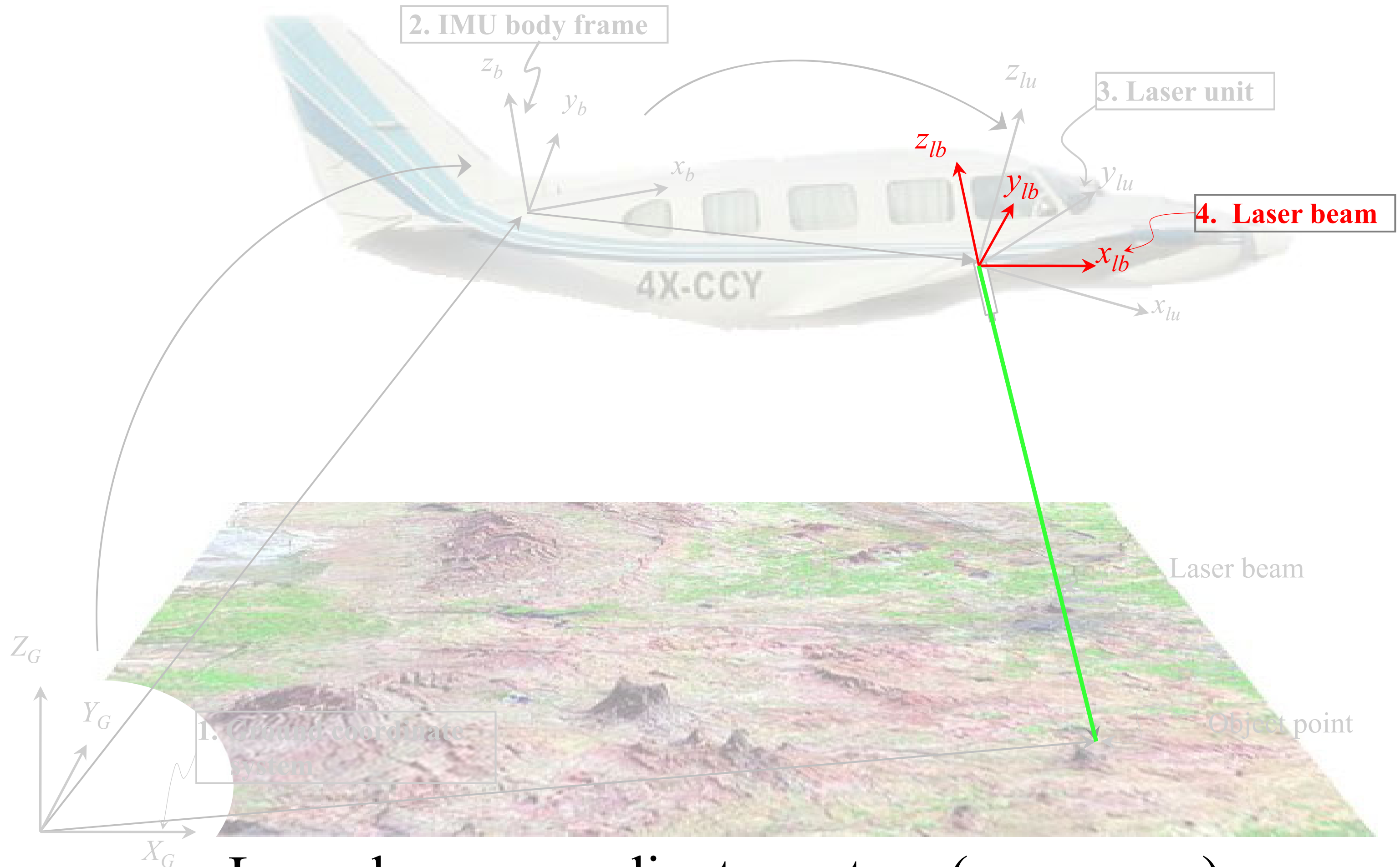
LiDAR Equation: Coordinate Systems



Laser unit coordinate system (x_{lu}, y_{lu}, z_{lu})

- Origin at the laser beam firing point

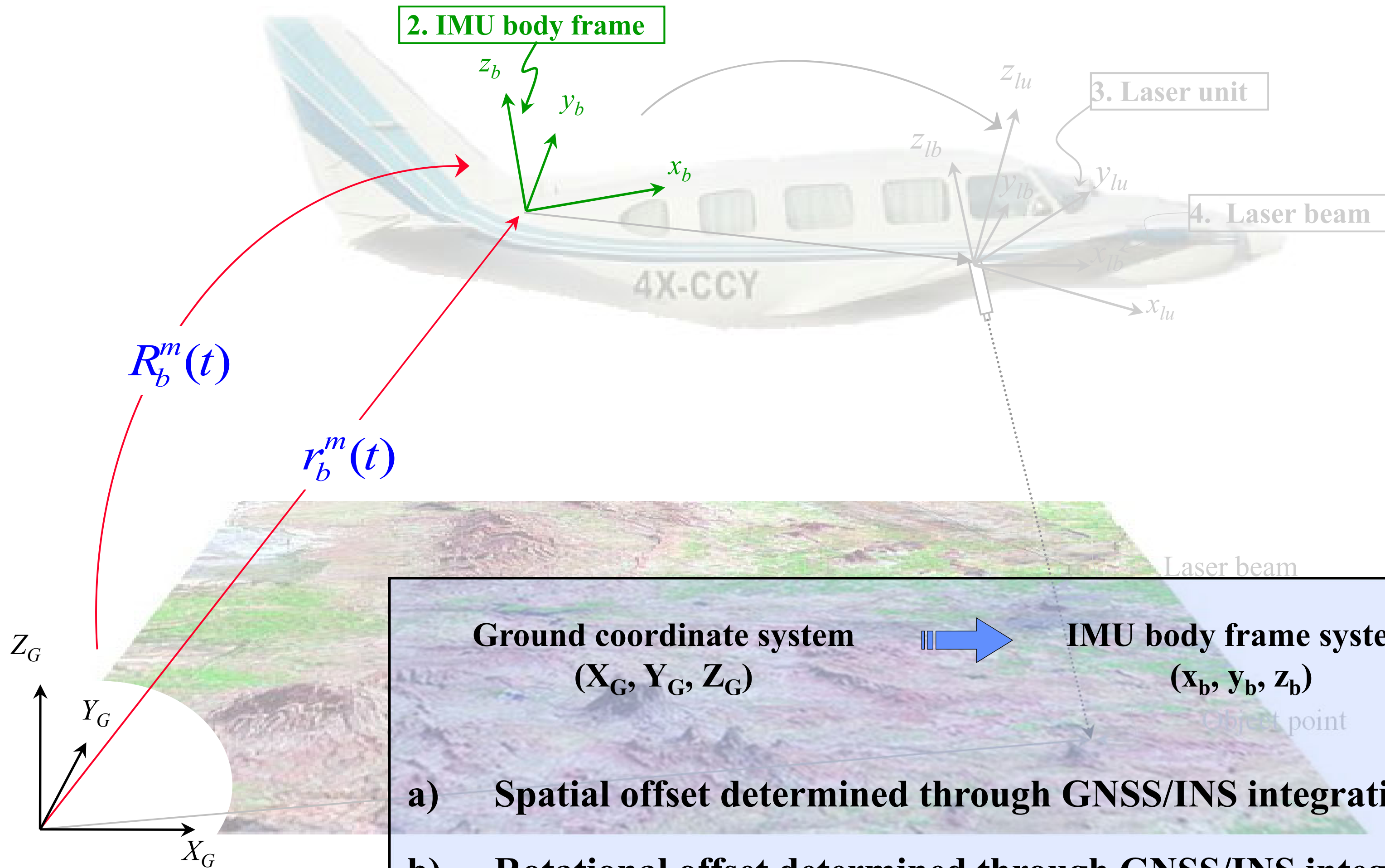
LiDAR Equation: Coordinate Systems



Laser beam coordinate system (x_{lb}, y_{lb}, z_{lb})

- Origin at the laser beam firing point
  - z-axis (z_{lb}) along the laser beam

LiDAR Equation: Coordinate Systems

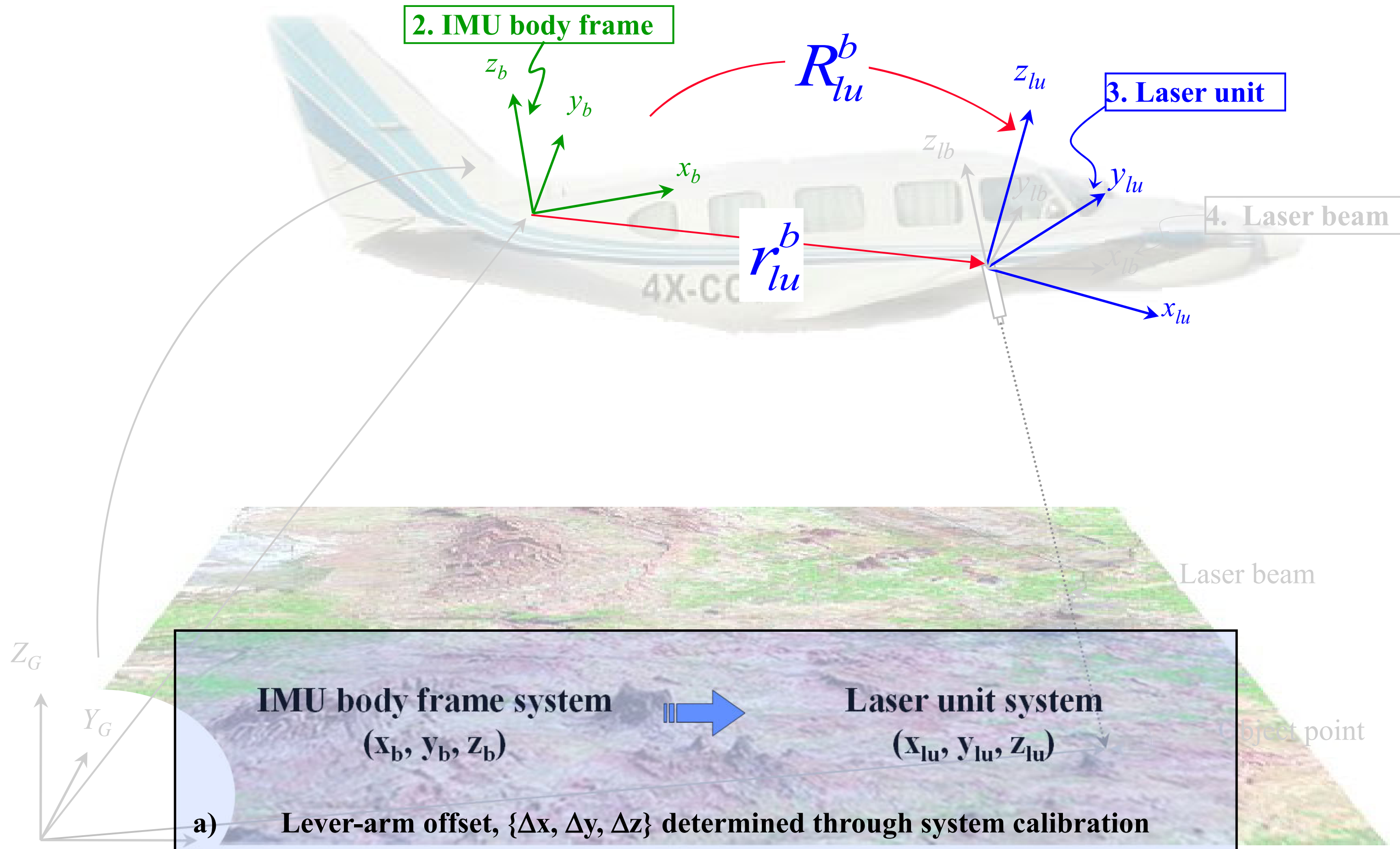


Ground coordinate system (X_G, Y_G, Z_G) → **IMU body frame system (x_b, y_b, z_b)**

a) Spatial offset determined through GNSS/INS integration

b) Rotational offset determined through GNSS/INS integration

LiDAR Equation: Coordinate Systems

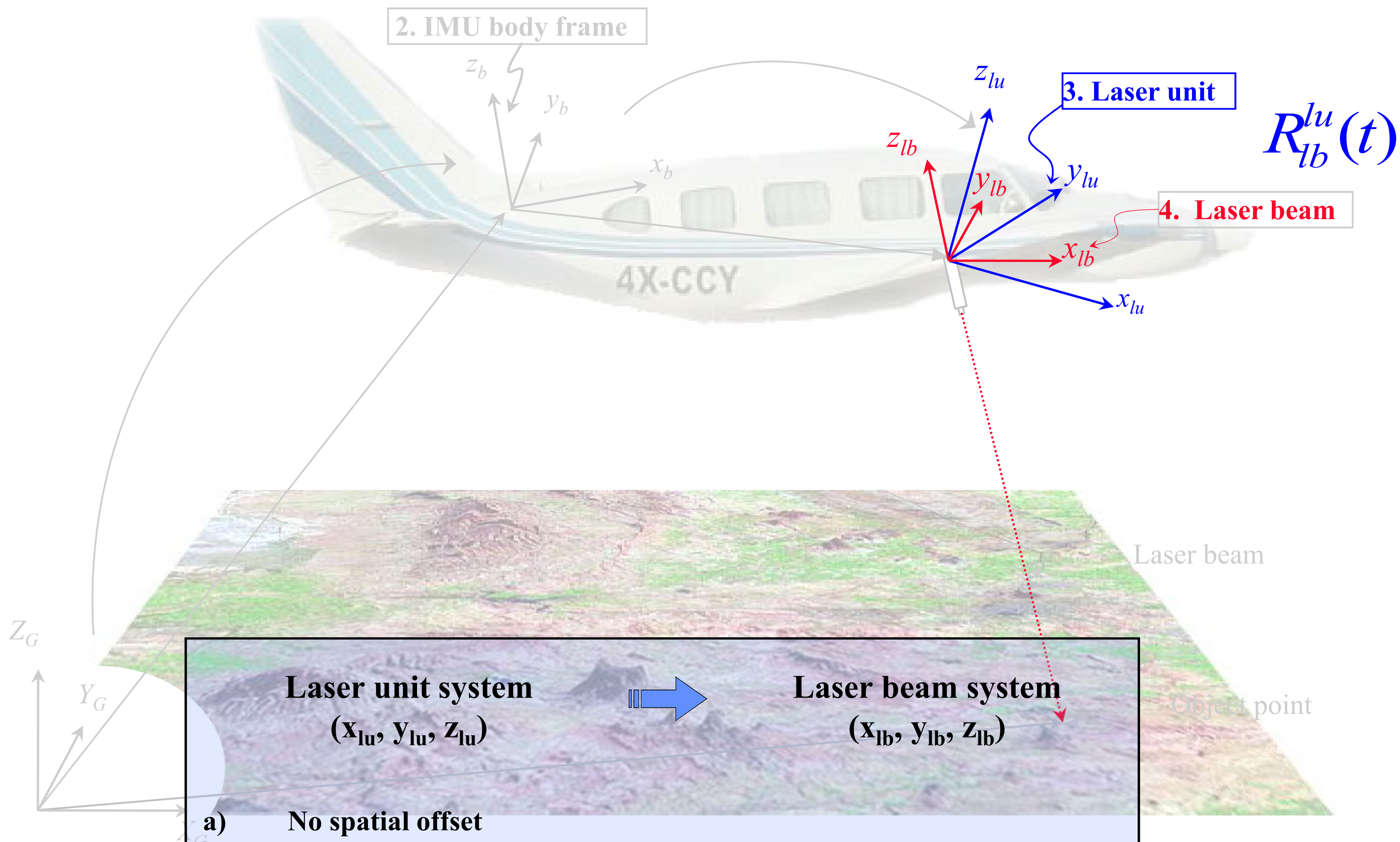


IMU body frame system (x_b, y_b, z_b) ⇨ **Laser unit system** (x_{lu}, y_{lu}, z_{lu})

a) Lever-arm offset, {Δx, Δy, Δz} determined through system calibration

b) Boresight angles, (Δω, Δϕ, Δκ) determined through system calibration

LiDAR Equation: Coordinate Systems

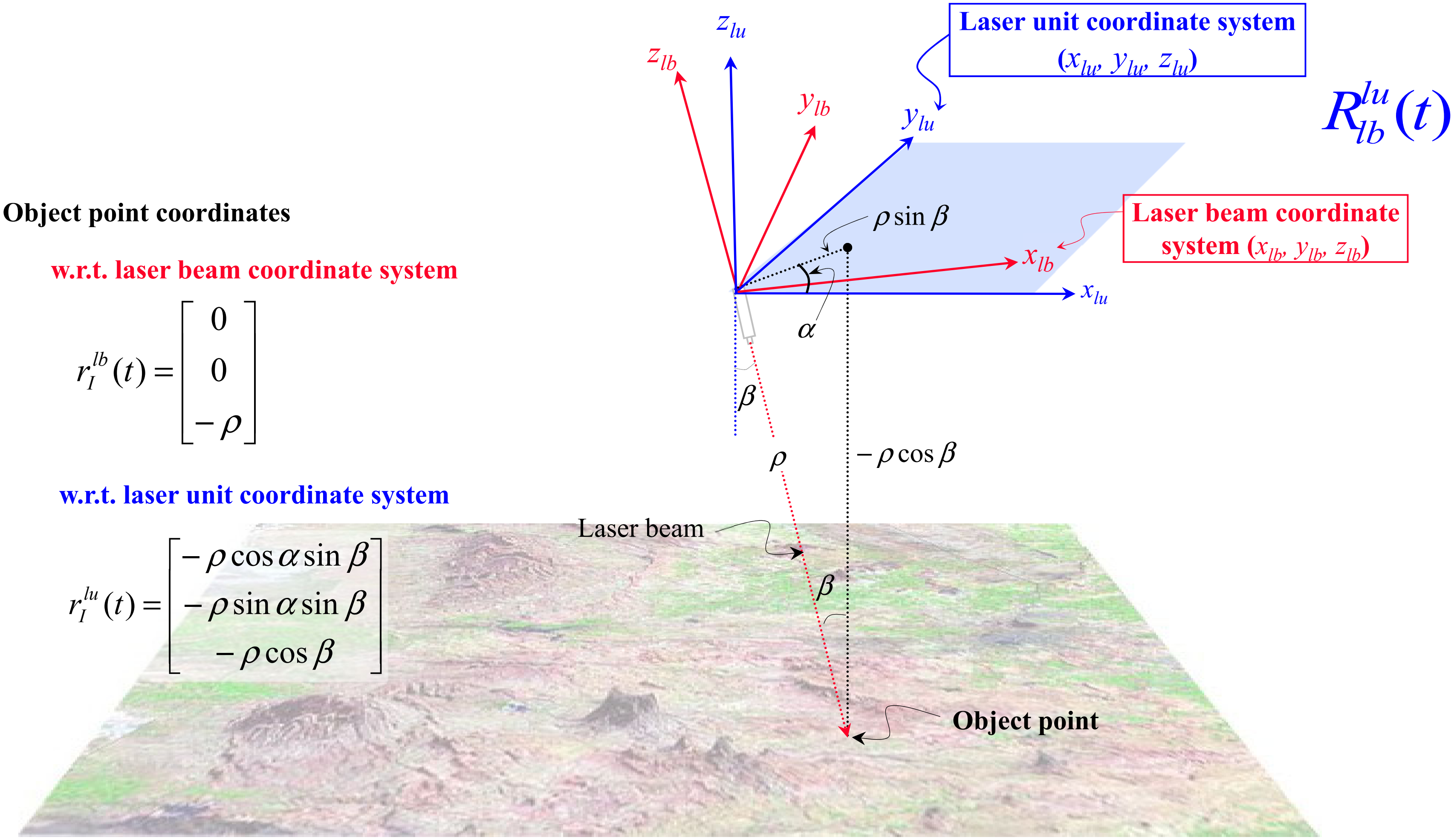


Laser unit system (x_{lu}, y_{lu}, z_{lu}) ➡ **Laser beam system** (x_{lb}, y_{lb}, z_{lb})

a) No spatial offset

b) Rotational offset, (α, β) defined by the laser system mirror

LiDAR Equation: Coordinate Systems



Object point coordinates

w.r.t. laser beam coordinate system

$$r_I^{lb}(t) = \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix}$$

w.r.t. laser unit coordinate system

$$r_I^{lu}(t) = \begin{bmatrix} -\rho \cos\alpha \sin\beta \\ -\rho \sin\alpha \sin\beta \\ -\rho \cos\beta \end{bmatrix}$$

LiDAR Equation: Coordinate Systems

Transformation between laser unit and laser beam coordinate systems can be established through the following rotations:

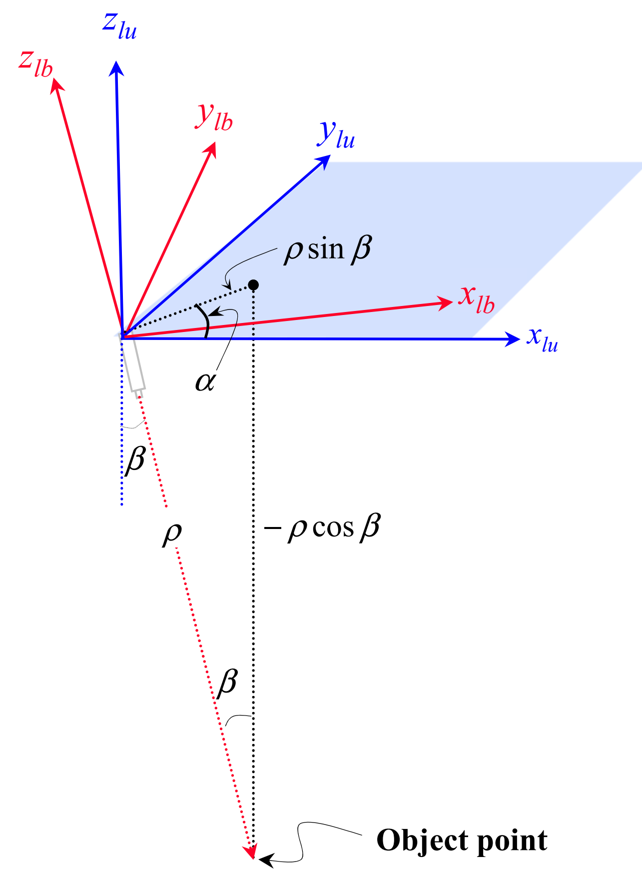
- Rotation (α) around z_{lu}
- Rotation (β) around y_{lu_α}

$$\begin{bmatrix} x_{lu} \\ y_{lu} \\ z_{lu} \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix}$$

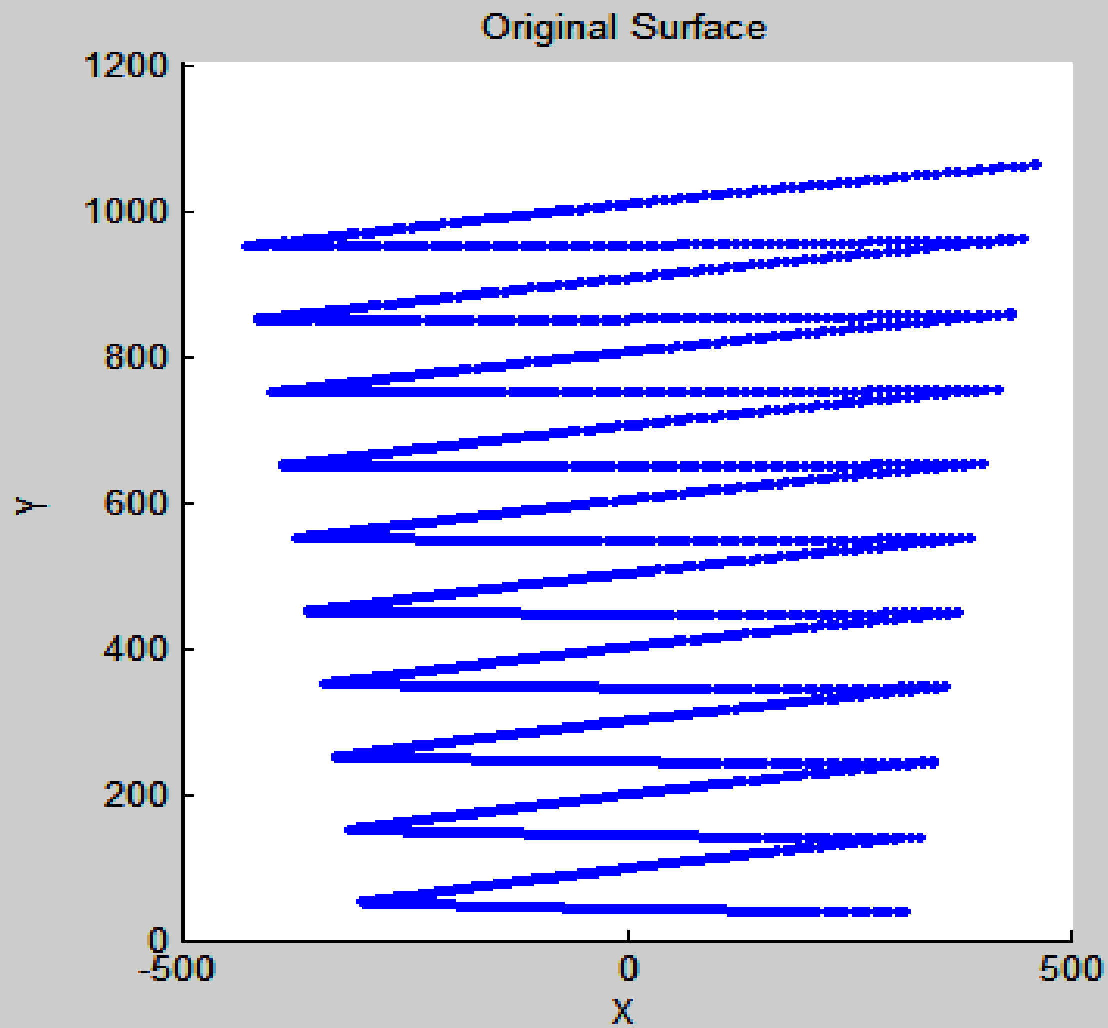
$$\begin{bmatrix} x_{lu} \\ y_{lu} \\ z_{lu} \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & -\sin\alpha & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta & \cos\alpha & \sin\alpha\sin\beta \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix}$$

$$\begin{bmatrix} x_{lu} \\ y_{lu} \\ z_{lu} \end{bmatrix} = \begin{bmatrix} -\rho\cos\alpha\sin\beta \\ -\rho\sin\alpha\sin\beta \\ -\rho\cos\beta \end{bmatrix}$$

Object point coordinates relative to the laser unit coordinate system

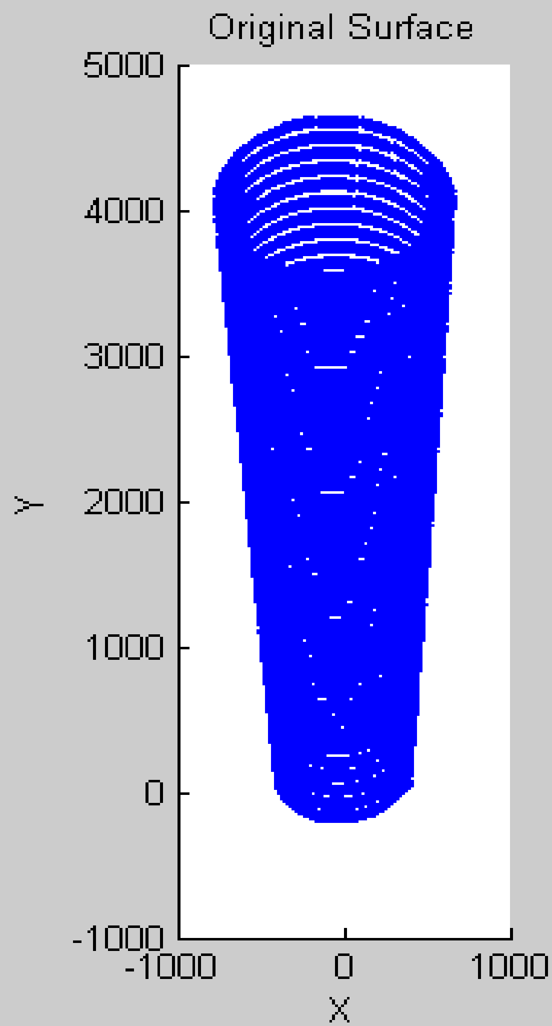


Linear Scanner



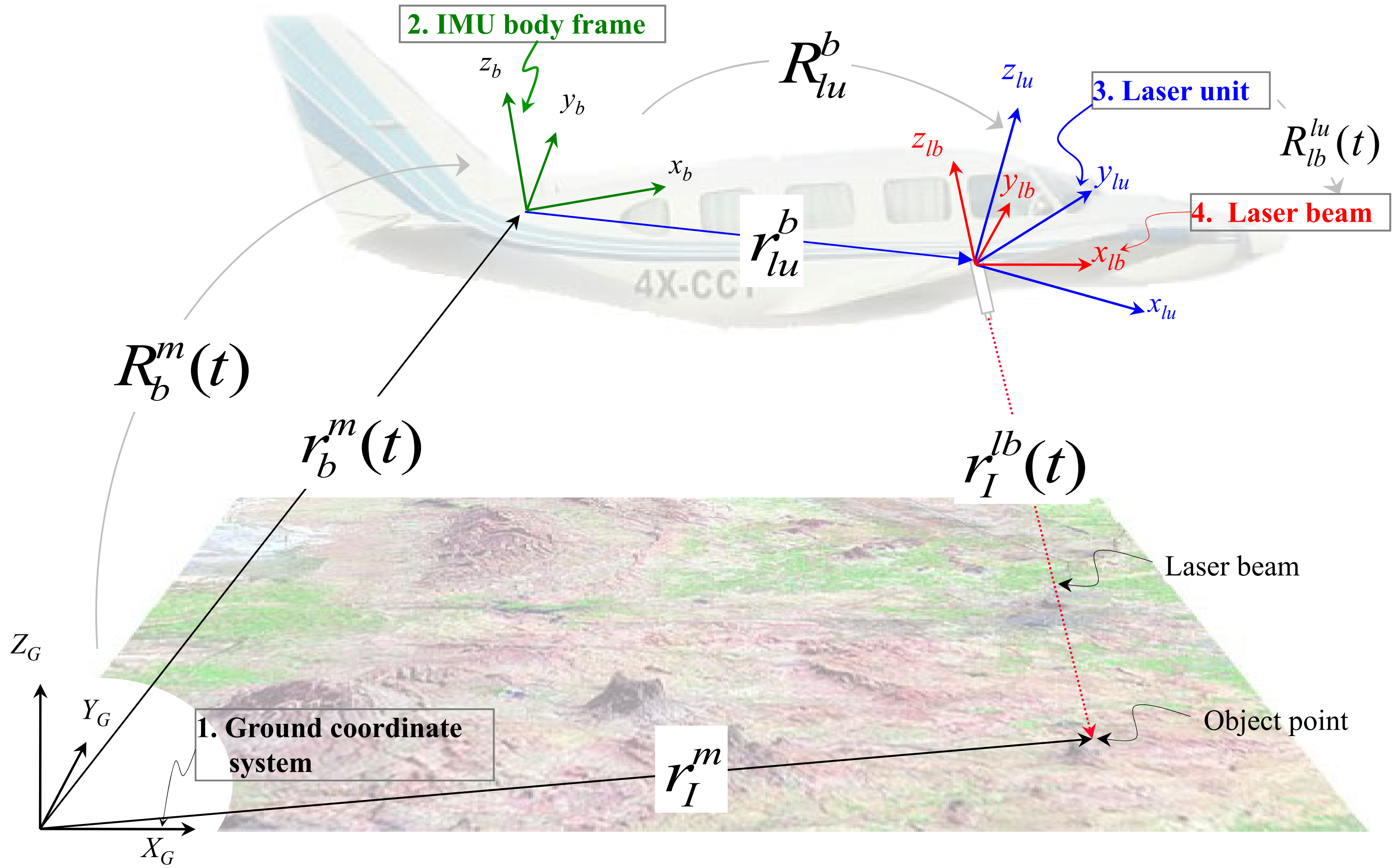
$\alpha = 0$

Elliptical Scanner



$\alpha \neq 0$

LiDAR Equation & Coordinate Systems



- LiDAR equation is a vector summation procedure.

LiDAR Equation (Mobile Systems)

$$r_I^m = r_b^m(t) + R_b^m(t)\ r_{lu}^b + R_b^m(t)\ R_{lu}^b\ R_{lb}^{lu}(t)\, r_I^{lb}(t)$$

r_I^m	ground coordinates of the object point under consideration
$r_b^m(t)$	ground coordinates of the origin of the IMU coordinate system
$R_b^m(t)$	rotation matrix relating the ground and IMU coordinate systems
r_{lu}^b	offset between the laser unit and IMU coordinate systems (lever arm offset)
R_{lu}^b	rotation matrix relating the IMU and laser unit coordinate systems (boresight matrix)
$R_{lb}^{lu}(t)$	rotation matrix relating the laser unit and laser beam coordinate systems
$r_I^{lb}(t)$	coordinates of the object point relative to the laser beam coordinate system

- Note: There is no redundancy in the surface reconstruction process.

LiDAR Equation (Static Systems)

$$r_I^{lu} = R_{lb}^{lu}(t)\, r_I^{lb}(t)$$

r_I^{lu} ground coordinates of the object point under consideration

$R_{lb}^{lu}(t)$ rotation matrix relating the laser unit and laser beam coordinate systems at a given epoch (t)

$r_I^{lb}(t)$ coordinates of the object point relative to the laser beam coordinate system at a given epoch (t)

- We are only dealing with Laser beam and laser unit coordinate systems

- Note: There is no redundancy in the surface reconstruction process.

LiDAR Equation

$$\vec{X}^{True} = f(\vec{x}, \vec{y}_{nf})$$

$$\vec{X}^{True} \equiv \textit{True coordinates of the LiDAR point}$$

$$\vec{x} \equiv \textit{System parameters } (\Delta X, \Delta Y, \Delta Z, \Delta\omega, \Delta\varphi, \Delta\kappa, \Delta\rho, S_\alpha, S_\beta)$$

$$\vec{y}_{nf} \equiv \textit{Noise - free system measurements } (\vec{X}_o(t), \omega(t), \varphi(t), \kappa(t), \rho(t), \alpha(t), \beta(t))$$

$$\vec{y}_{nf} = \vec{y} - \vec{e}$$

$$\vec{y} \equiv \textit{System measurements}$$

$$\vec{e} \equiv \textit{Random noise contaminating the system measurements}$$

$$\vec{\tilde{X}}^{True} = f(\vec{x}, \vec{y})$$

$$\vec{\tilde{X}}^{True} \equiv \text{Predicted true coordinates of the LiDAR point}$$

LiDAR Equation

$$\vec{\tilde{X}}^{True} = f(\vec{x}, \vec{y})$$

$$\vec{\tilde{X}}^{True} = f(\vec{x}, \vec{y}_{nf} + \vec{e}) \approx f(\vec{x}, \vec{y}_{nf}) + \frac{\partial f}{\partial \vec{y}} \vec{e}$$

$$\vec{\tilde{X}}^{True} = \vec{X}^{True} + B\vec{e} = \vec{X}^{True} + \vec{\bar{e}}$$

$\vec{\tilde{X}}^{True}$ Predicted true coordinates of the LiDAR point

$\vec{X}^{True}$ True coordinates of the LiDAR point

$\vec{\bar{e}}$ Noise vector contaminating the predicted true coordinates of the LiDAR point

LiDAR Output

- 232802.510 319978.600 44.300 41.0 9 First
- 232802.510 319978.600 44.300 41.0 9 Last
- 232802.360 319979.590 44.460 38.0 9 First
- 232802.360 319979.590 44.460 38.0 9 Last
- 232802.250 319980.340 44.550 41.0 9 First
- 232802.250 319980.340 44.550 41.0 9 Last
- 232802.100 319981.420 44.470 37.0 9 First
- 232802.100 319981.420 44.470 37.0 9 Last
- ………….
- …………….

Black Box (non-transparent model)

LiDAR Error Budget

- The quality of the derived point cloud from a LiDAR system depends on:
  - **<u>Systematic errors</u>** in the system parameters:
    - Biases in the Lever-arm offset parameters ($\delta\Delta X$, $\delta\Delta Y$, $\delta\Delta Z$)
    - Biases in the angular boresight parameters ($\delta\Delta\omega$, $\delta\Delta\phi$, $\delta\Delta\kappa$)
    - Biases in the measured ranges ($\delta\Delta\rho$)
    - Scale bias in the mirror angles (δS_{α}, δS_{β})
  - **<u>Random errors</u>** in the system measurements:
    - Position and orientation information from the GNSS/INS unit
    - Ranges between the laser beam firing point and its footprints
    - Mirror angles
- We would like to investigate the impact of systematic and random errors on the quality of the derived LiDAR surface.

Systematic Errors

- Objective: Show the effect of systematic errors/biases in the LiDAR parameters on the reconstructed object space
- The effects will be derived through mathematical analysis of the LiDAR equation.
- The effects will be also analyzed through a simulation process:
  - Simulated surface & trajectory → LiDAR measurements → Add biases → Reconstructed surface
  - The effects will be shown through the differences between the reconstructed footprints and the simulated surface (i.e., ground truth).
- These effects will be shown for linear & elliptical LiDAR systems (with more emphasis on linear scanners).

Impact of Systematic Biases

- Mathematical Analysis of the LiDAR Equation

$$r_I^m = f(\vec{x})$$

where:

– $\vec{x} = (\Delta X, \Delta Y, \Delta Z, \Delta\omega, \Delta\varphi, \Delta\kappa, \Delta\rho, S).$

$$\delta r_I^m = \frac{\partial r_I^m}{\partial \vec{x}} \delta \vec{x}$$

Impact of systematic biases

where:

– $\delta\vec{x} = (\delta\Delta X, \delta\Delta Y, \delta\Delta Z, \delta\Delta\omega, \delta\Delta\varphi, \delta\Delta\kappa, \delta\Delta\rho, \delta S)$

Impact of Systematic Biases

Mathematical Analysis of the LiDAR Equation

$$r_I^m = r_b^m(t) + R_b^m(t)\, r_{lu}^b + R_b^m(t)\, R_{lu}^b\, R_{lb}^{lu}(t)\, r_I^{lb}(t)$$

- Assuming small boresight angles and vertical linear scanner:

$$r_I^m = r_b^m(t) + \begin{bmatrix} \cos k & -\sin k & 0 \\ \sin k & \cos k & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} \cos k & -\sin k & 0 \\ \sin k & \cos k & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\Delta\kappa & \Delta\varphi \\ \Delta\kappa & 1 & -\Delta\omega \\ -\Delta\varphi & \Delta\omega & 1 \end{bmatrix} \begin{bmatrix} -(\rho+\Delta\rho)\sin(S\beta) \\ 0 \\ -(\rho+\Delta\rho)\cos(S\beta) \end{bmatrix}$$

- Assuming heading (κ) angles of 0° and 180° for the forward and backward flight lines, respectively:

$$r_I^m = r_b^m(t) + \begin{bmatrix} \pm\Delta X \\ \pm\Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} \pm 1 & \mp\Delta\kappa & \pm\Delta\varphi \\ \pm\Delta\kappa & \pm 1 & \mp\Delta\omega \\ -\Delta\varphi & \Delta\omega & 1 \end{bmatrix} \begin{bmatrix} x \\ 0 \\ z \end{bmatrix}$$

$r_I^{lu}(t)$

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

Impact of Systematic Biases

Mathematical Analysis of the LiDAR Equation

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	$\mathbf{0}$
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	$\mathbf{0}$
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\,\delta\Delta\omega$	$\mathbf{0}$
$\delta\Delta\varphi$	$\pm z\,\delta\Delta\varphi$	0	$-x\,\delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\,\delta\Delta\kappa$	$\mathbf{0}$
$\delta\Delta\rho$	$\mp sin(S\beta)\,\delta\Delta\rho$	0	$-cos(S\beta)\,\delta\Delta\rho$
δS	$\pm z\,\beta\,\delta S$	$\mathbf{0}$	$-x\,\beta\,\delta S$

- Y- axis is along the flight direction.

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

Impact of Systematic Biases

Mathematical Analysis of the LiDAR Equation

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\,\delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\,\delta\Delta\varphi$	0	$-x\,\delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\,\delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\,\delta\Delta\rho$	0	$-cos(S\beta)\,\delta\Delta\rho$
δS	$\pm z\,\beta\,\delta S$	**0**	$-x\,\beta\,\delta S$

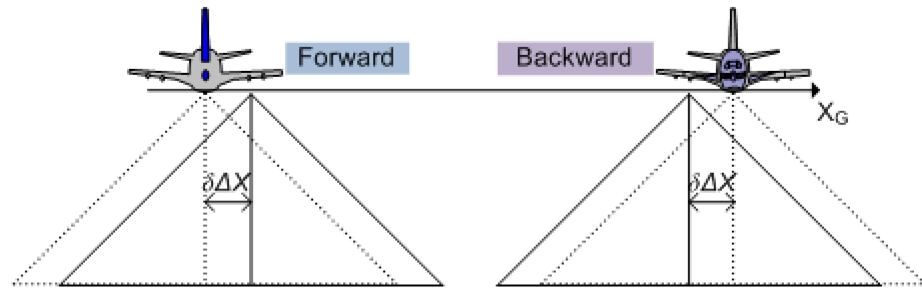
- Y- axis is along the flight direction.

