Chapter 1: Overview

- Photogrammetry: Introduction & Applications
- Photogrammetric tools:
  - Rotation matrices
  - Photogrammetric point positioning
  - Photogrammetric bundle adjustment

- This chapter will cover the incorporation of GNSS/INS position and attitude information for photogrammetric and LiDAR-based reconstruction procedures.

Chapter 2

PHOTOGRAMMETRIC & LIDAR GEOREFERENCING

Overview

- Introduction
- Georeferencing Alternatives:
  - Indirect georeferencing – Photogrammetric mapping & static laser scanners (specially designed targets)
  - Integrated Sensor Orientation (ISO) – Photogrammetric mapping
  - Direct georeferencing – Photogrammetric and LiDAR mapping
- Direct Georeferencing: Operational Example
  - Terrestrial Mobile Mapping Systems (MMS) – Photogrammetric-based system
- Accuracy Analysis of Different Georeferencing Techniques – Photogrammetric mapping
- Concluding Remarks

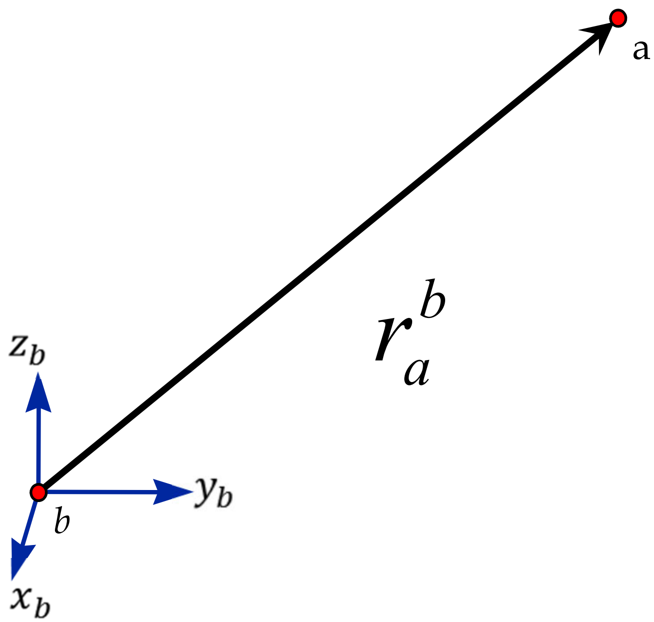
Notation

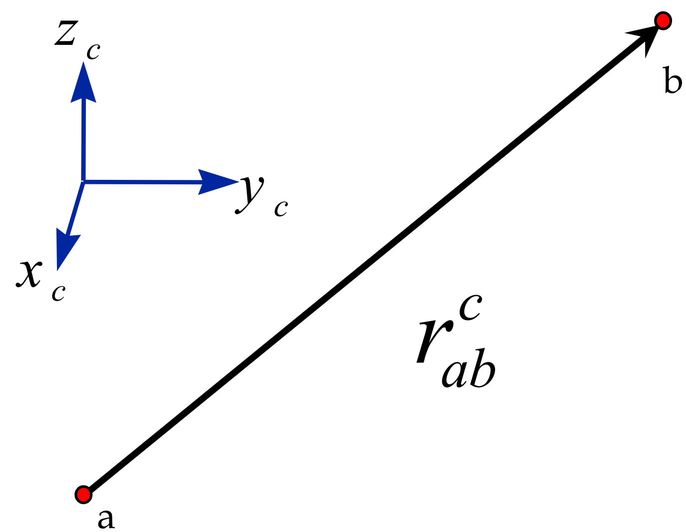
- r_a^b Stands for the coordinates of point ***a*** relative to point ***b*** – this vector is defined relative to the coordinate system associated with point ***b***.

- r_{ab}^c Stands for the components of the vector $\overrightarrow{\boldsymbol{ab}}$ relative to the coordinate system denoted by ***c***.

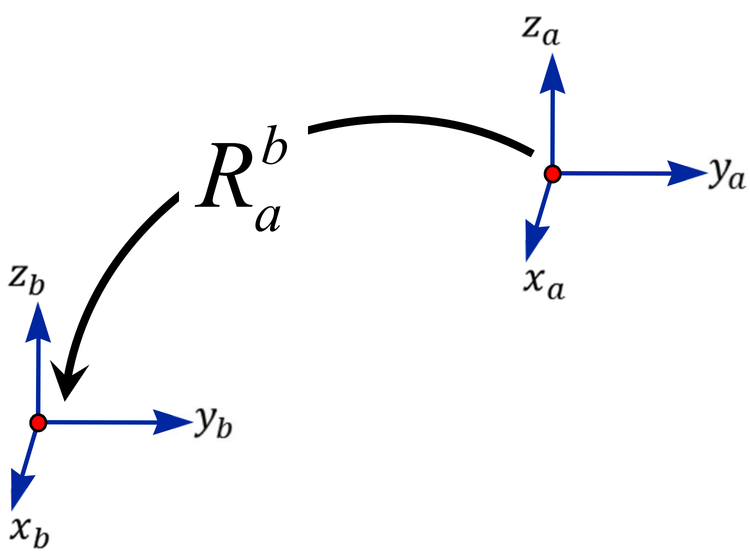
- R_a^b Stands for the rotation matrix that transforms a vector defined relative to the coordinate system denoted by ***a*** into a vector defined relative to the coordinate system denoted by ***b***.

Notation

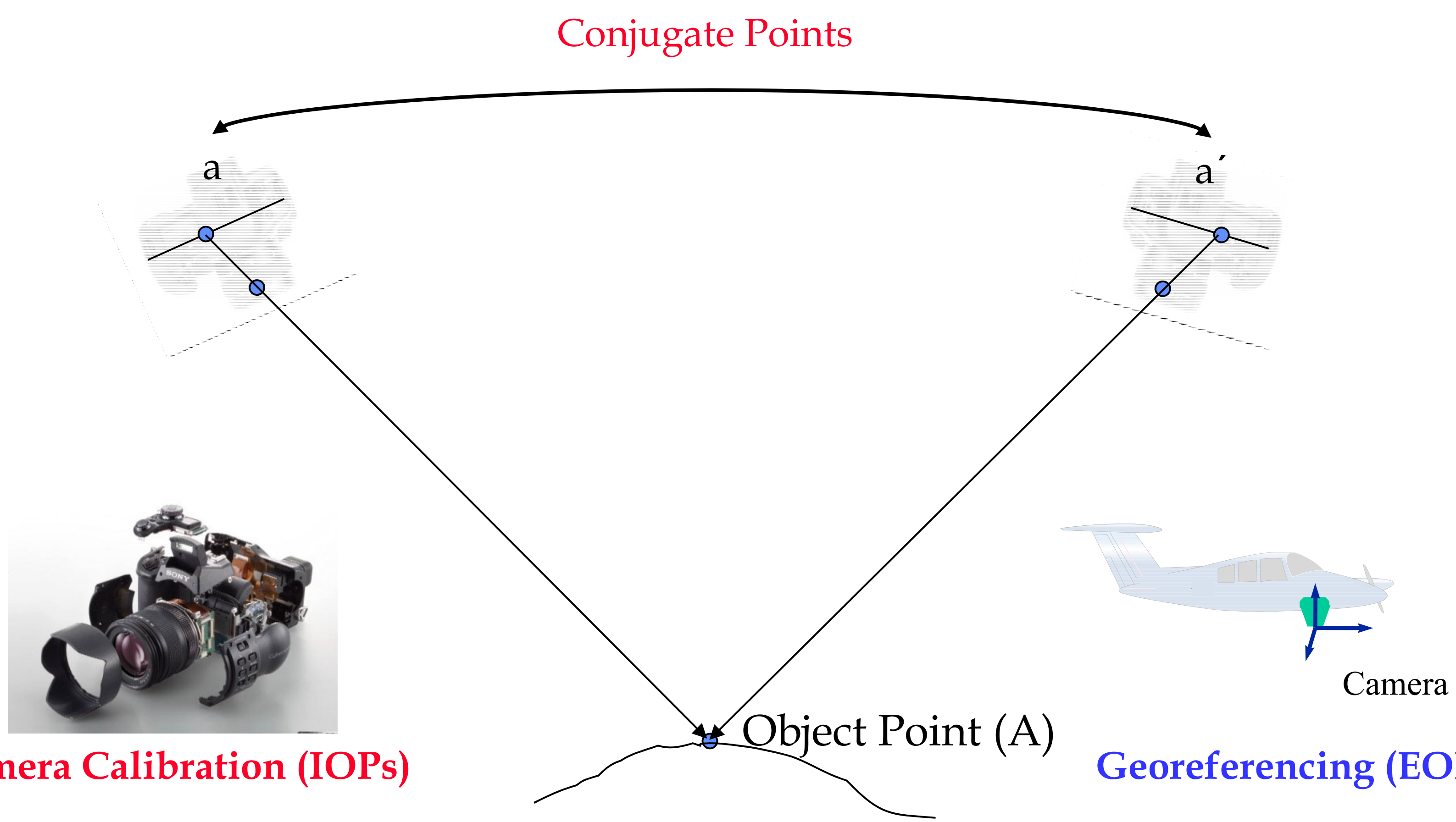




Notation



Photogrammetric Reconstruction



Camera Calibration (IOPs)

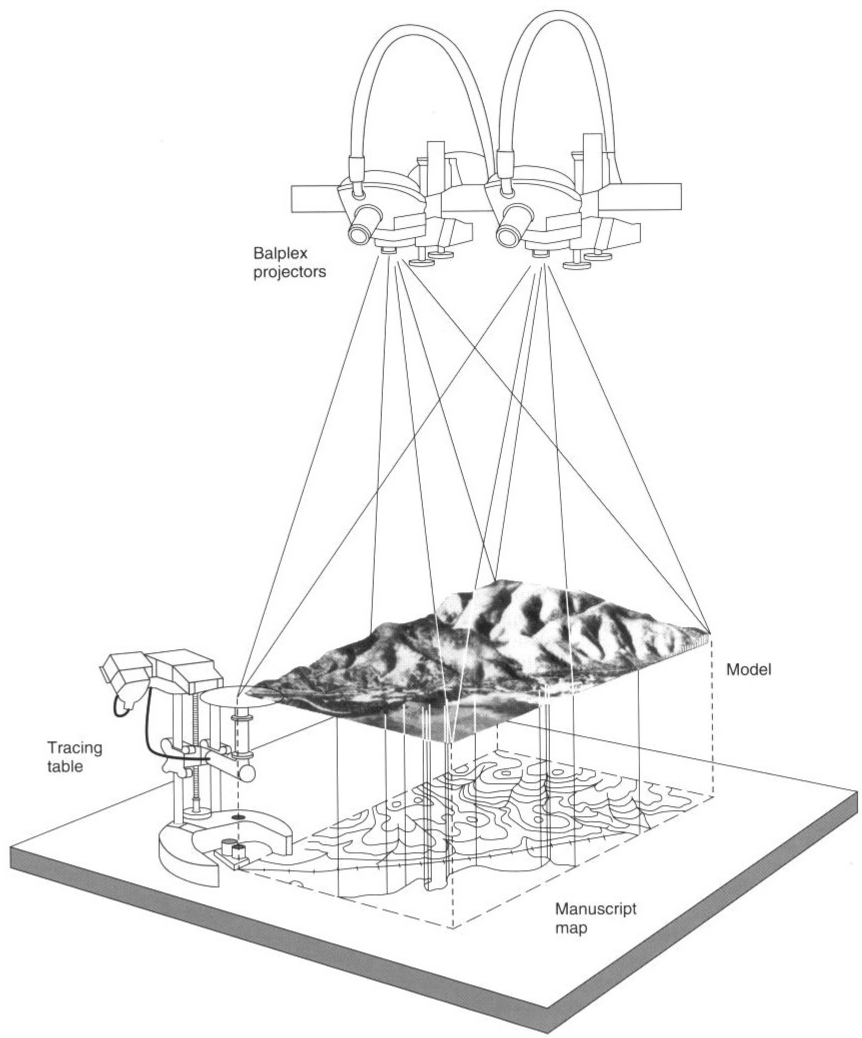
Georeferencing (EOPs)

- The interior orientation parameters of the involved cameras have to be known.
- The position and the orientation of the camera stations have to be known.