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

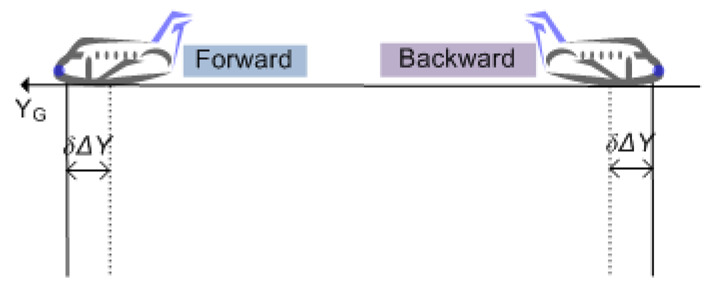
Lever-Arm Offset Bias (δΔX, δΔY, δΔZ)

Linear & Elliptical Scanner

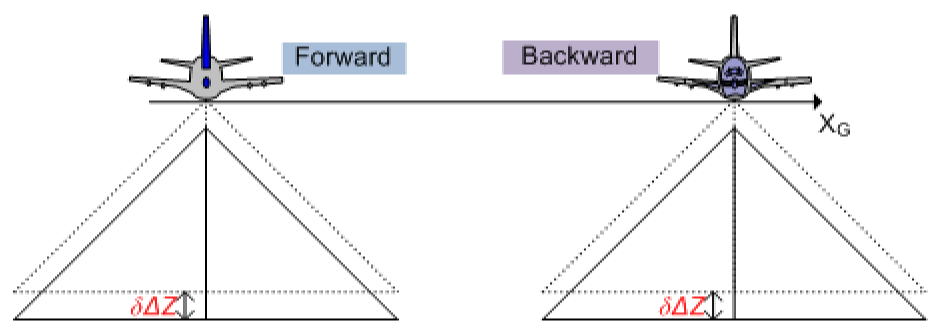
Lever-Arm Offset Bias (δΔX)



Lever-Arm Offset Bias (δΔY)



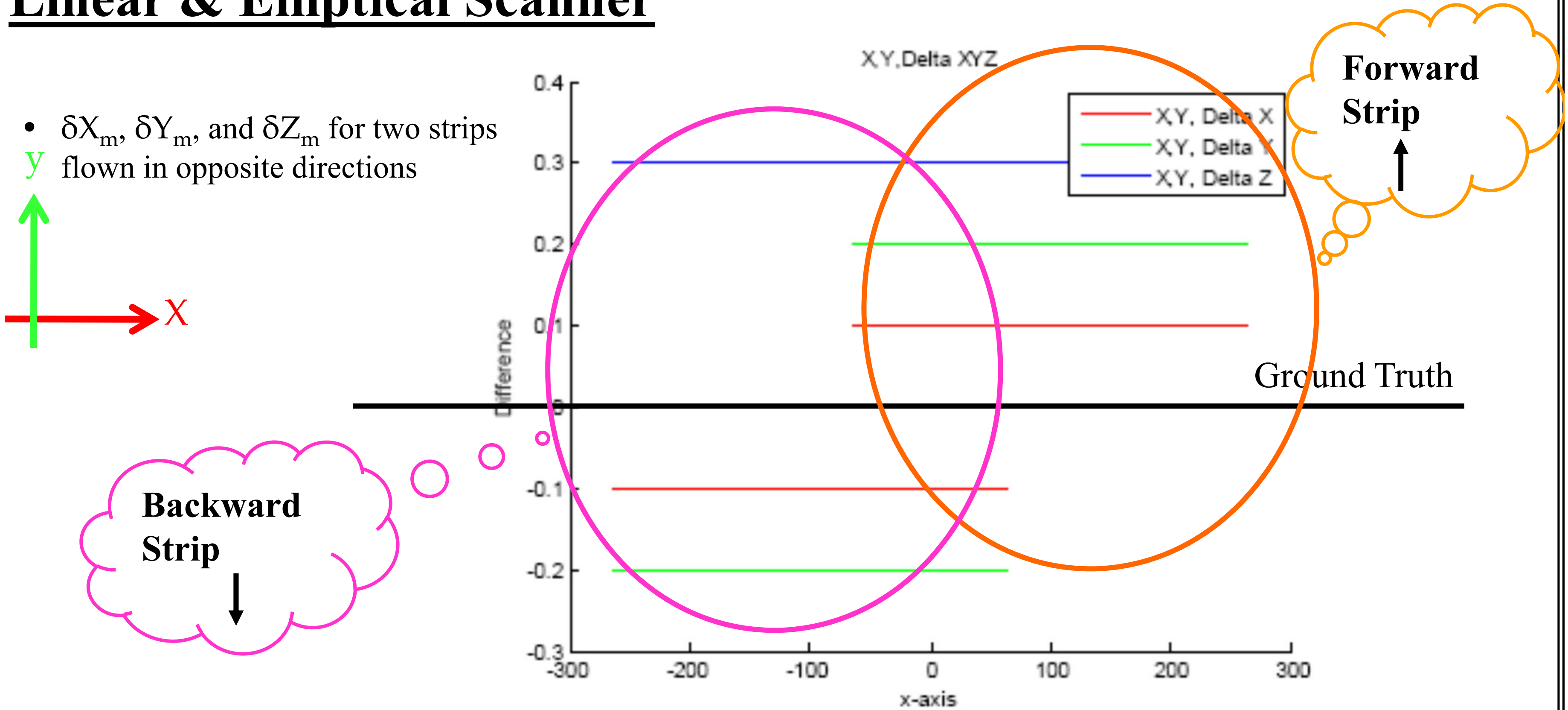
Lever-Arm Offset Bias (δΔZ)



Lever-Arm Offset Bias (δΔX, δΔY, δΔZ)

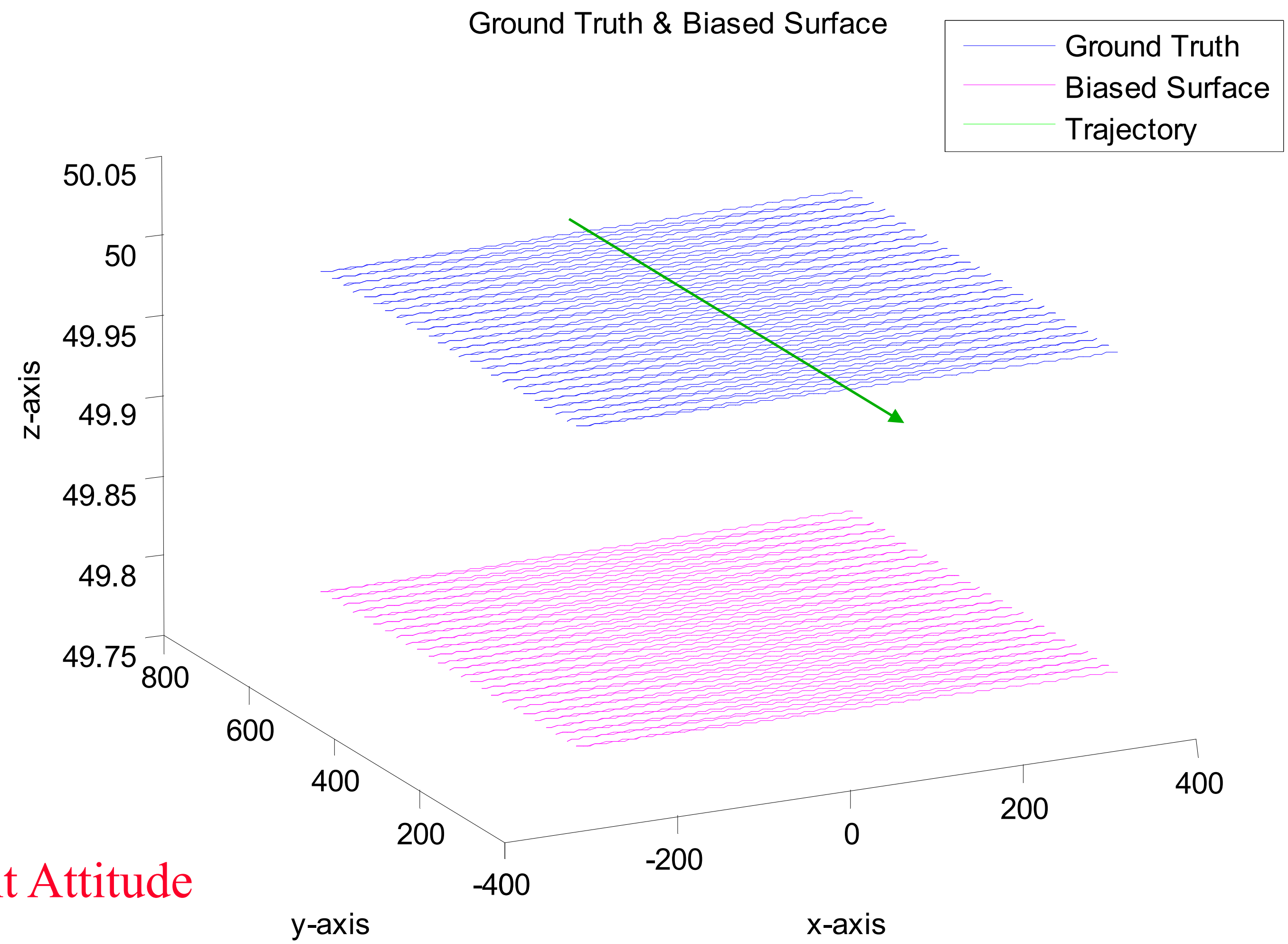
Linear & Elliptical Scanner

- δX_m, δY_m, and δZ_m for two strips flown in opposite directions



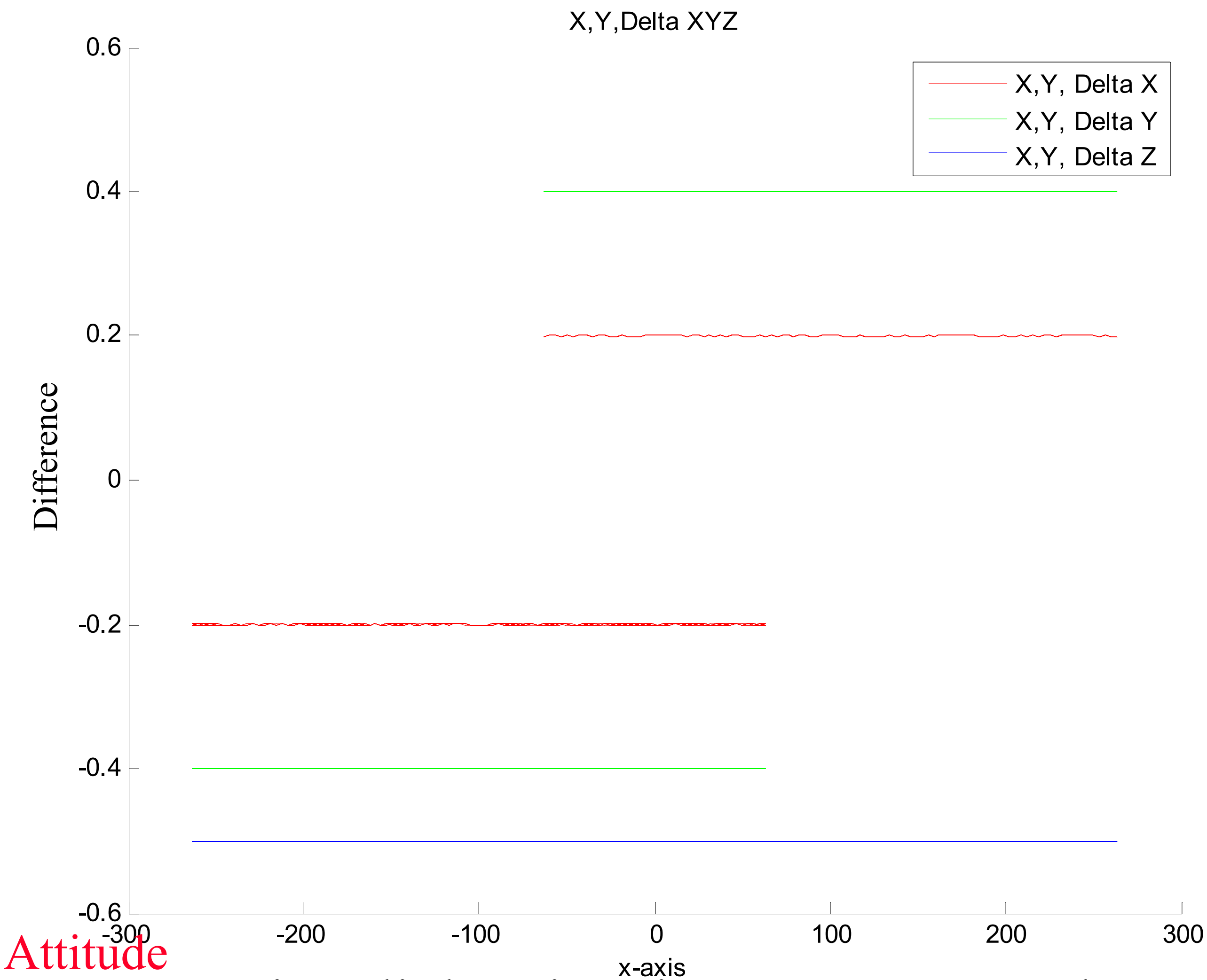
	Flying Height	Flying Direction	Look Angle
Lever-arm Offset Bias	• Effect is independent of the flying height	• Planimetric effect is dependent on the flying direction • Vertical effect is independent of the flying direction	• Effect is independent of the look angle

Linear Scanner & Lever-Arm Offset Bias



Constant Attitude

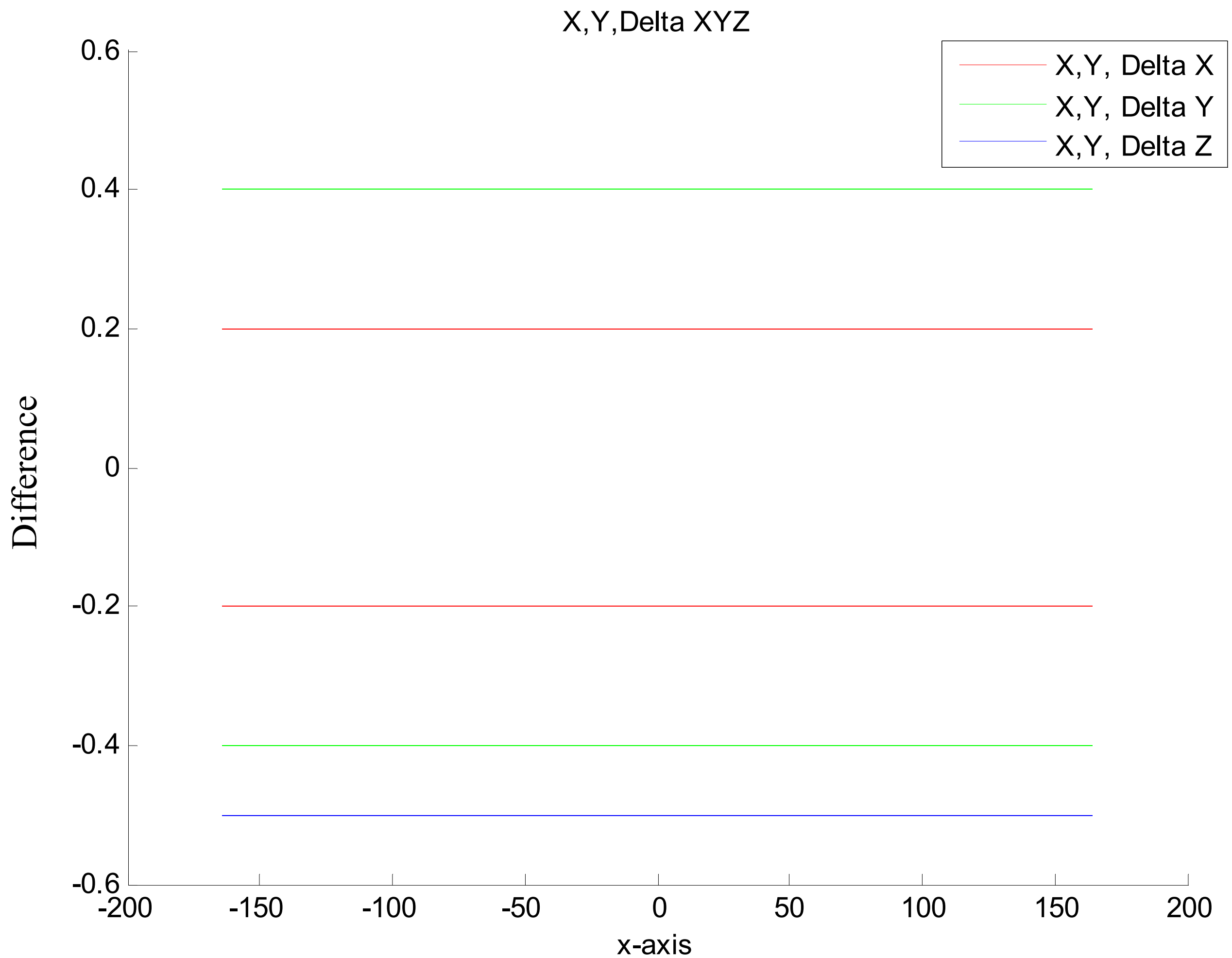
Linear Scanner & Lever-Arm Offset Bias



Constant Attitude

Opposite Flight Directions & 30% Overlap

Linear Scanner & Lever-Arm Offset Bias

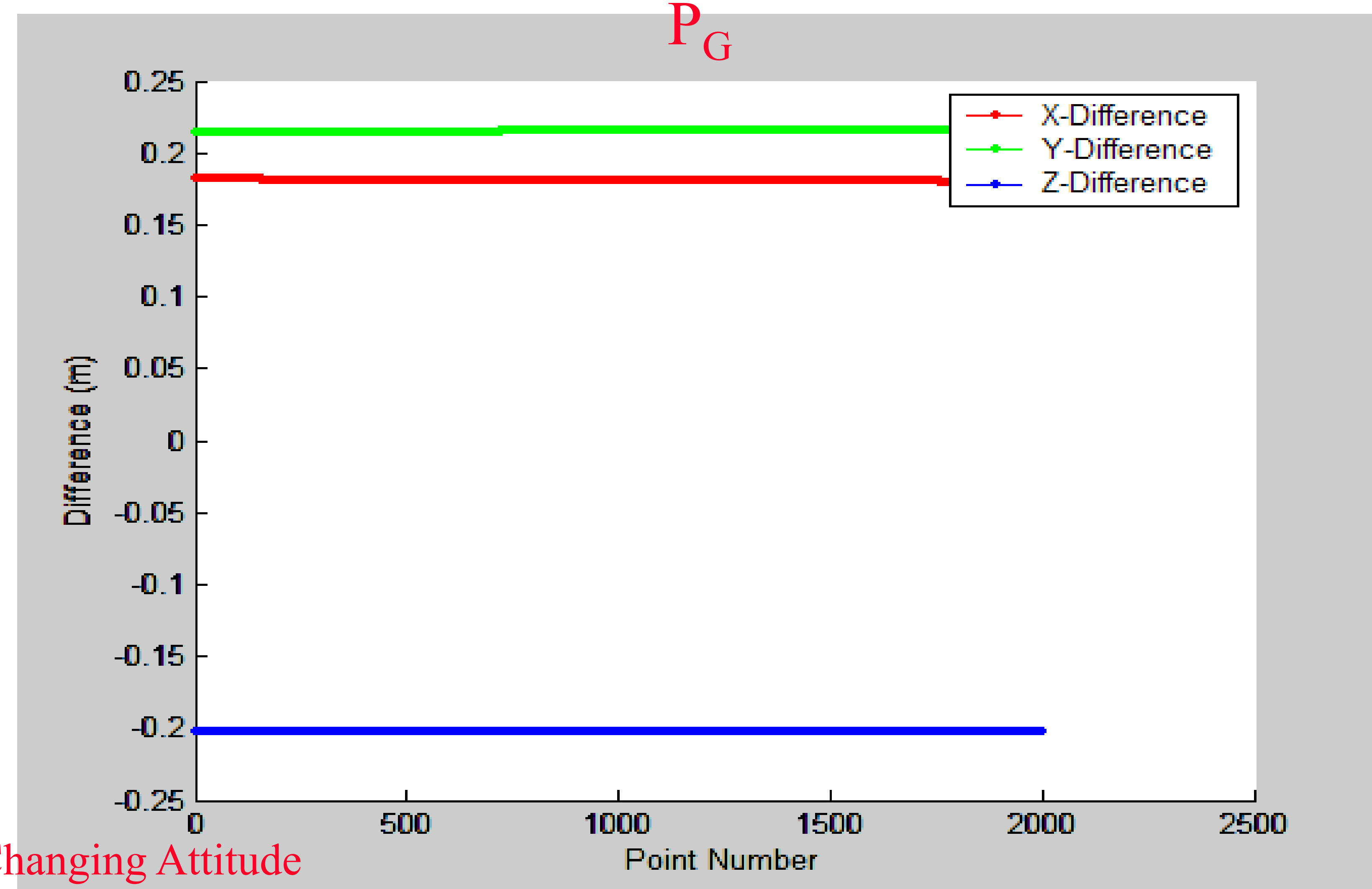


Constant Attitude

Opposite Flight Directions & 100% Overlap

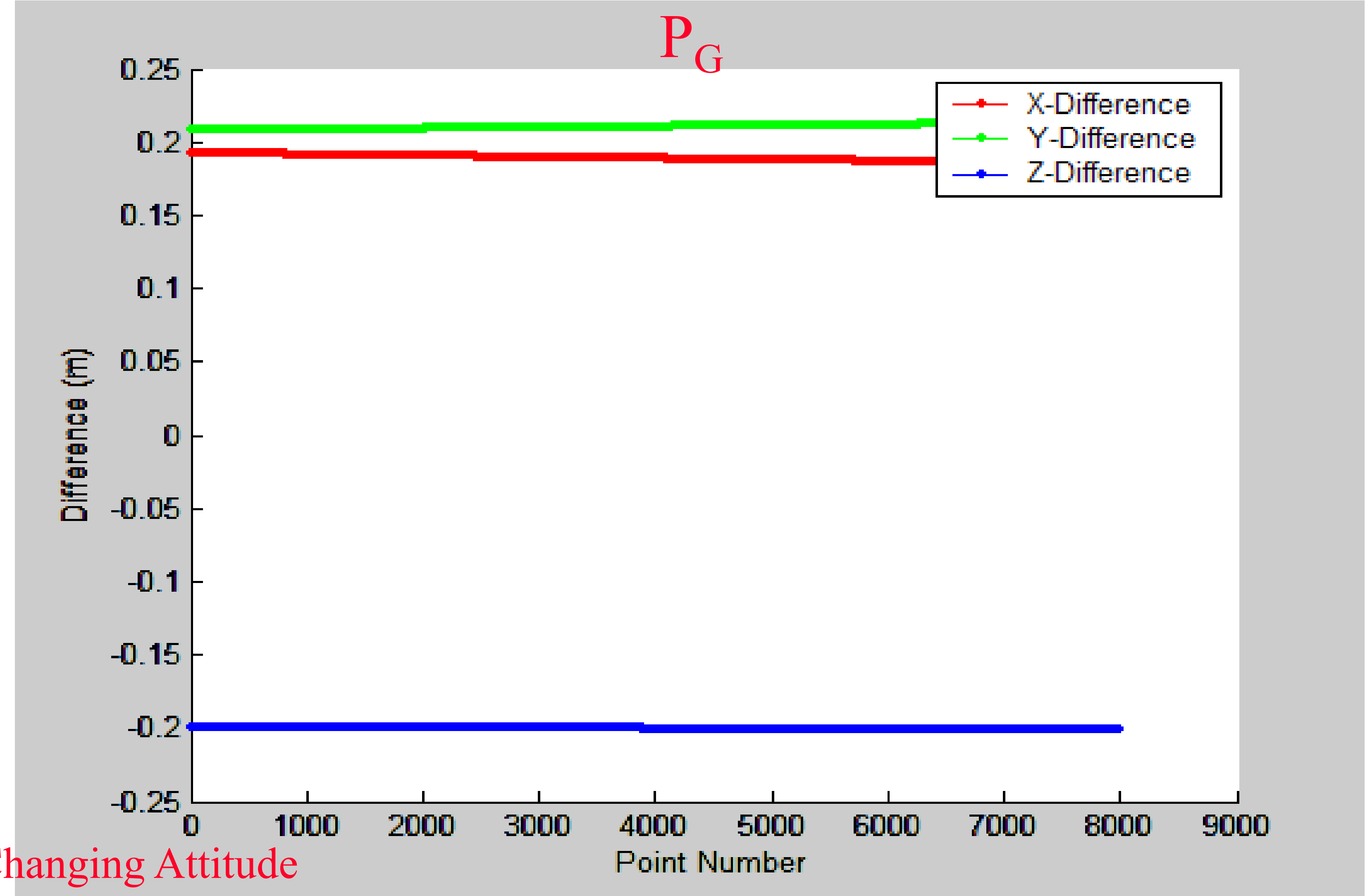
Linear Scanner & Lever-Arm Offset Bias

P_G



Changing Attitude

Elliptical Scanner & Lever-Arm Offset Bias



Changing Attitude

Impact of Systematic Biases

Mathematical Analysis of the LiDAR Equation

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	$\mathbf{0}$
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	$\mathbf{0}$
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\,\delta\Delta\omega$	$\mathbf{0}$
$\delta\Delta\varphi$	$\pm z\,\delta\Delta\varphi$	0	$-x\,\delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\,\delta\Delta\kappa$	$\mathbf{0}$
$\delta\Delta\rho$	$\mp sin(S\beta)\,\delta\Delta\rho$	0	$-cos(S\beta)\,\delta\Delta\rho$
δS	$\pm z\,\beta\,\delta S$	$\mathbf{0}$	$-x\,\beta\,\delta S$

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

- Y- axis is along the flight direction.

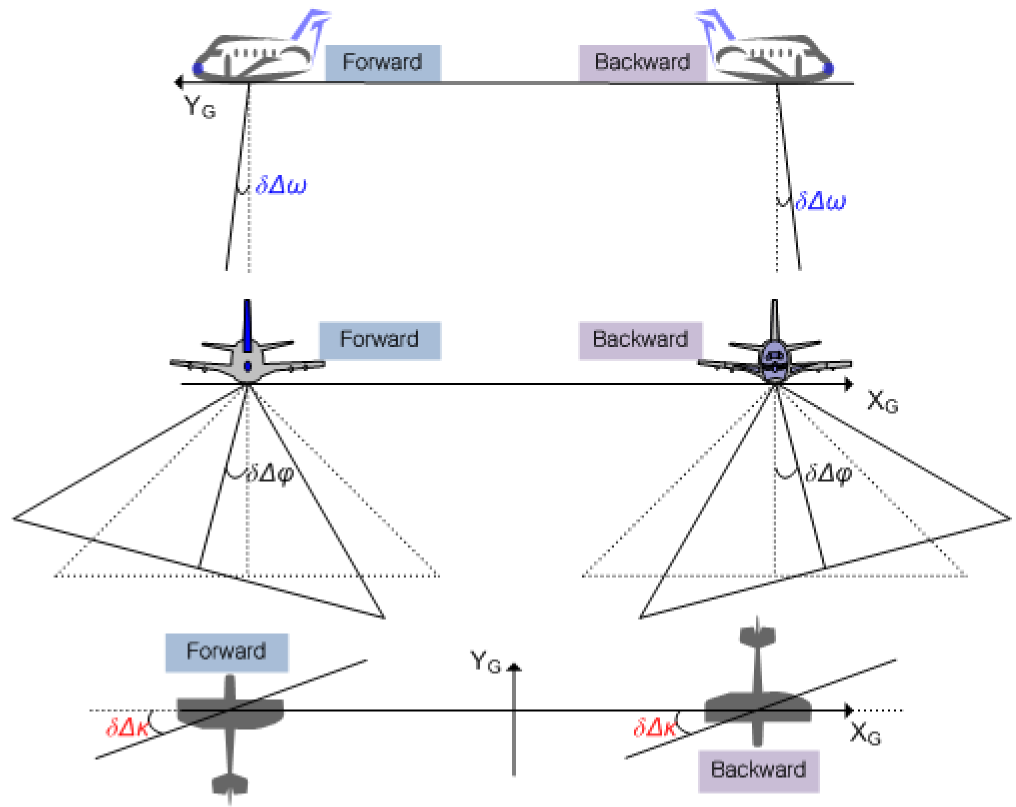
Boresight Angular Bias ($\delta\Delta\omega$, $\delta\Delta\phi$, $\delta\Delta\kappa$)

Linear Scanner

Boresighting Pitch Bias ($\delta\Delta\omega$)

Boresighting Roll Bias ($\delta\Delta\phi$)

Boresighting Heading Bias ($\delta\Delta\kappa$)



Boresight Pitch Bias (δΔω)

Mathematical Analysis of the LiDAR Equation

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\,\delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\,\delta\Delta\varphi$	0	$-x\,\delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\,\delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\,\delta\Delta\rho$	0	$-cos(S\beta)\,\delta\Delta\rho$
δS	$\pm z\,\beta\,\delta S$	**0**	$-x\,\beta\,\delta S$

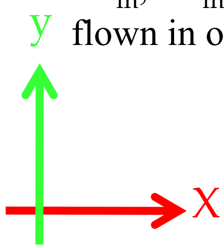
- Y- axis is along the flight direction.

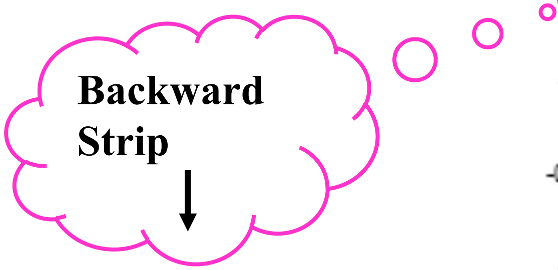
Top sign refers to the forward flight and the bottom sign refers to the backward flight.

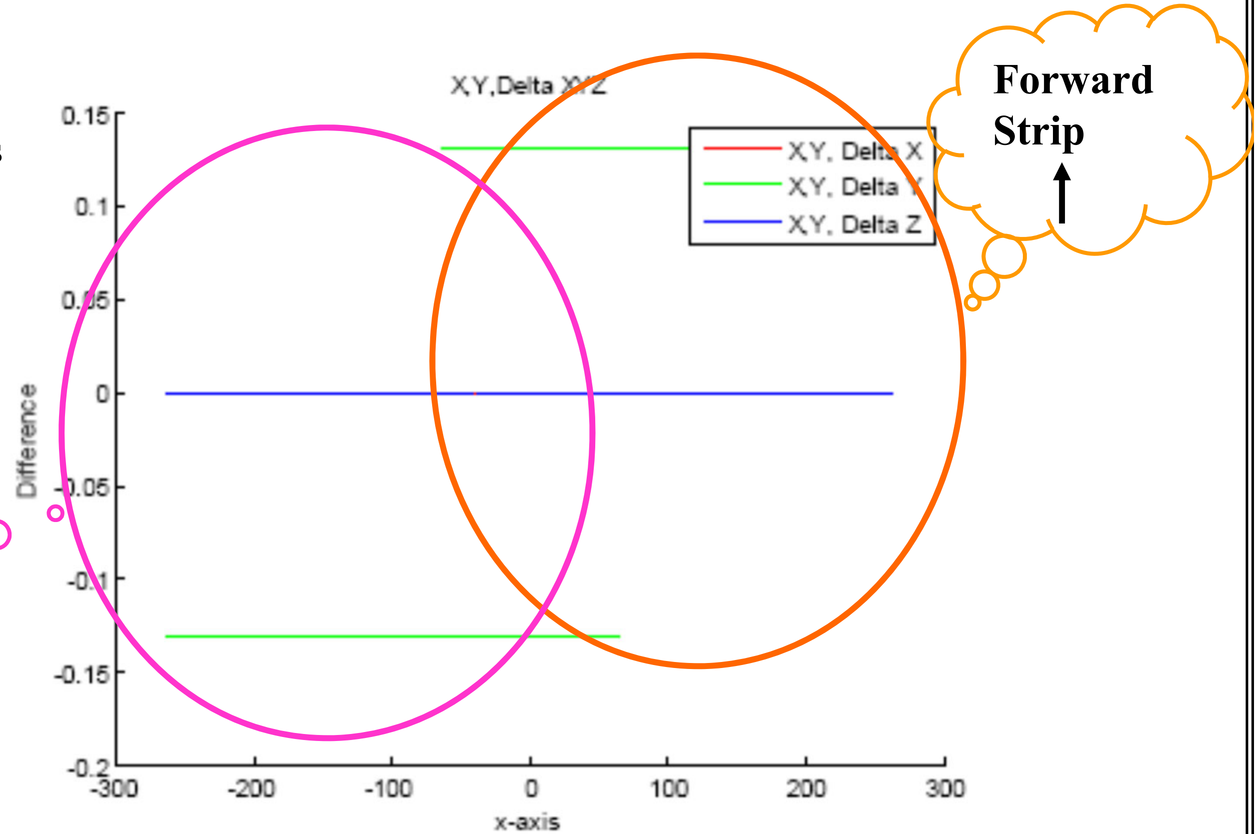
Boresight Pitch Bias ($\delta\Delta\omega$)

Linear Scanner

- δX_m, δY_m, and δZ_m for two strips flown in opposite directions







The pitch bias only affects the planimetric component along the flight direction (Y-Axis in this example).

	Flying Height	Flying Direction	Look Angle
Boresight Pitch Bias	• Effect is dependent on the flying height.	• Planimetric effect along the flight direction is dependent on the flying direction.	• Effect is independent of the look angle.

Boresight Roll Bias (δΔϕ)

Mathematical Analysis of the LiDAR Equation

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\,\delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\,\delta\Delta\varphi$	0	$-x\,\delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\,\delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\,\delta\Delta\rho$	0	$-\,cos(S\beta)\,\delta\Delta\rho$
δS	$\pm z\,\beta\,\delta S$	**0**	$-x\,\beta\,\delta S$

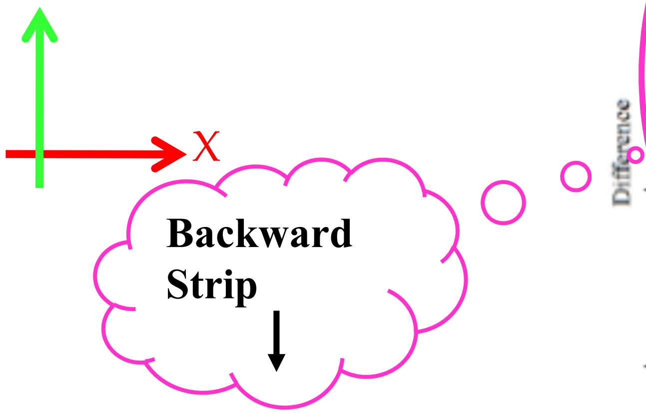
Top sign refers to the forward flight and the bottom sign refers to the backward flight.

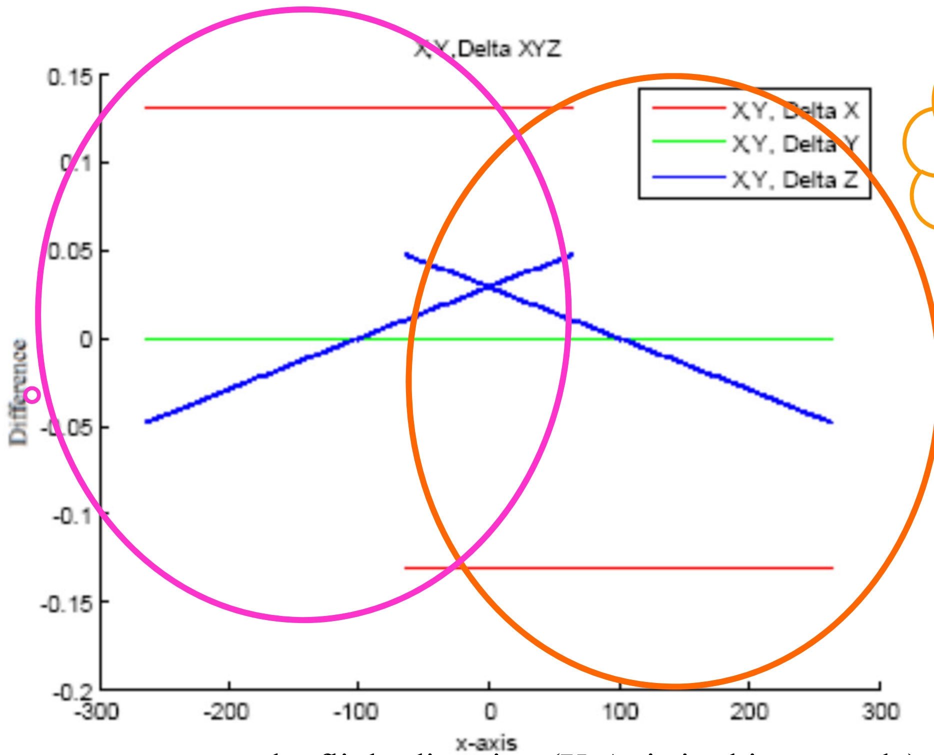
- Y- axis is along the flight direction.

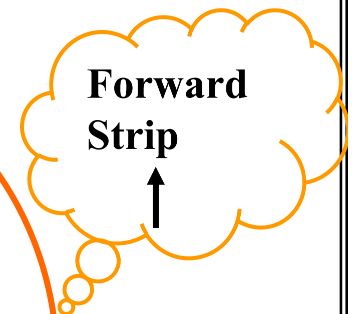
Boresight Roll Bias ($\delta\Delta\phi$)

Linear Scanner

- δX_m, δY_m, and δZ_m for two strips flown in opposite directions







The roll bias affects the planimetric component across the flight direction (X-Axis in this example) and the height component.

	Flying Height	Flying Direction	Look Angle
Boresight Roll Bias	• Planimetric effect across the flight direction is dependent on the flying height. • Vertical effect is independent of the flying height.	• Planimetric effect across the flight direction and vertical effect are dependent on the flying direction.	• Planimetric effect across the flight direction is independent of the look angle. • Vertical effect is dependent on the look angle.

Boresight Heading Bias (δΔκ)

Mathematical Analysis of the LiDAR Equation

	δX_m	δY_m	δZ_m
$\delta \Delta X$	$\pm\delta\Delta X$	0	$\mathbf{0}$
$\delta \Delta Y$	0	$\pm\delta\Delta Y$	$\mathbf{0}$
$\delta \Delta Z$	0	0	$\delta\Delta Z$
$\delta \Delta \omega$	0	$\mp z\,\delta\Delta\omega$	$\mathbf{0}$
$\delta \Delta \varphi$	$\pm z\,\delta\Delta\varphi$	0	$-x\,\delta\Delta\varphi$
$\delta \Delta \kappa$	0	$\pm x\,\delta\Delta\kappa$	$\mathbf{0}$
$\delta \Delta \rho$	$\mp sin(S\beta)\,\delta\Delta\rho$	0	$-\,cos(S\beta)\,\delta\Delta\rho$
δS	$\pm z\,\beta\,\delta S$	$\mathbf{0}$	$-x\,\beta\,\delta S$

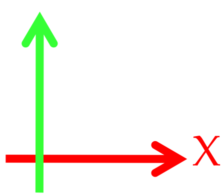
Top sign refers to the forward flight and the bottom sign refers to the backward flight.

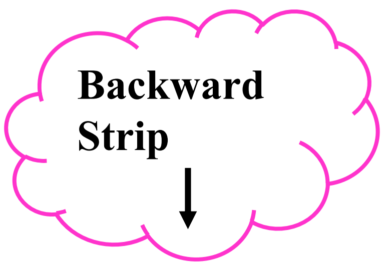
- Y- axis is along the flight direction.