Photogrammetry

- The objective of photogrammetry is to transform centrally projected images into a three-dimensional model, which can be used to plot an orthogonal map.
- The three-dimensional model can be obtained through:
  - Interior Orientation
    - Defined through a calibration procedure
  - Exterior Orientation
    - Defined through a georeferencing procedure

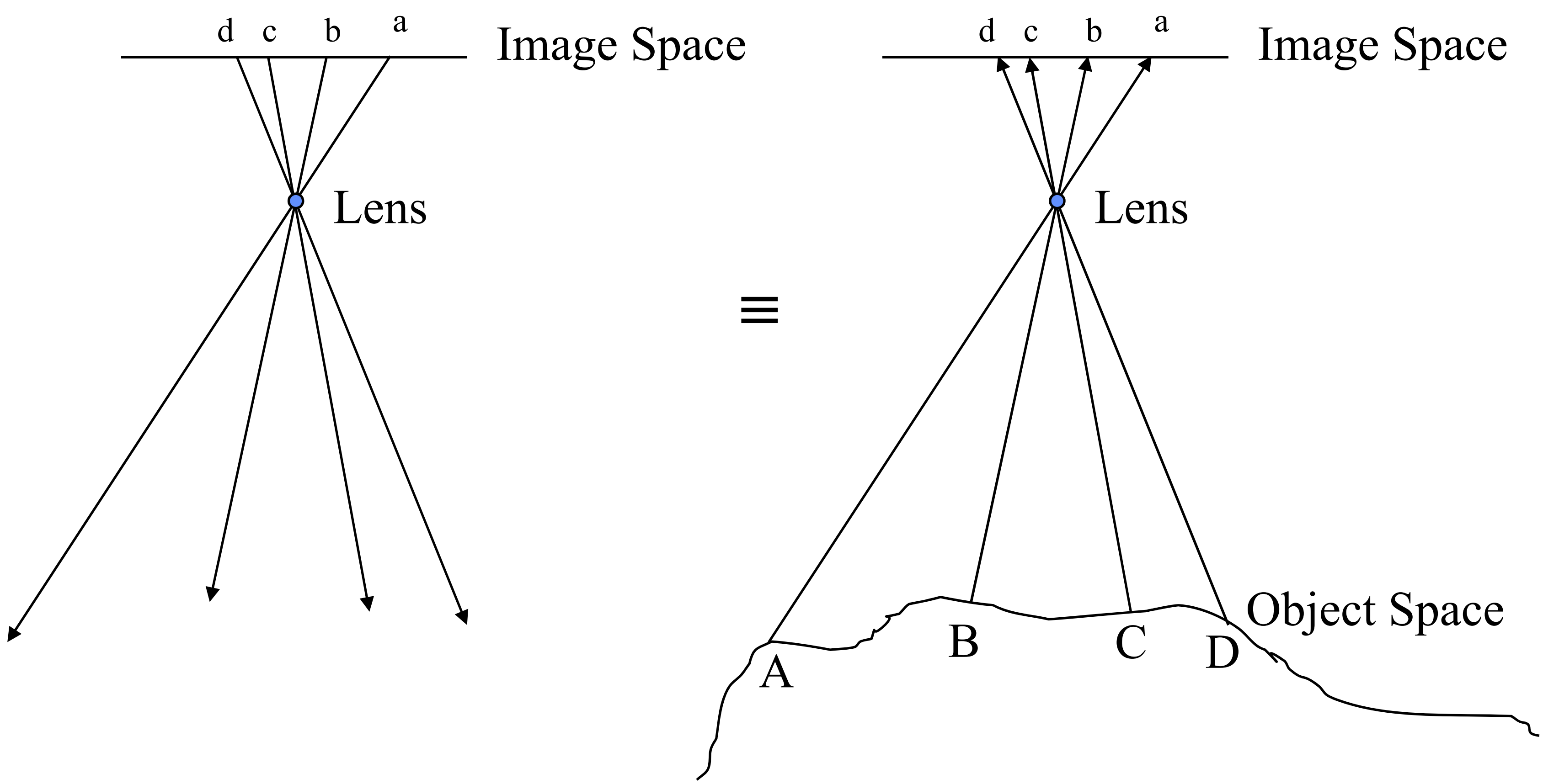
Photogrammetry



Interior Orientation

- Purpose: Reconstruct the bundle of light rays (as defined by the perspective center and the image points) in such a way that it is similar to the incident bundle onto the camera at the moment of exposure.
- Interior orientation is defined by the position of the perspective center w.r.t. the image coordinate system (x_p, y_p, c).
- Another component of the interior orientation is the distortion parameters.

Interior Orientation



IO: Target Function

Interior Orientation

- Alternative procedures for estimating the Interior Orientation Parameters (IOPs) include:
  - Laboratory camera calibration (Multi-collimators),
  - Indoor camera calibration, and
  - In-situ camera calibration.

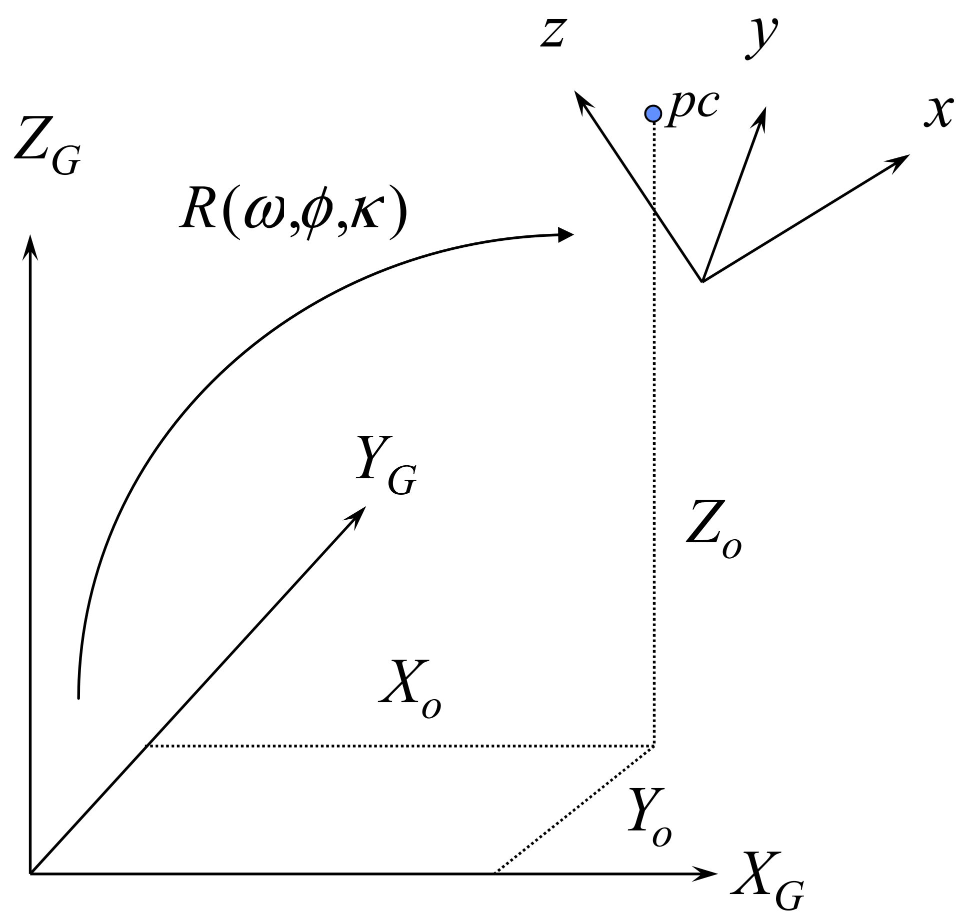
Analytical Camera Calibration

Georeferencing

- Georeferencing: the process of relating the sensor (i.e., camera or LiDAR unit) and ground coordinate systems.
- Defines the position and orientation information of the camera (image bundle) or the LiDAR unit (laser beam) at the moment of exposure.
- Traditionally, the georeferencing parameters for photogrammetric systems are obtained using Ground Control Points (GCPs) through a bundle adjustment procedure.
  - Indirect georeferencing