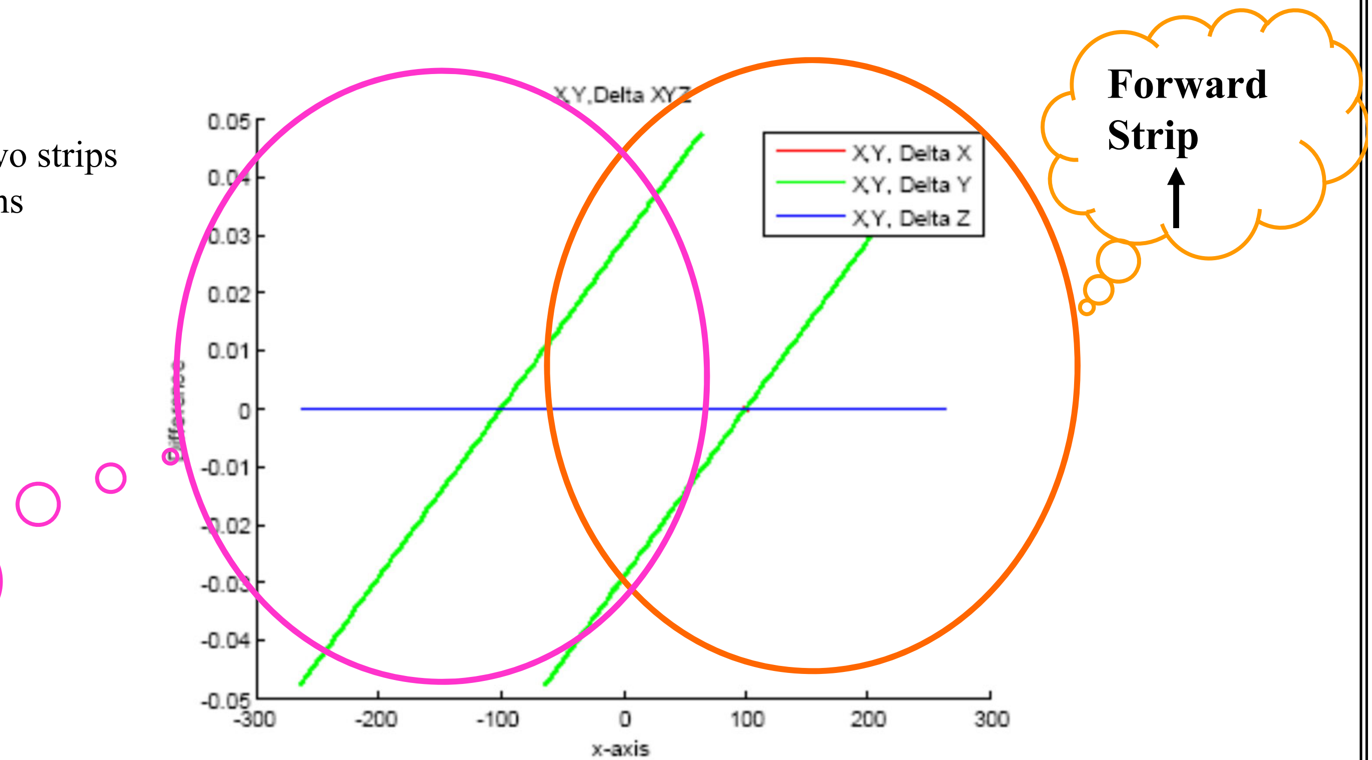
Boresight Heading Bias (δΔκ)

Linear Scanner

- δX_m, δY_m, and δZ_m for two strips flown in opposite directions



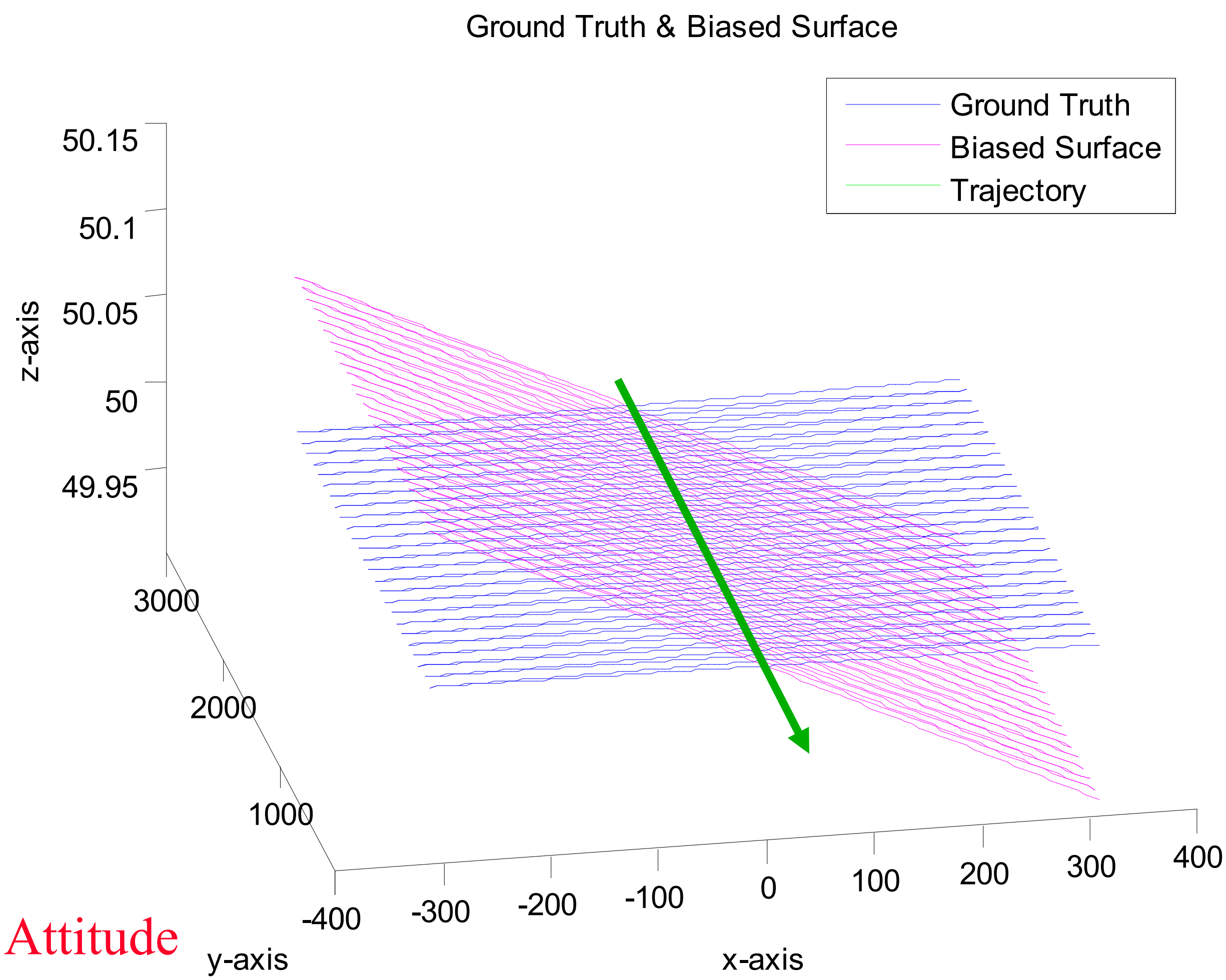




The heading bias only affects the planimetric component along the flight direction (Y-Axis in this example).

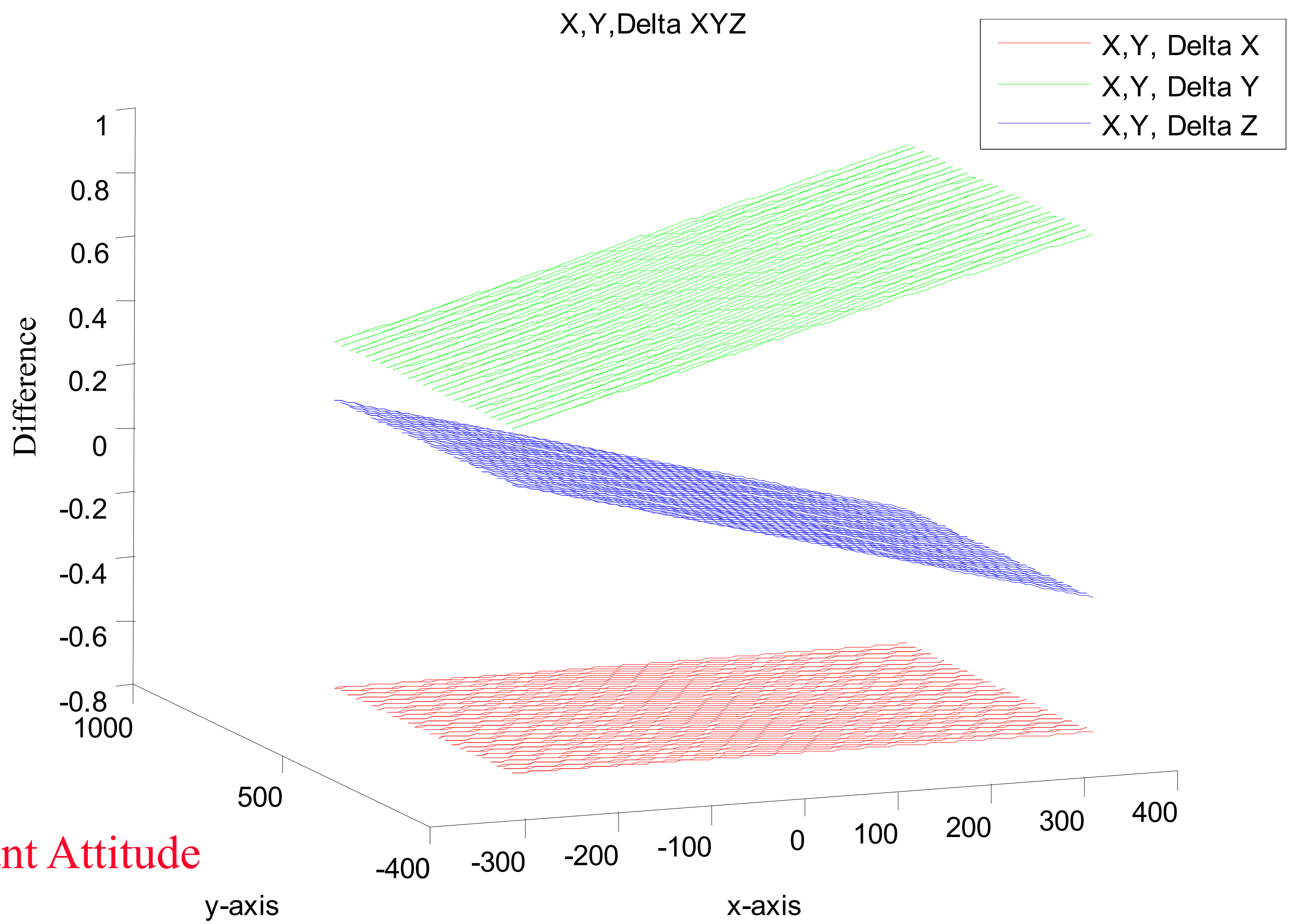
	Flying Height	Flying Direction	Look Angle
Boresight Heading Bias	• Effect is independent of the flying height.	• Planimetric effect along the flight direction is independent of the flying direction.	• Planimetric effect along the flight direction is dependent on the look angle.

Linear Scanner & Boresight Angular Bias



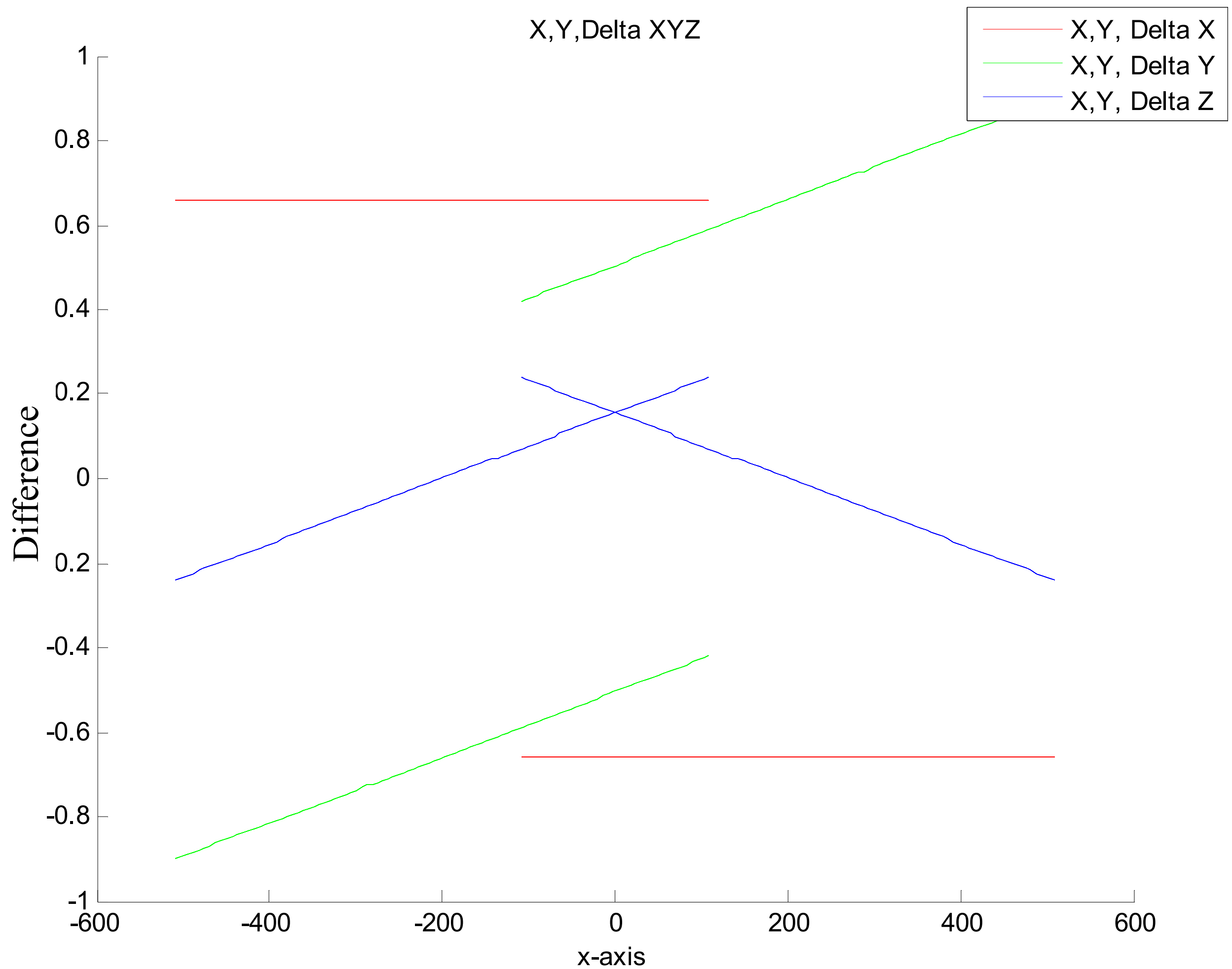
Constant Attitude

Linear Scanner & Boresight Angular Bias



Constant Attitude

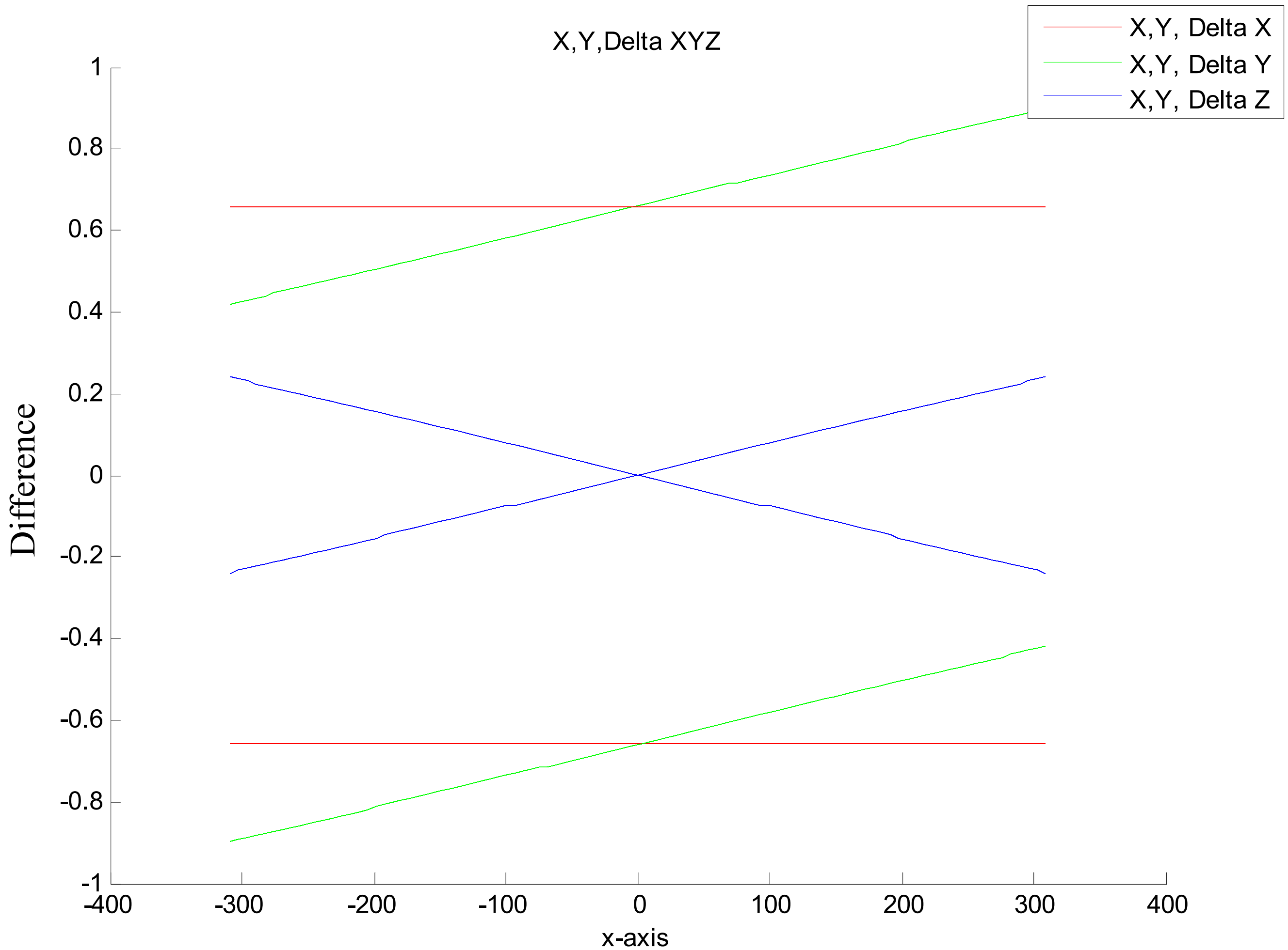
Linear Scanner & Boresight Angular Bias



Constant Attitude

Opposite Flight Directions & 30% Overlap

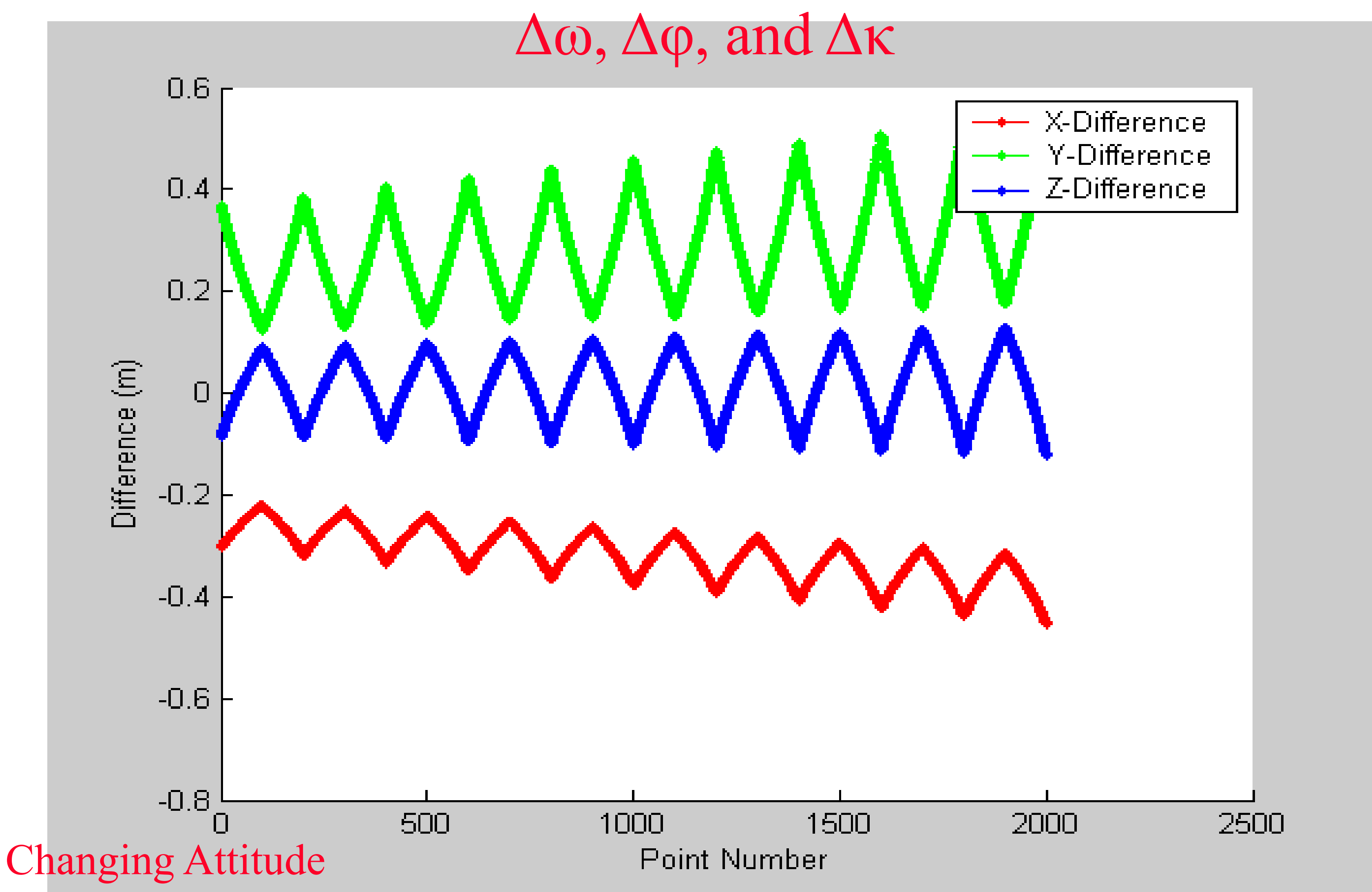
Linear Scanner & Boresight Angular Bias



Constant Attitude

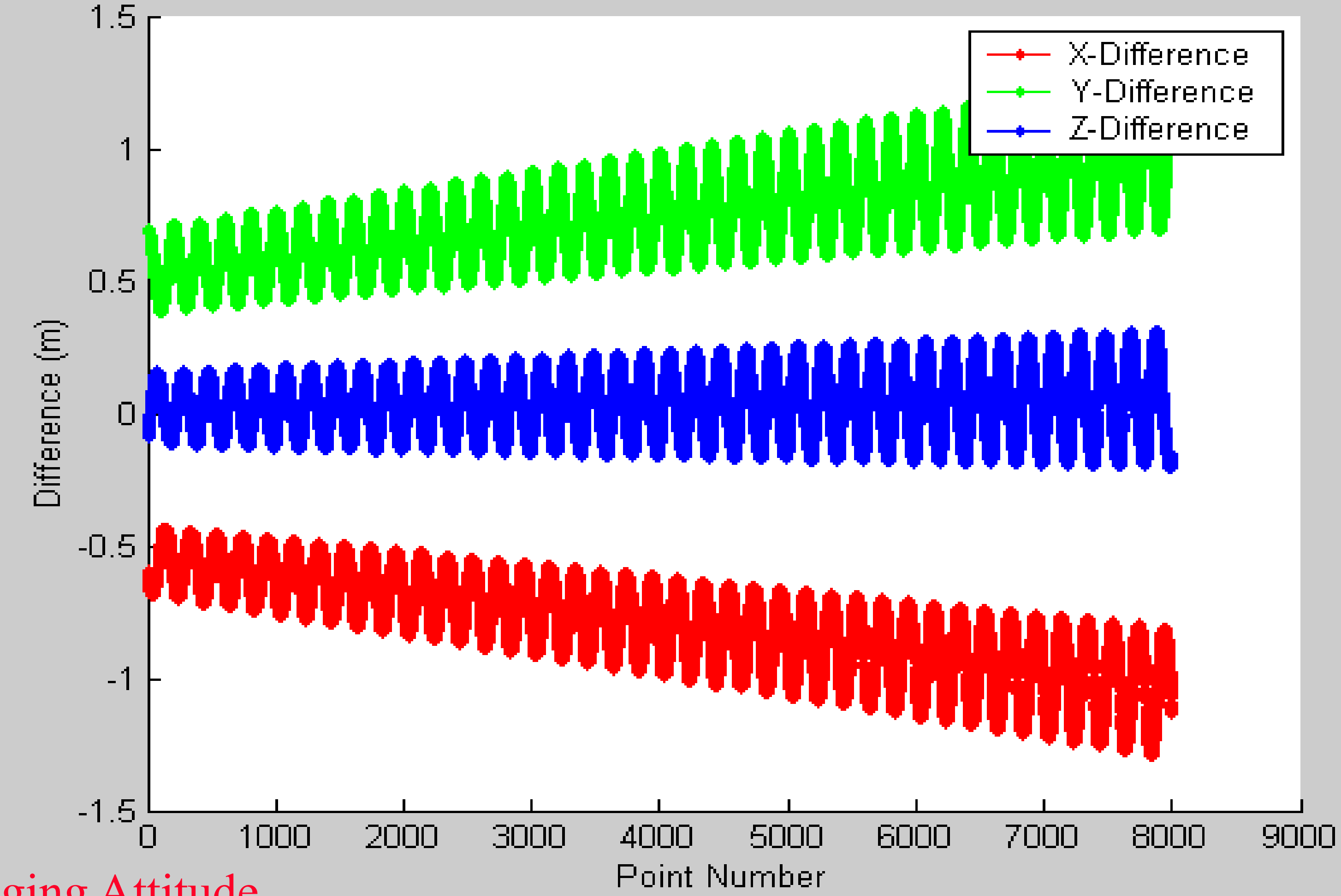
Opposite Flight Directions & 100% Overlap

Linear Scanner & Boresight Angular Bias



Elliptical Scanner & Boresight Angular Bias

Δω, Δφ, and Δκ



Changing Attitude

Laser Beam Range Bias (δΔρ)

Mathematical Analysis of the LiDAR Equation

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\,\delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\,\delta\Delta\varphi$	0	$-x\,\delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\,\delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\,\delta\Delta\rho$	0	$-cos(S\beta)\,\delta\Delta\rho$
δS	$\pm z\,\beta\,\delta S$	**0**	$-x\,\beta\,\delta S$

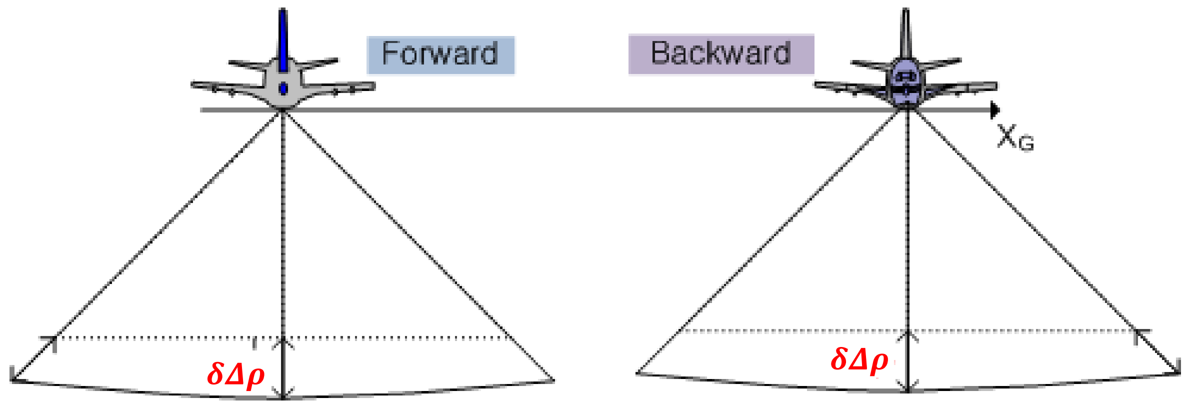
Top sign refers to the forward flight and the bottom sign refers to the backward flight.

- Y- axis is along the flight direction.

Laser Beam Range Bias ($\delta\Delta\rho$)

Linear Scanner

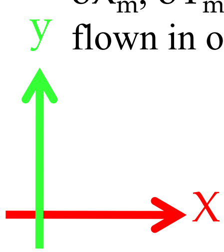
Range Bias ($\delta\rho$)

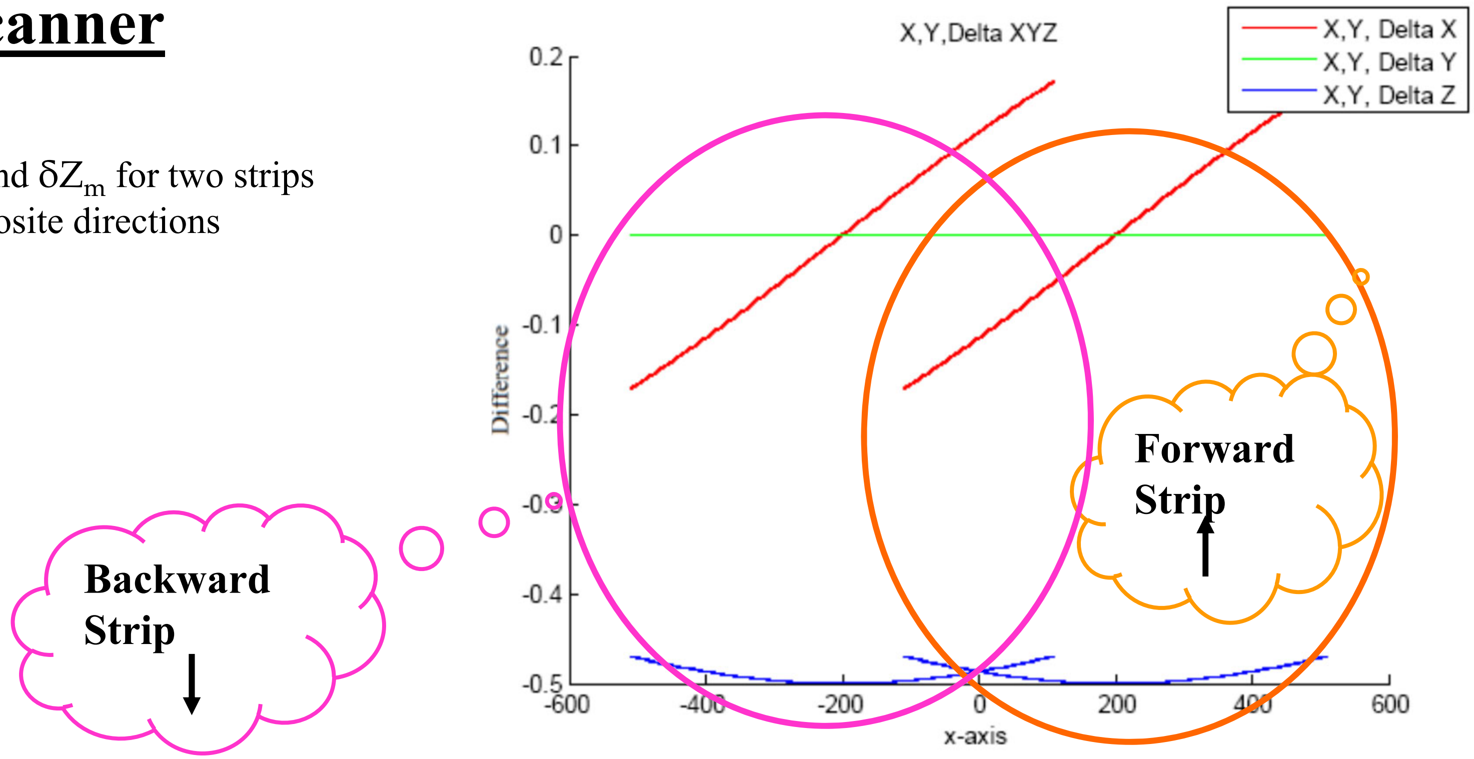


Laser Beam Range Bias ($\delta\Delta\rho$)

Linear Scanner

- δX_m, δY_m, and δZ_m for two strips flown in opposite directions

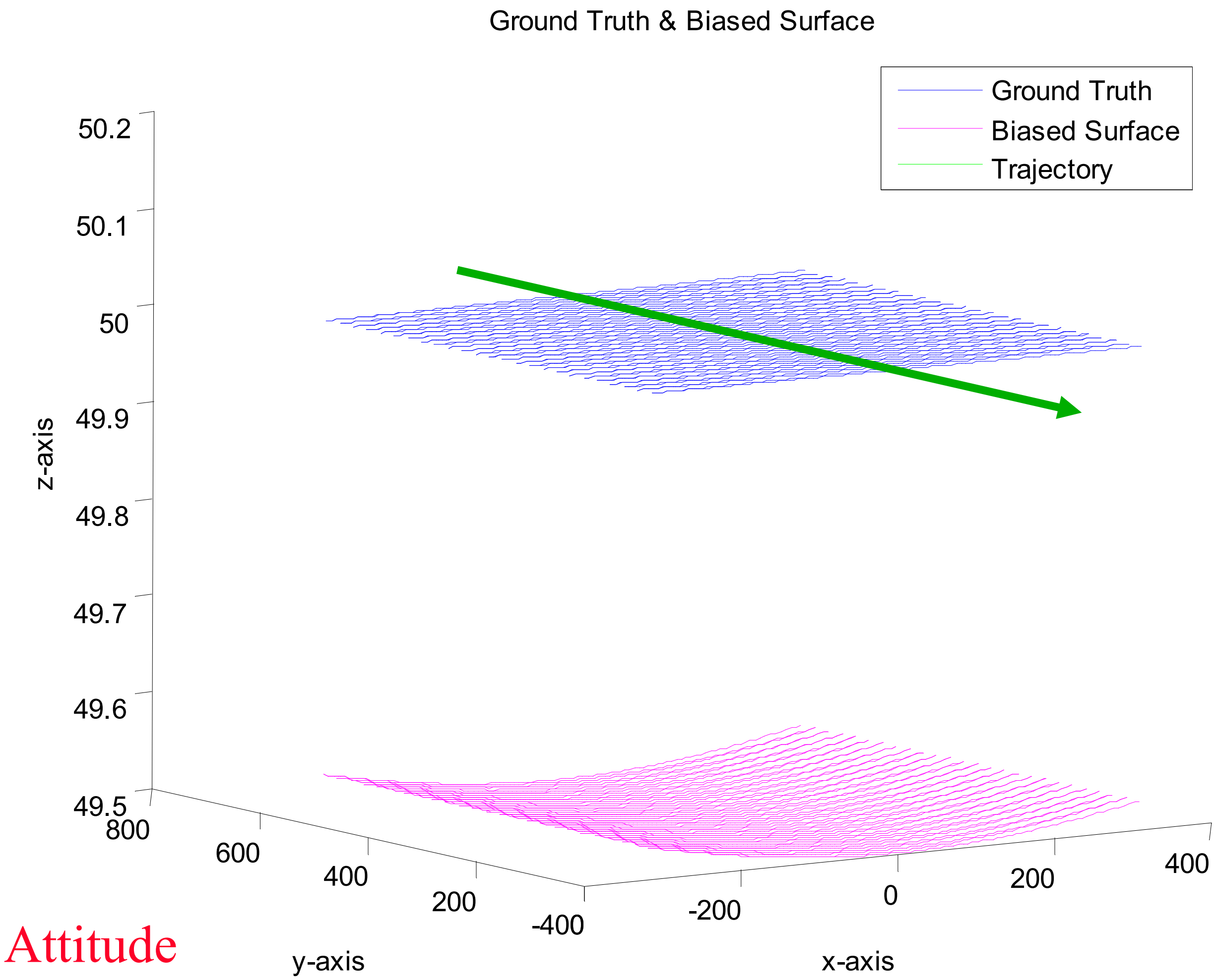




The range bias affects the planimetric component across the flight direction (X-Axis in this example) and the height component.

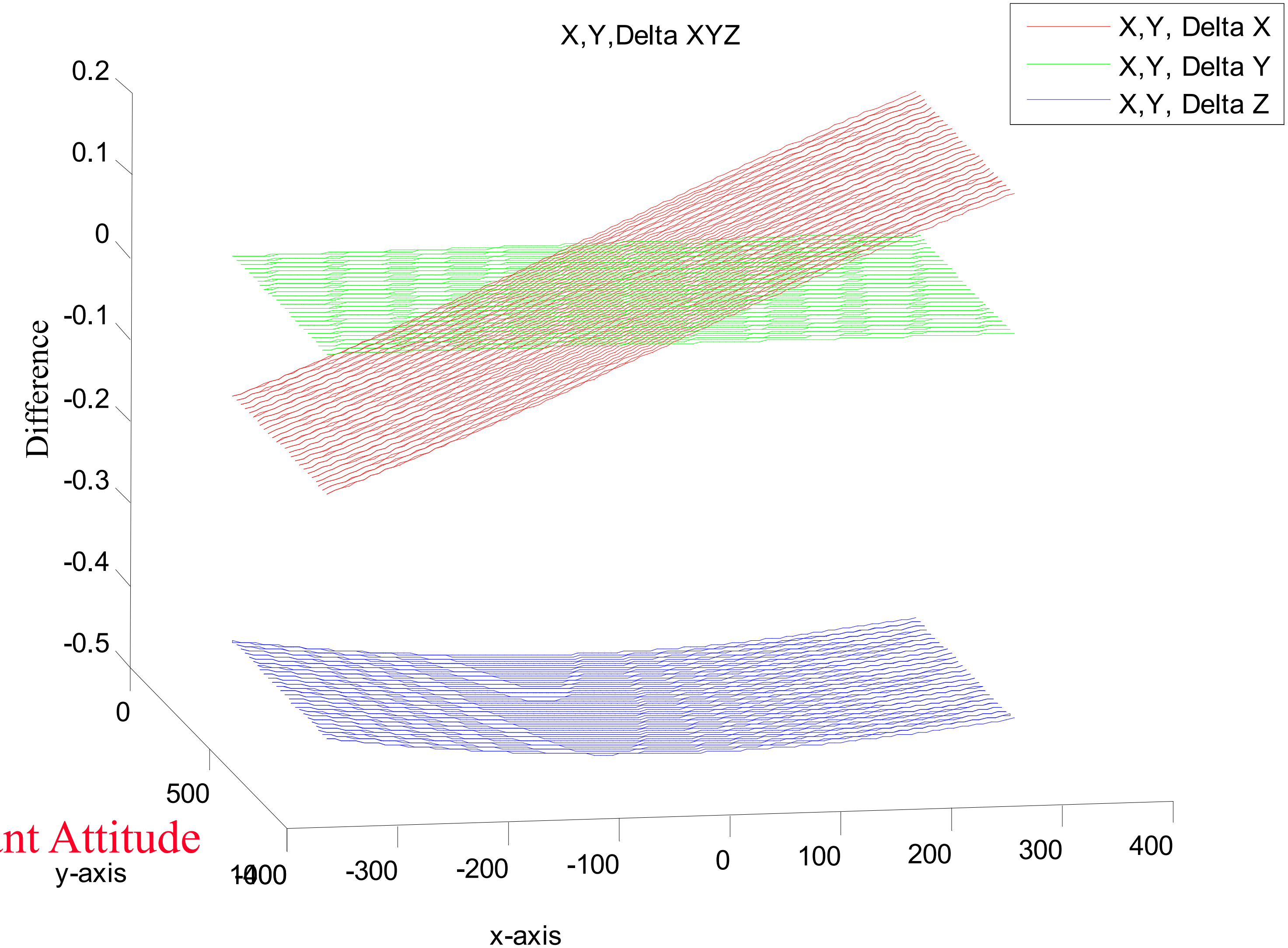
	Flying Height	Flying Direction	Look Angle
Range Bias	• Effect is independent of the flying height.	• Planimetric effect across the flight direction and vertical effect are independent of the flying direction.	• Planimetric effect across the flight direction and vertical effect are dependent on the look angle (DX more than DZ).