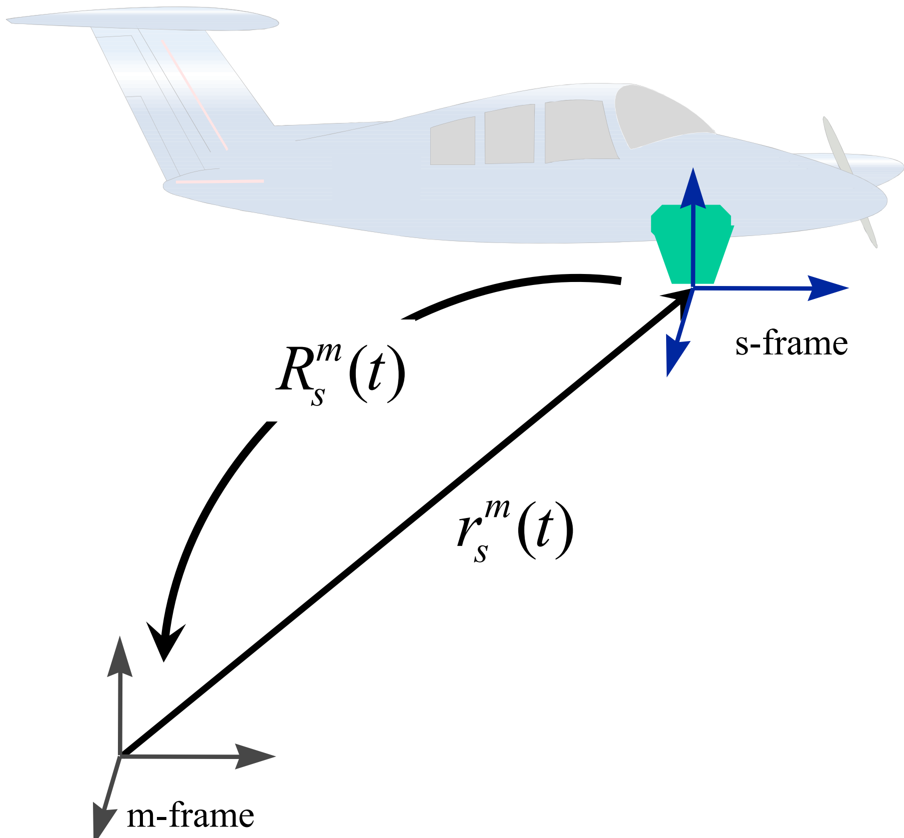
Georeferencing

Photogrammetric Mapping

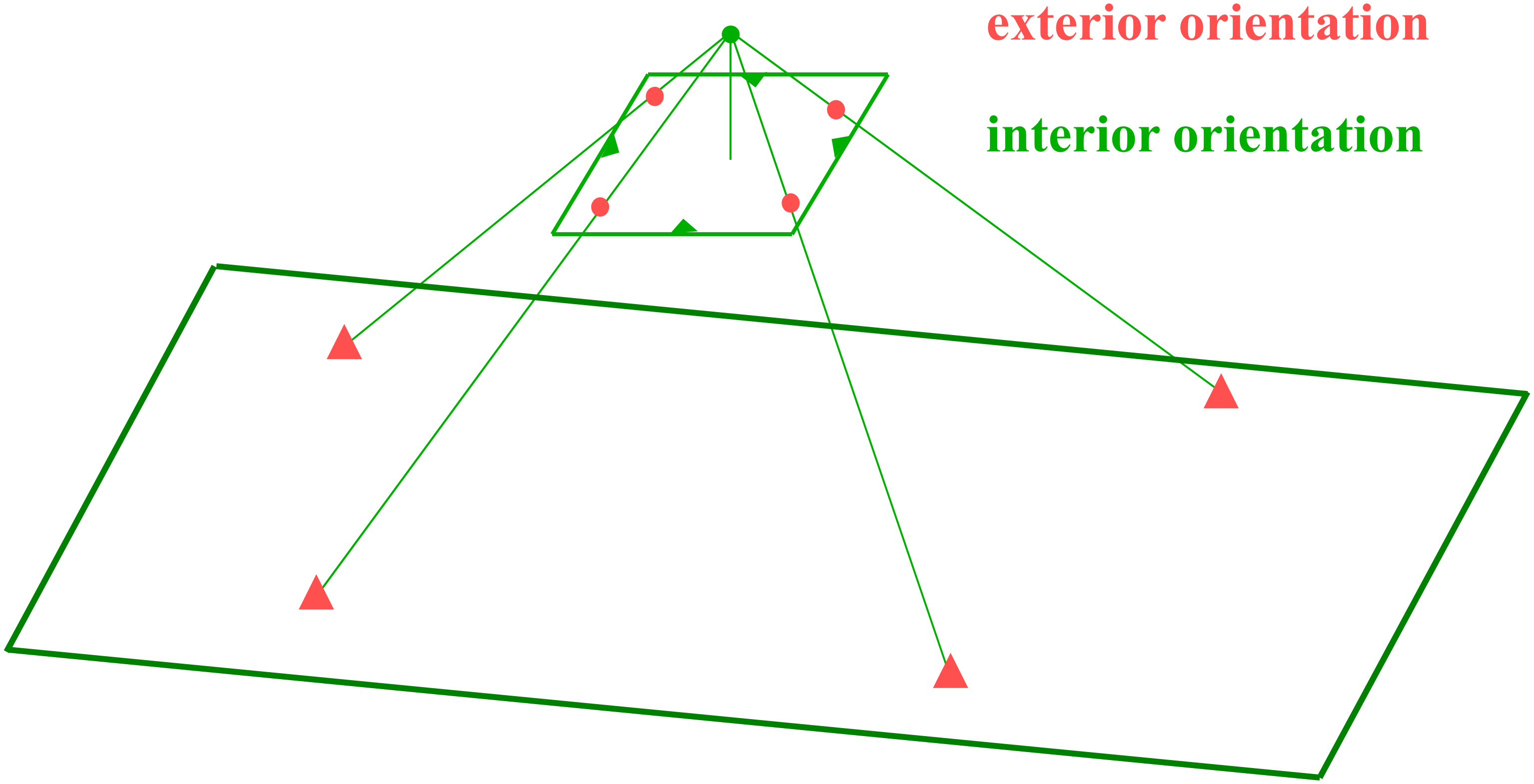


Georeferencing

- Exterior Orientation Parameters (EOPs) – georeferencing parameters – define the position, $r_s^m(t)$, and orientation $R_s^m(t)$, of the sensor coordinate system relative to the mapping reference frame at the moment of exposure.

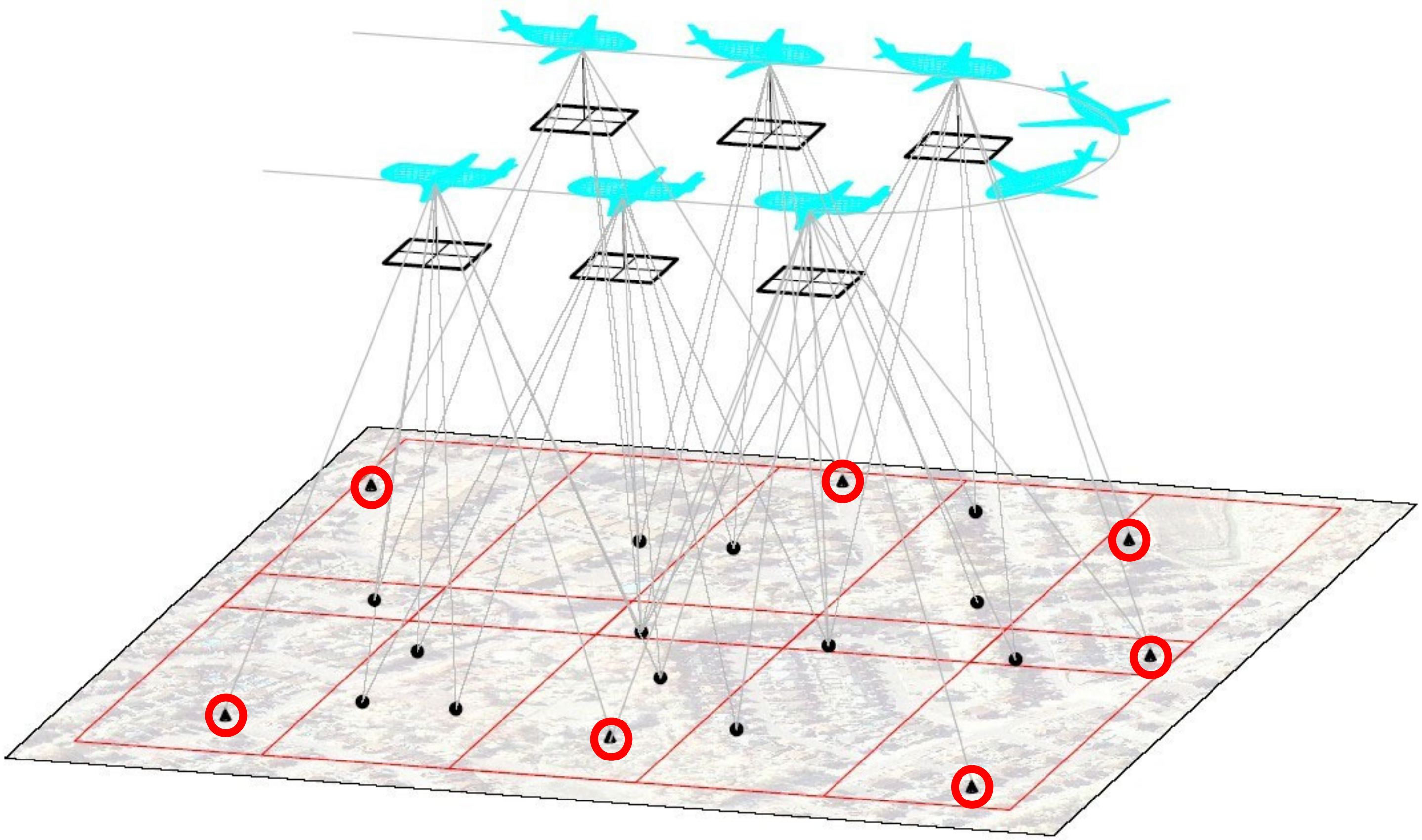


Indirect Georeferencing: Single Image



Single Photo Resection Procedure

Indirect Georeferencing: Image Block



Ground Control Points
- Tie Points

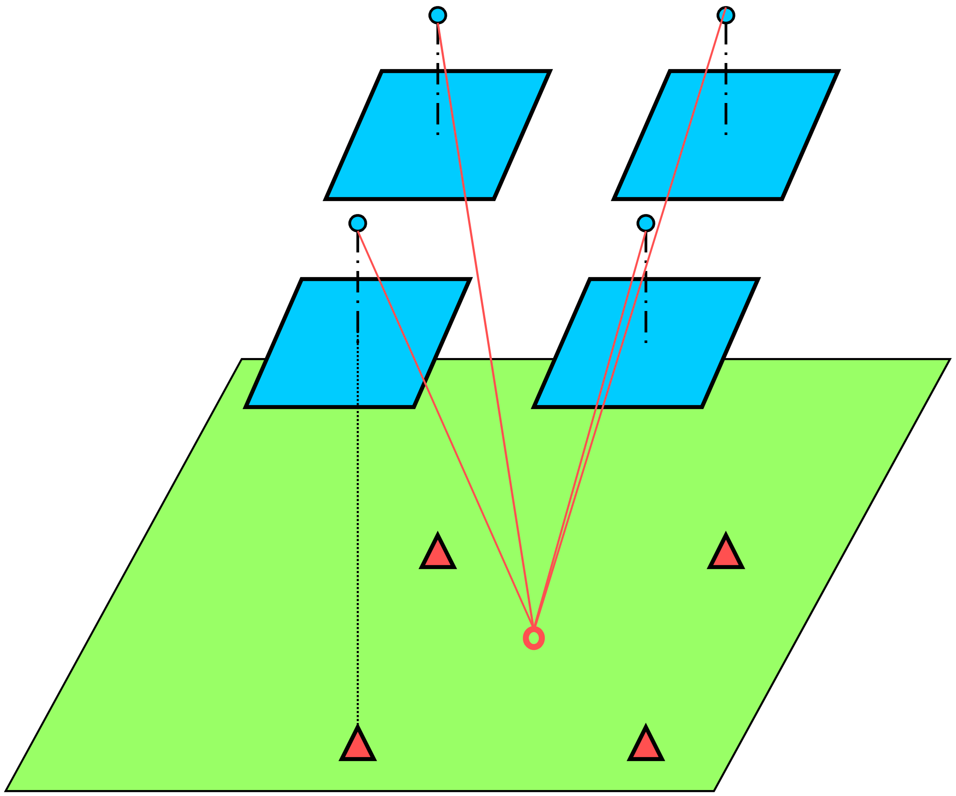
Bundle Adjustment Procedure

Photogrammetric Indirect Georeferencing

- Within the indirect georeferencing procedure, the Exterior Orientation Parameters (EOPs) are determined in such a way that:
  - Conjugate light rays intersect as well as possible, and
  - Light rays, which correspond to ground control points, pass as close as possible to their object space locations.
- In other words, the EOPs are indirectly determined to satisfy the above mentioned objectives.

Reconstruction with Indirect Georeferencing

Photogrammetric Mapping



Interpolation Process

LiDAR Indirect Georeferencing

- LiDAR georeferencing can be performed using signalized targets that can be identified in the point clouds).

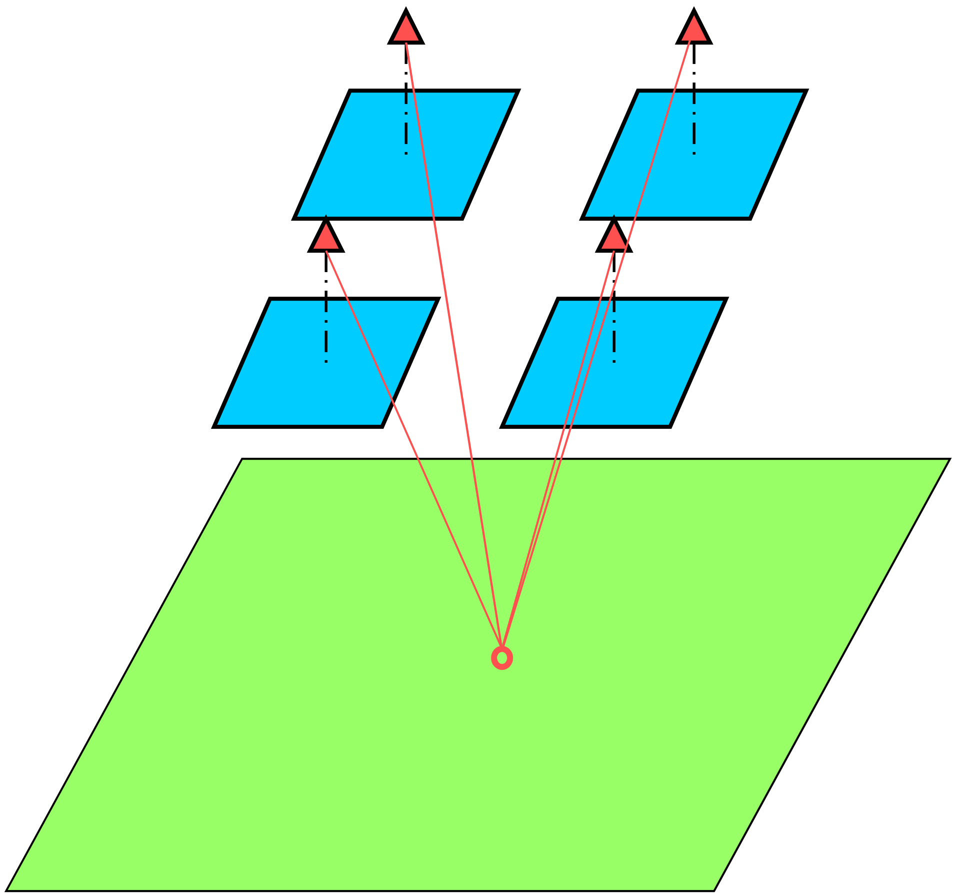
Example of targets used for the georeferncing & registration of terrestrial laser scans (photos courtesy of leica-geosystems)

Direct Georeferencing

- Nowadays, direct georeferencing is possible using an integrated DGNSS/INS.
- The position and orientation of each image/laser pulse is directly determined using on-board sensors without the need for GCPs.
  - Photogrammetric Systems: Economic advantages, especially in areas with poor or sparse control
  - Mobile LiDAR Systems: This is mandatory.
- Precaution:
  - Consider the spatial and temporal relationship between the involved sensors and derived measurements, respectively
  - Calibrating the entire system is essential.
    - System calibration encompass the individual sensor calibration as well as the mounting parameters (lever arm and boresight components) among the different sensors.

Direct Georeferencing

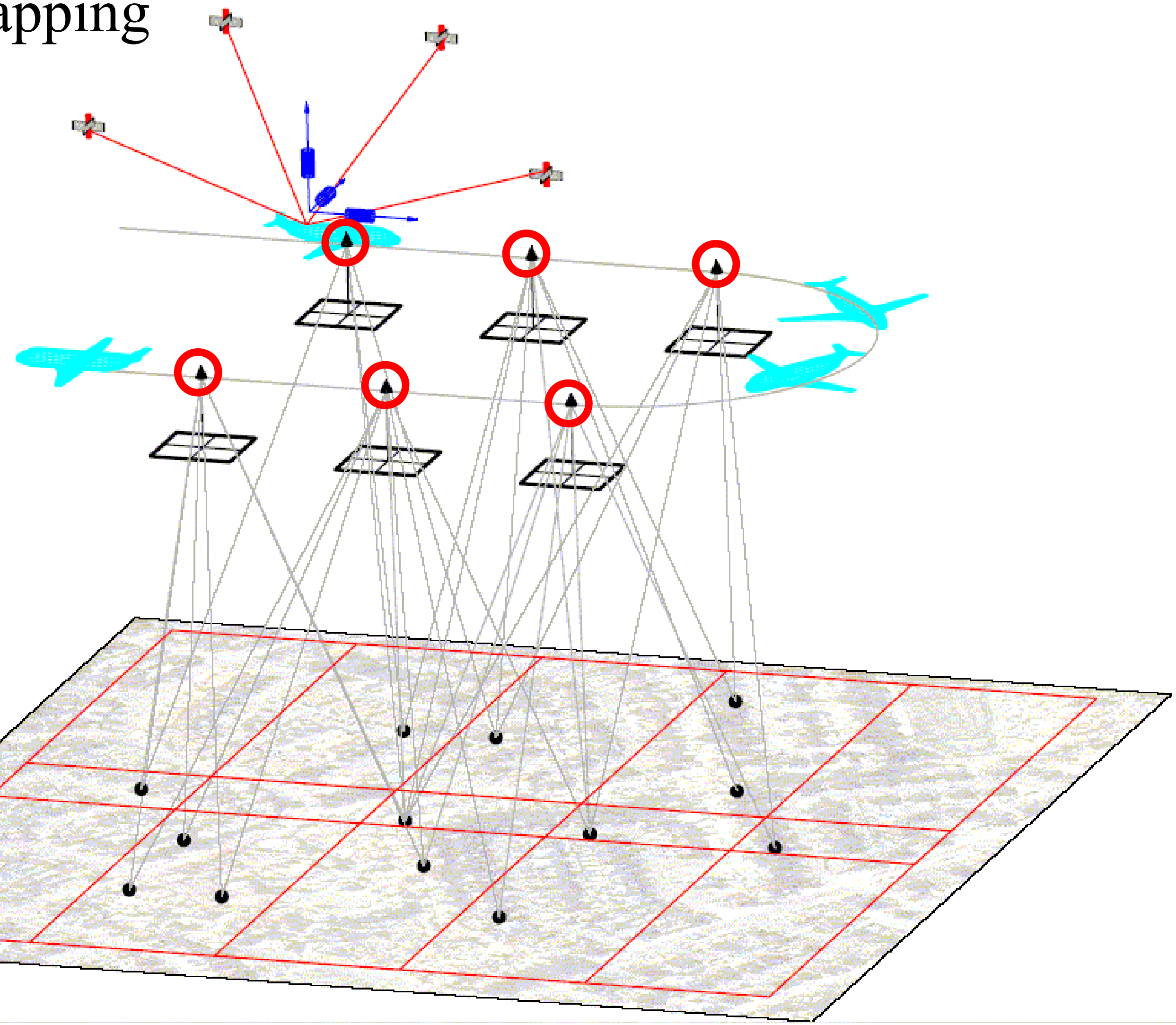
Photogrammetric Mapping



Extrapolation Process

Direct Georeferencing

Photogrammetric Mapping



- Ground Control Points
- Tie Points

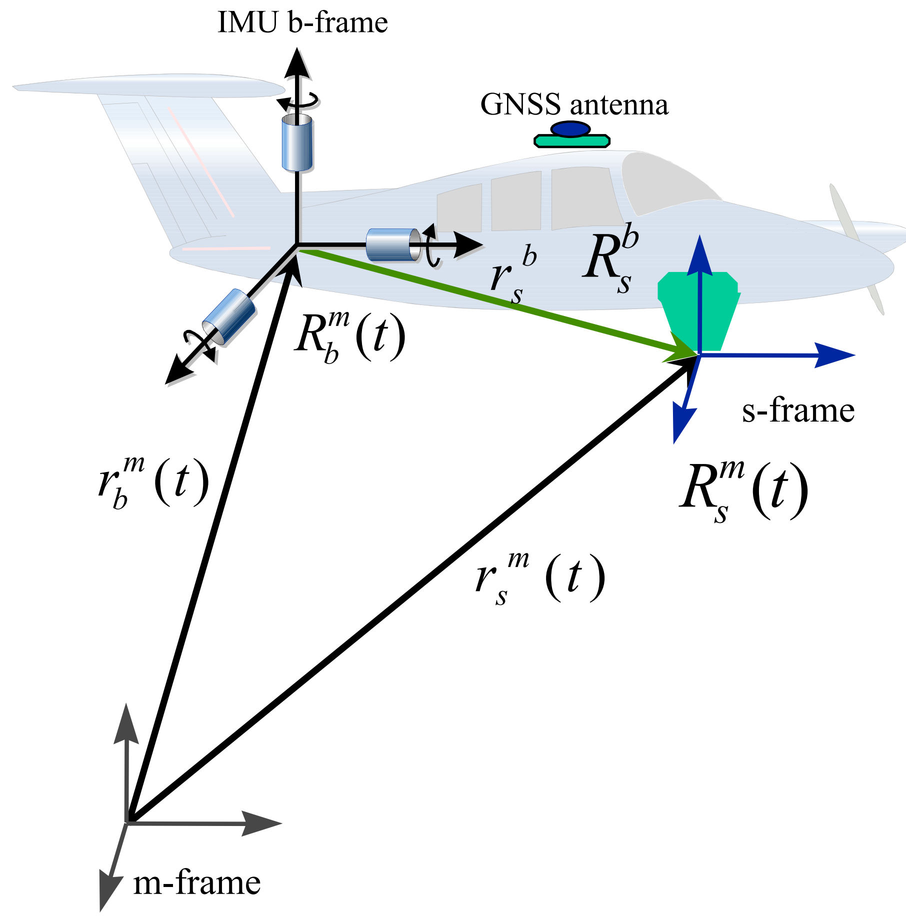
Direct Georeferencing

$$r_s^m(t) = r_b^m(t) + R_b^m(t)\, r_s^b$$

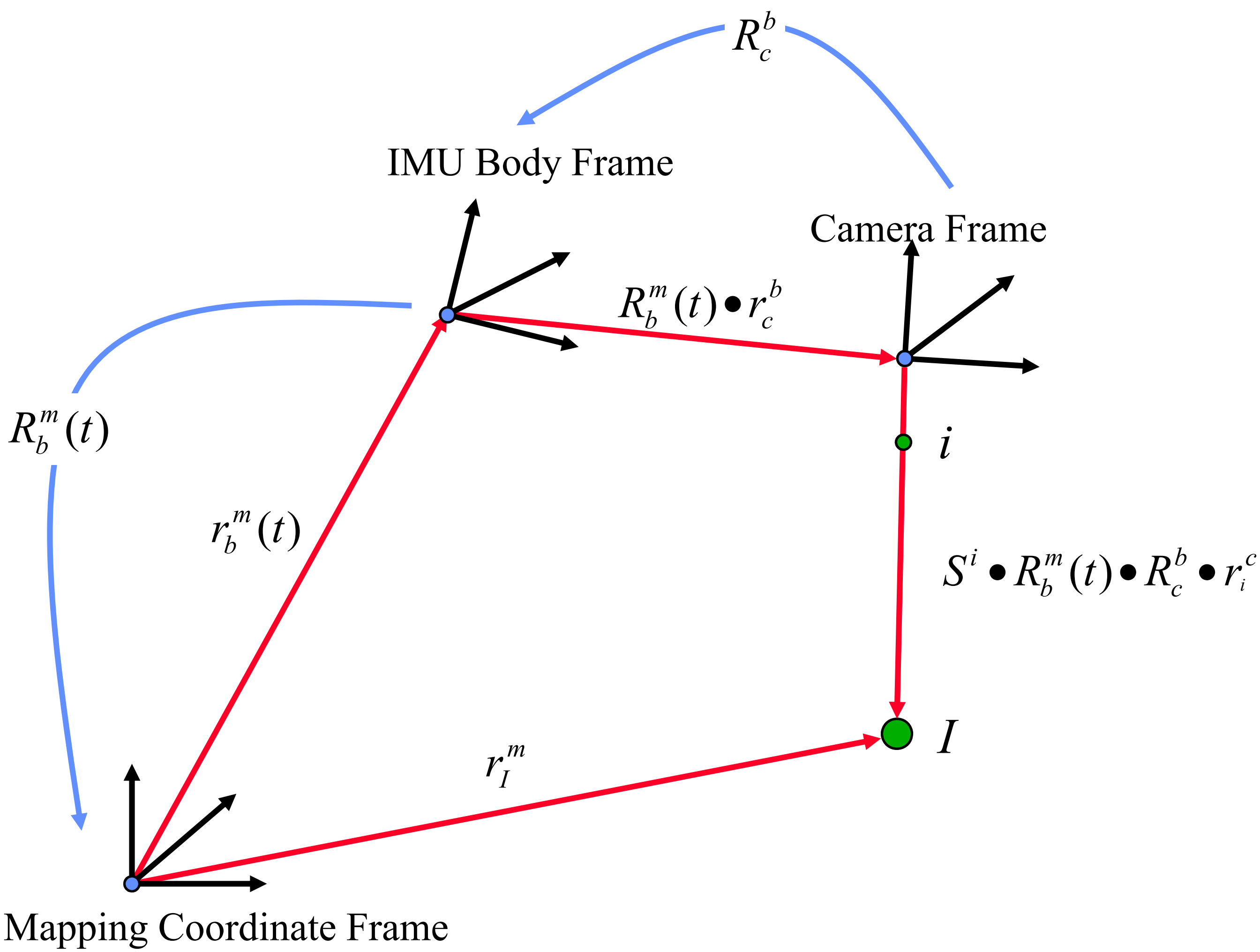
Sensor position — GNSS/INS position — GNSS/INS attitude — Calibration

$$R_s^m(t) = R_b^m(t)\; R_s^b$$

Sensor attitude — GNSS/INS attitude — Calibration



Direct Georeferencing: Single Image



- **With direct georeferencing, can we reconstruct the object space from a single image?**