Linear Scanner & Laser Beam Range Bias



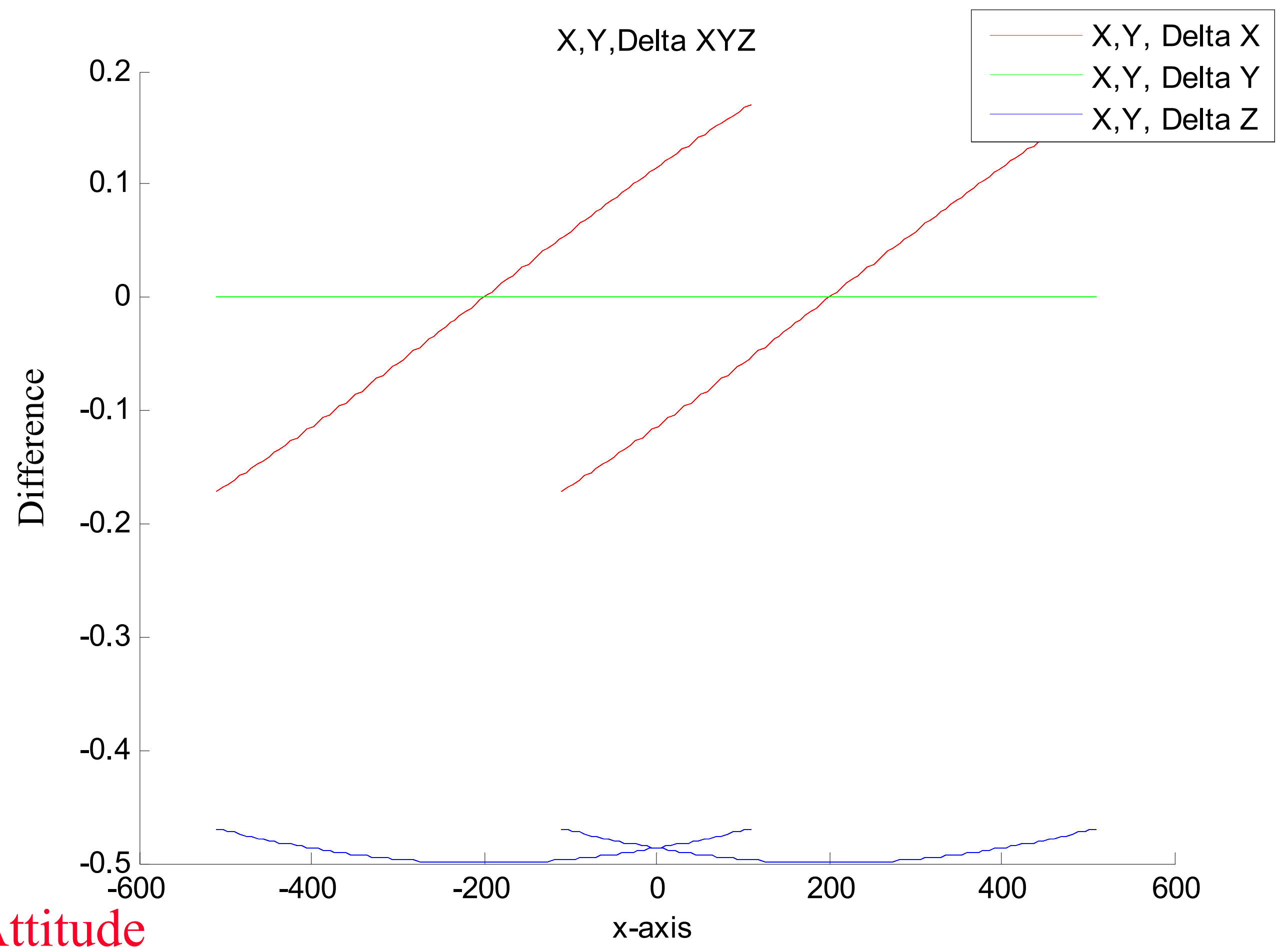
Constant Attitude

Linear Scanner & Laser Beam Range Bias



Constant Attitude

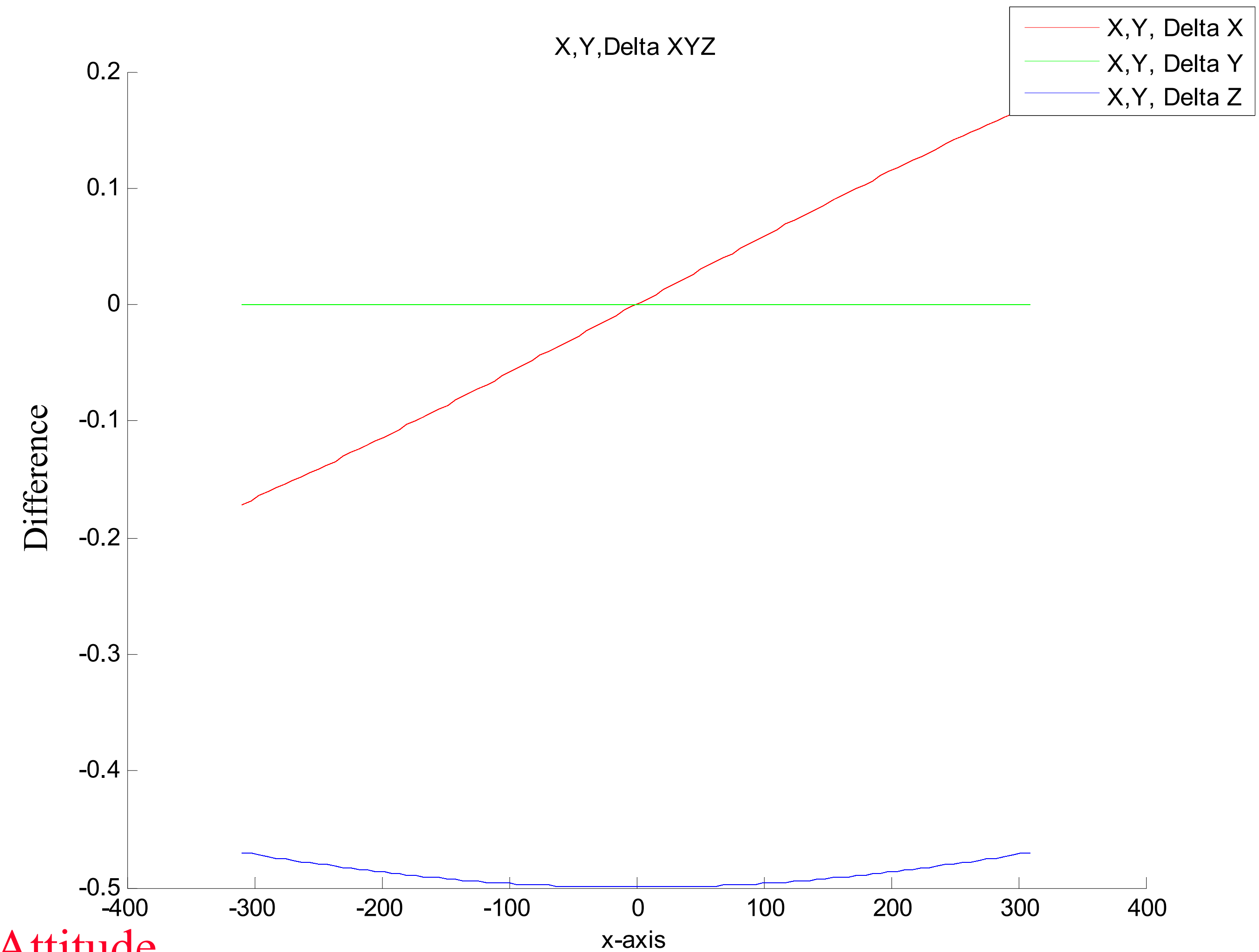
Linear Scanner & Laser Beam Range Bias



Constant Attitude

Opposite Flight Directions & 30% Overlap

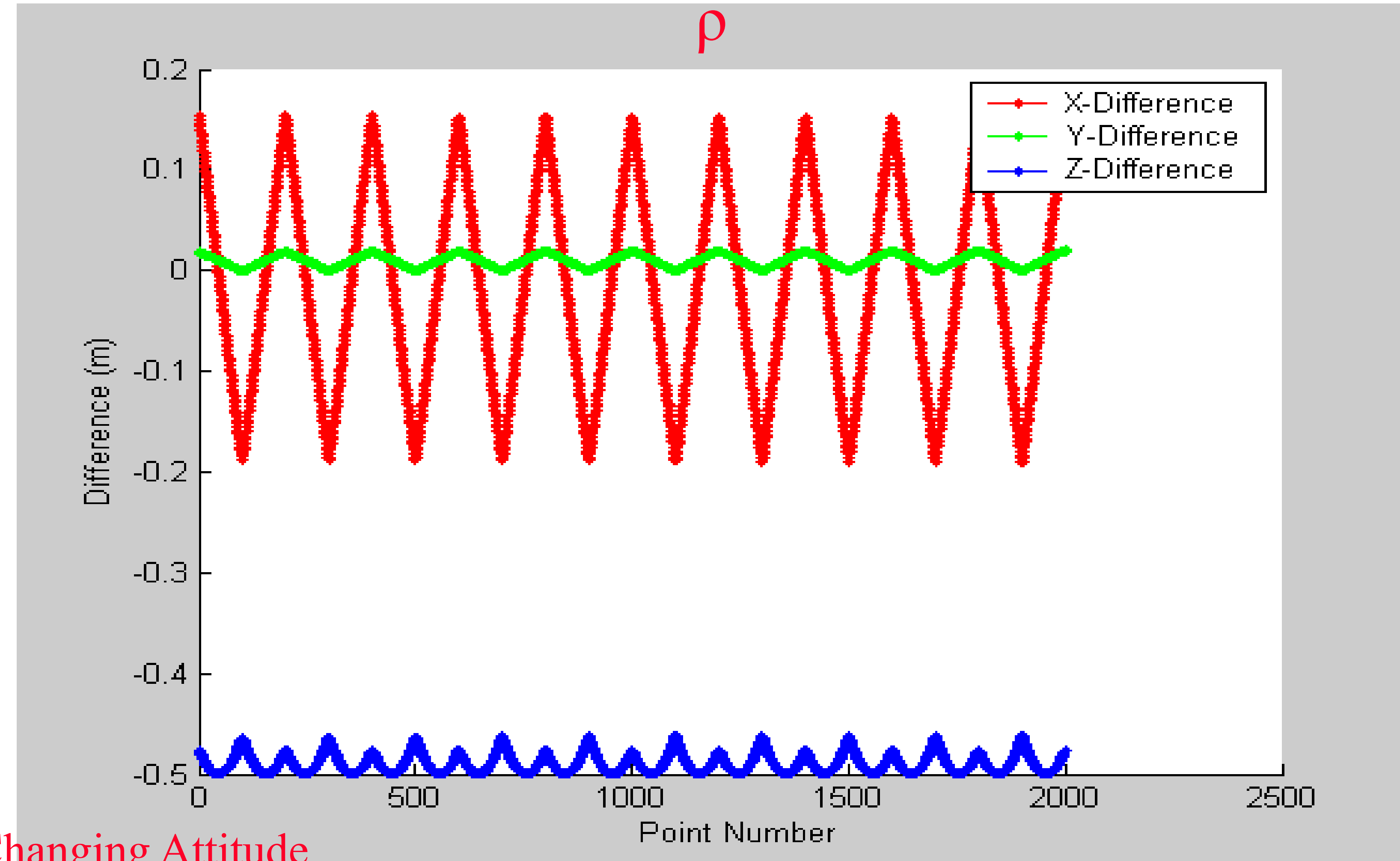
Linear Scanner & Laser Beam Range Bias



Constant Attitude

Opposite Flight Directions & 100% Overlap

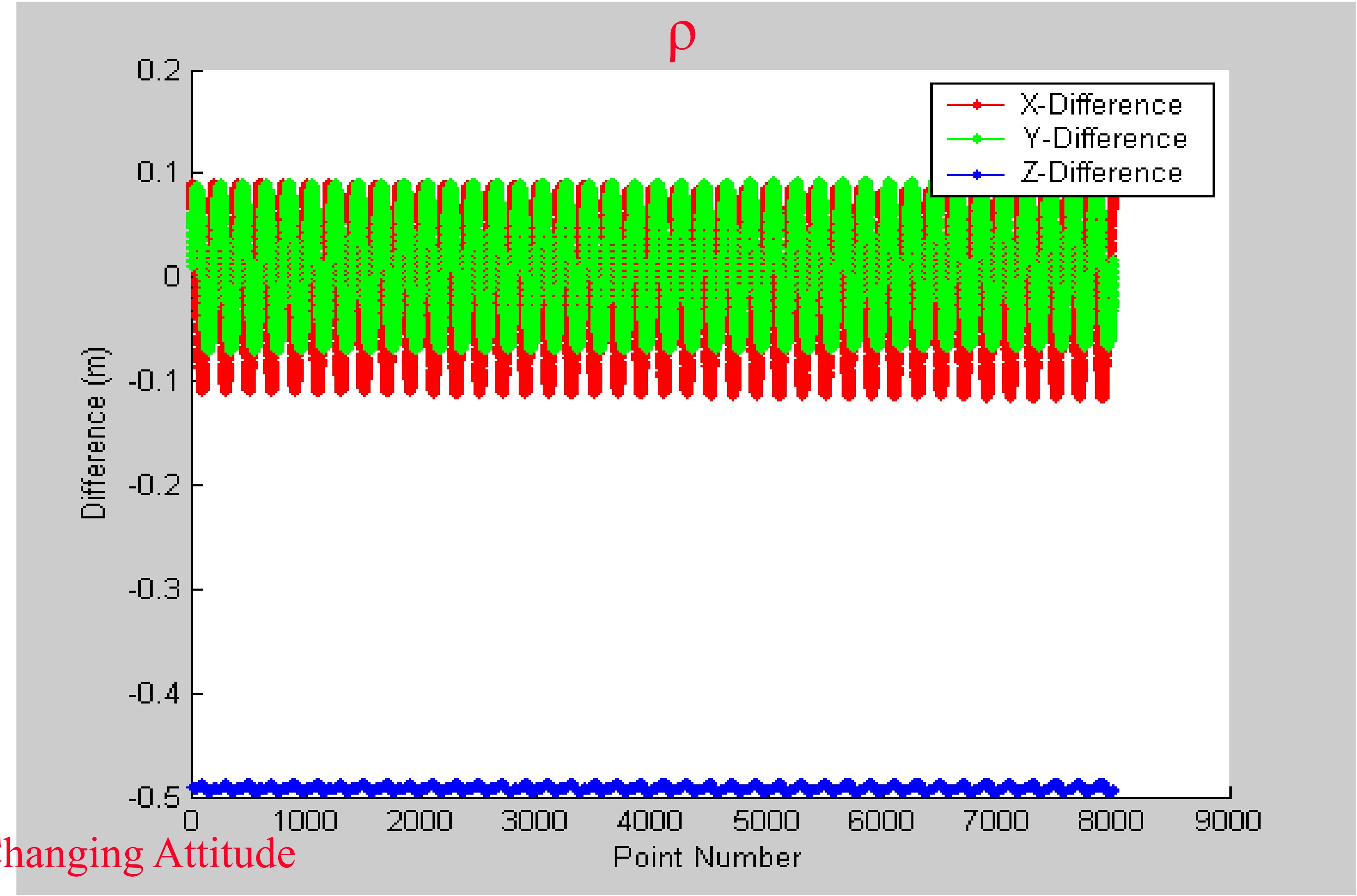
Linear Scanner & Laser Beam Range Bias



Changing Attitude

- Flight direction parallel to the Y-axis of the ground coordinate system.

Elliptical Scanner & Laser Beam Range Bias



Changing Attitude

Laser Beam Angular Bias (δS)

Mathematical Analysis of the LiDAR Equation

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\,\delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\,\delta\Delta\varphi$	0	$-x\,\delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\,\delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\,\delta\Delta\rho$	0	$-\,cos(S\beta)\,\delta\Delta\rho$
δS	$\pm z\,\beta\,\delta S$	**0**	$-x\,\beta\,\delta S$

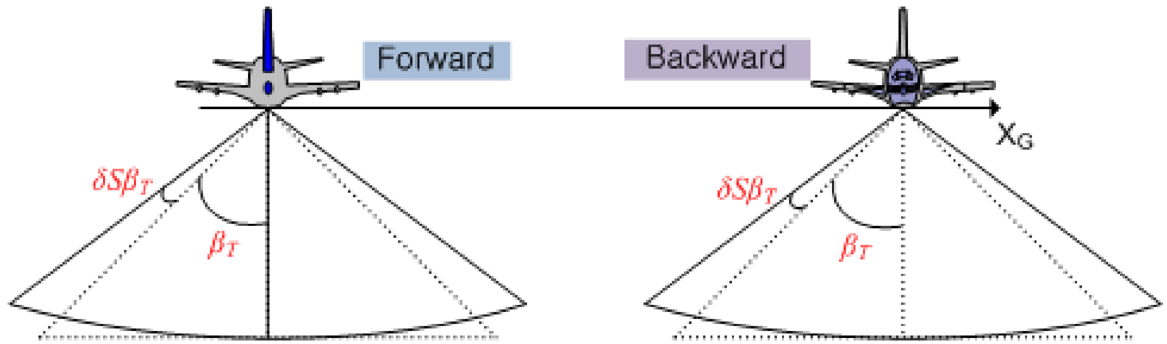
Top sign refers to the forward flight and the bottom sign refers to the backward flight.

- Y- axis is along the flight direction.

Laser Beam Angular Bias (δS)

Linear Scanner

Mirror Angle Scale Bias (δS)

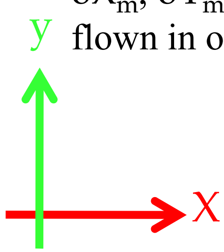


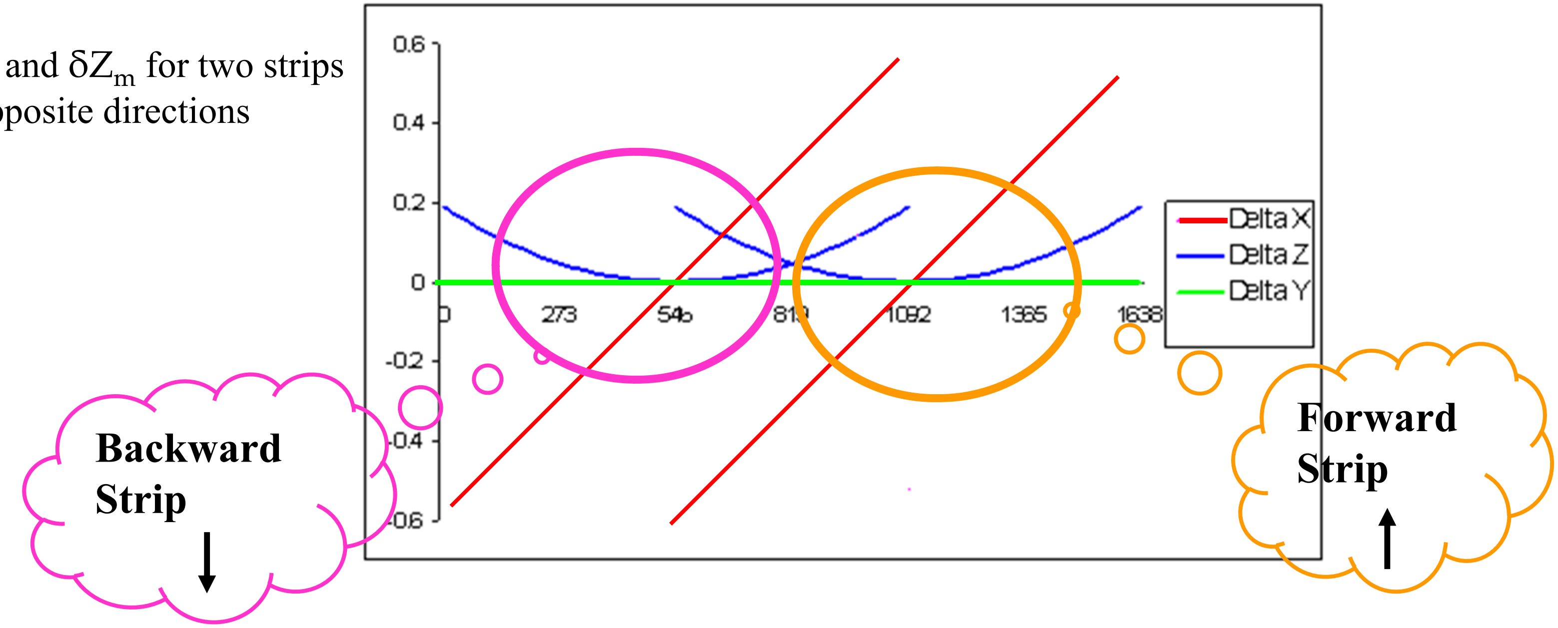
Laser Beam Angular Bias (δS)

Linear Scanner

$$\beta_{true} = \beta_{meas.} * (S + \delta S)$$

- δX_m, δY_m, and δZ_m for two strips flown in opposite directions

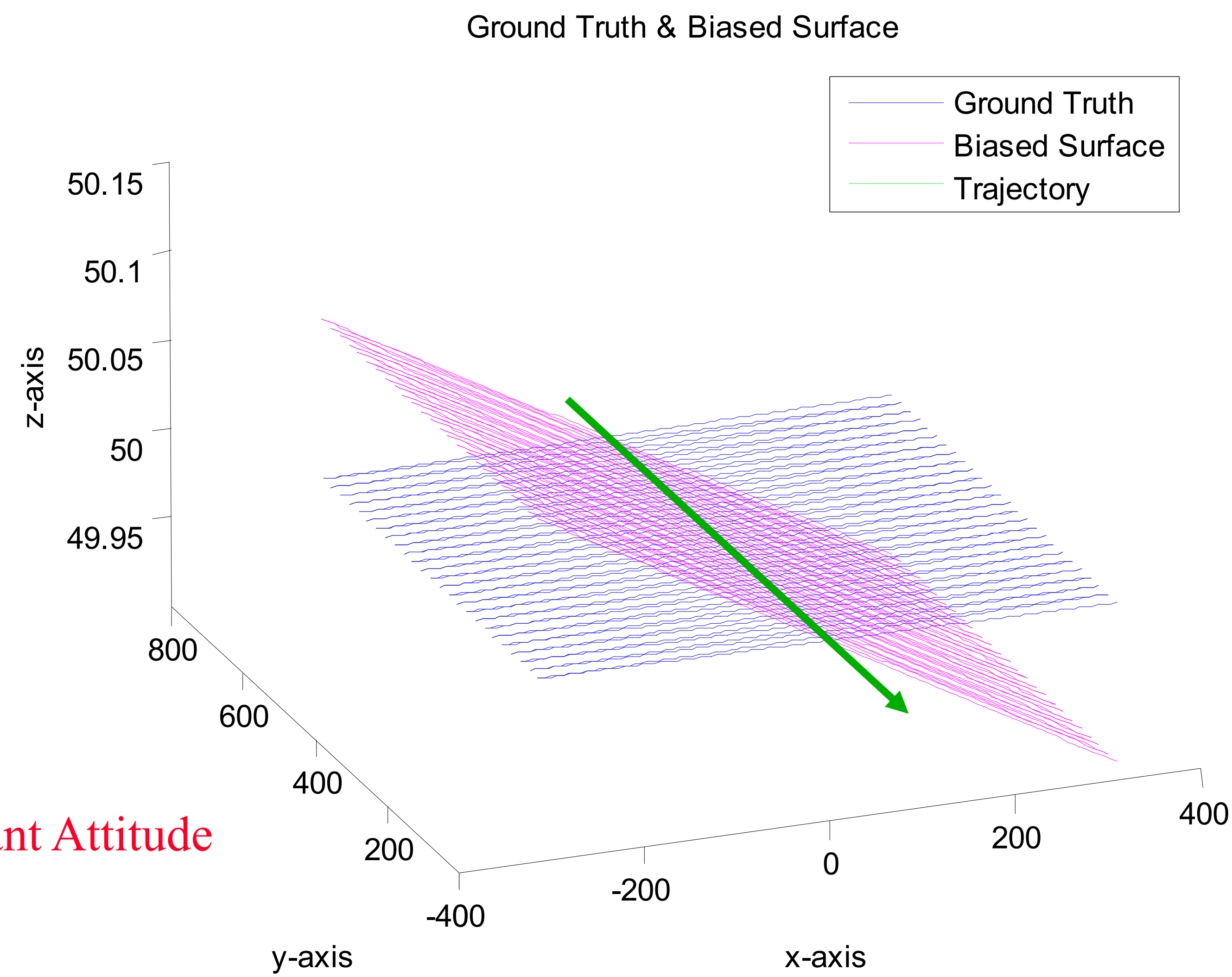




The mirror angle scale bias affects the planimetric component across the flight direction (X-Axis in this example) and the height component.

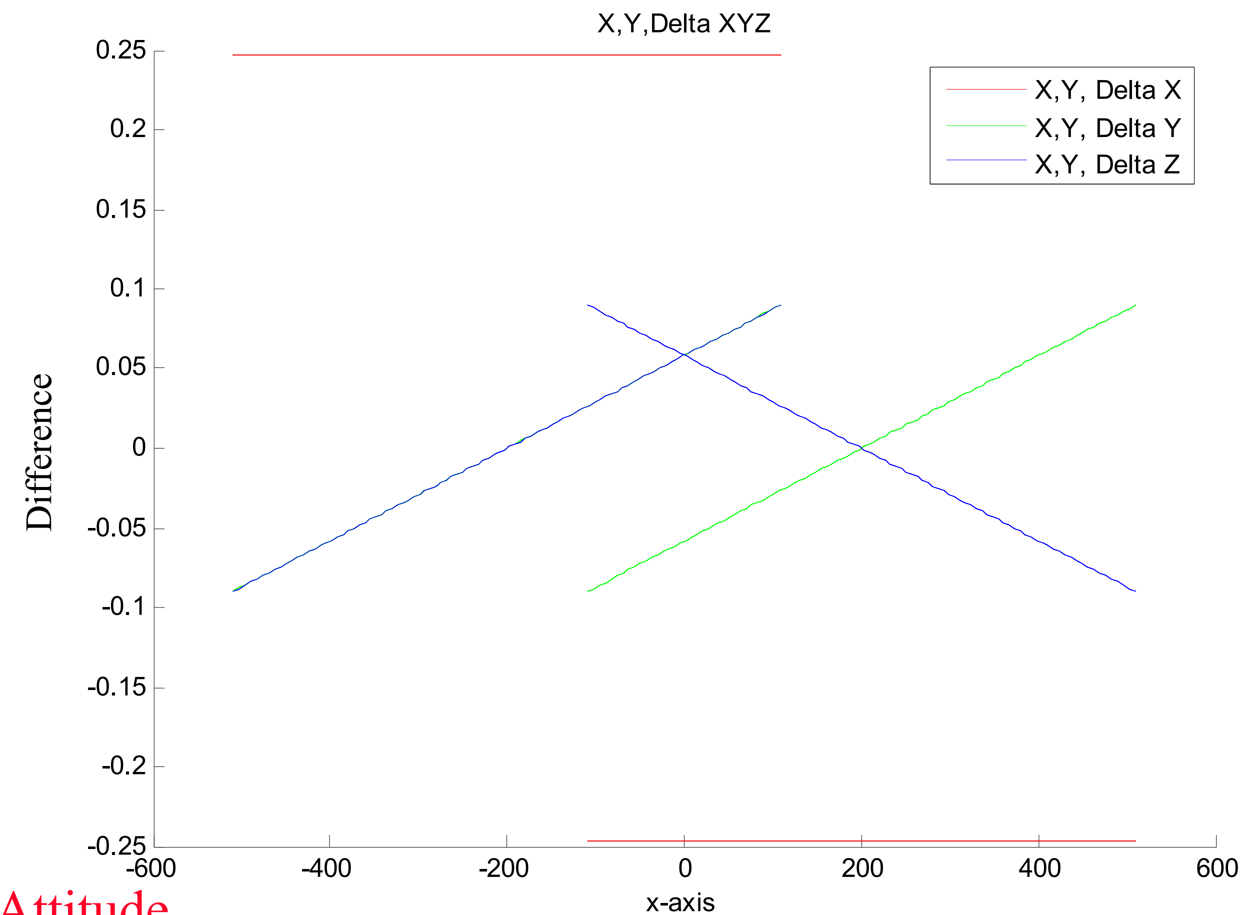
	Flying Height	Flying Direction	Look Angle
Mirror Angle Scale	• Effect is dependent on the flying height.	• Planimetric effect across the flight direction and vertical effect are independent of the flying direction.	• Planimetric effect across the flight direction and vertical effect are dependent on the look angle.

Linear Scanner & Laser Beam Angular Bias



Constant Attitude

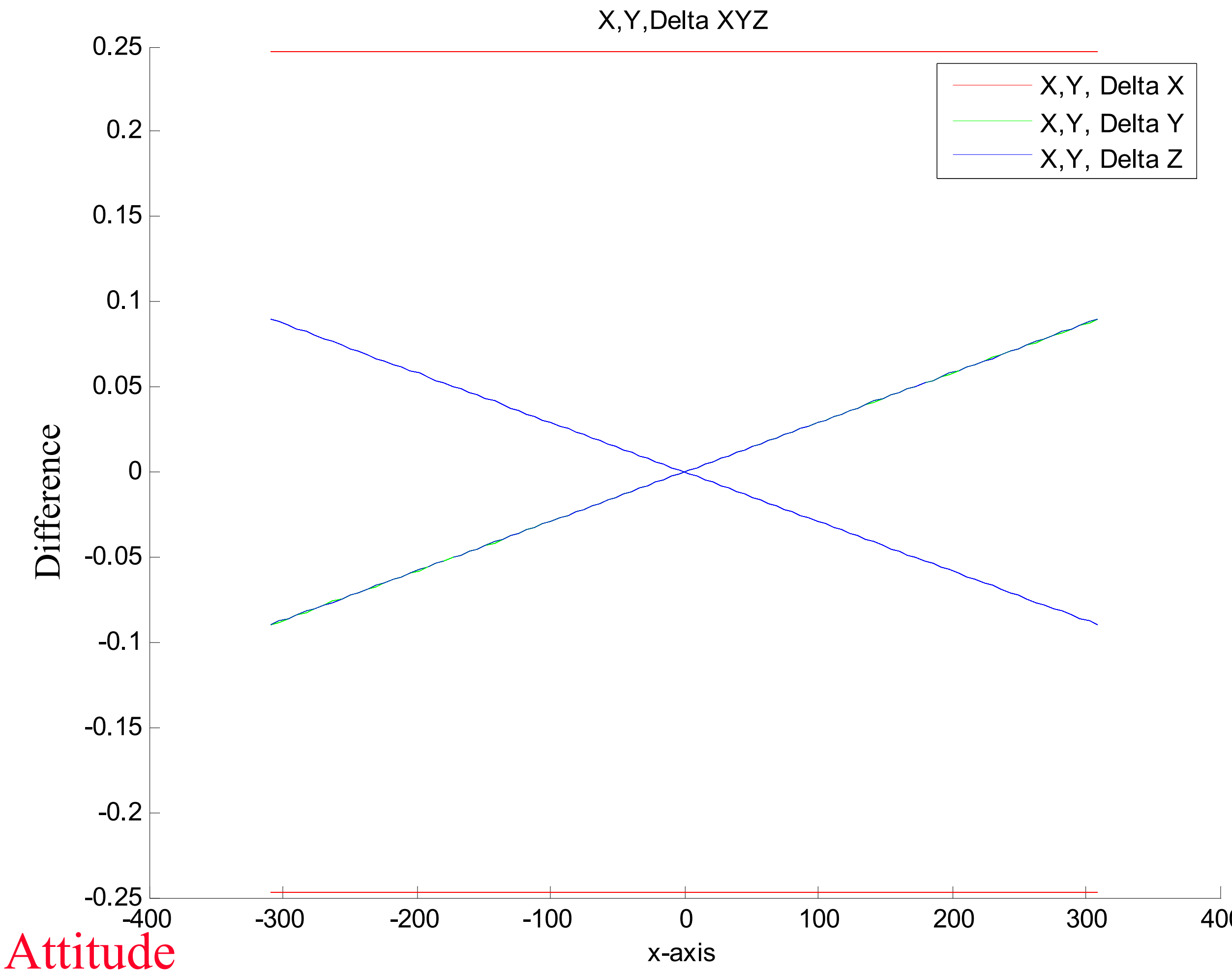
Linear Scanner & Laser Beam Angular Bias



Constant Attitude

- Opposite Flight Directions & 30% Overlap
- Delta Y is independent of the flight direction.

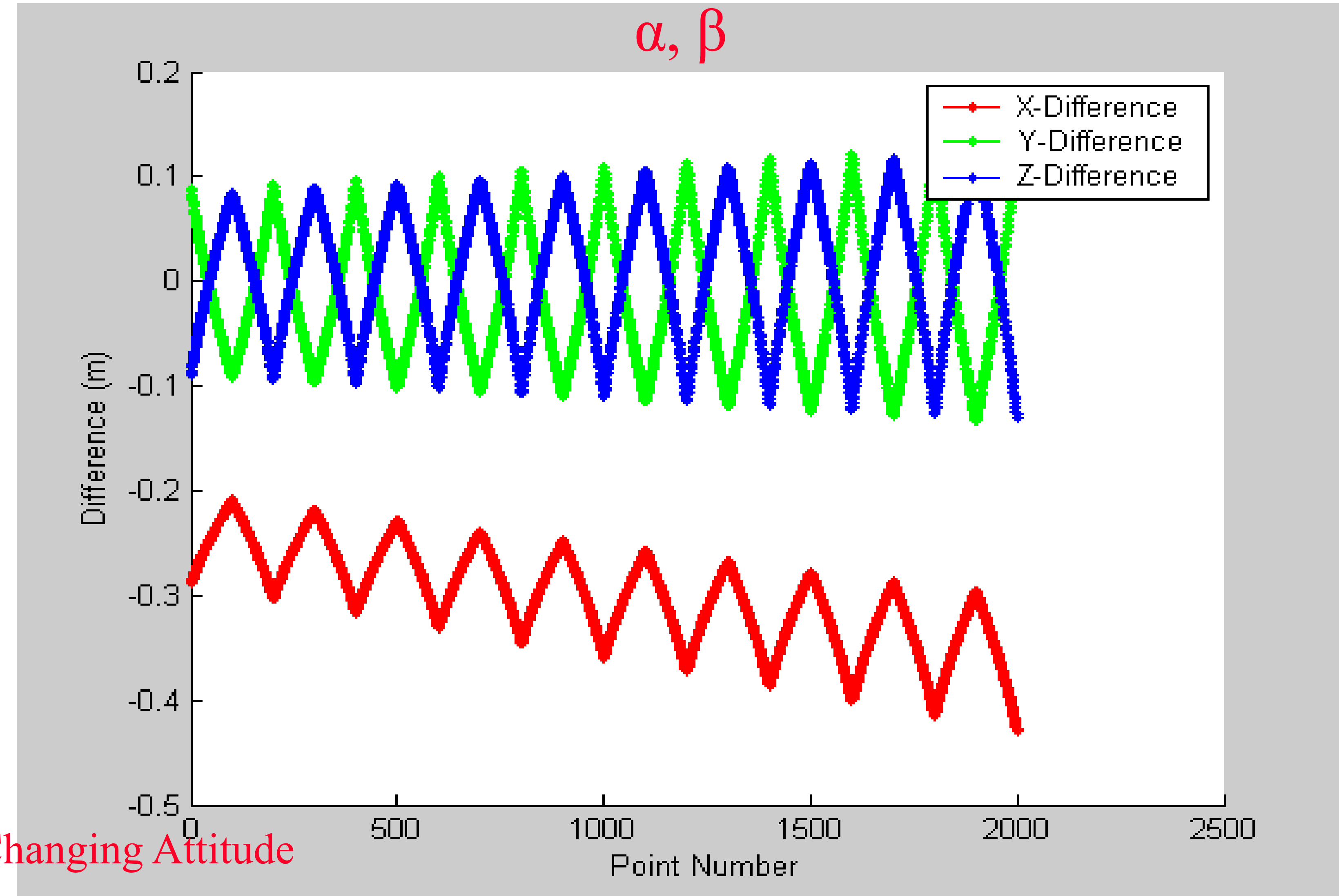
Linear Scanner & Laser Beam Angular Bias



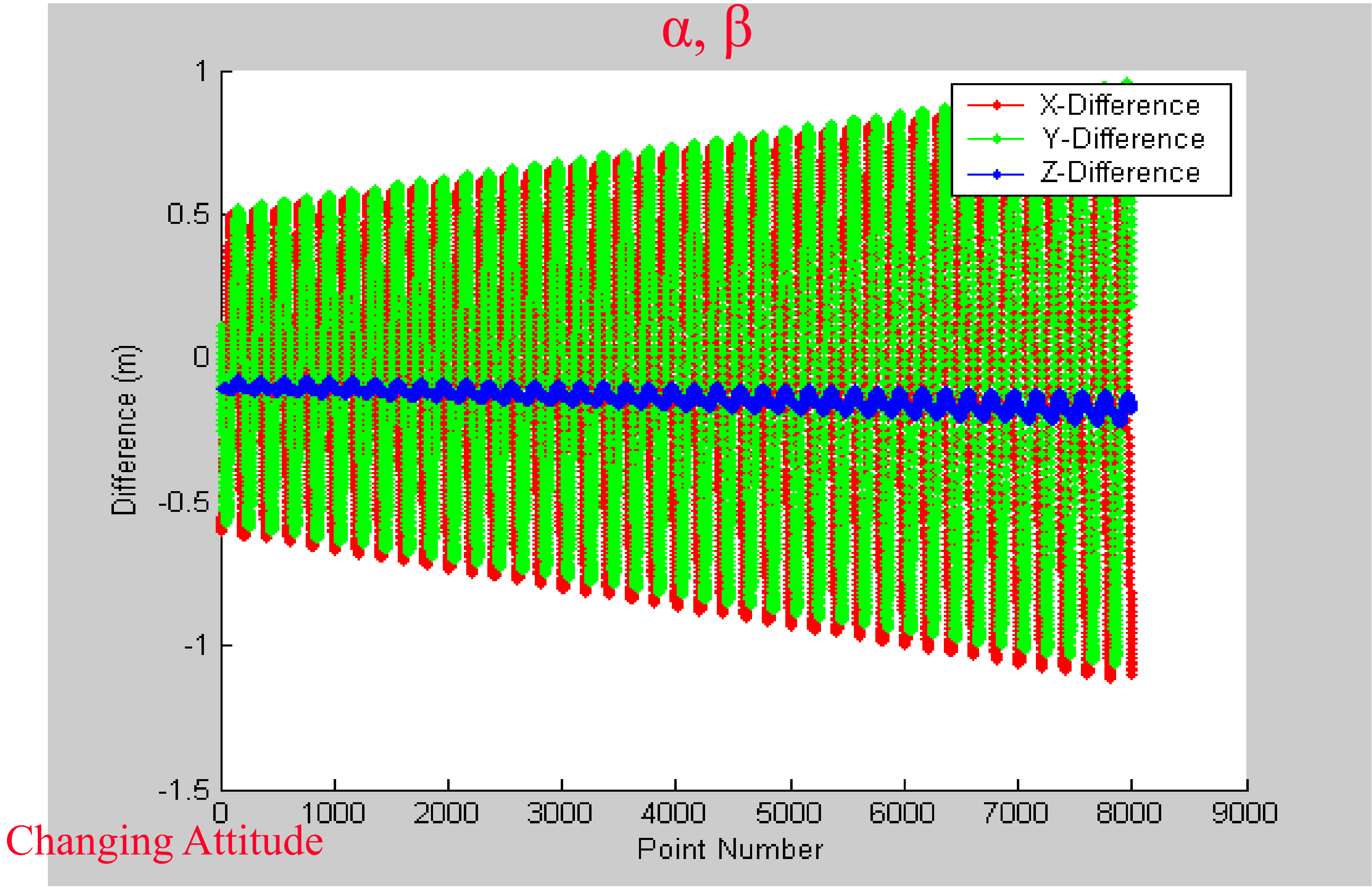
Constant Attitude

- Opposite Flight Directions & 100% Overlap
- Delta Y is independent of the flight direction.