Direct Georeferencing: Single Image

$$r_I^m = r_b^m(t) + R_b^m(t)[S^i \bullet R_c^b \bullet r_i^c + r_c^b]$$

r_I^m is the position vector of point (*I*) in the mapping frame (m-frame),

$r_b^m(t)$ is the interpolated position vector of the IMU b-frame in the m-frame,

S^i is a scale factor specific to one-image/one-point combination,

$R_b^m(t)$ is the interpolated rotation matrix between the IMU b-frame and the m-frame,

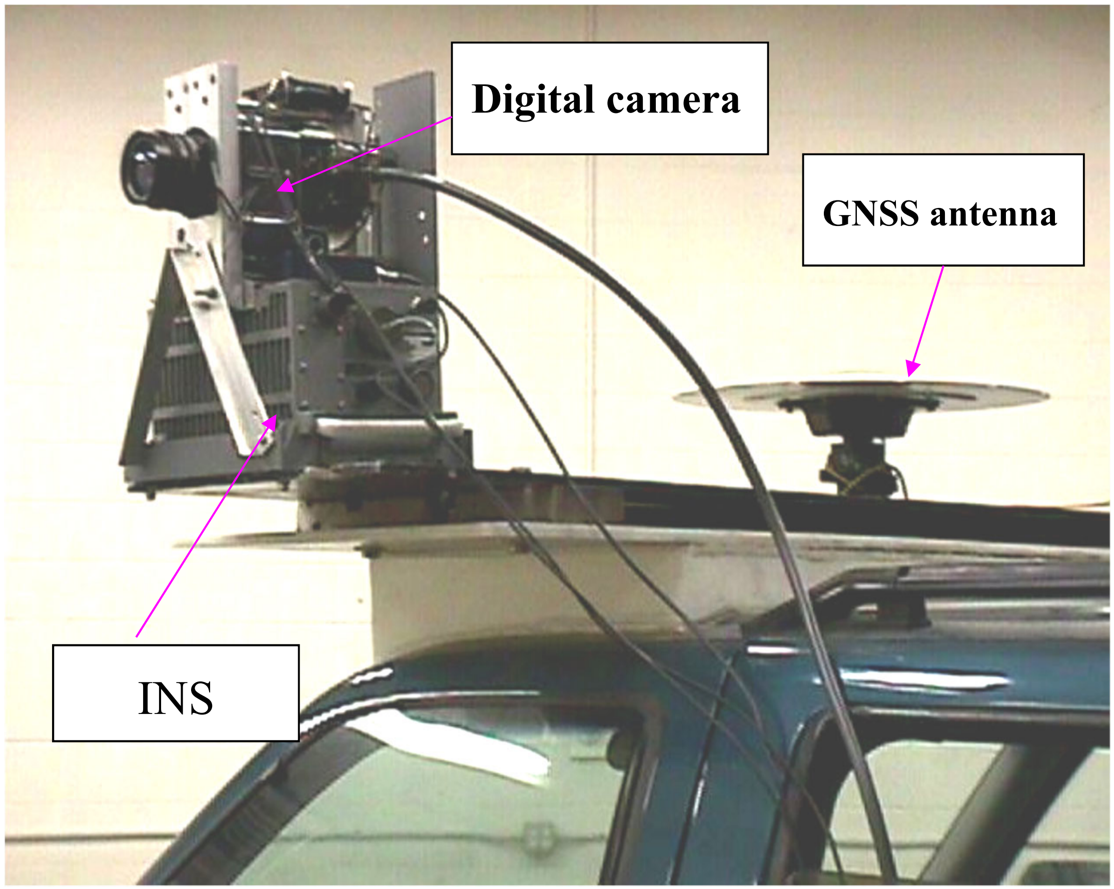
(t) is the time of exposure (i.e., the time of capturing the image),

R_c^b is the differential rotation between the camera frame (c-frame) and the b-frame,

r_i^c is the position vector of point (*i*) in the camera frame (c-frame), and

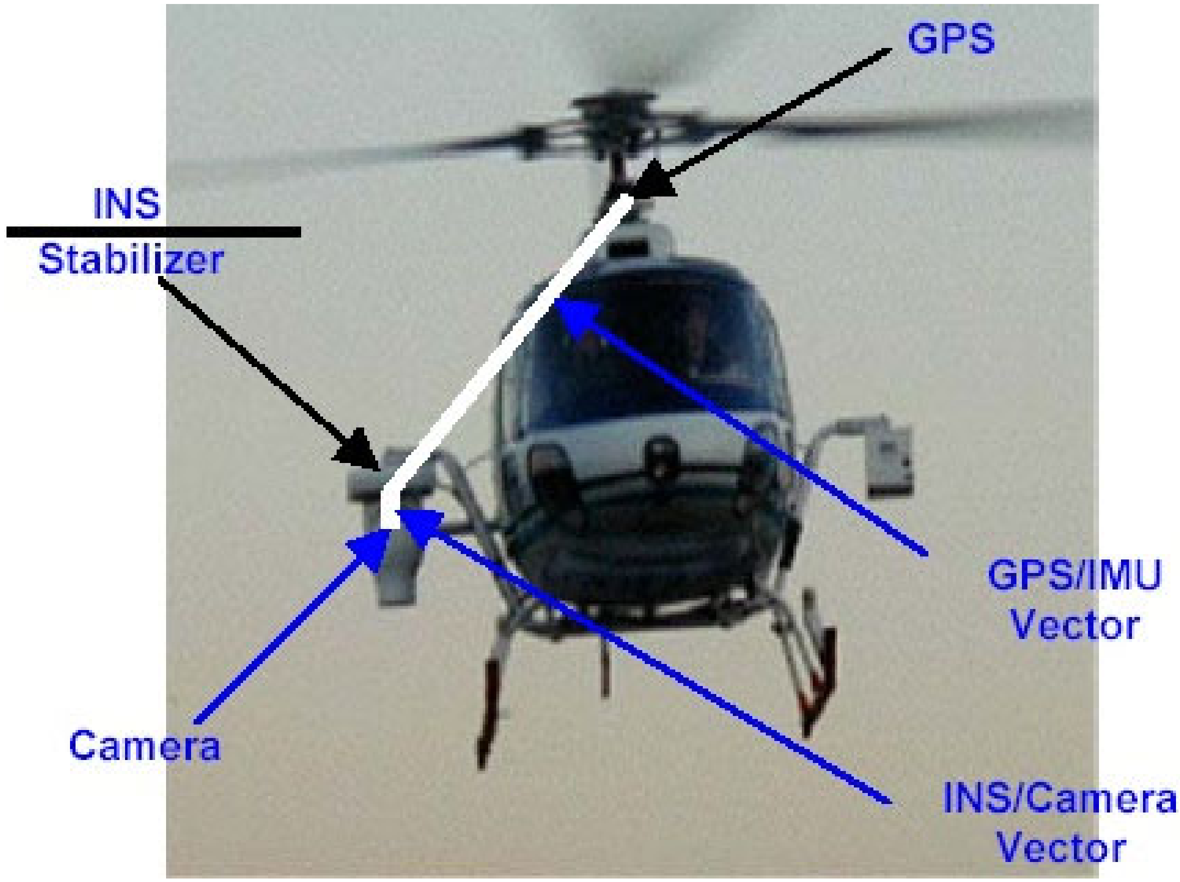
r_c^b is the offset between the camera and the IMU in the b-frame.

Direct Georeferencing: Land-based System



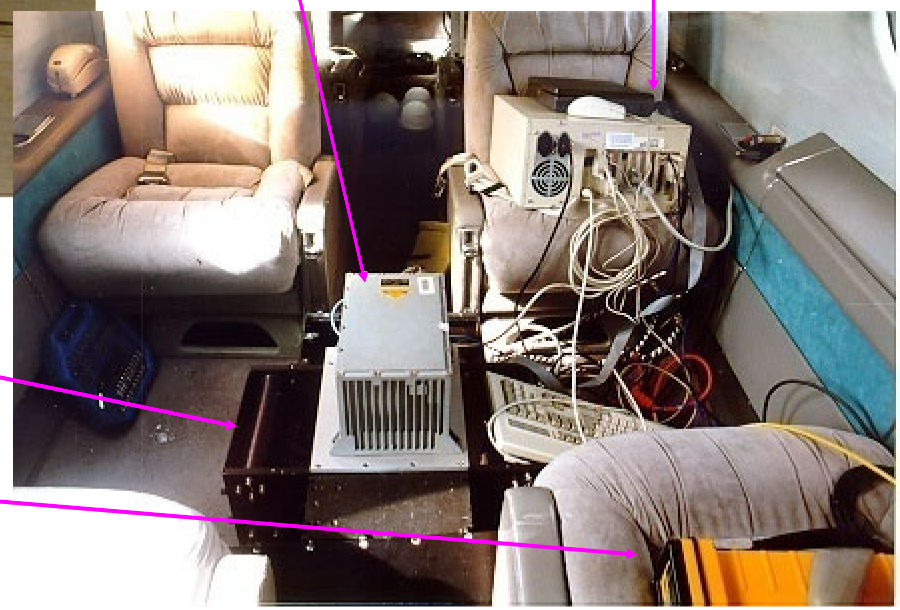
Direct georeferencing in practice

Direct Georeferencing: Airborne System

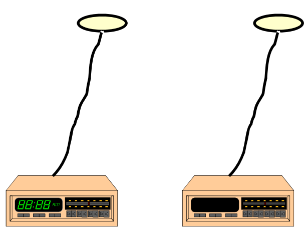


Direct georeferencing in practice

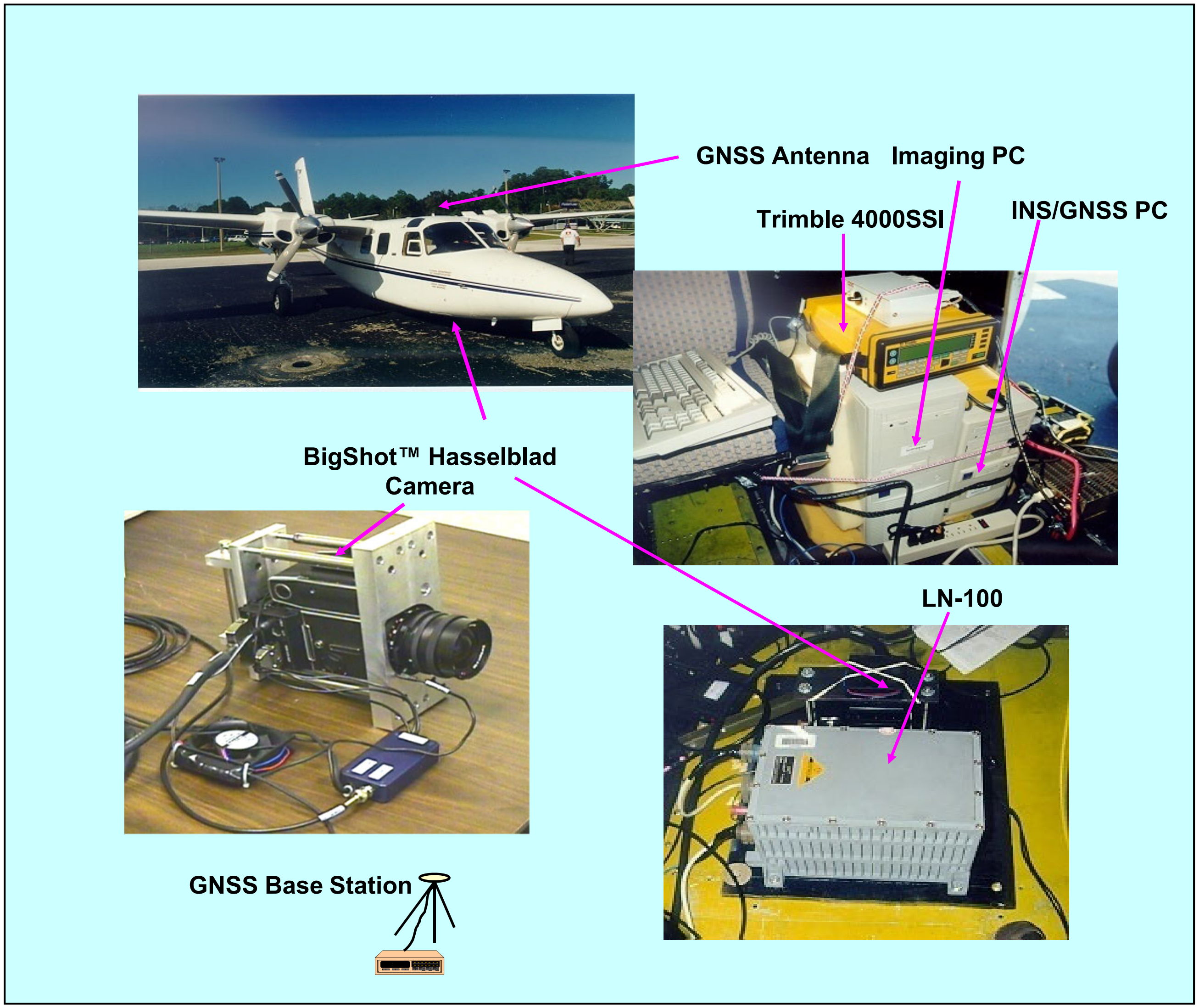
Direct Georeferencing: Airborne System



Two Base Stations



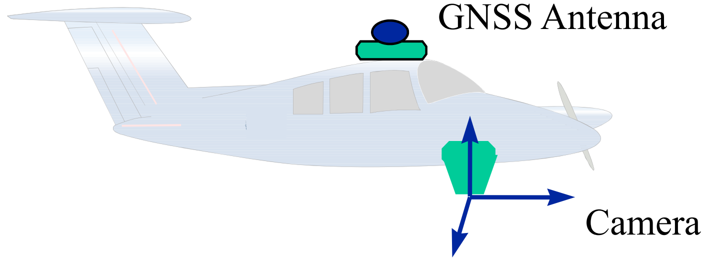
Direct Georeferencing



Hardware Configuration

Direct Georeferencing

Mobile Terrestrial Laser Scanning System



Integrated Sensor Orientation (ISO)

GNSS-Controlled Aerial Triangulation

Photogrammetric Mapping

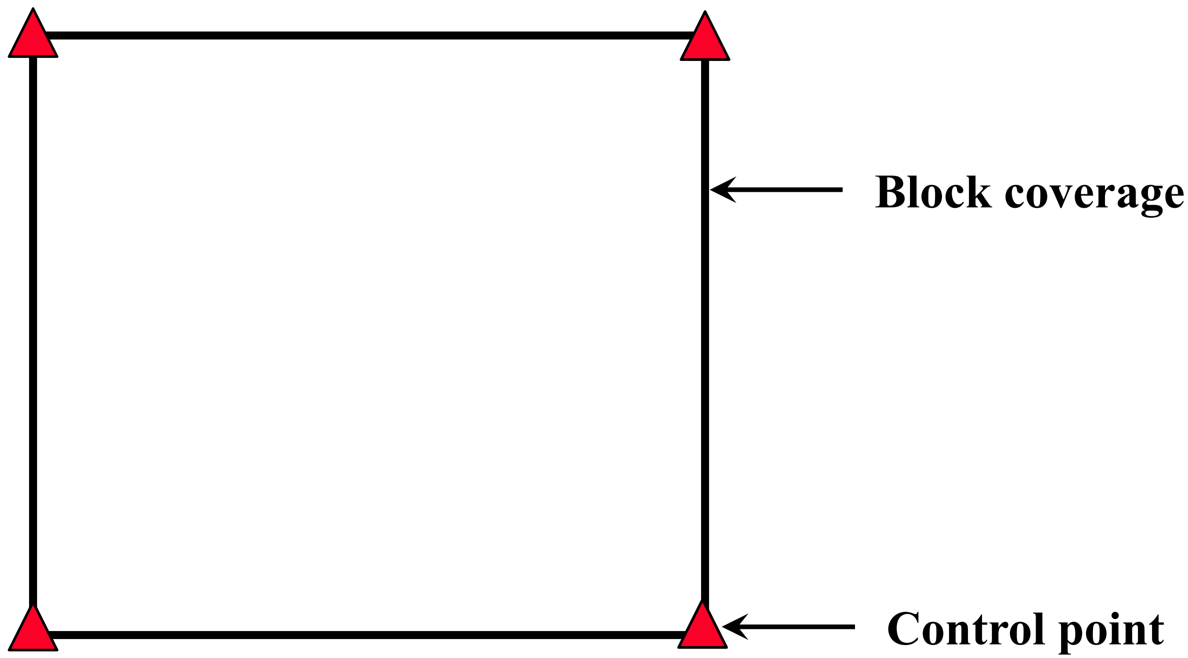
GNSS and Photogrammetry

- Role of GNSS in various photogrammetric activities:
  – Provide ground coordinates for control points
  – Pin-point photography to precisely execute a flight mission
  – Provide direct observations of the position of the projection center for bundle block adjustment
- The following slides will be concentrating on the last item, namely:
  – Derive the ground coordinates of the perspective center at the moment of exposure
    - GNSS-controlled aerial triangulation

GNSS-Controlled Aerial Triangulation

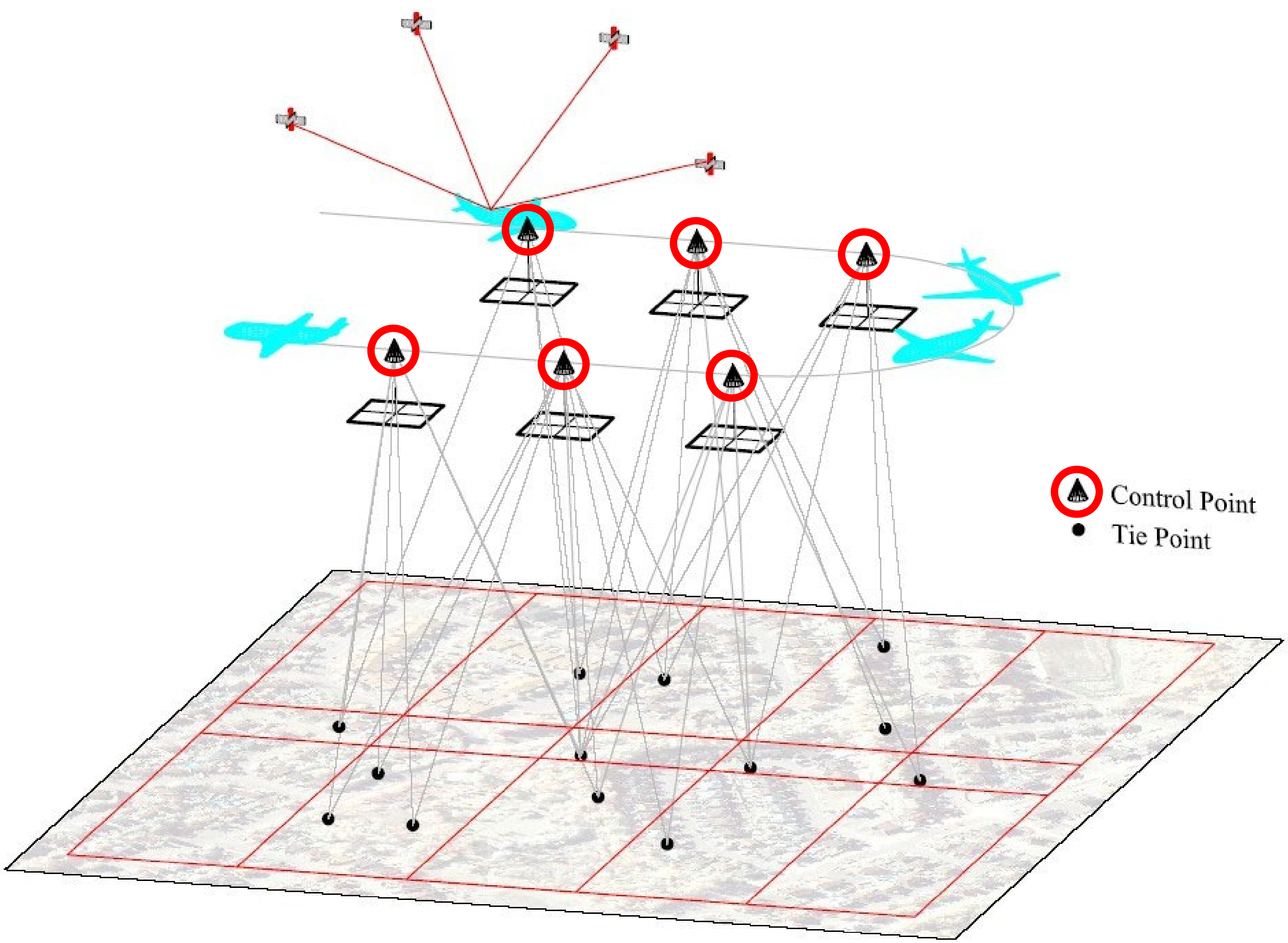
- Advantages:
  - GNSS observations at the aircraft can stabilize the heights along as well as across the strips.
  - GNSS observations at the aircraft would reduce (or even eliminate) the need for ground control points.
  - For normal-case photography over flat terrain, GNSS observations at the aircraft would decouple the correlation between the principal distance and the flying height (if we are performing self calibration).

GNSS-Controlled Aerial Triangulation



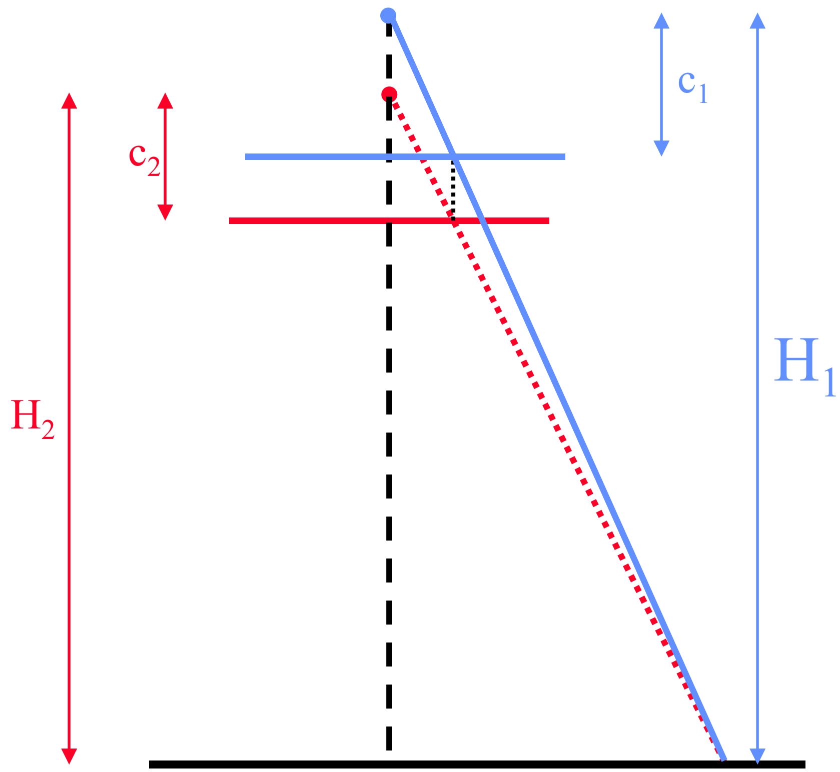
- The vertical accuracy within a block, which has control only at its corners, is worse at the center of the block.
- The vertical accuracy will deteriorate as the size of the block increases.
- Incorporating the GNSS observations at the exposure stations in the bundle adjustment procedure (GNSS-controlled aerial triangulation) would improve the vertical accuracy within the block.