Linear Scanner & Laser Beam Angular Bias



Elliptical Scanner & Laser Beam Angular Bias



Error Sources: Systematic Biases

- As expected, systematic biases will lead to systematic errors in the derived point cloud.
- Diagnostic hints:
  - Lever-arm offset error:
    - Constant shift in the object space assuming constant attitude
    - Independent of the system parameters (height & look angle)
    - Planimetric effects depend on flight direction
  - Angular biases (attitude or mirror angles):
    - Planimetric coordinates are affected more than vertical coordinates.
    - Dependent on the system parameters (height & look angle, flight direction)
  - Range bias:
    - Mainly affects the vertical component
    - Independent of the system height and flight direction
    - Dependent on the system look angle

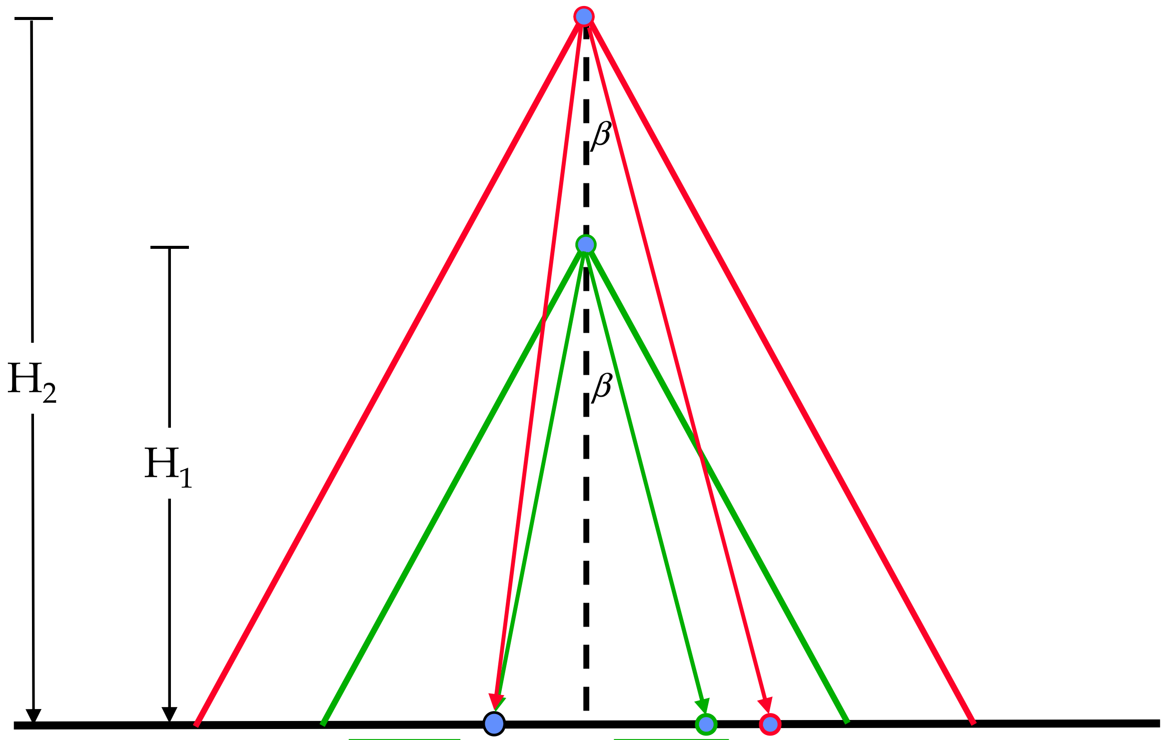
Error Sources: Systematic Biases

Mathematical Analysis of the LiDAR Equation

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	$\mathbf{0}$
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	$\mathbf{0}$
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\,\delta\Delta\omega$	$\mathbf{0}$
$\delta\Delta\varphi$	$\pm z\,\delta\Delta\varphi$	0	$-x\,\delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\,\delta\Delta\kappa$	$\mathbf{0}$
$\delta\Delta\rho$	$\mp sin(S\beta)\,\delta\Delta\rho$	0	$-\,cos(S\beta)\,\delta\Delta\rho$
δS	$\pm z\,\beta\,\delta S$	$\mathbf{0}$	$-x\,\beta\,\delta S$

Error Sources: Systematic Biases

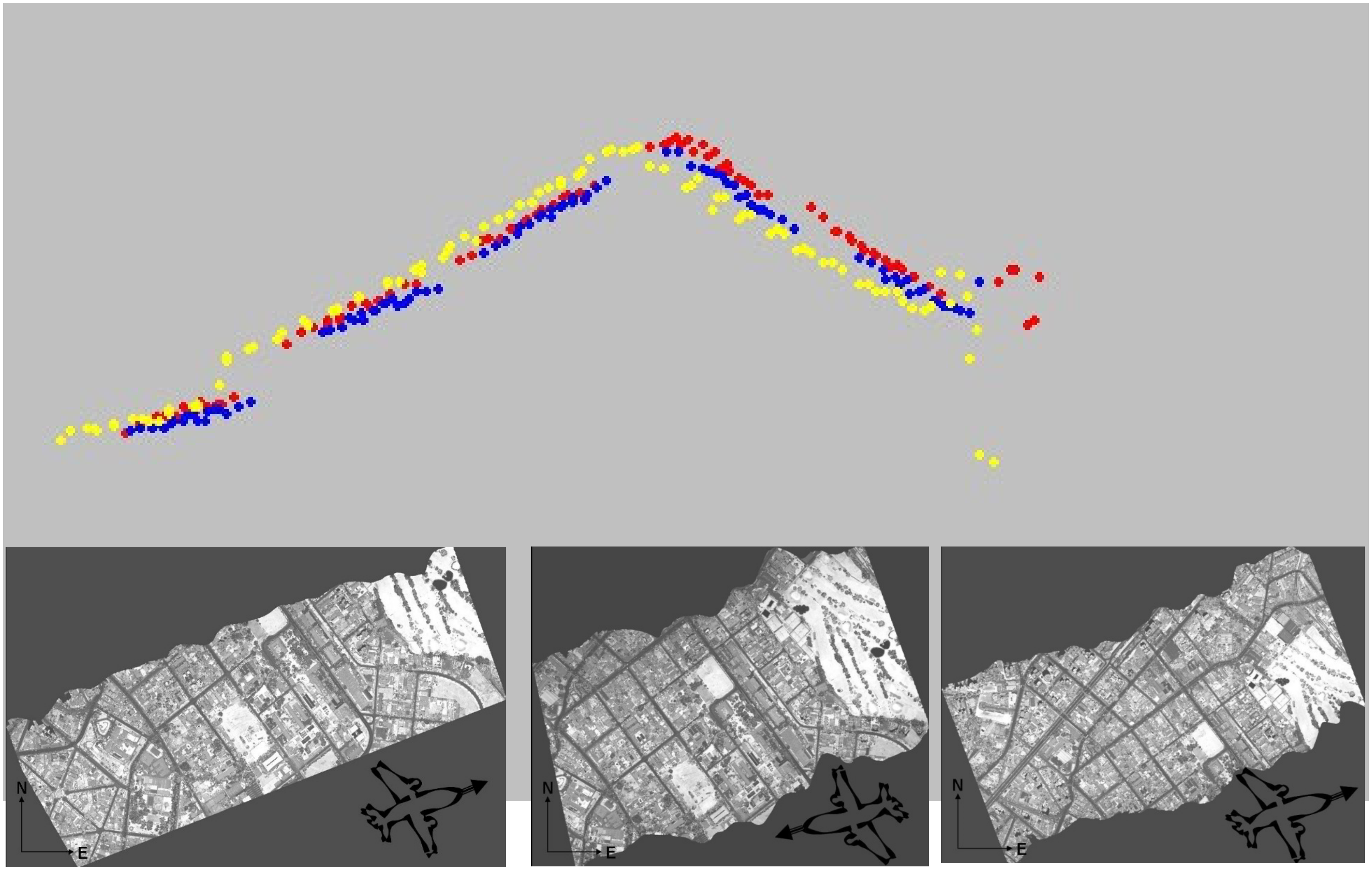
Mathematical Analysis of the LiDAR Equation



$$\delta XZ^{H_1}_{\delta\Delta\rho} \neq \delta XZ^{H_2}_{\delta\Delta\rho} \qquad \delta XZ^{H_1}_{\delta\Delta\rho} = \delta XZ^{H_2}_{\delta\Delta\rho}$$

Impact of the range bias on strips captured at different flying heights

Error Sources: Systematic Biases



Error Sources: Systematic Biases

	Flying Height	Flying Direction	Look Angle
Lever-Arm Offset Bias	Effect is independent of the Flying Height	Effect is dependent on the Flying Direction (Except ΔZ)	Effect is independent of the Look Angle
Boresight Angular Bias	Effect increases with the Flying Height	Effect is dependent on the Flying Direction	Effect changes with the Look Angle (Except ΔX)
Laser Beam Range Bias	Effect is independent of the Flying Height	Effect is independent of the Flying Direction	Effect changes with the Look Angle (Except ΔY)
Laser Beam Angular Bias	Effect increases with the Flying Height	Effect is dependent on the Flying Direction (Except ΔY)	Effect changes with the Look Angle (Except ΔX)

- Assumption:
  - Linear Scanner
  - Constant Attitude & Straight Line Trajectory
  - Flying Direction Parallel to the Y axis
  - Flat horizontal terrain

Error Sources: Random Errors

- The effect of random errors can be analyzed in one of two different ways:
  - Approach # I: **Simulation**
    - Simulated surface & trajectory → LiDAR measurements → Add noise → Reconstructed surface
    - Evaluate the difference between the reconstructed footprints and the simulated surface (i.e., ground truth)
  - Approach # II: **variance-covariance propagation**
    - Use the law of error propagation to evaluate the accuracy (noise level) of the derived point cloud as it is determined by the accuracy (noise level) in the LiDAR measurements

Error Sources: Random Errors

Approach II: Law of Error Propagation:

$$r_I^m = r_b^m(t) + R_b^m(t)\, r_{lu}^b + R_b^m(t)\, R_{lu}^b\, R_{lb}^{lu}(t)\, r_I^{lb}(t)$$

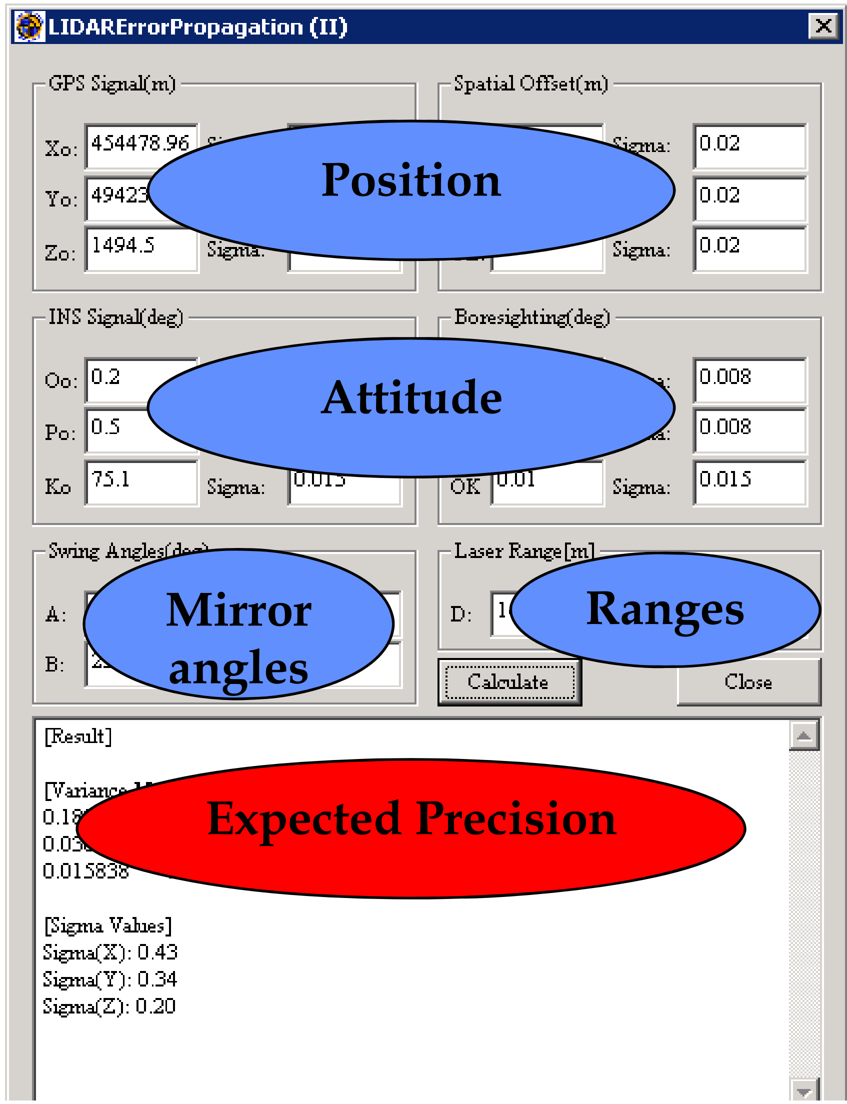
$$r_I^m = f(\vec{y})$$

$$\vec{y} = (r_b^m(t), R_b^m(t), \alpha(t), \beta(t), \rho(t))$$

$$B = {\partial f}/{\partial \vec{y}}$$

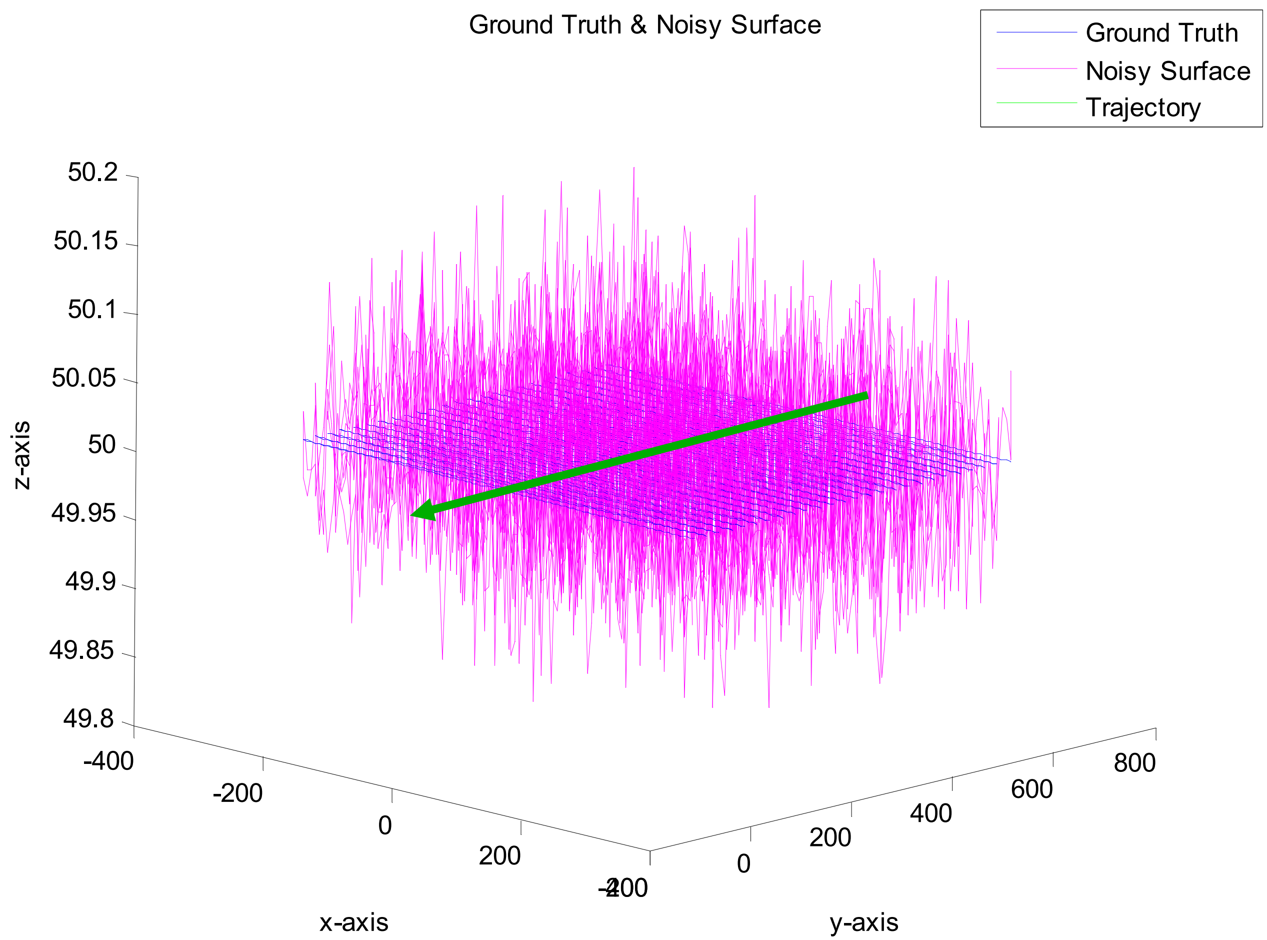
$$\sum r_I^m = B \sum \vec{y}\, B^T$$

LiDAR Error Propagation Calculator (# II)

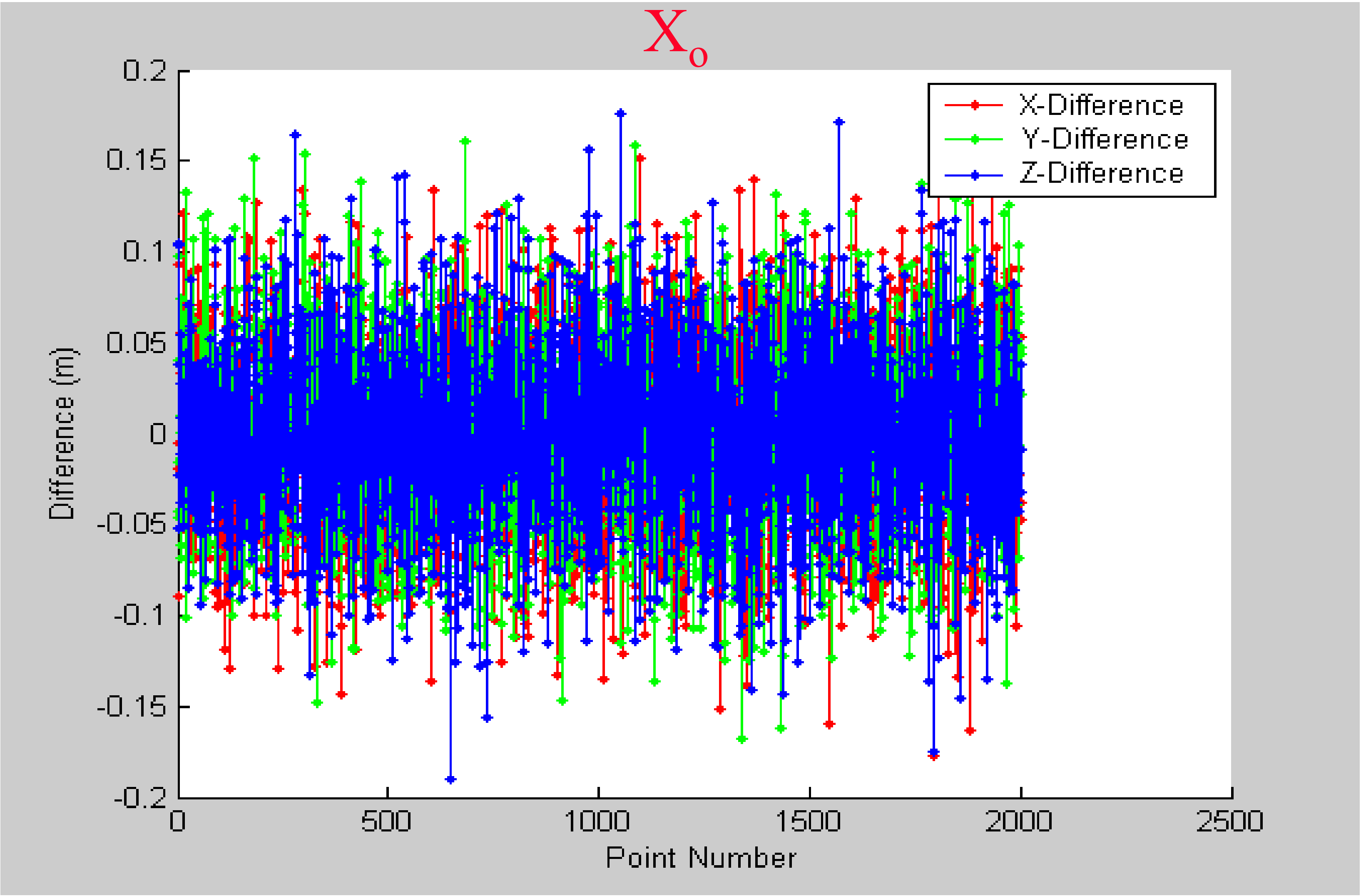


http://ilmbwww.gov.bc.ca/bmgs/pba/trim/specs

Linear Scanner & GNSS/INS-Pos. Noise (I)

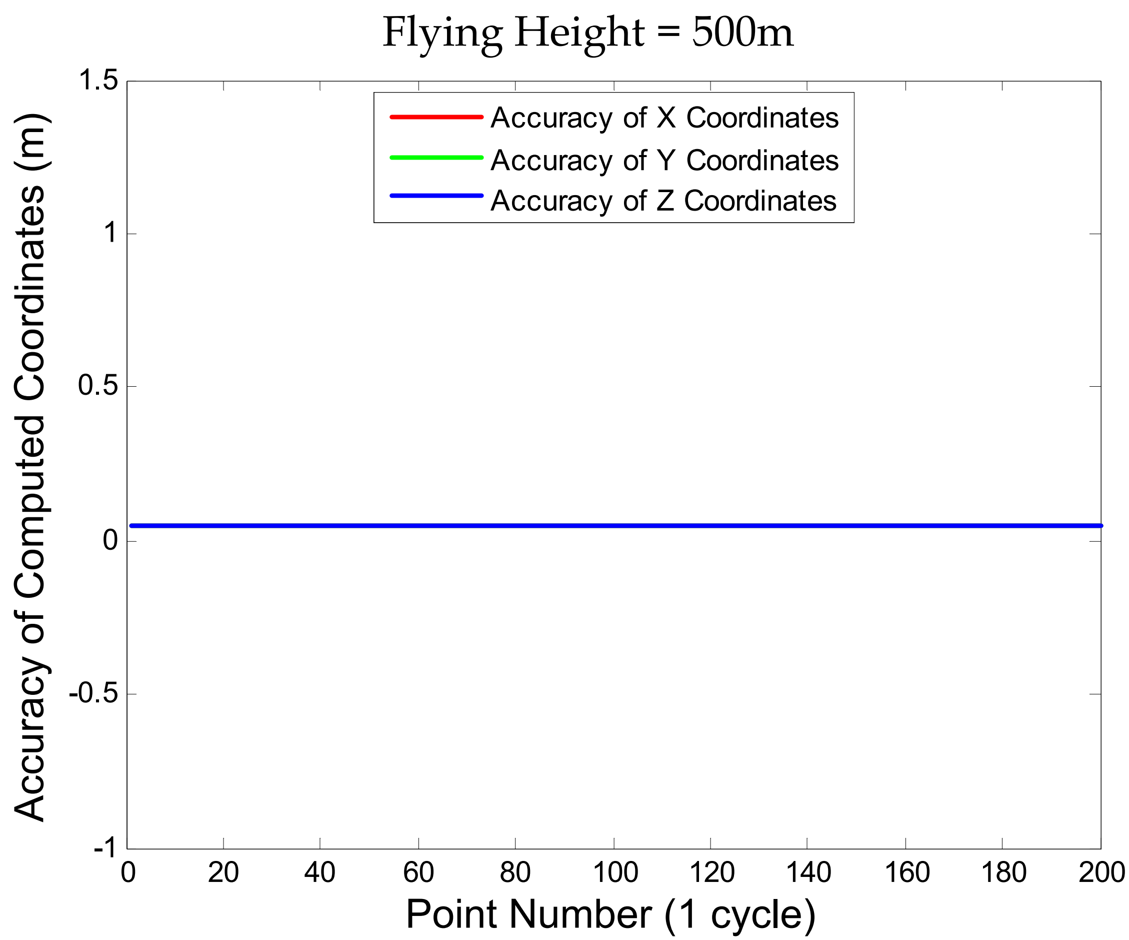


Linear Scanner & GNSS/INS-Pos. Noise (I)



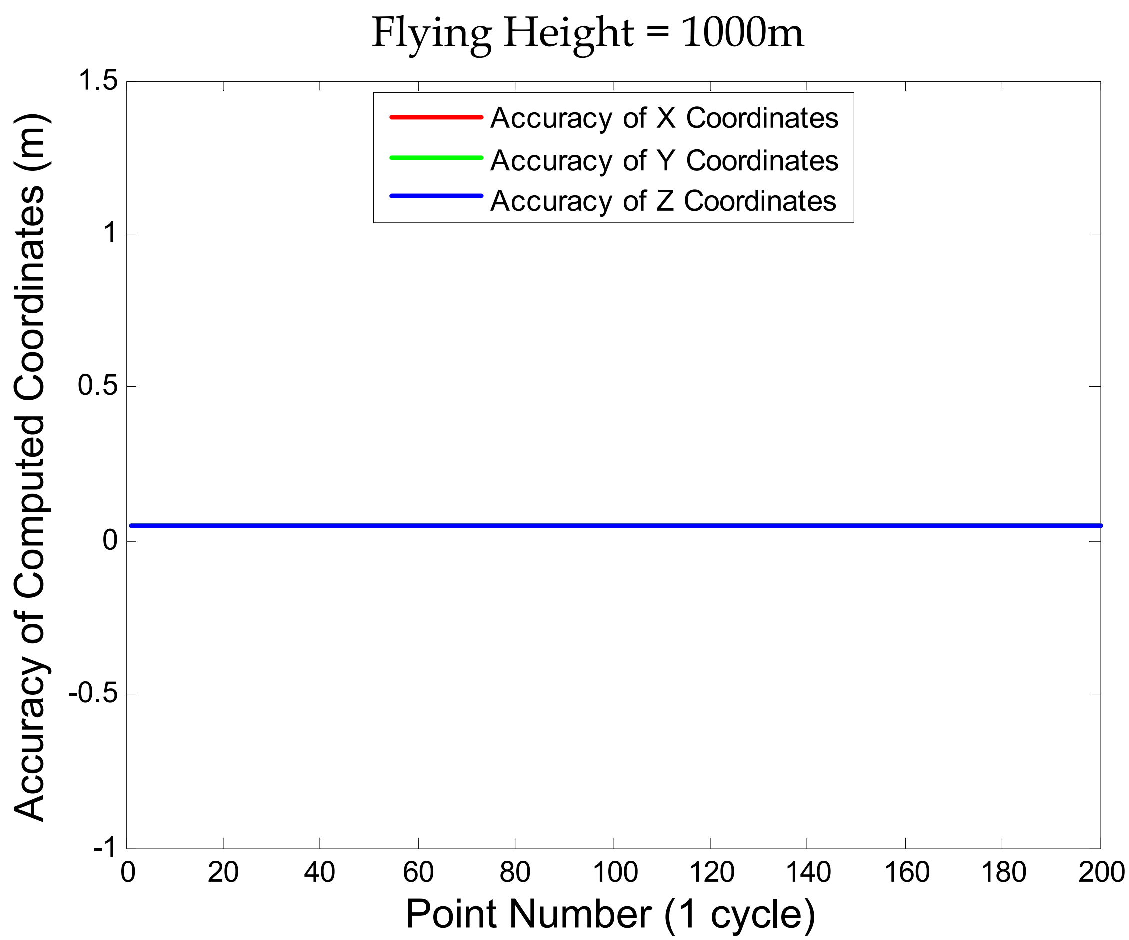
- Independent of the flying height and look angle

Linear Scanner & GNSS/INS-Pos. Noise (II)



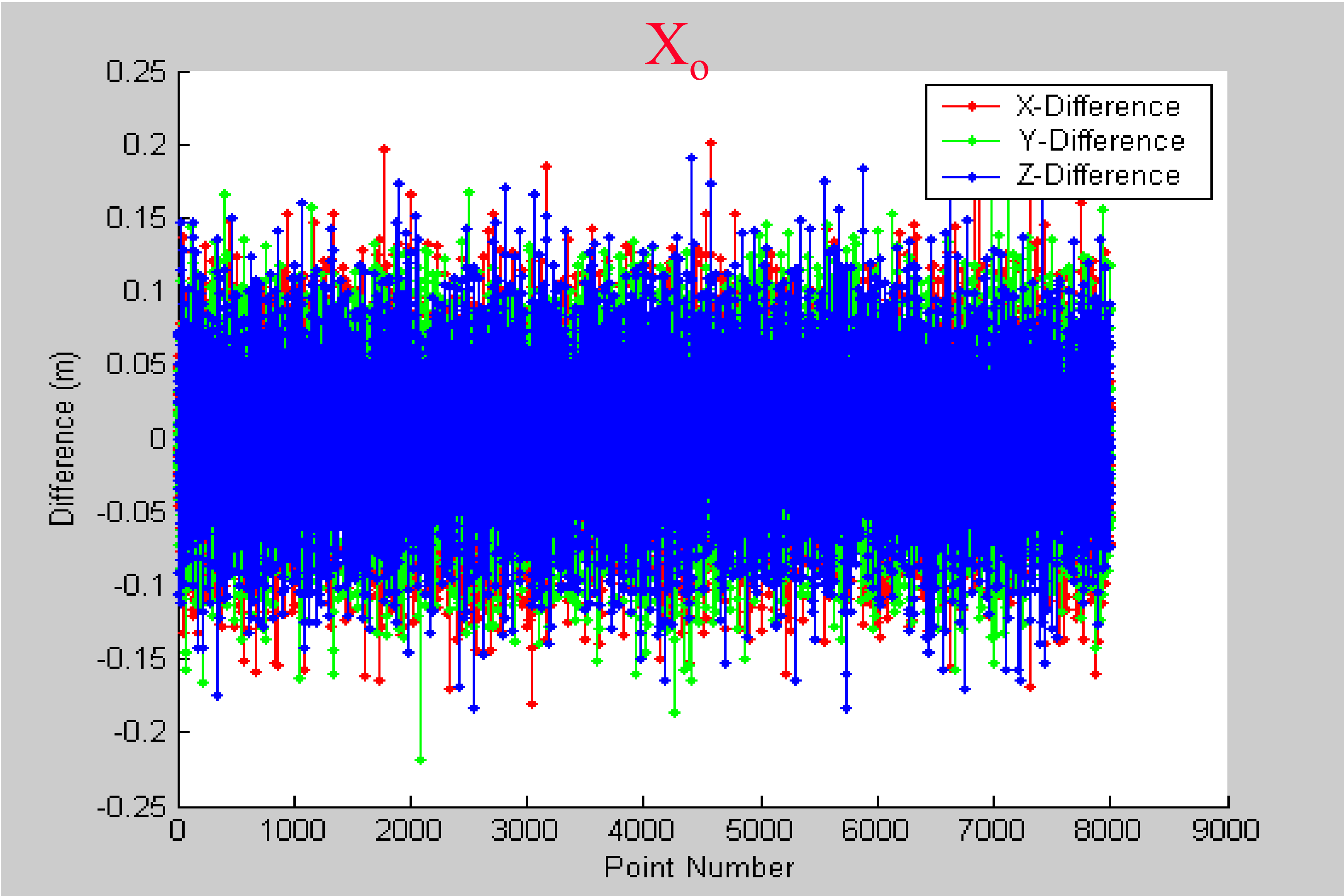
- Independent of the flying height and look angle

Linear Scanner & GNSS/INS-Pos. Noise (II)



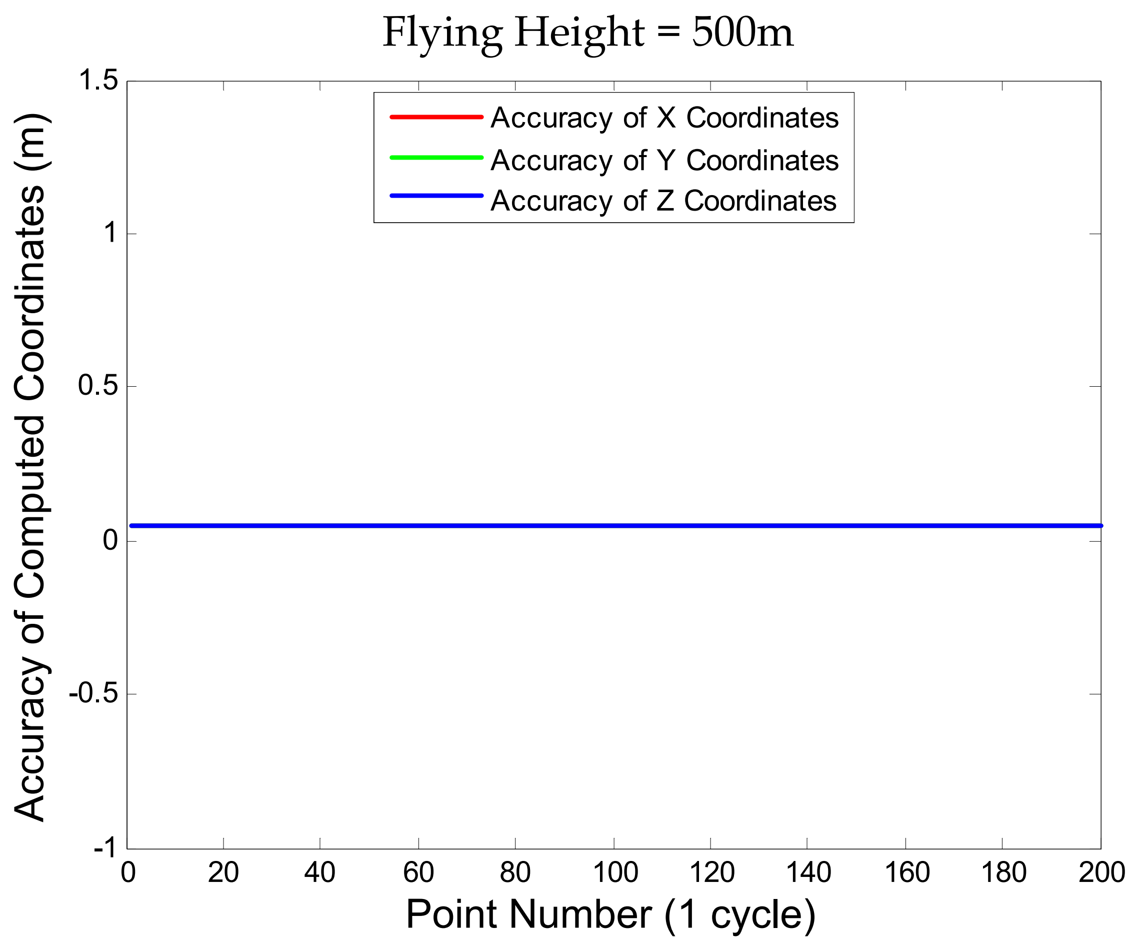
- Independent of the flying height and look angle

Elliptical Scanner & GNSS/INS-Pos. Noise (II)



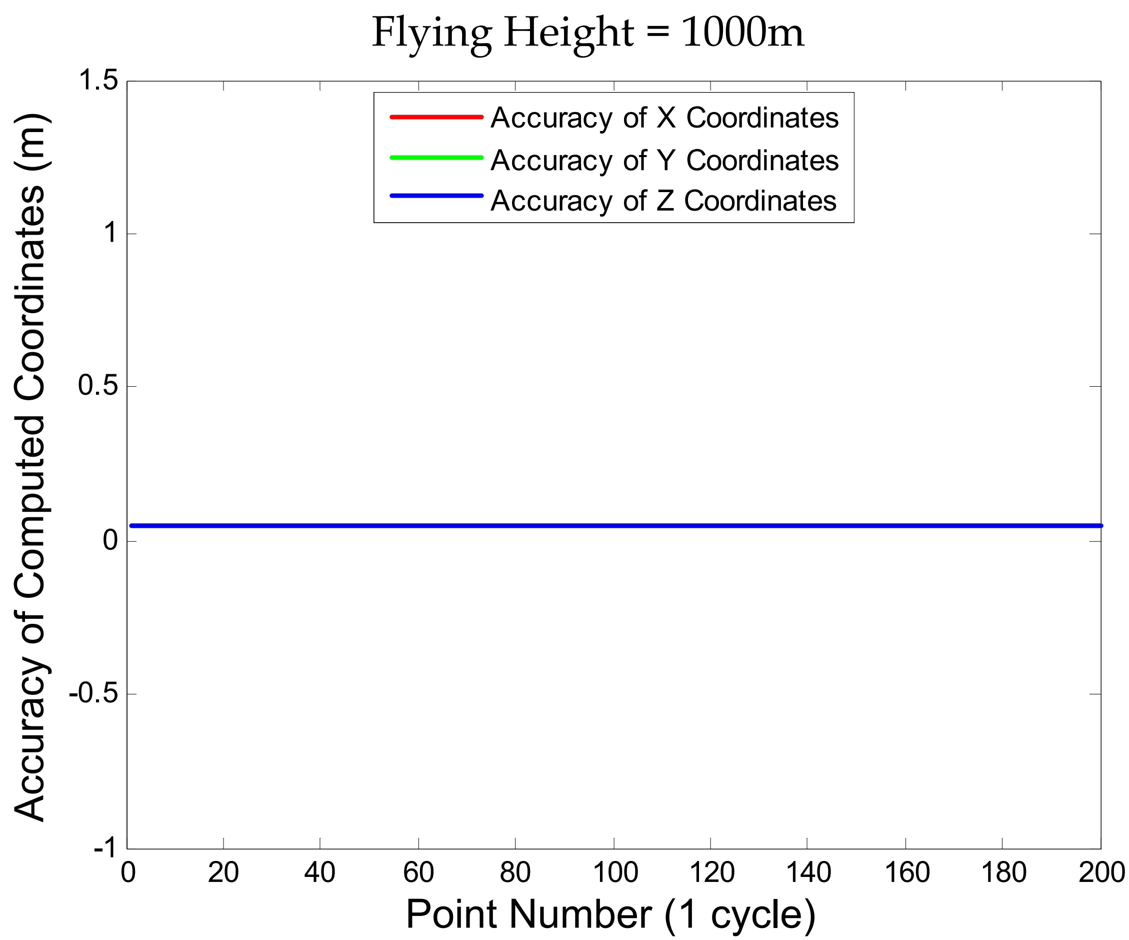
- Independent of the flying height and look angle

Elliptical Scanner & GNSS/INS-Pos. Noise (II)



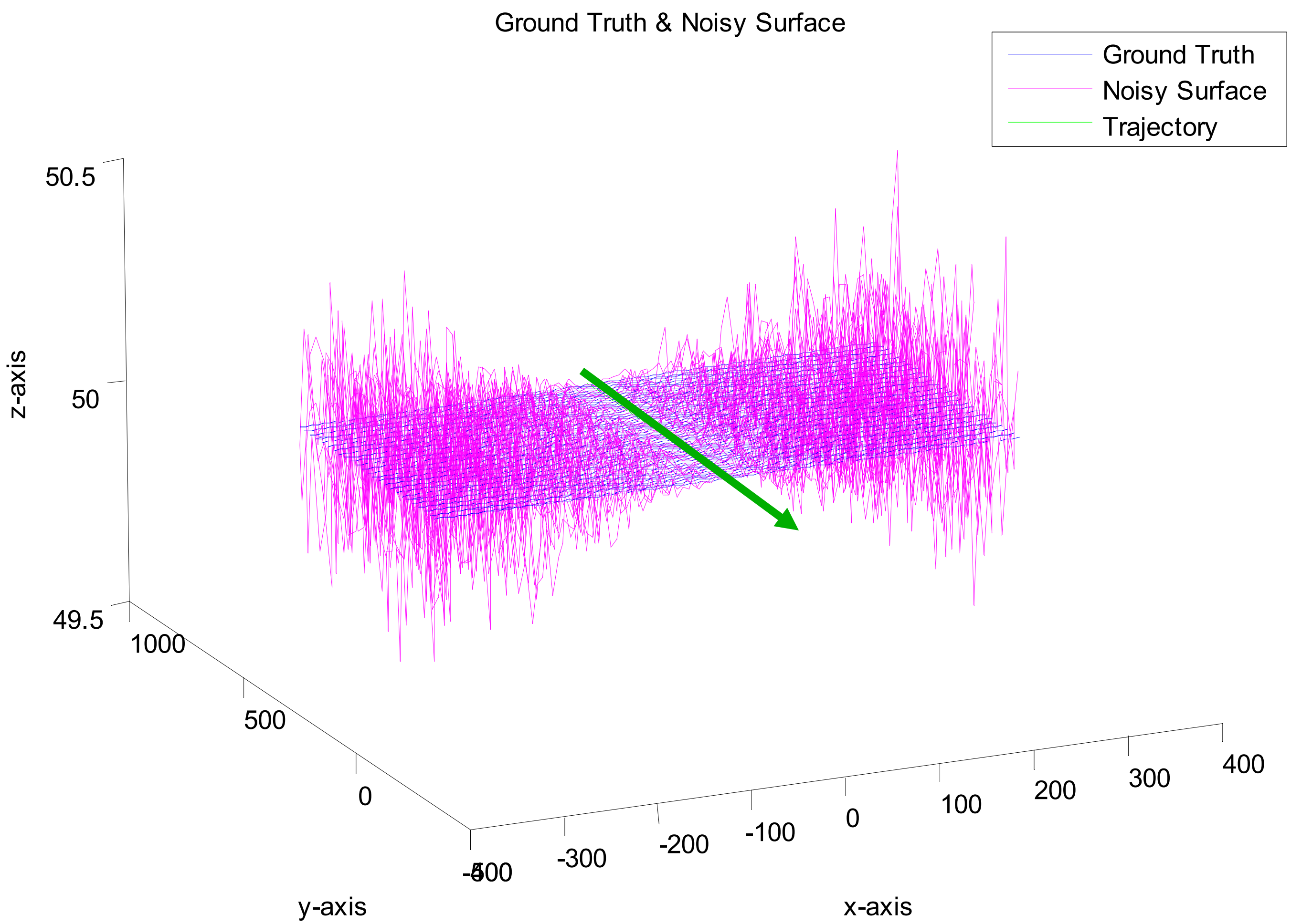
- Independent of the flying height and look angle

Elliptical Scanner & GNSS/INS-Pos. Noise (II)

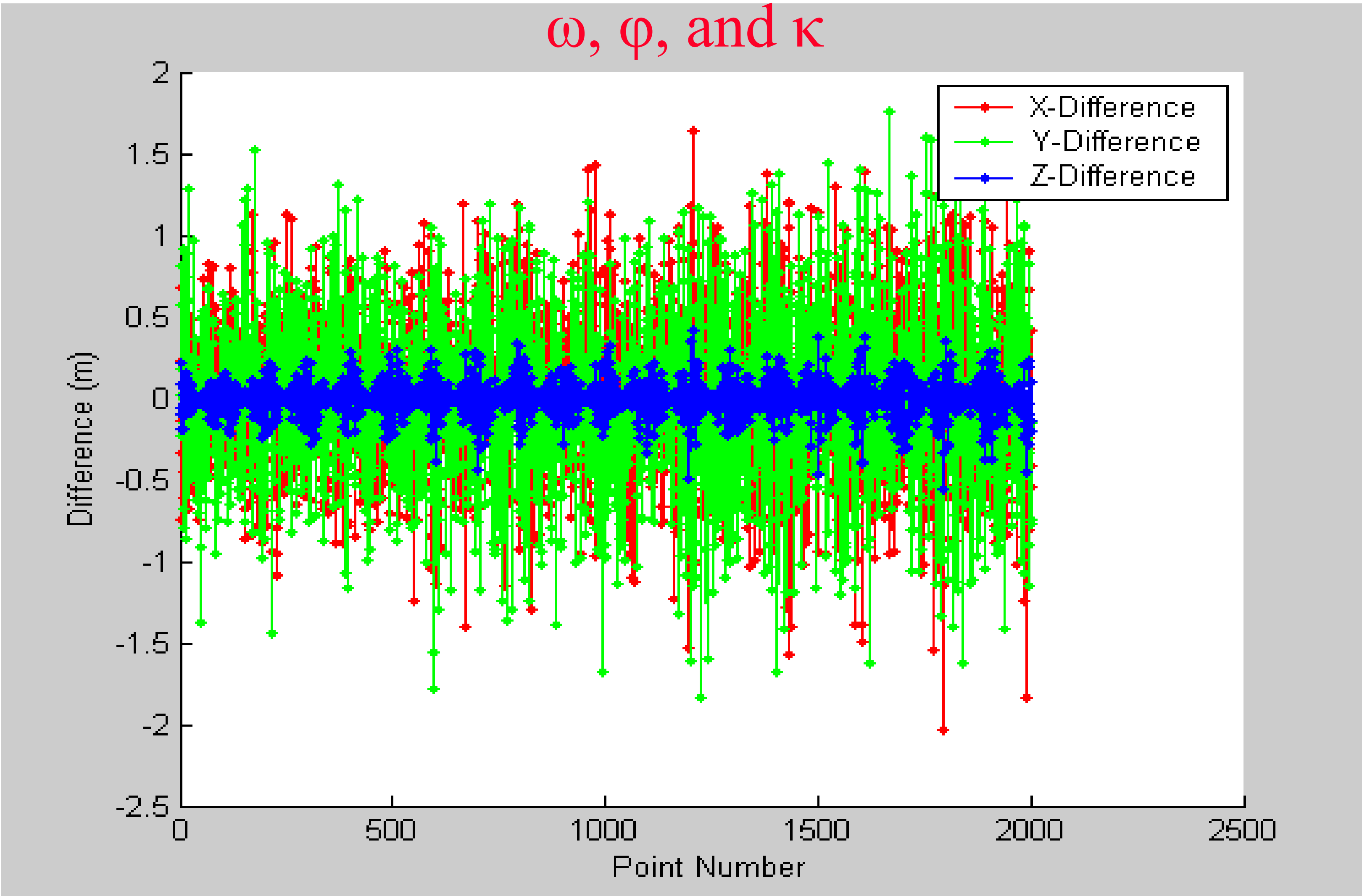


- Independent of the flying height and look angle

Linear Scanner & GNSS/INS-Attitude Noise (I)