GNSS-Controlled Aerial Triangulation



- GNSS position information at the exposure stations acts as control points which, if well-distributed, will define the datum.

GNSS-Controlled Aerial Triangulation



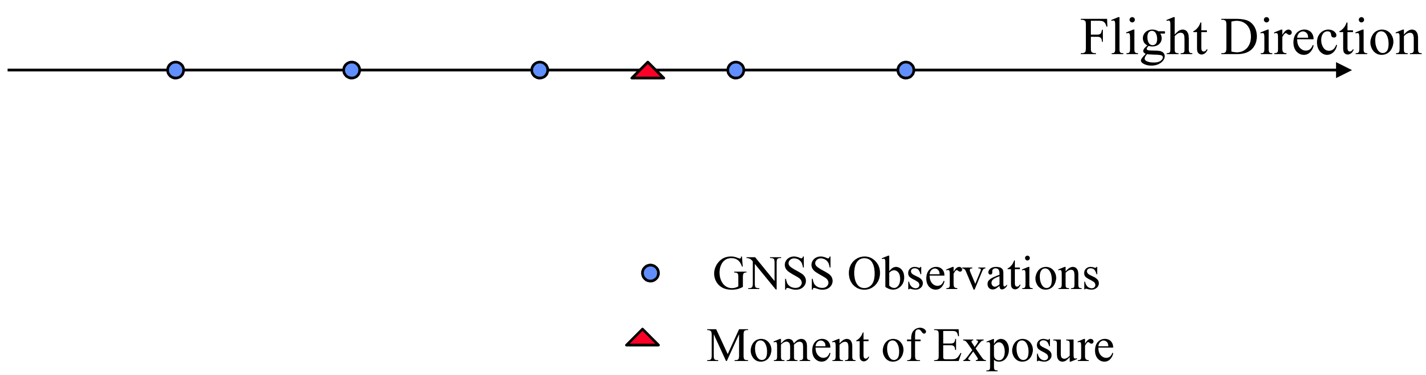
$$c_1/H_1 = c_2/H_2$$

- GNSS position information at the exposure stations will decouple the principal distance and the flying height.

GNSS-Controlled Aerial Triangulation

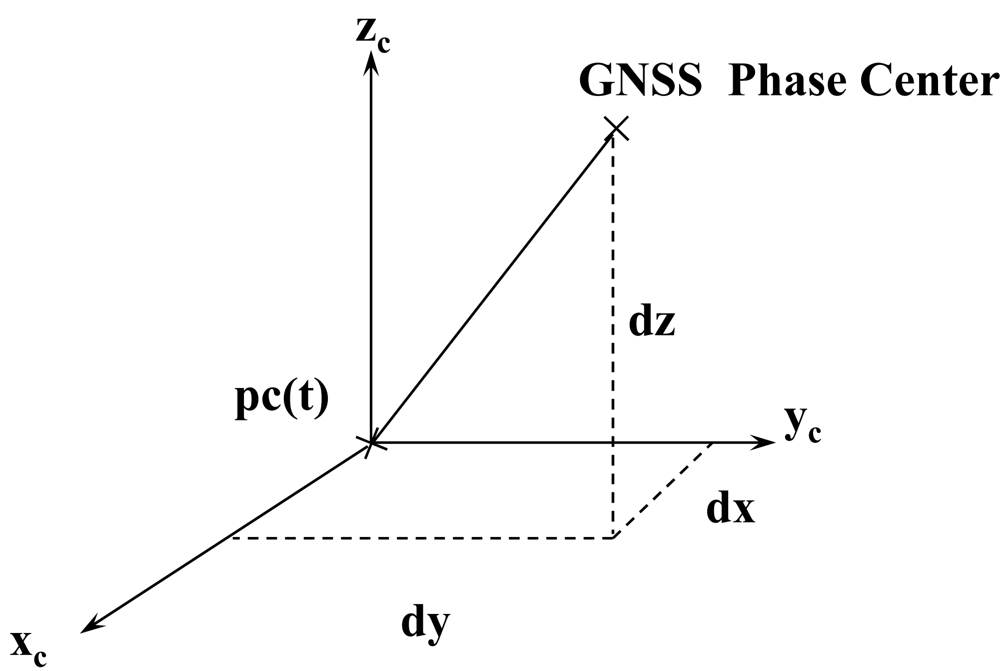
- Special Considerations:
  - Time offset between the epochs at which GNSS observations are collected and the moment of exposure
  - Spatial offset between the GNSS antenna phase center and the camera perspective center
  - Datum problem:
    - GNSS provides latitude, longitude, and ellipsoidal height.
    - GCPs might be represented by latitude, longitude, and orthometric height.
  - GNSS-controlled strip triangulation:
    - The roll angle across the flight direction cannot be determined without GCPs.
    - What is the minimum number of required GCPs?

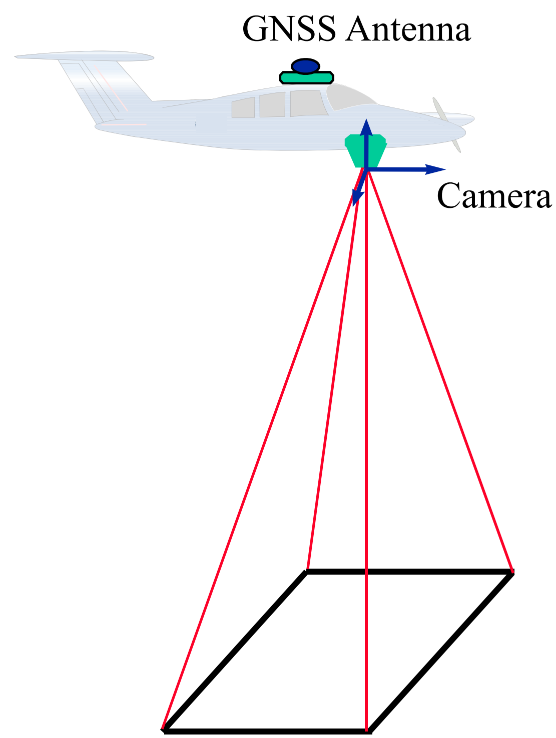
Time Offset



- The GNSS position has to be interpolated to the moment of exposure.
- In modern systems, there is a direct link between the camera and the GNSS receiver:
  - The camera is instructed to capture an image exactly at an epoch when GNSS observations are collected.

Spatial Offset



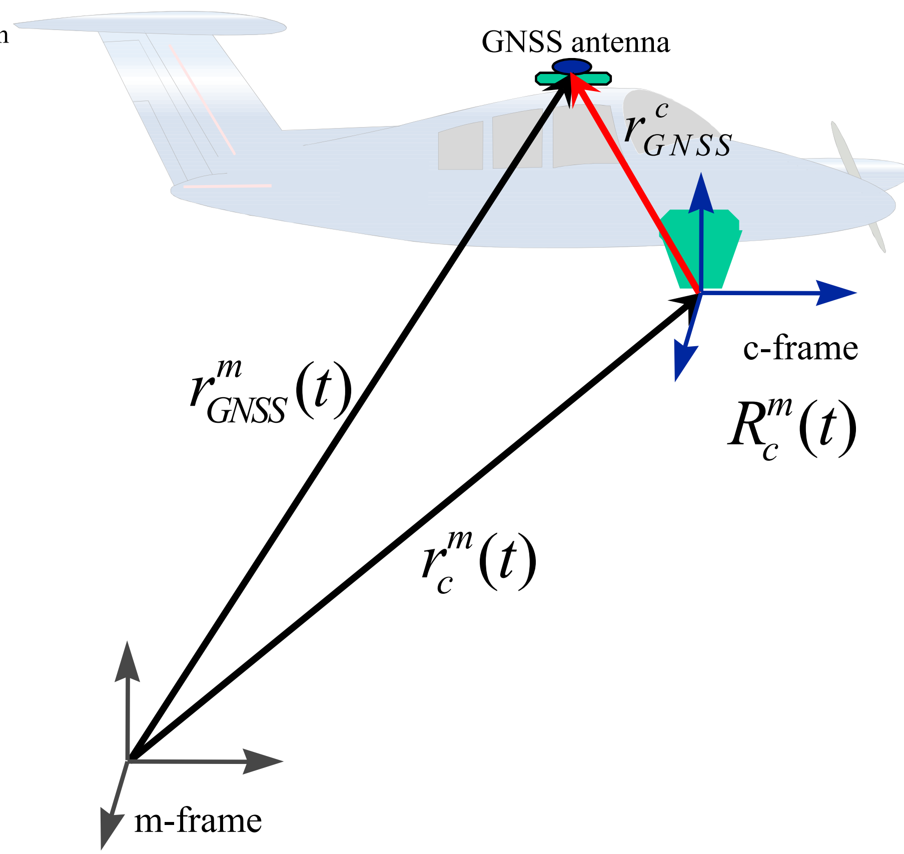


- The spatial offset has to be measured relative to the camera coordinate system.
  - The offset components do not change as the aircraft attitude changes.

Spatial Offset

$$r^m_{GNSS}(t) = r^m_c(t) + R^m_c(t)\, r^c_{GNSS}$$

GNSS position — Camera position — Camera attitude — Lever arm



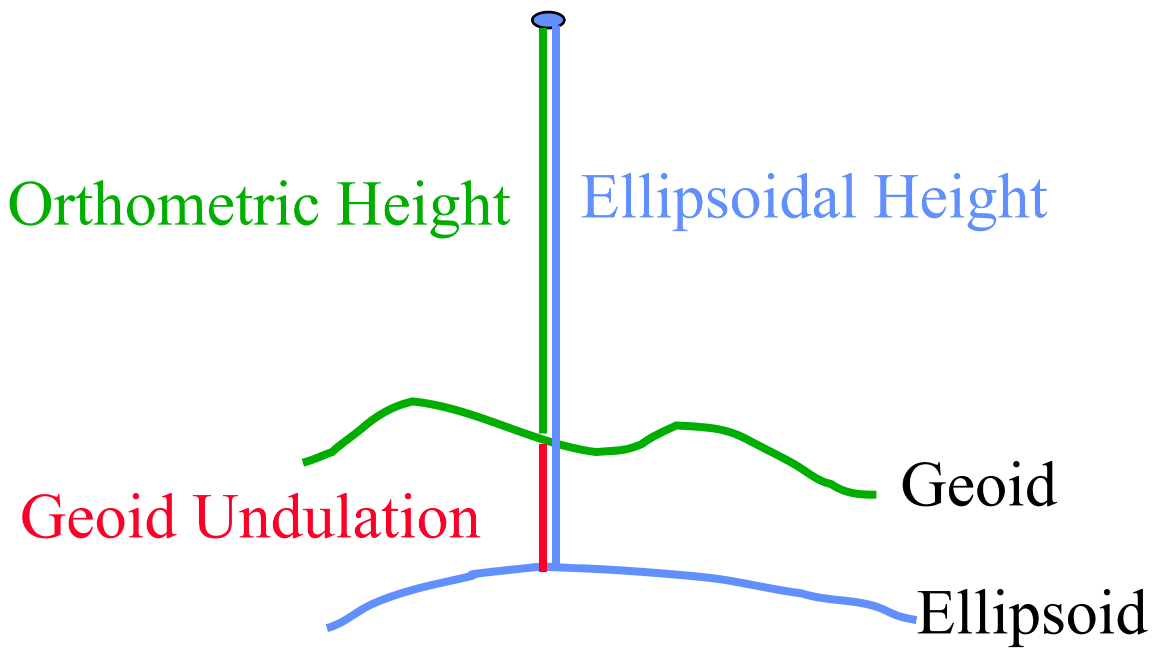
Spatial Offset

$$r^m_{GNSS}(t) = r^m_c(t) + R^m_c(t)\, \underbrace{r^c_{GNSS}}_{\text{Lever arm}} + e^m_{GNSS}(t)$$

$$\begin{bmatrix} X^t_{GNSS} \\ Y^t_{GNSS} \\ Z^t_{GNSS} \end{bmatrix} = \begin{bmatrix} X^t_o \\ Y^t_o \\ Z^t_o \end{bmatrix} + R(\omega_t, \varphi_t, \kappa_t) \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} + \begin{bmatrix} e_{x_{GNSS}} \\ e_{y_{GNSS}} \\ e_{z_{GNSS}} \end{bmatrix}$$

$$\begin{bmatrix} e_{x_{GNSS}} \\ e_{y_{GNSS}} \\ e_{z_{GNSS}} \end{bmatrix} \sim (\underline{0},\ \Sigma_{GNSS})$$