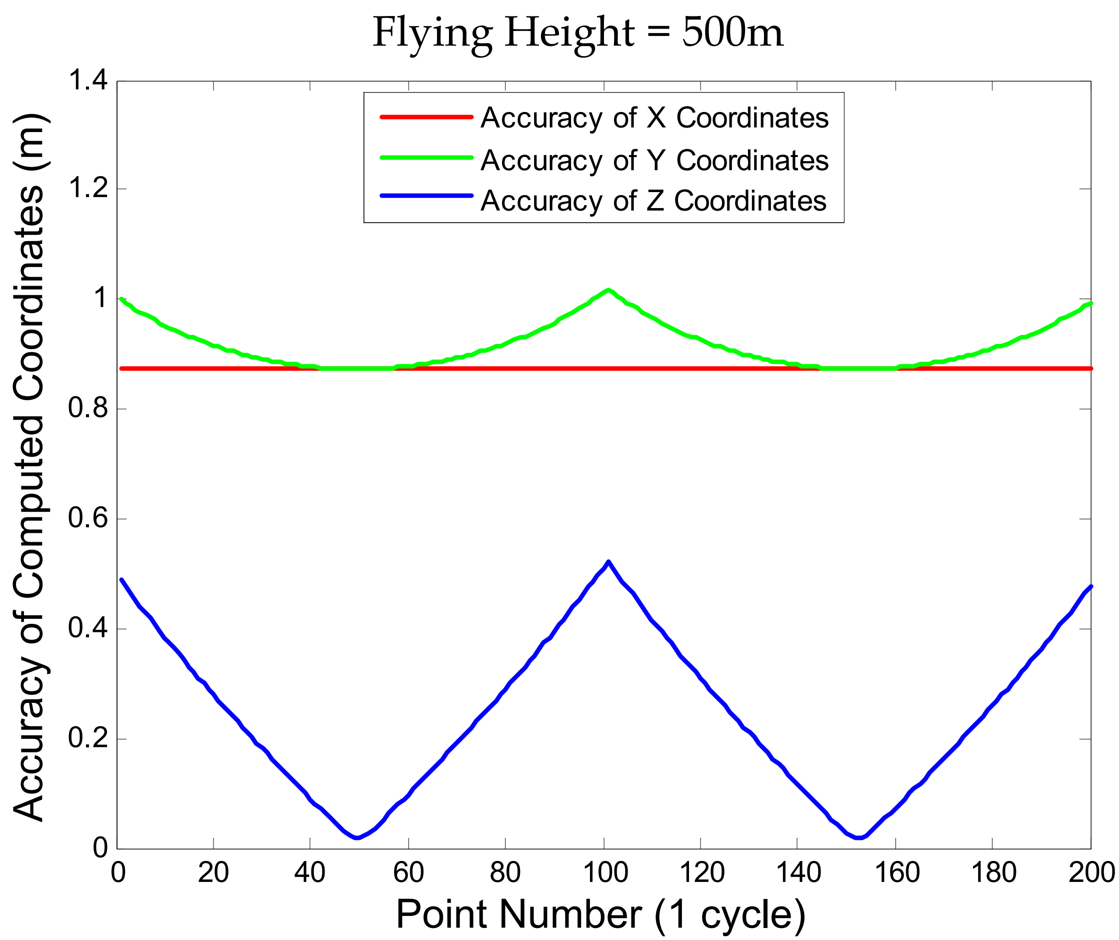


Linear Scanner & GNSS/INS-Attitude Noise (I)



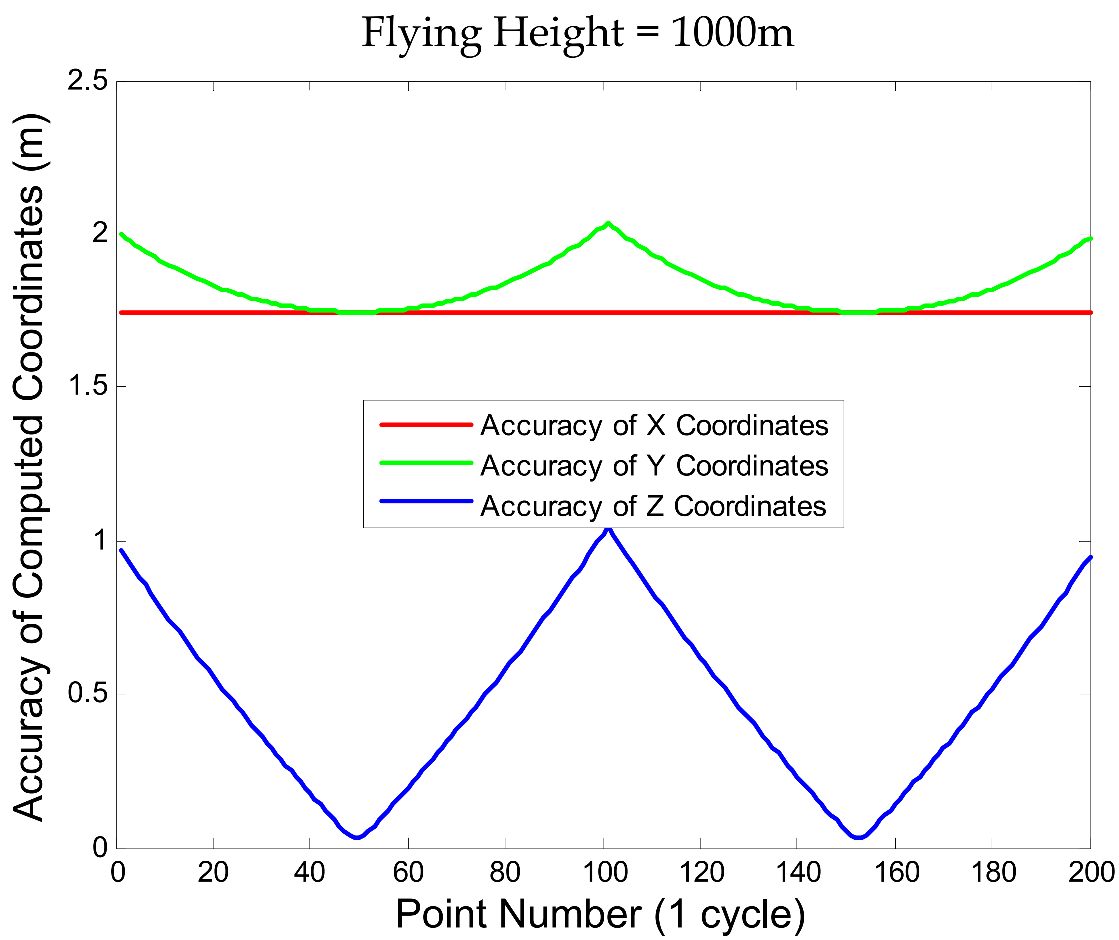
Propagates with the flying height

Linear Scanner & GNSS/INS-Attitude Noise (II)



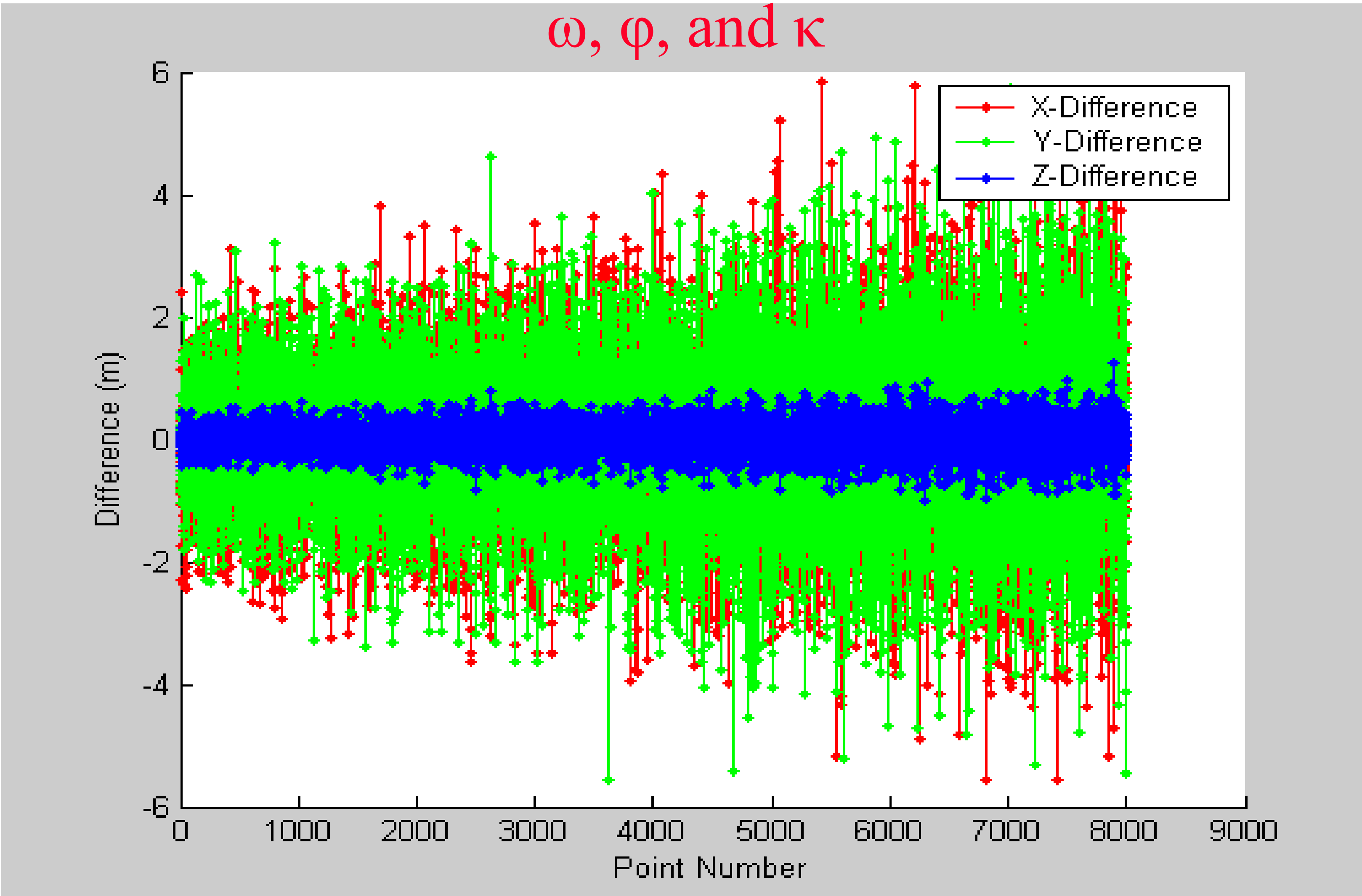
- Propagates with the flying height
- Dependent on the look angle

Linear Scanner & GNSS/INS-Attitude Noise (II)



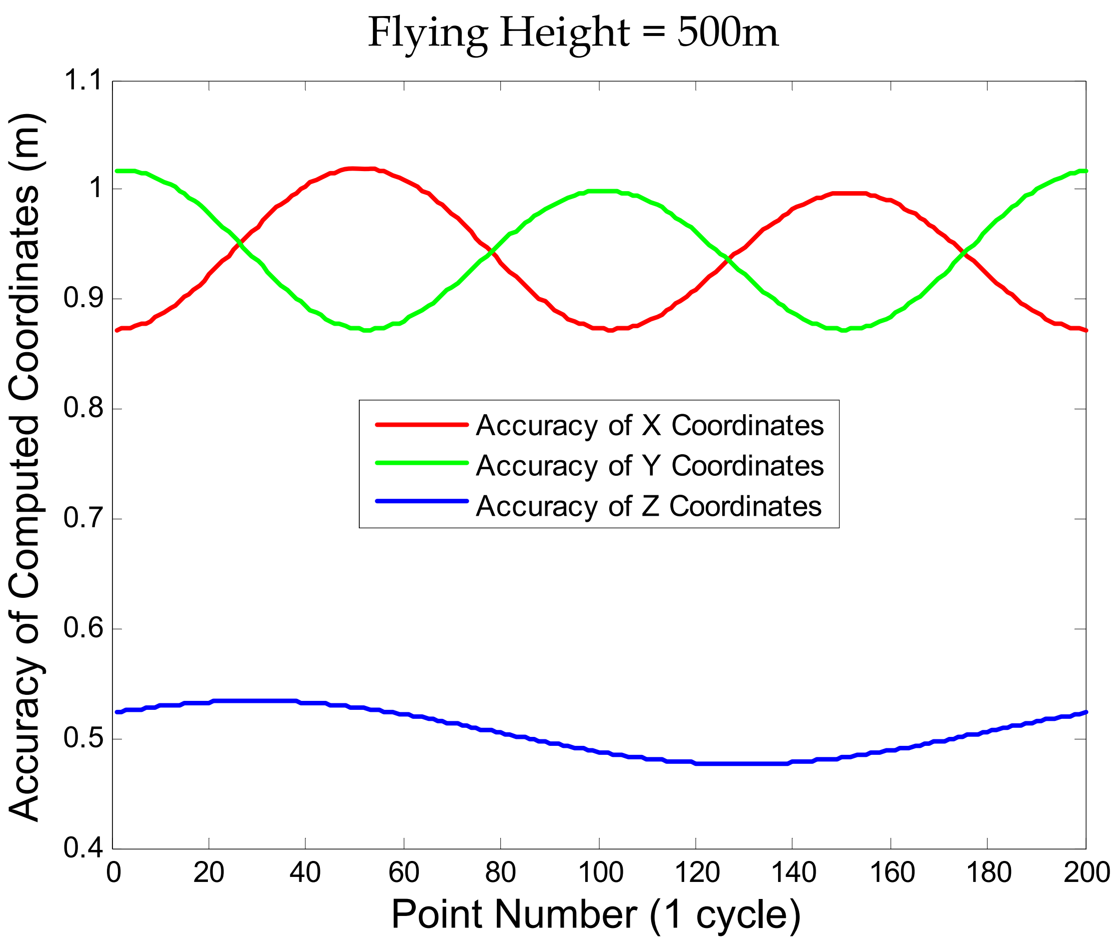
- Propagates with the flying height
- Dependent on the look angle

Ellipt. Scanner & GNSS/INS-Attitude Noise (I)



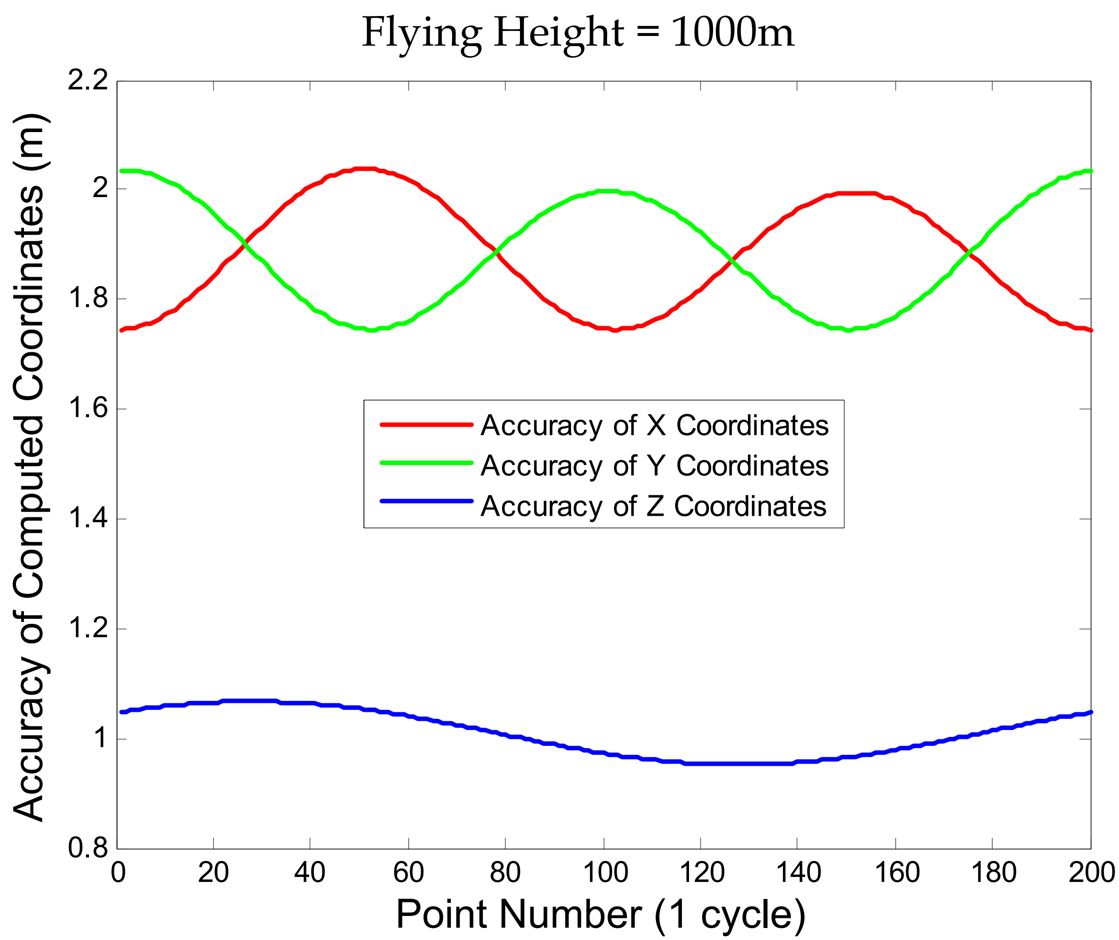
Propagates with the flying height

Ellipt. Scanner & GNSS/INS-Attitude Noise (II)



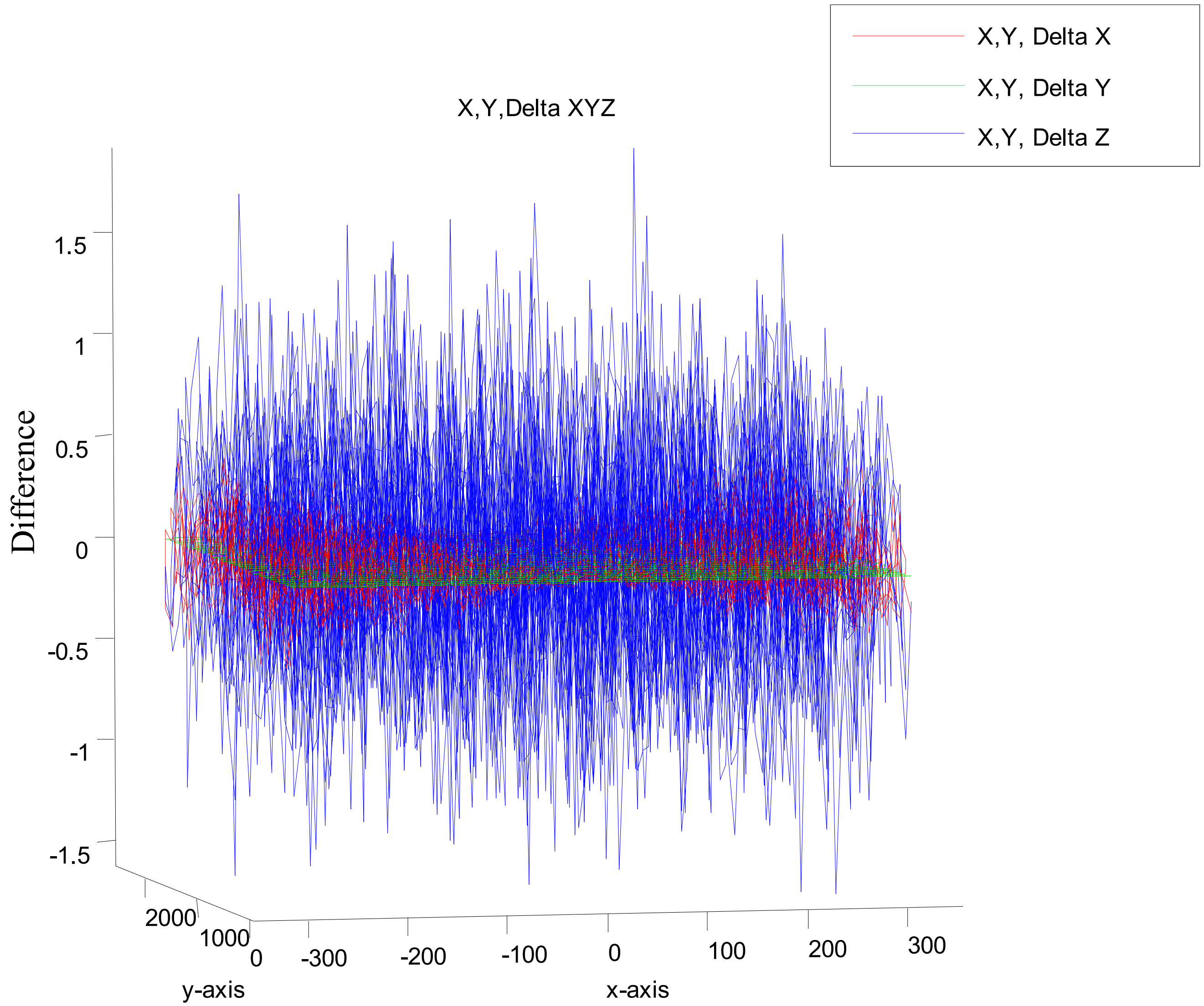
- Propagates with the flying height
- Dependent on the look angle

Ellipt. Scanner & GNSS/INS-Attitude Noise (II)

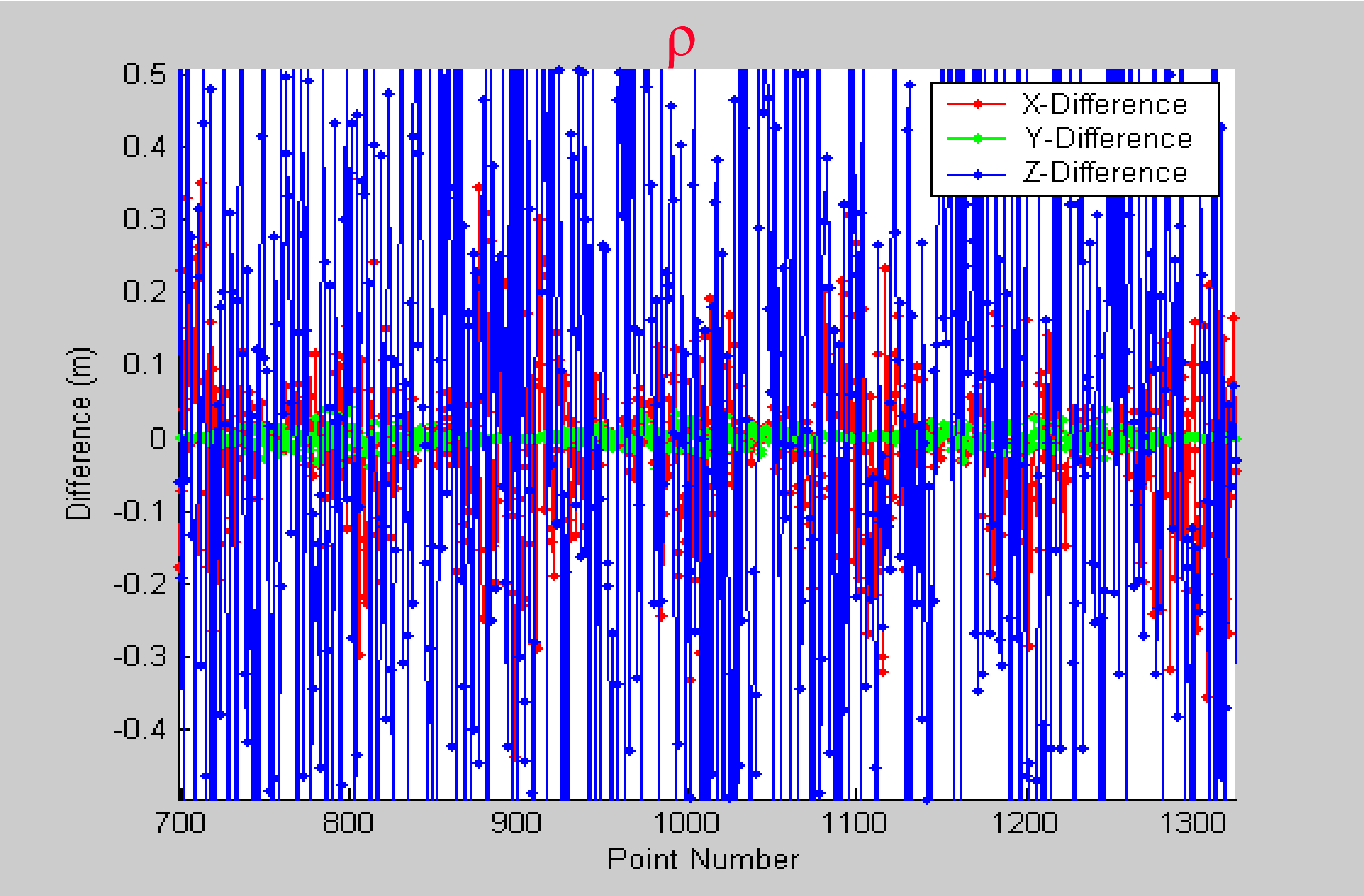


- Propagates with the flying height
- Dependent on the look angle

Linear Scanner & Laser Beam Range Noise (I)

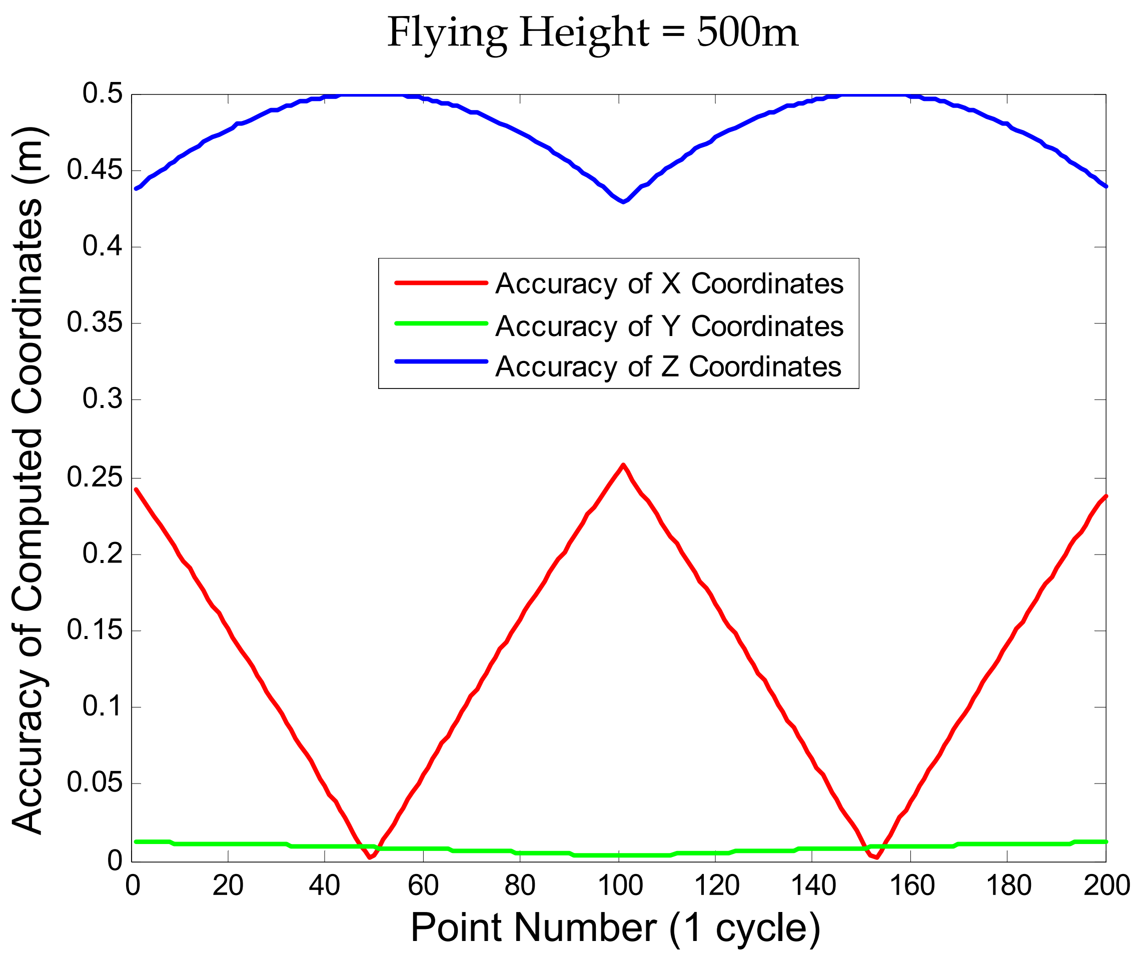


Linear Scanner & Laser Beam Range Noise (I)



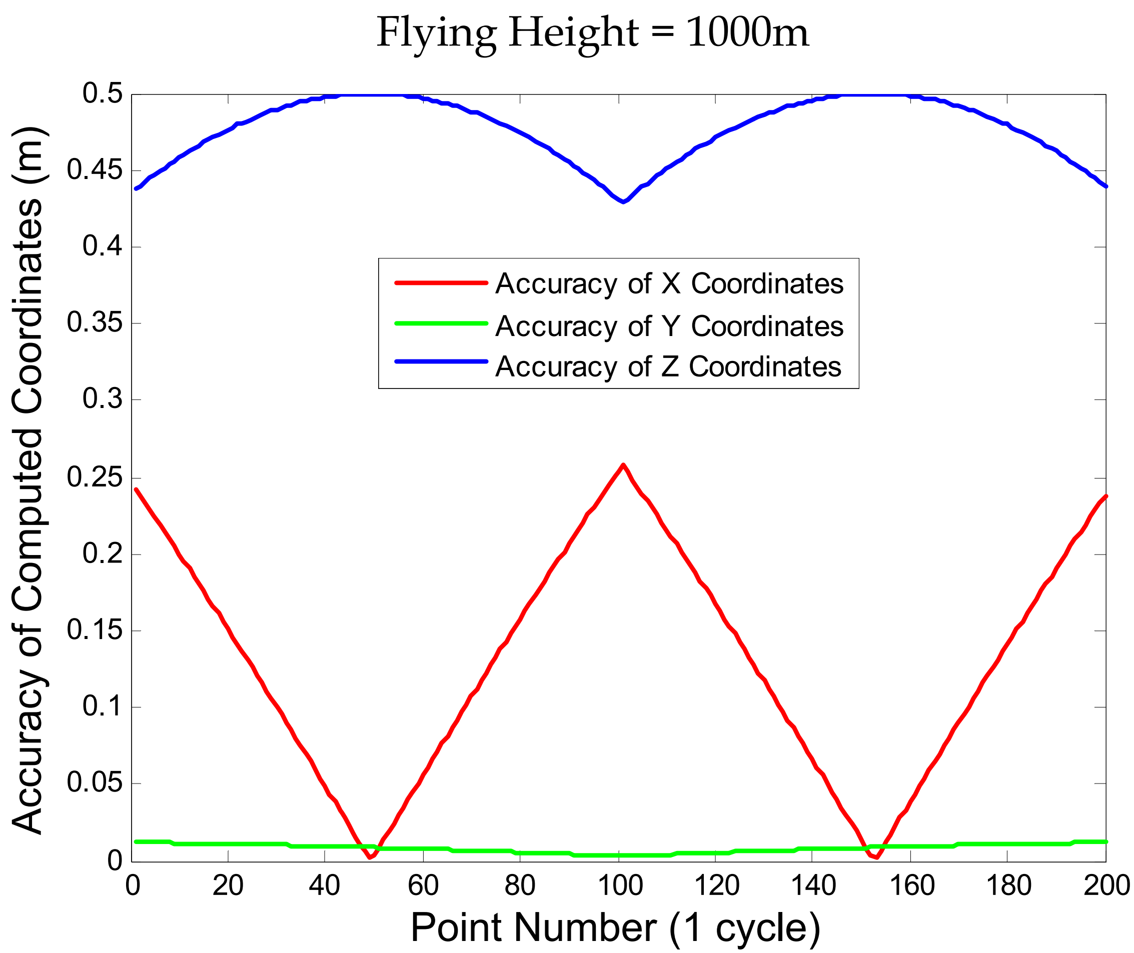
- Flight direction parallel to the Y-axis of the ground coordinate system

Linear Scanner & Laser Beam Range Noise (II)



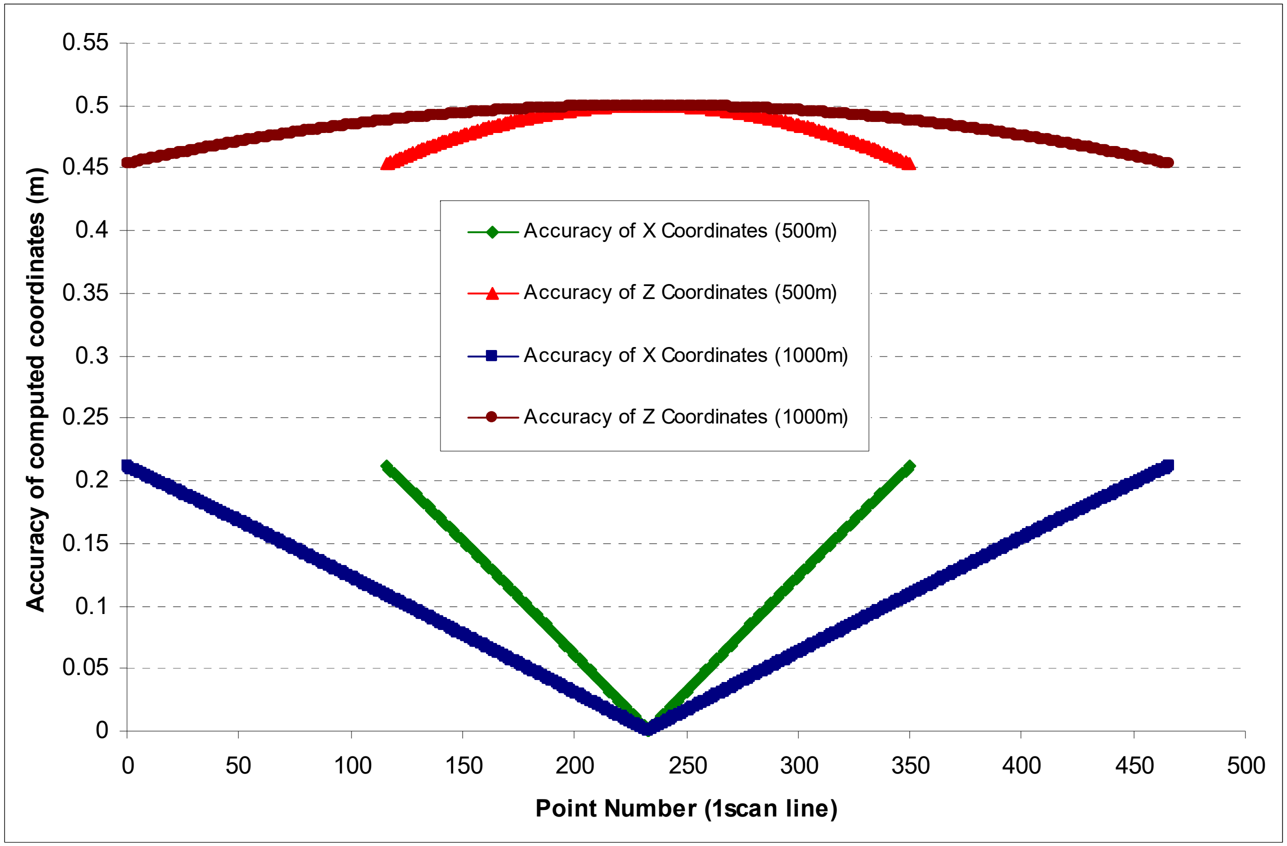
- Independent of the flying height
- Dependent on the look/scan angle

Linear Scanner & Laser Beam Range Noise (II)



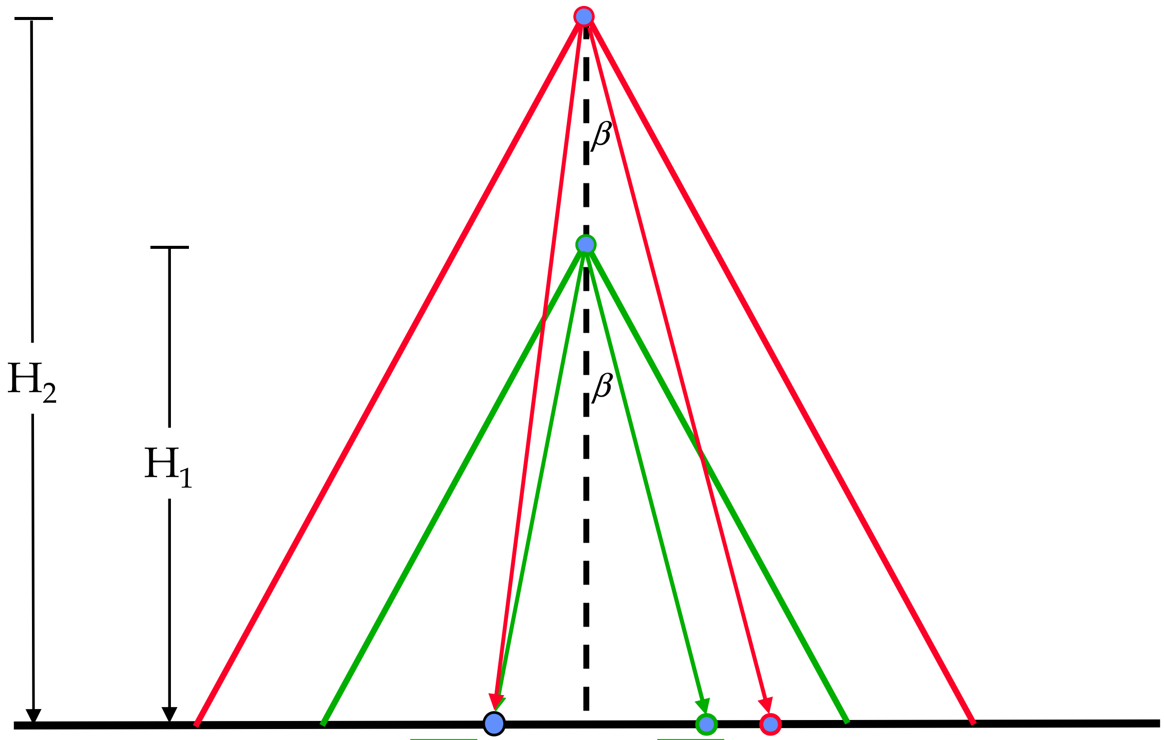
- Independent of the flying height
- Dependent on the look/scan angle

Linear Scanner & Laser Beam Range Noise (II)



- Independent of the flying height
- Dependent on the look/scan angle

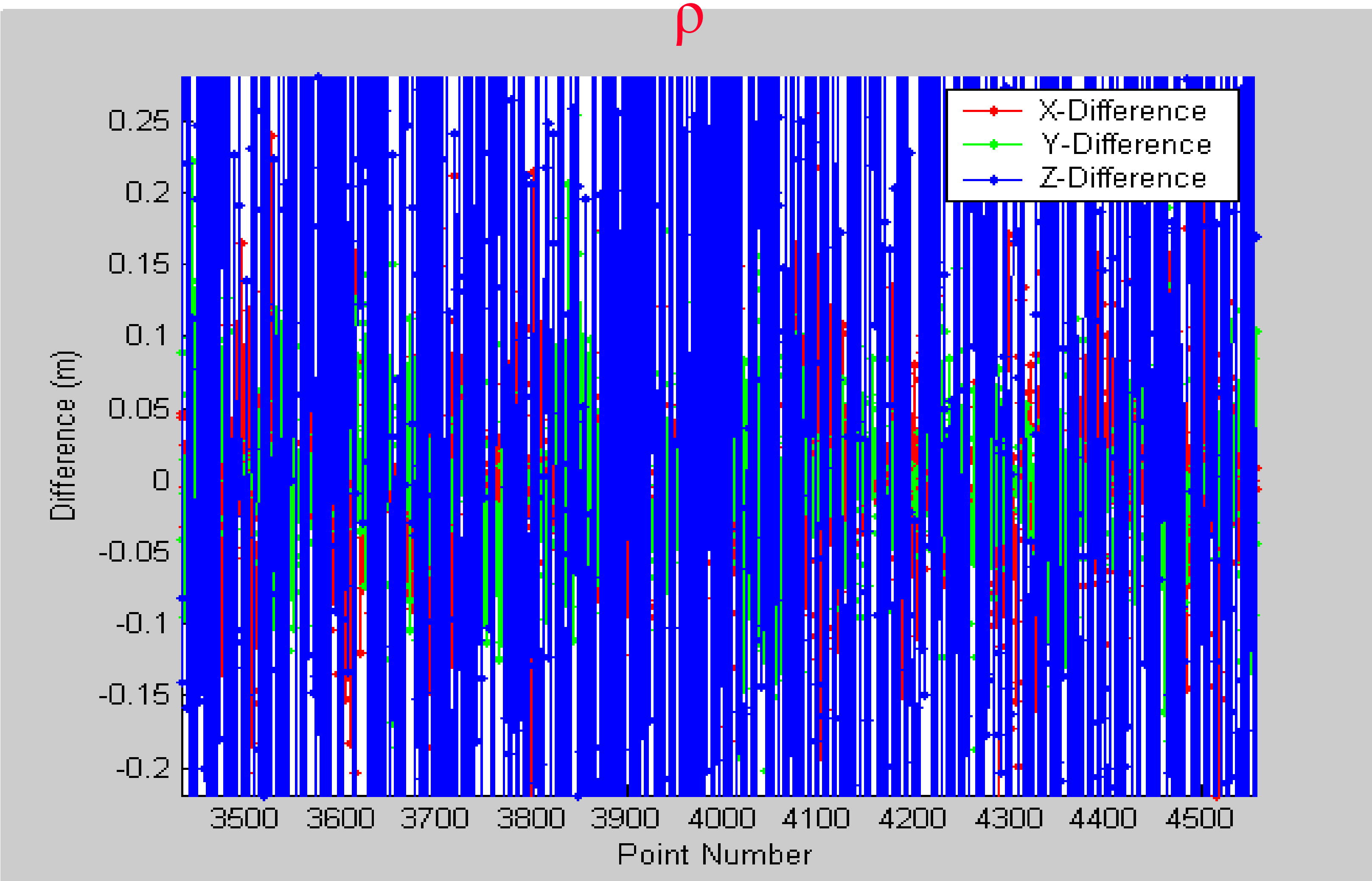
Linear Scanner & Laser Beam Range Noise (II)



$\sigma_{\rho}^{H_1} \neq \sigma_{\rho}^{H_2}$ $\sigma_{\rho}^{H_1} = \sigma_{\rho}^{H_2}$

- Independent of the flying height
- Dependent on the look/scan angle

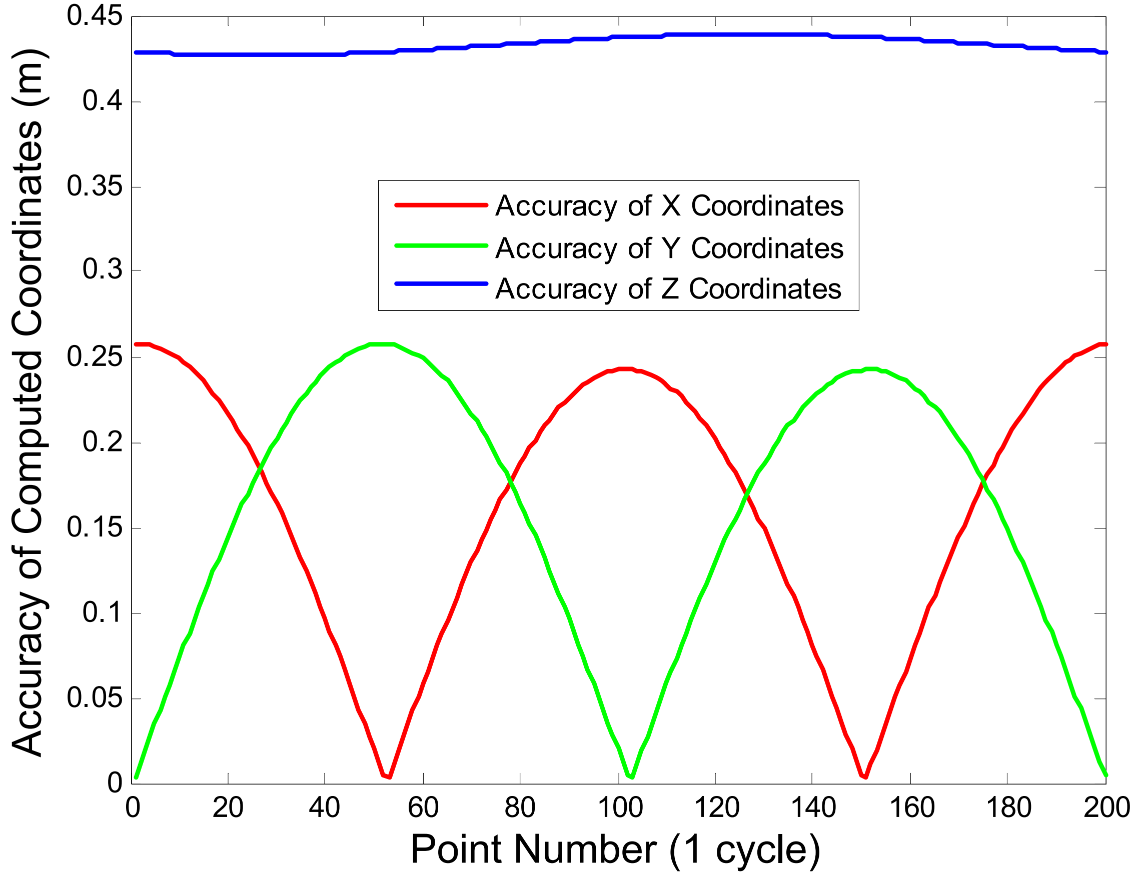
Elliptical Scanner & Laser Beam Range Noise (II)



Independent of the flying height

Elliptical Scanner & Laser Beam Range Noise (II)

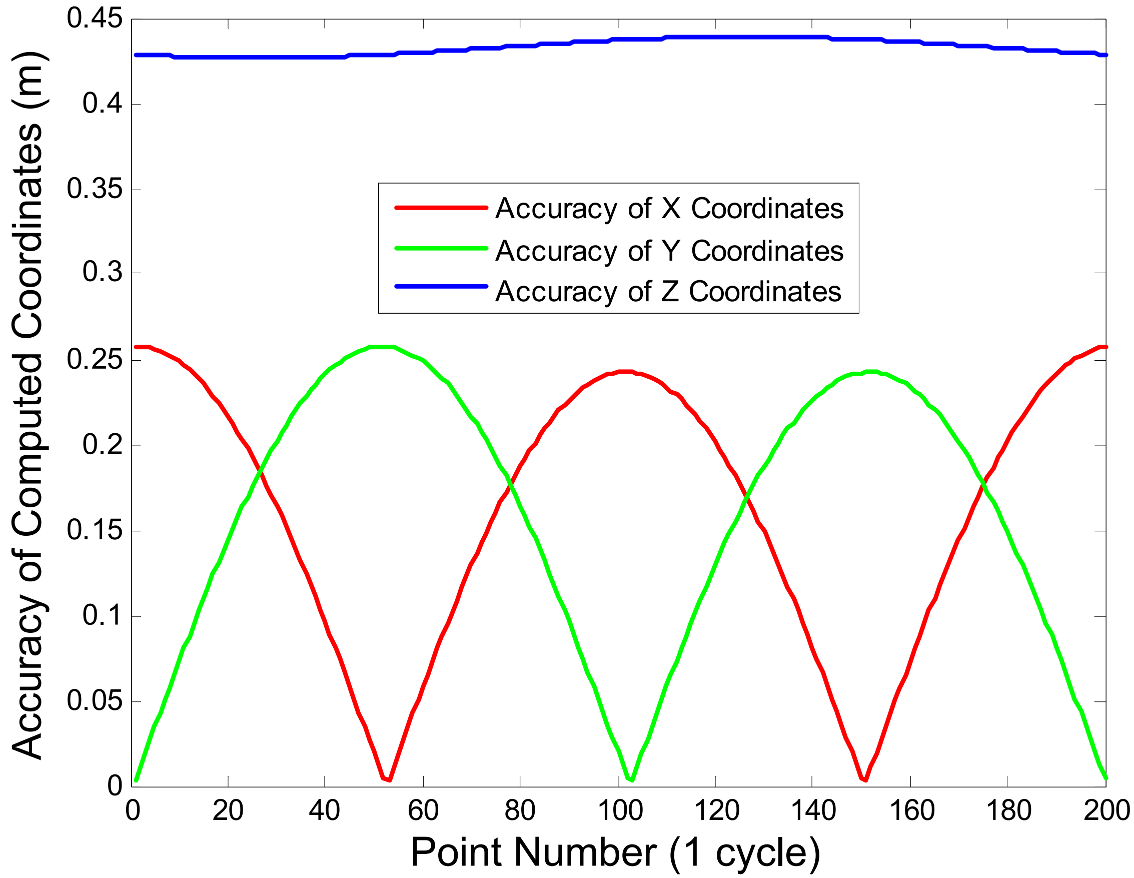
Flying Height = 500m



- Independent of the flying height
- Dependent on the look/scan angle

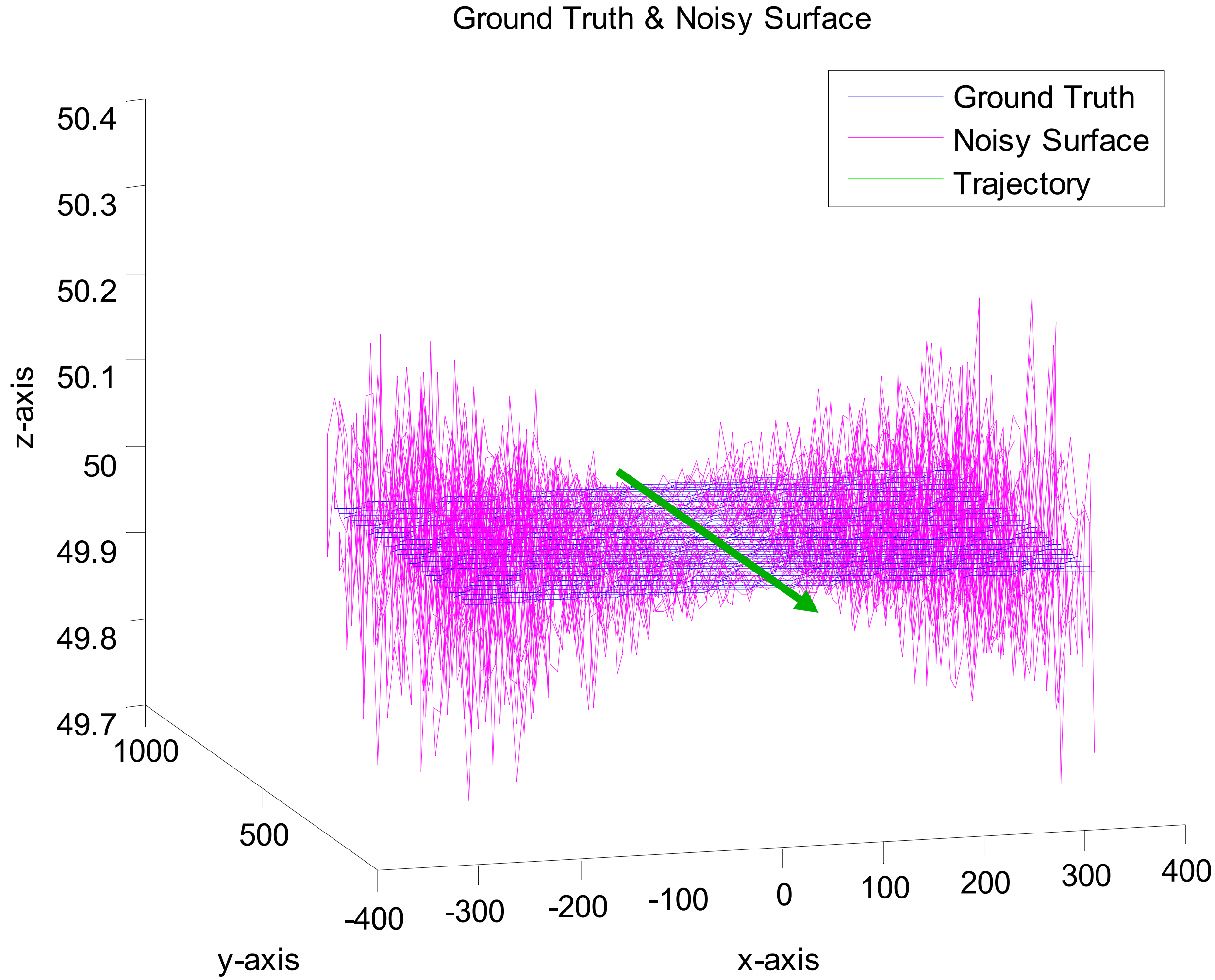
Elliptical Scanner & Laser Beam Range Noise (II)

Flying Height = 1000m

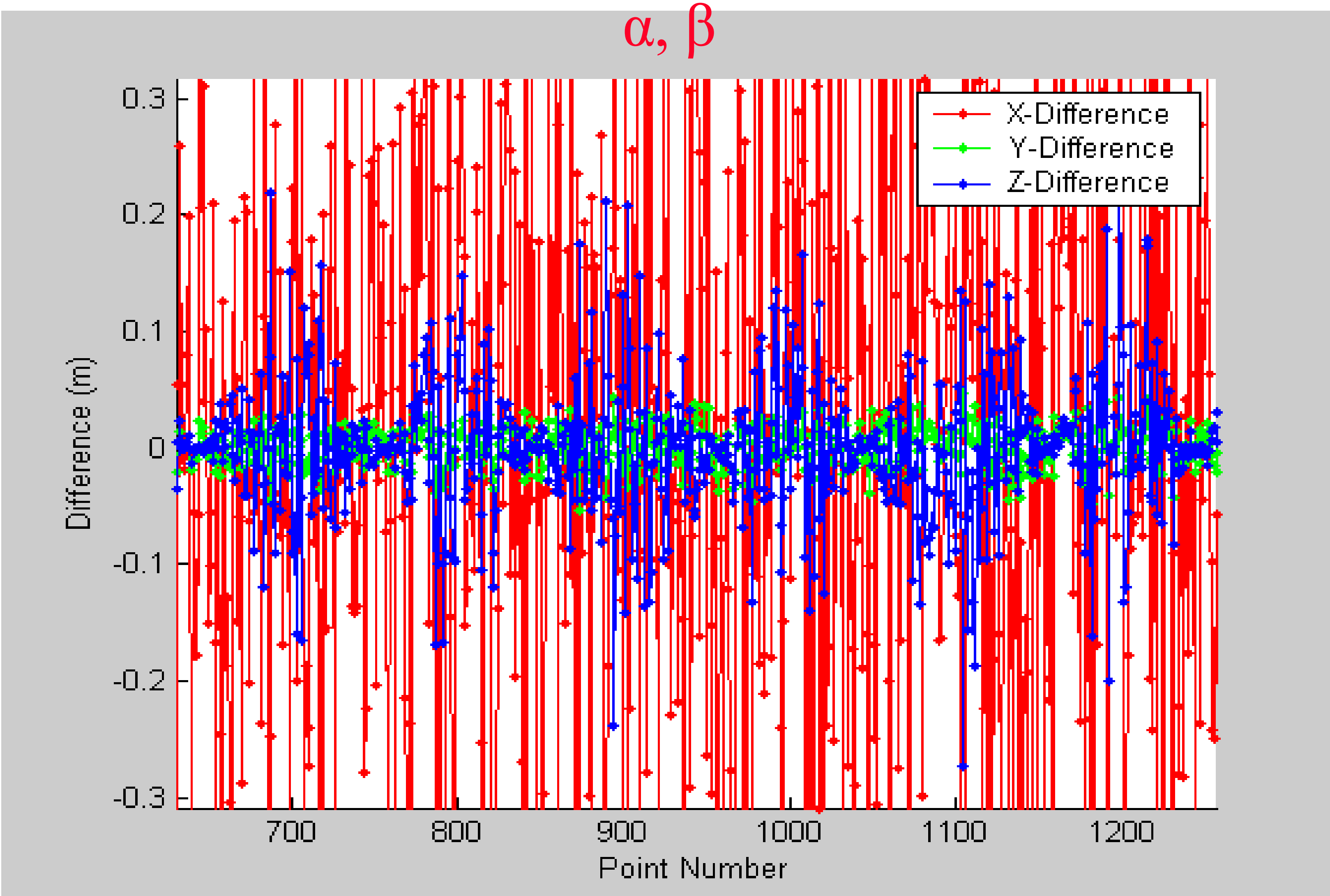


- Independent of the flying height
- Dependent on the look/scan angle

Linear Scanner & Laser Beam Angular Noise (I)

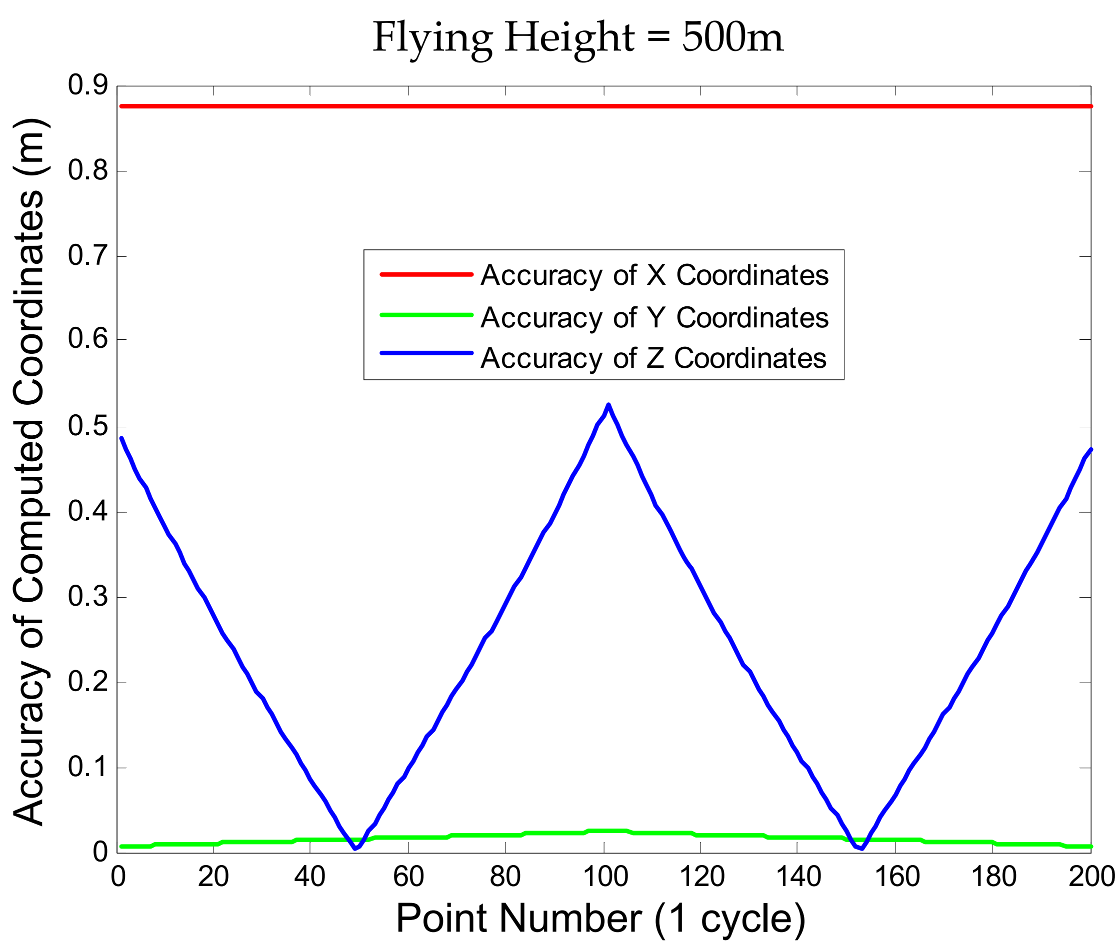


Linear Scanner & Laser Beam Angular Noise (I)



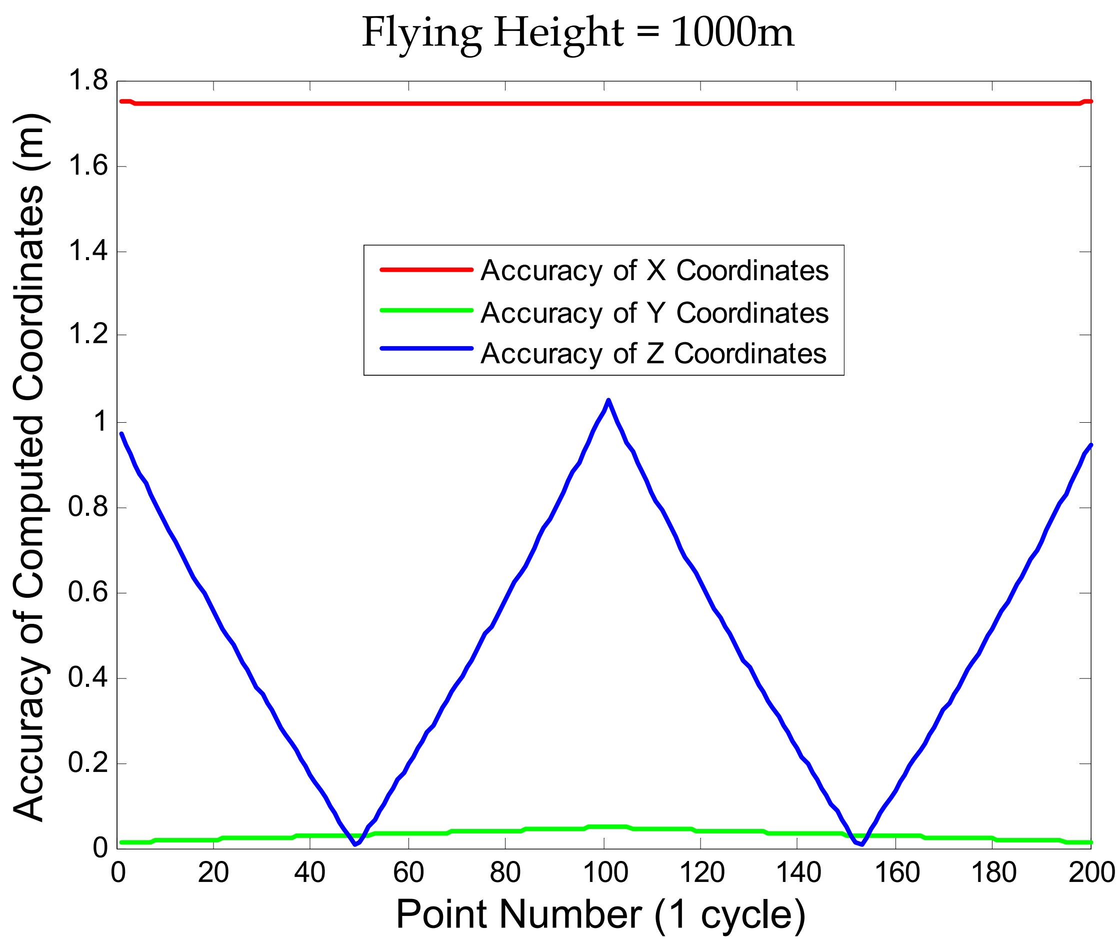
Propagates with the flying height

Linear Scanner & Laser Beam Angular Noise (II)



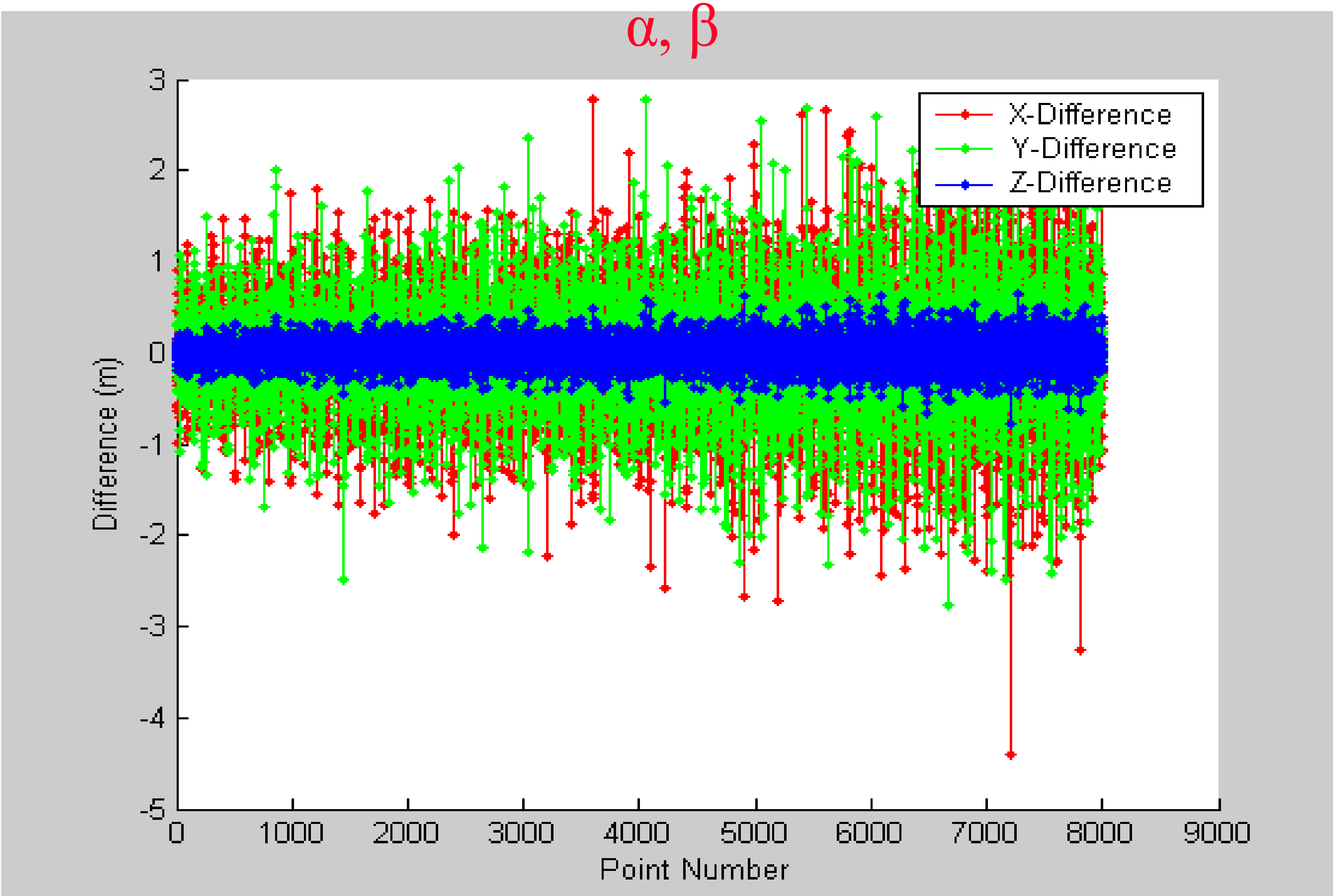
- Propagates with the flying height
- Dependent on the look angle

Linear Scanner & Laser Beam Angular Noise (II)



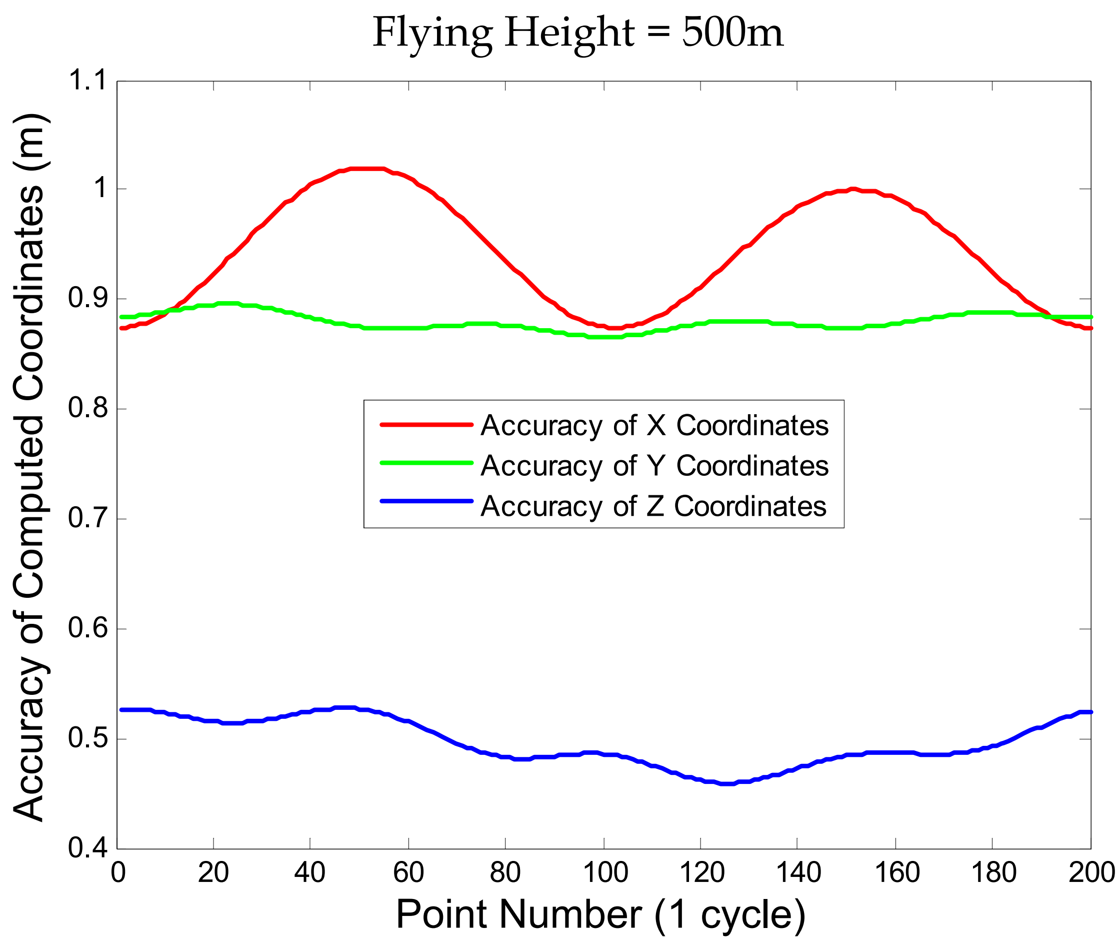
- Propagates with the flying height
- Dependent on the look angle

Ellipt. Scanner & Laser Beam Angular Noise (I)



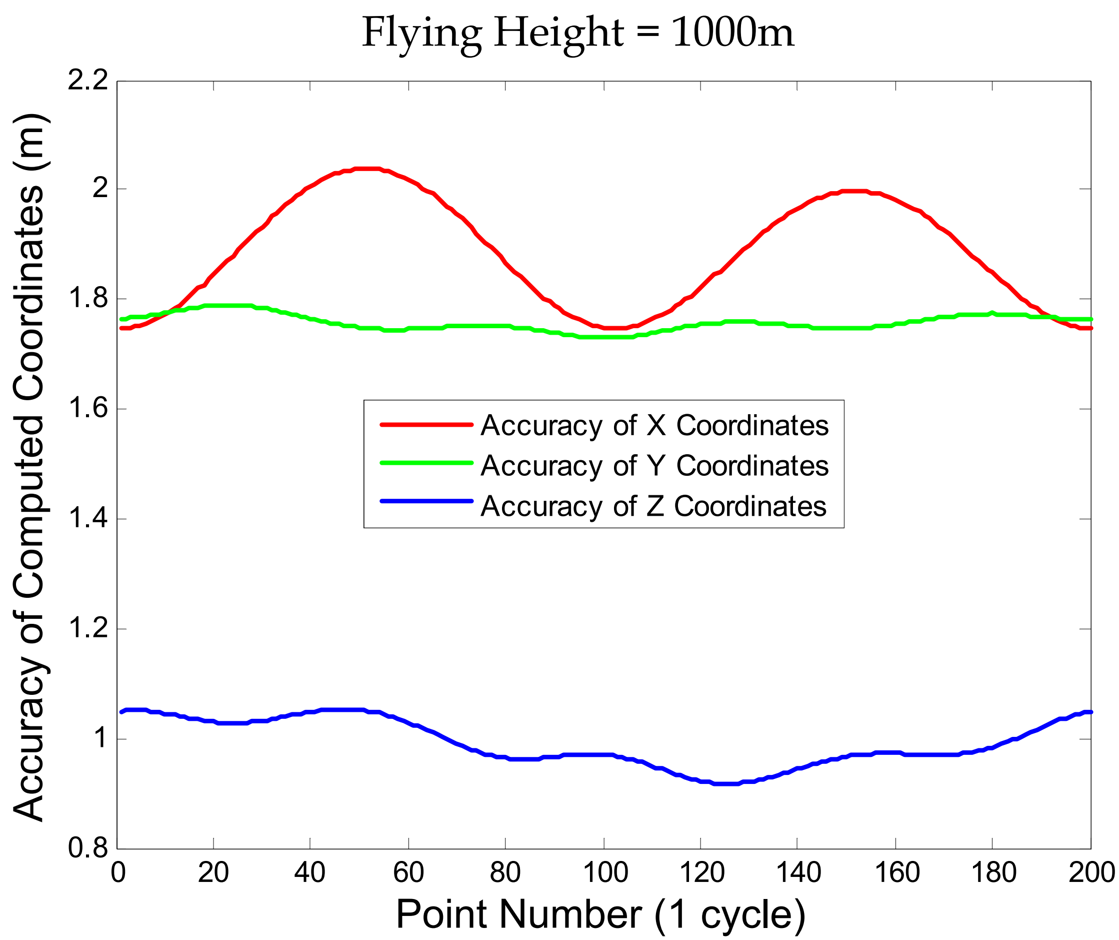
Propagates with the flying height

Ellipt. Scanner & Laser Beam Angular Noise (II)



- Propagates with the flying height
- Dependent on the look angle

Ellipt. Scanner & Laser Beam Angular Noise (II)

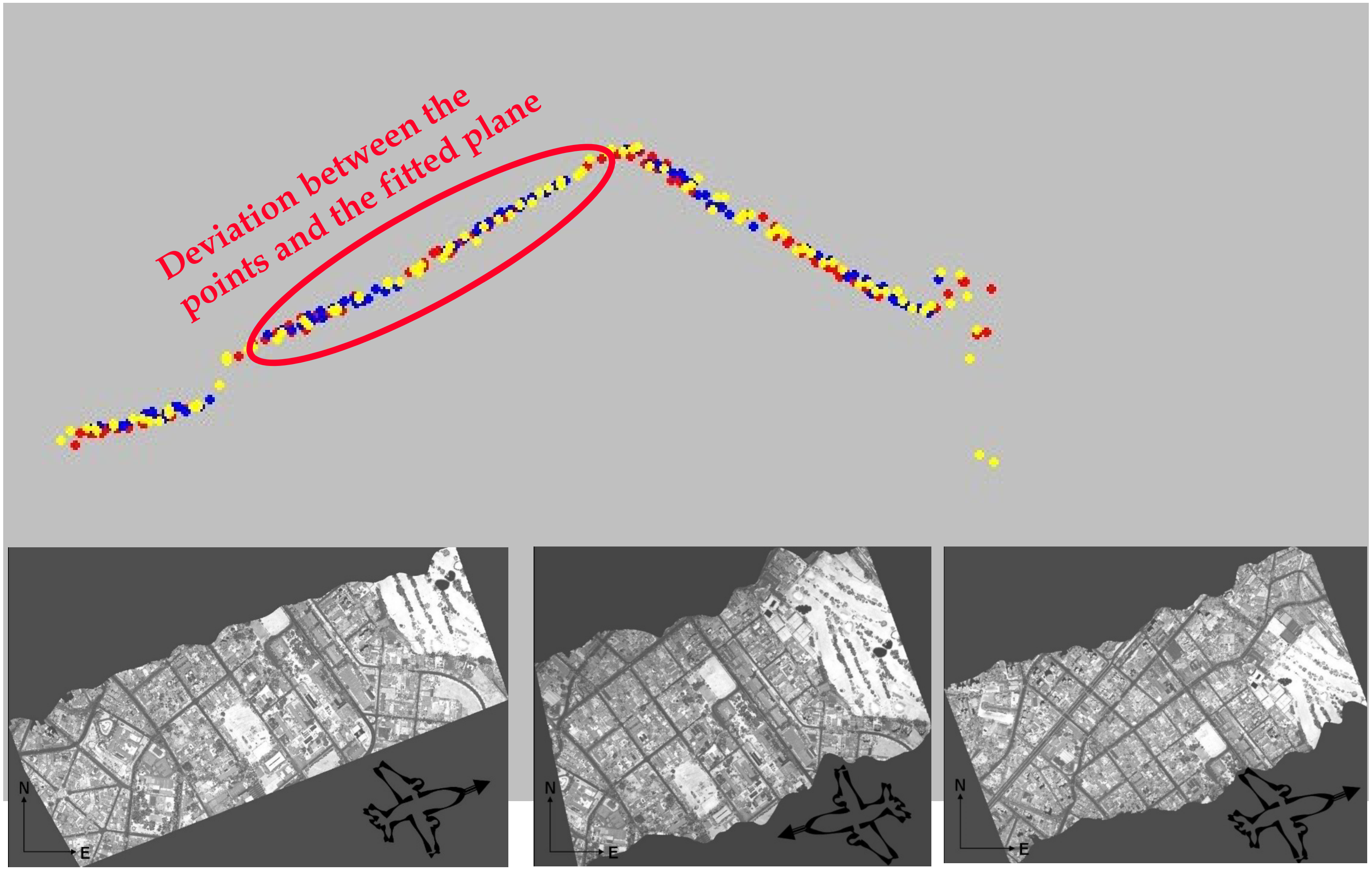


- Propagates with the flying height
- Dependent on the look angle

Error Sources: Random Noise

- As expected, random noise will lead to random errors in the derived point cloud (refer to the outcome of approach I).
- Diagnostic hints:
  - GNSS/INS-position noise:
    - Similar noise level in derived point cloud
    - Independent of the system parameters (height & look angle)
  - Angular noise (GNSS/INS-attitude or mirror angles):
    - Planimetric coordinates are affected more than vertical coordinates.
    - Dependent on the system parameters (height & look angle)
    - The magnitude of the introduced noise increases with an increase in the flying height and off-nadir angle.
  - Range noise:
    - Mainly affects the vertical component
    - Independent of the system height
    - Dependent on the system look angle

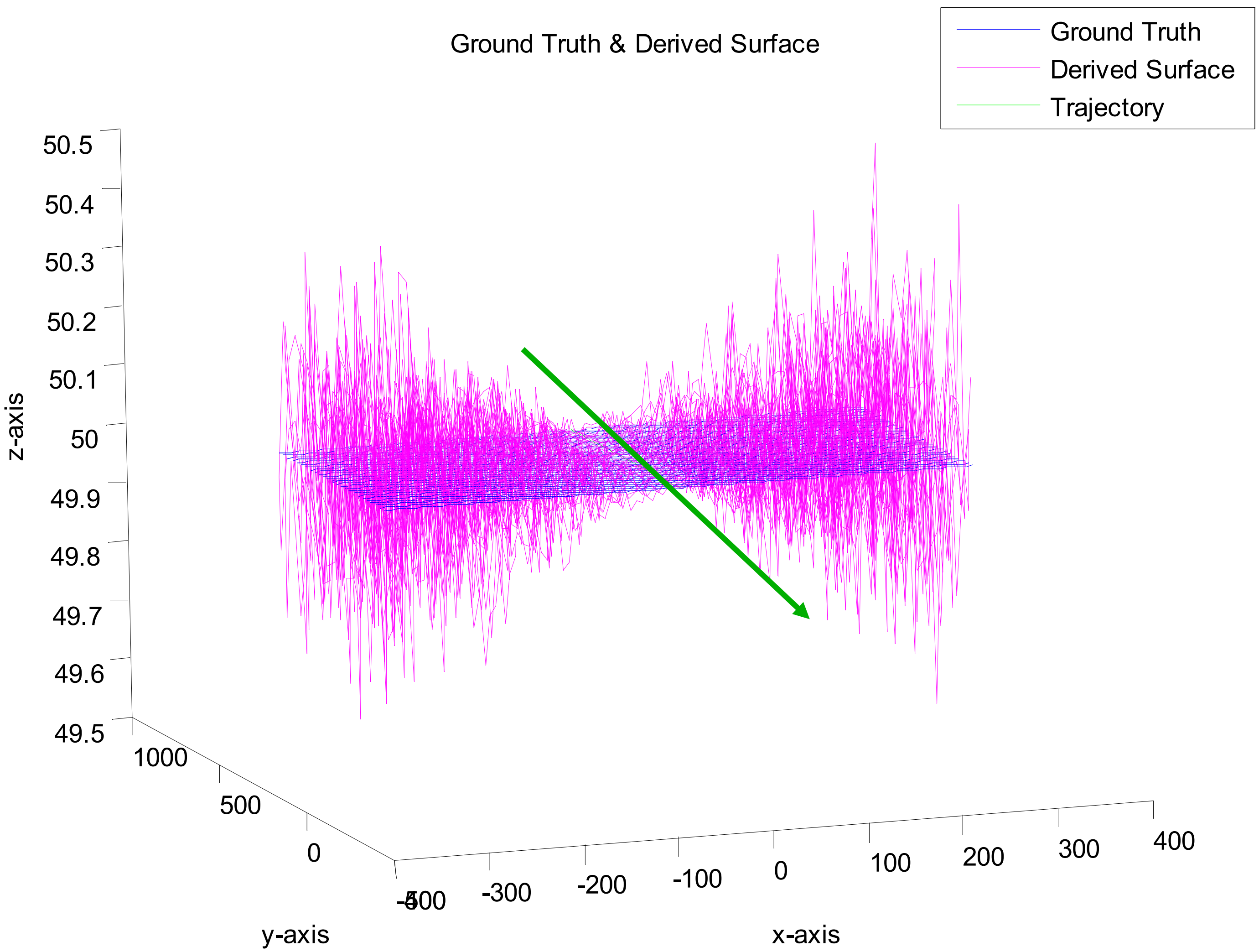
Error Sources: Random Noise



Error Sources: Final Remarks

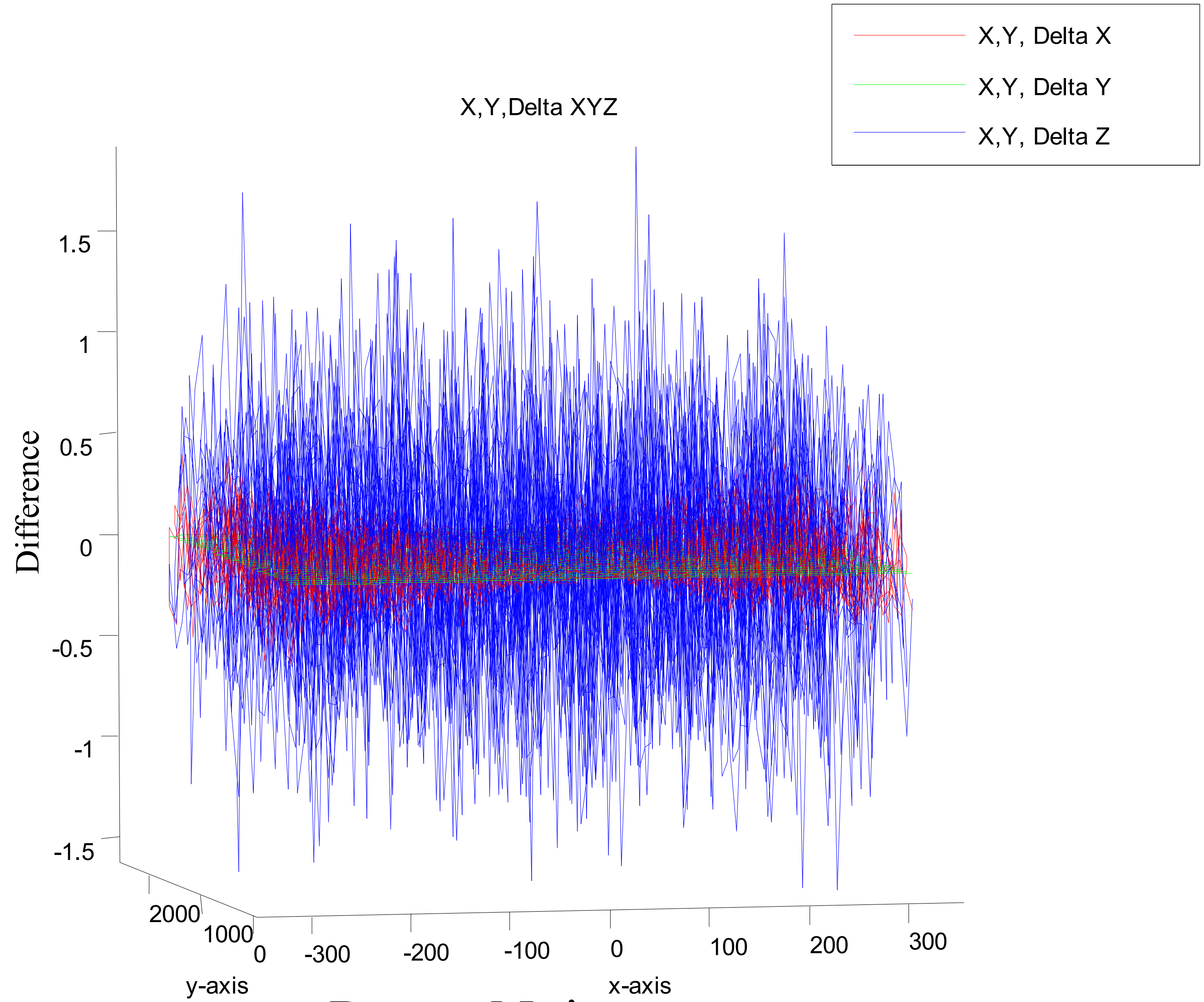
- Systematic errors → systematic biases
- Random noise → random errors
- It is believed that random noise will not affect the relative accuracy.
  - However, this is not the case for LiDAR systems.
  - Random errors will affect the relative accuracy of the derived point cloud.
  - Depending on the considered parameter, the relative effect of the corresponding noise level will not be the same.

Random Noise & Relative Accuracy



GNSS/INS-Attitude Noise

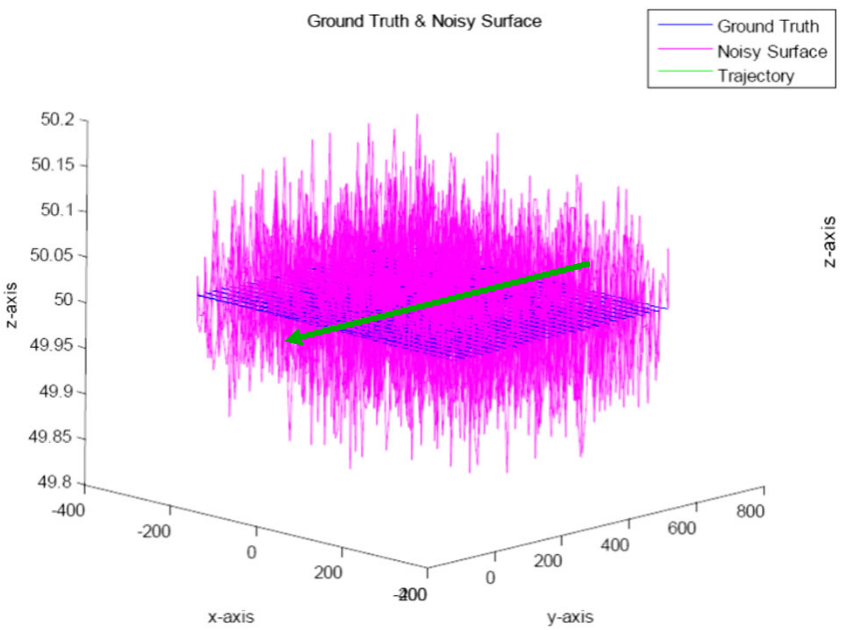
Random Noise & Relative Accuracy



Range Noise

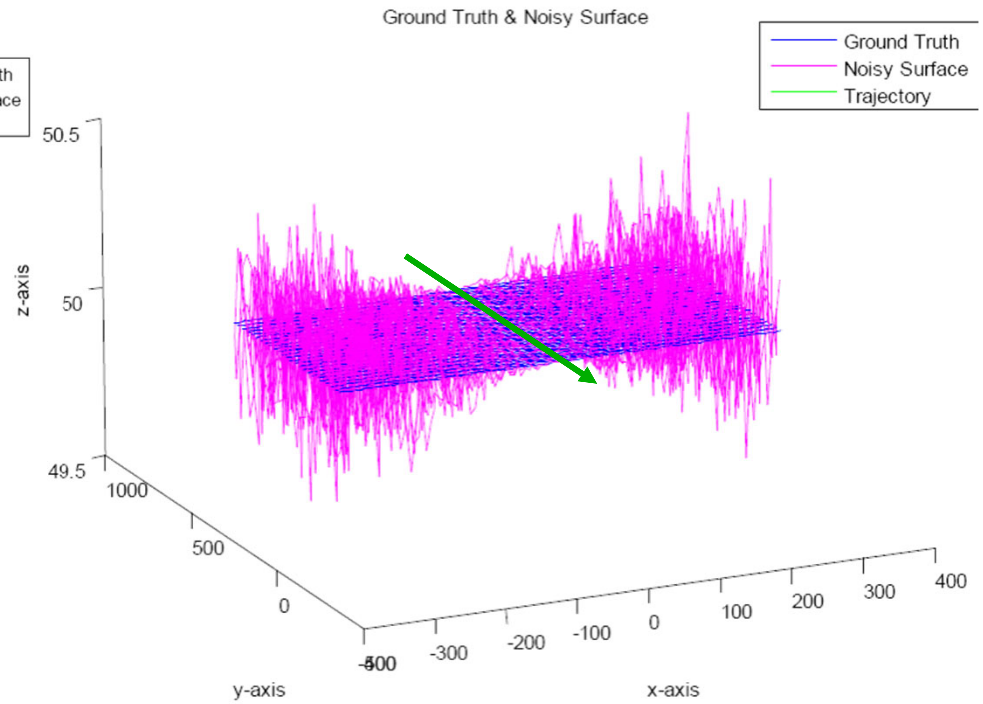
Random Noise & Relative Accuracy

Random Noise added to the Position Measurements



Relative precision is not affected

Random Noise added to the Orientation Measurements

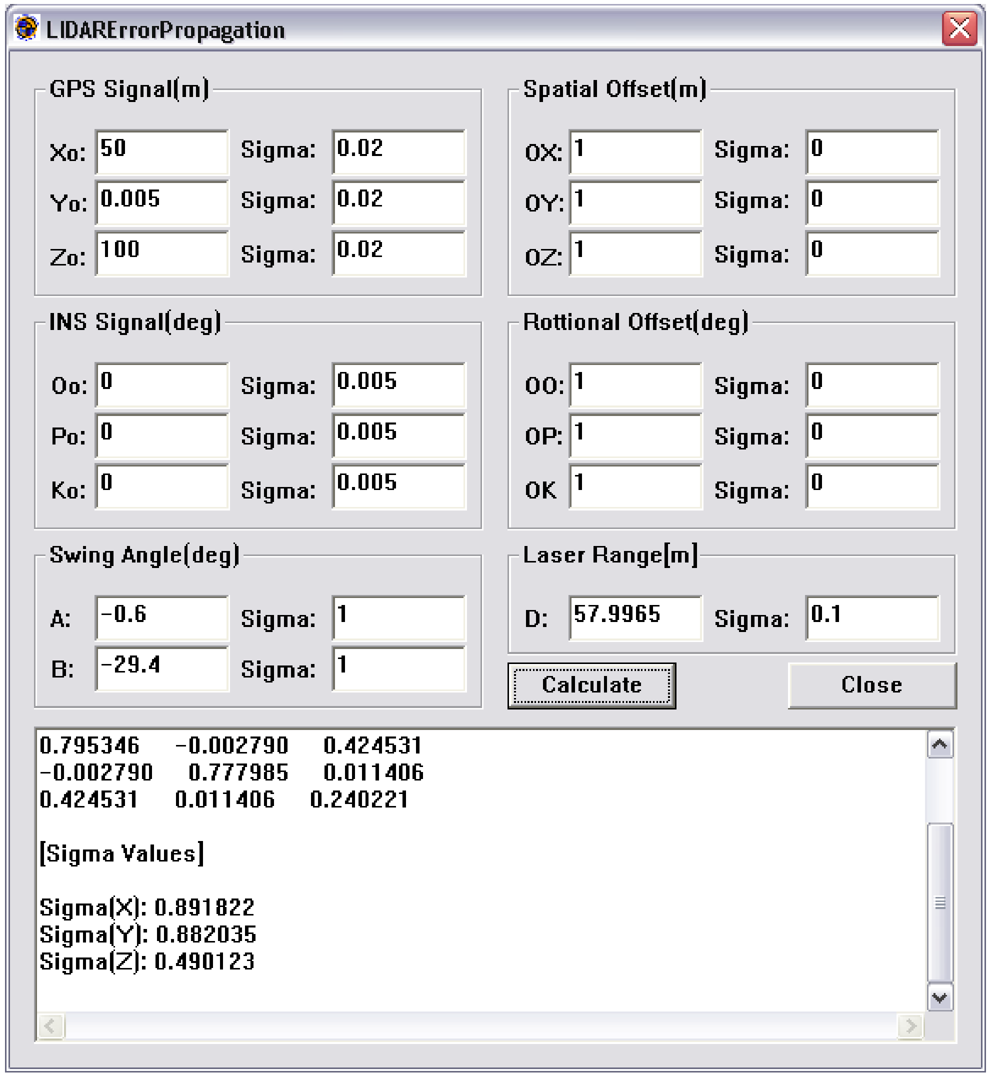


Relative precision is affected

LiDAR Error Propagation Calculator

- The calculator allows one to enter specific values for each of the input measurements/parameters for a certain LiDAR point and to enter the noise level for each of the measurements/parameters.
- The program then determines the accuracy of the ground coordinates of the point.
- Conversely, if the user requires a specific accuracy in the final ground coordinates, the program can be used to determine the accuracies that would be required for the input components through a trial and error process.

LiDAR Error Propagation Calculator



LiDAR Error Propagation Calculator

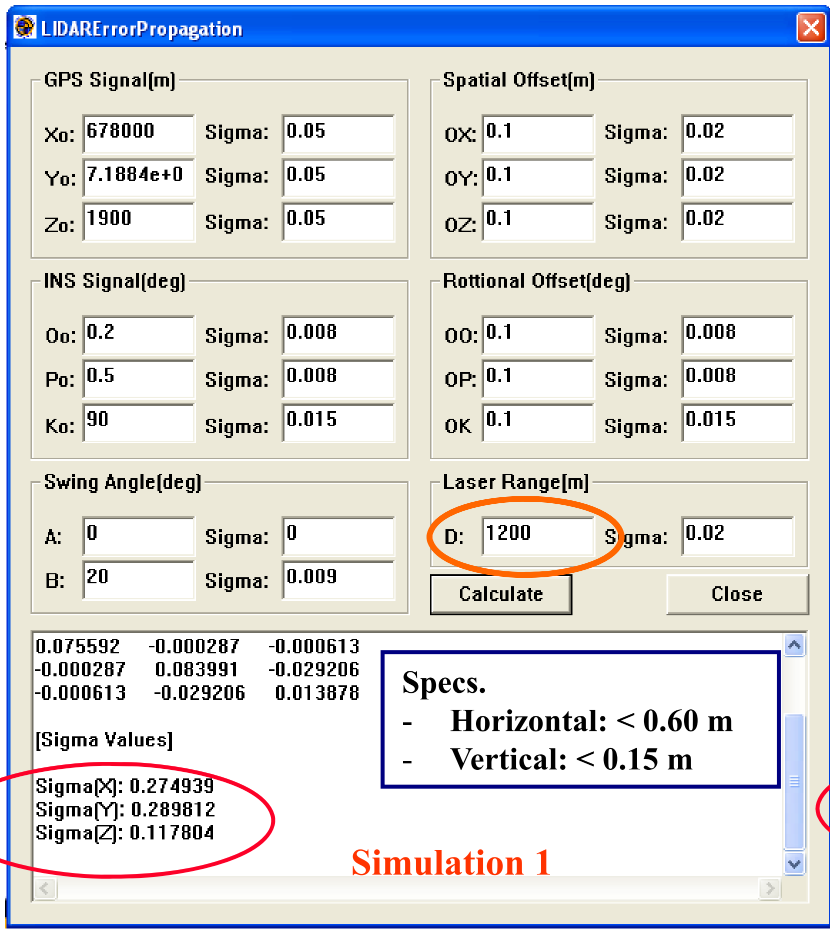
- Accuracy of the system components

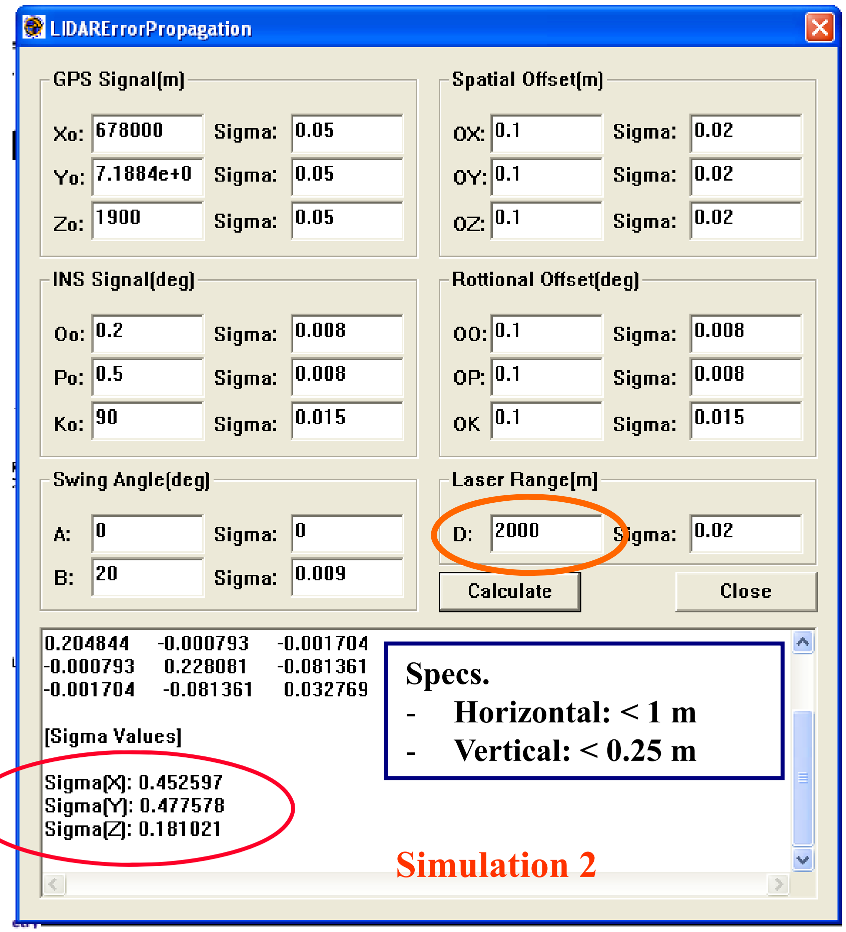
System Model	GNSS (m) Post-Processed	IMU (deg) Post-Processed			Scan Angle (deg)	Laser Range (cm)
		Roll	Pitch	Heading		
ALTM 2050	0.05 – 0.3	0.008	0.008	0.015	0.009	~ 2
ALTM 3100	0.05 – 0.3	0.005	0.005	0.008	0.009	~ 2

- System Manufacturer Specification (Optech: ALTM 2050 and ALTM 3100)
  - Horizontal accuracy : 1/2000 x altitude
  - Vertical accuracy : <15 cm at 1200 m
    : <25 cm at 2000 m

LiDAR Error Propagation Calculator

- Expected accuracy (assuming flat solid surface) of the ground coordinates as derived from the error propagation – ALTM 2050

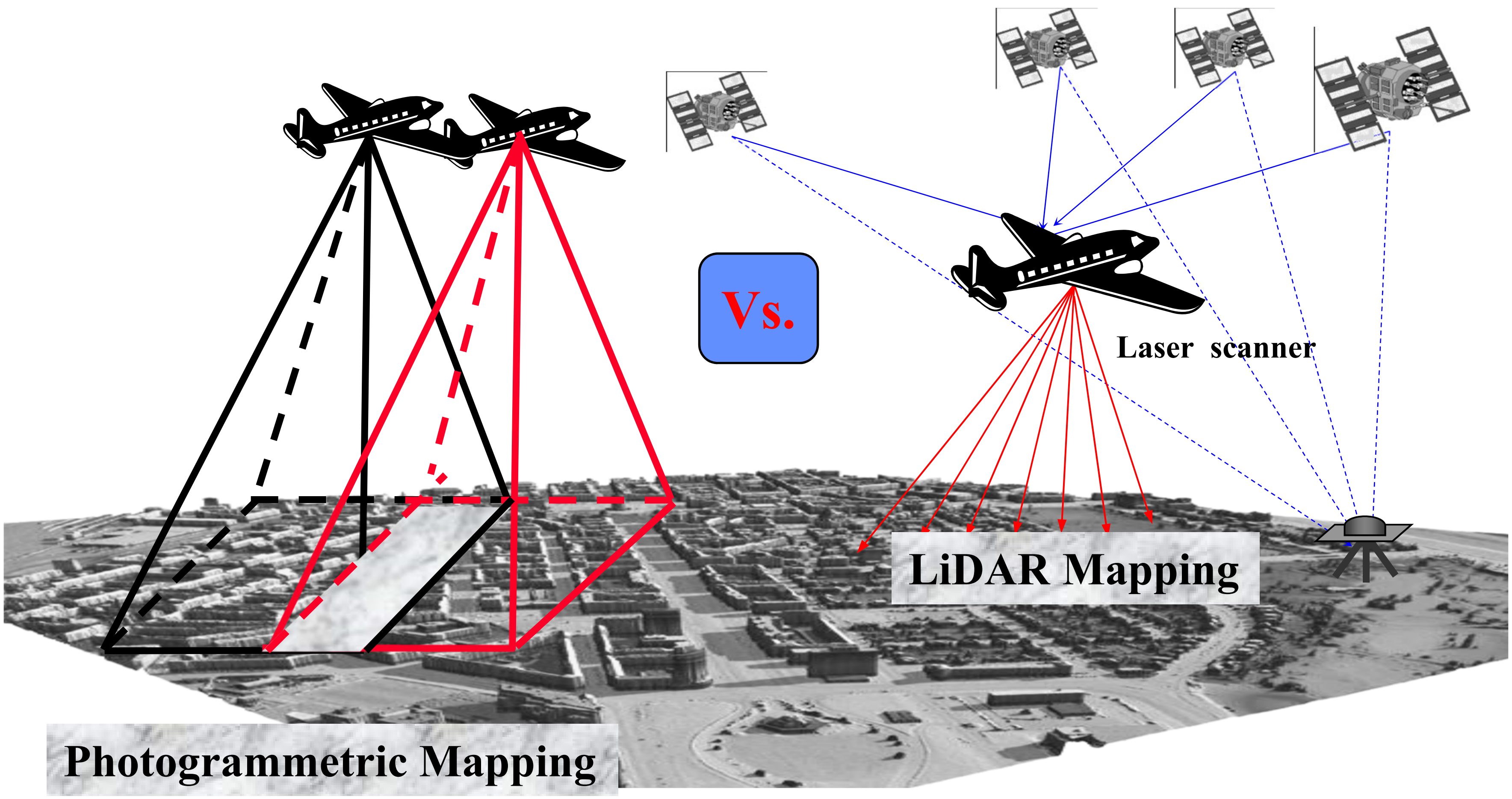




LiDAR Vs. Photogrammetric Mapping

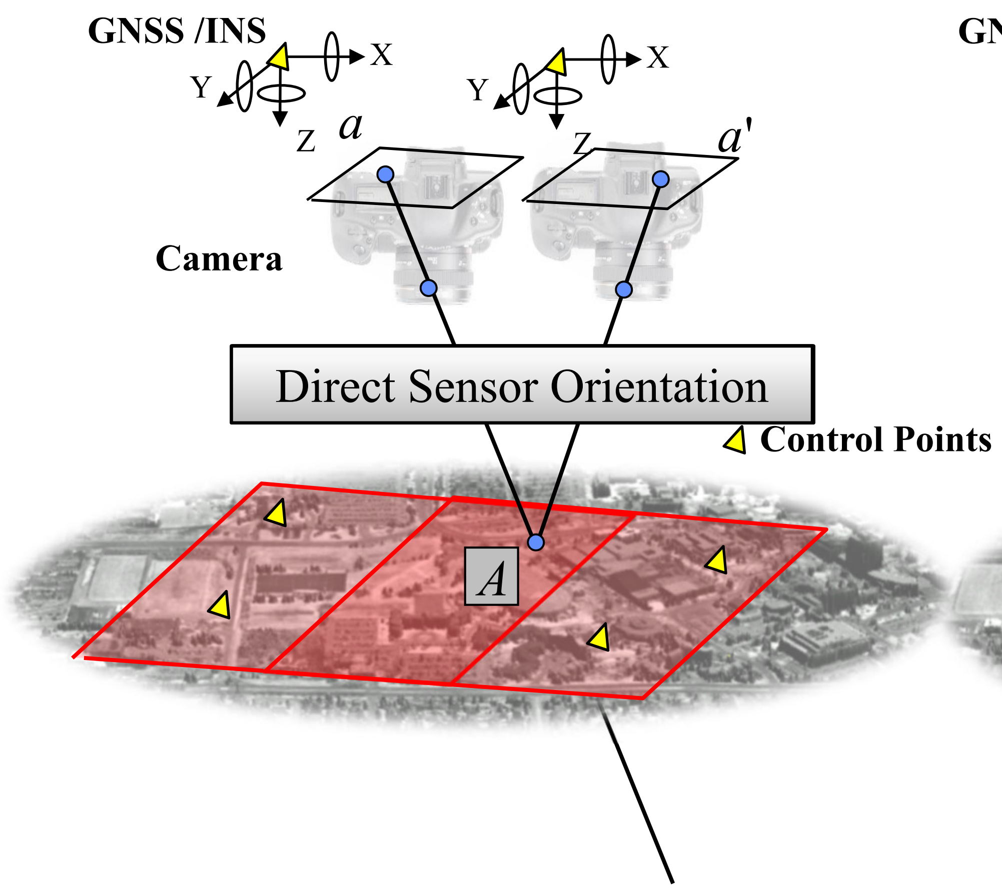
	LiDAR	Photogrammetry
Georeferencing	Direct Only (Mobile Systems)	Indirect/Direct
Reconstruction	Single Pulse	Intersection of Conjugate Light Rays
System Calibration	Mounting Parameters + Laser Ranging/ Scanning Unit	Mounting Parameters + Camera Calibration
Point Positioning Equation	Vector Summation Procedure	Vector Summation Procedure

LiDAR Vs. Photogrammetric Mapping

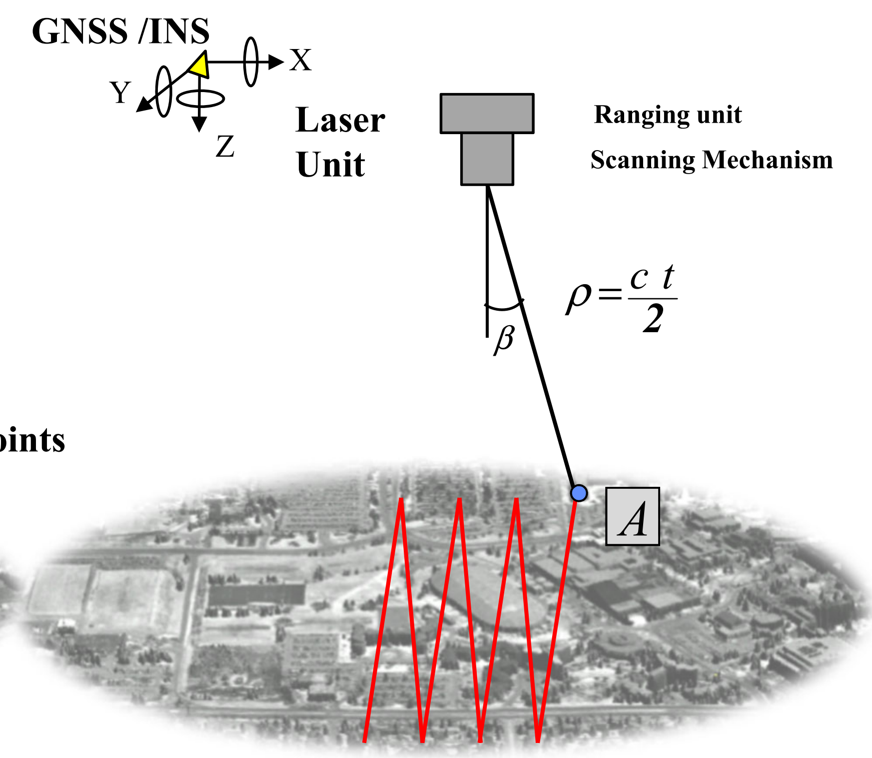


LiDAR Vs. Photogrammetric Mapping

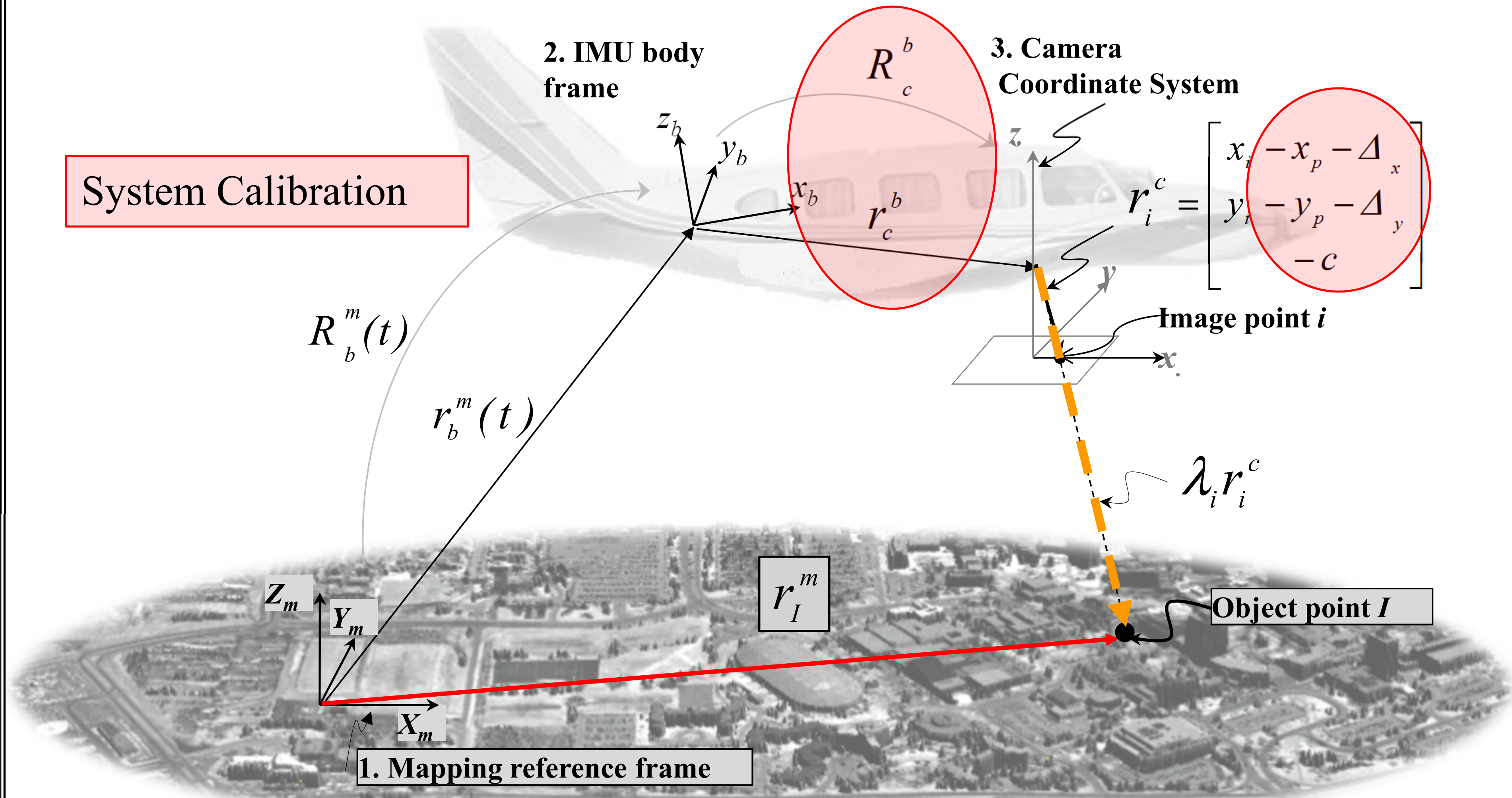
Photogrammetric Mapping



LiDAR Mapping

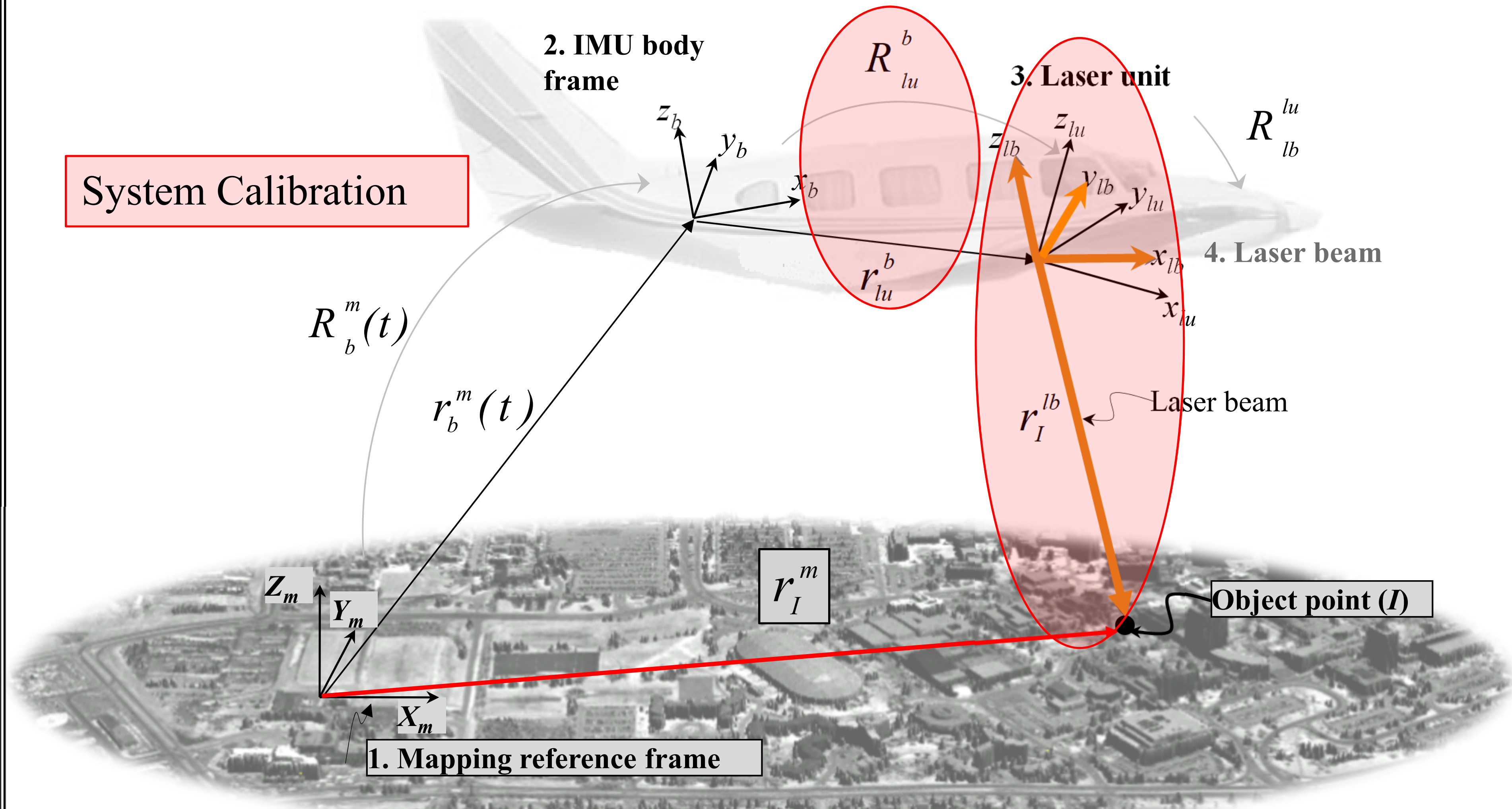


Photogrammetric System Calibration



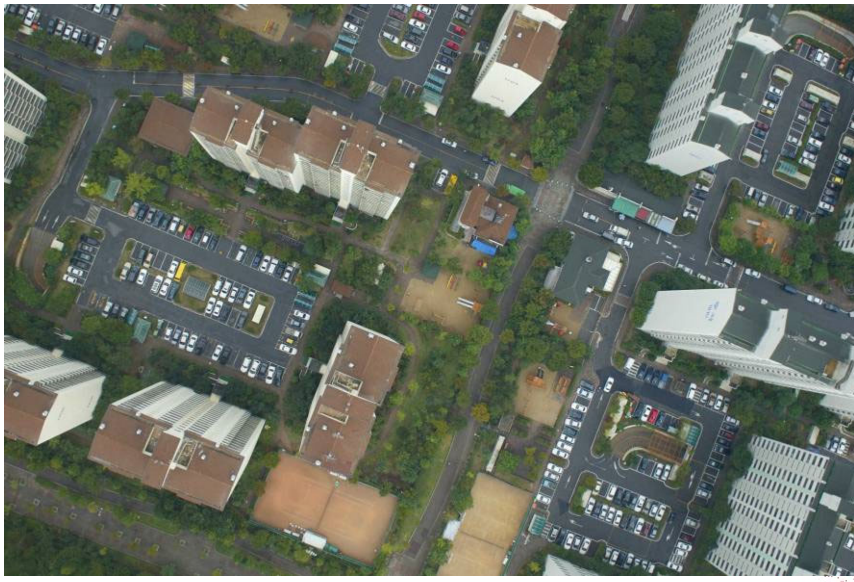
$$r_I^m = r_b^m(t) + R_b^m(t)\, r_c^b + \lambda_i\, R_b^m(t)\, R_c^b\, r_i^c$$

LiDAR System Calibration



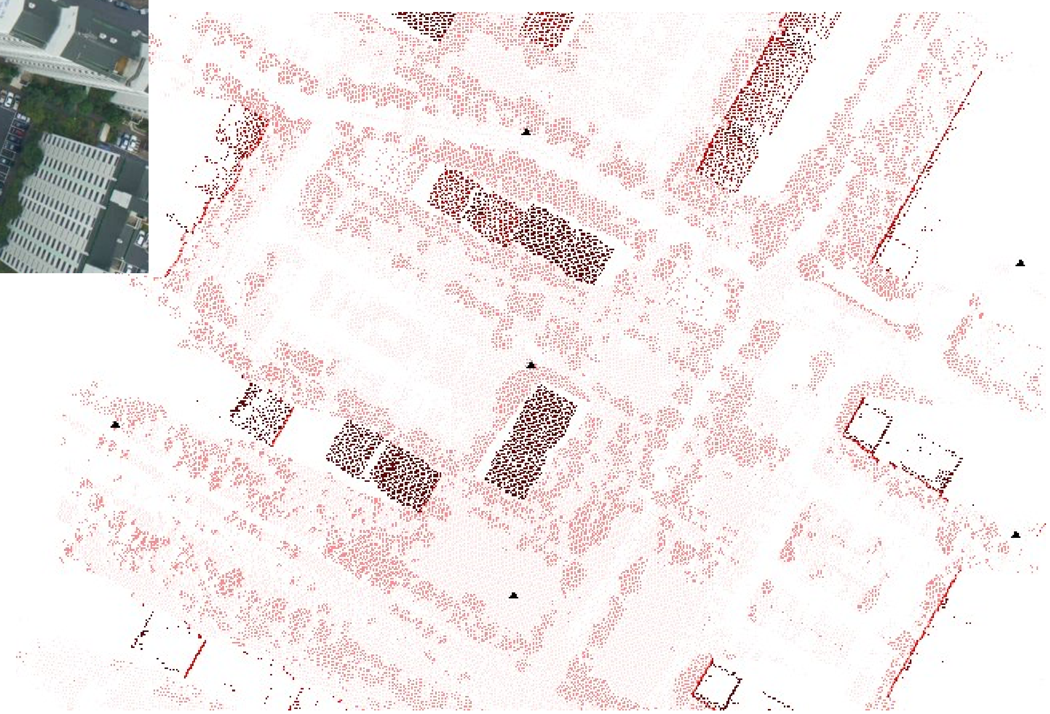
$$r^{m}_{I} = r^{m}_{b}(t) + R^{m}_{b}(t)\, r^{b}_{lu} + R^{m}_{b}(t)\, R^{b}_{lu} R^{lu}_{lb} r^{lb}_{I}$$

LiDAR Vs. Photogrammetric Mapping



Optical Imagery

LiDAR Range Image



LiDAR Vs. Photogrammetric Mapping

Optical Imagery

LiDAR Intensity Image

LiDAR Vs. Photogrammetric Mapping

LiDAR (Pros)	**Photogrammetry (Cons)**
Dense information along homogeneous surfaces	Almost no positional information along homogeneous surfaces
Day or night data collection	Day time data collection
Direct acquisition of 3D coordinates	Complicated and sometimes unreliable matching procedures
Vertical accuracy is better than the planimetric accuracy	Vertical accuracy is worse than the planimetric accuracy

LiDAR Vs. Photogrammetric Mapping

Photogrammetry (Pros)	LiDAR (Cons)
High redundancy	No inherent redundancy
Rich with semantic information	Positional; difficult to derive semantic information
Dense positional information along object space breaklines	Almost no information along breaklines
Planimetric accuracy is better than the vertical accuracy	Planimetric accuracy is worse than the vertical accuracy
Transparent Model	***Non-transparent model***

LiDAR Vs. Photogrammetric Mapping