Datum Problem



- Be careful when you have the following:
  - GNSS observations at the aircraft, and
  - Ground control points.

Incorporating GNSS Observations: Remarks

- For GNSS observations at the aircraft, we have to:
  - Interpolate the GNSS position at the moment of exposure (time offset)
  - Determine the spatial offset between the GNSS antenna phase center and the camera perspective center (spatial offset – lever arm)
  - If you have GCPs, make sure that GNSS and ground control coordinates are referenced to the same mapping frame (datum problem)
- Problem: Camera stabilization device
  - The camera is rotated within the aircraft to have the optical axis as close as possible to the plumb line.

GNSS Observations: Mathematical Model

$$r^m_{GNSS}(t) = r^m_c(t) + R^m_c(t)\, r^c_{GNSS} + e^m_{GNSS}(t)$$

$$\begin{bmatrix} X^t_{GNSS} \\ Y^t_{GNSS} \\ Z^t_{GNSS} \end{bmatrix} = \begin{bmatrix} X^t_o \\ Y^t_o \\ Z^t_o \end{bmatrix} + R(\omega_t, \varphi_t, \kappa_t) \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} + \begin{bmatrix} e_{x_{GNSS}} \\ e_{y_{GNSS}} \\ e_{z_{GNSS}} \end{bmatrix}$$

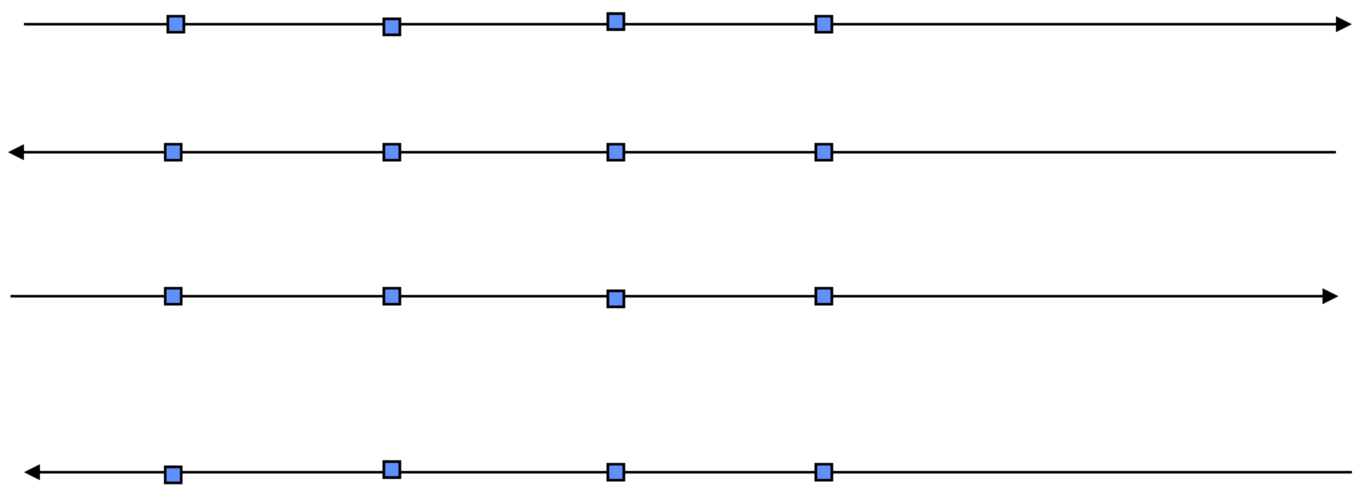
$$\begin{bmatrix} e_{x_{GNSS}} \\ e_{y_{GNSS}} \\ e_{z_{GNSS}} \end{bmatrix} \sim (\underline{0}, \Sigma_{GNSS})$$

- Used as additional observations in the bundle adjustment procedure

GNSS-Controlled Aerial Triangulation

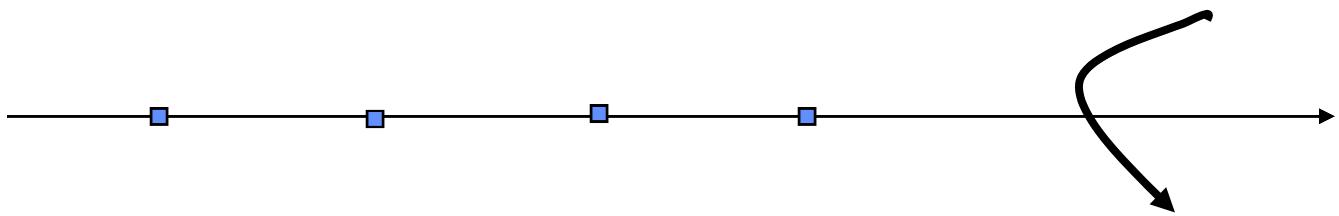
- We would like to investigate the possibility of carrying out GNSS-controlled aerial triangulation without the need for Ground Control Points (GCPs) when dealing with:
  - Block of images (multiple flight lines)
  - A single strip/flight line
- Remember: GNSS observations at the aircraft and/or GCPs are needed to establish the datum for the adjustment (AO).
  - We need at least three control points (either in the form of GNSS or GCPs) that are not collinear.

GNSS-Controlled Block Triangulation



- Theoretically, the adjustment can be carried out without the need for any GCPs.

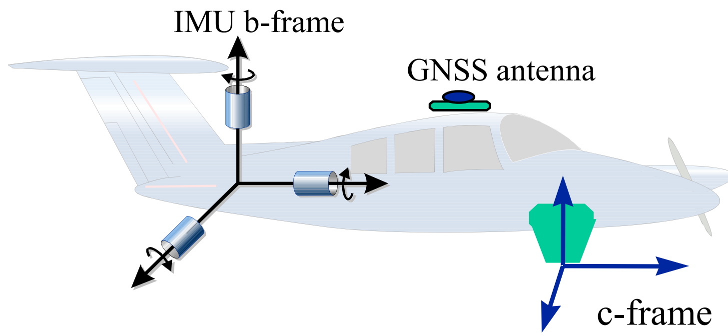
GNSS-Controlled Strip Triangulation



- The roll angle cannot be solved for.

GNSS-Controlled Aerial Triangulation

- Remarks:
  - GNSS onboard the imaging platform provides information about the position of the exposure station.
  - For photogrammetric reconstruction, the position and the attitude of the imaging system is required.
  - The attitude of the imaging system can be recovered through a GNSS-controlled aerial triangulation.
    - This is only possible for an image block.
    - For a single flight line, additional control is required to estimate the roll angle across the flight line.
      - The additional control can be provided using an Inertial Navigation System (INS) and/or Ground Control Points (GCPs).



Integrated Sensor Orientation (ISO)

GNSS/INS-Controlled Aerial Triangulation

Photogrammetric Mapping

GNSS/INS-Controlled Aerial Triangulation

- In such a case, we have a GNSS/INS unit onboard the mapping platform.
- The GNSS/INS-integrated position and attitude, which usually refer to the IMU body frame, can be used as an additional information in the triangulation procedure.
  - GNSS/INS-controlled aerial triangulation (Integrated Sensor Orientation)
- The following slides explain the procedure for the incorporation of the integrated GNSS/INS position and orientation information into the bundle adjustment procedure.

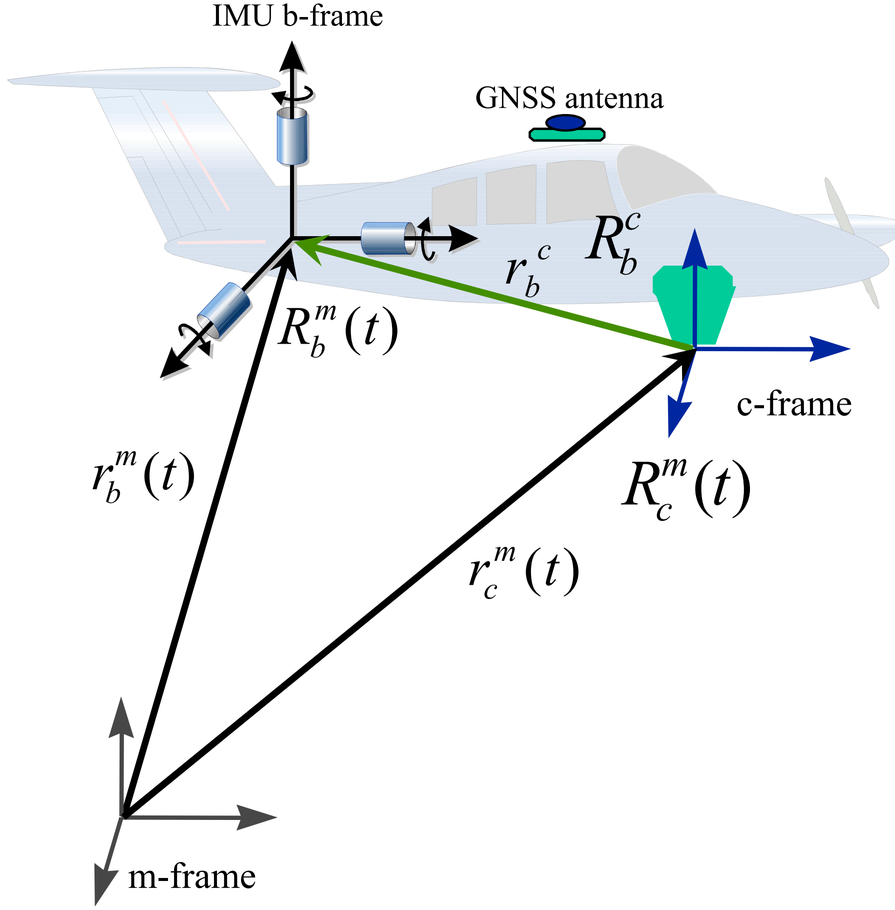
GNSS/INS-Controlled Aerial Triangulation

$$r_b^m(t) = r_c^m(t) + R_c^m(t)\, r_b^c$$

GNSS/INS position — Camera position — Camera attitude — Calibration

$$R_b^m(t) = R_c^m(t)\; R_b^c$$

GNSS/INS attitude — Camera attitude — Calibration



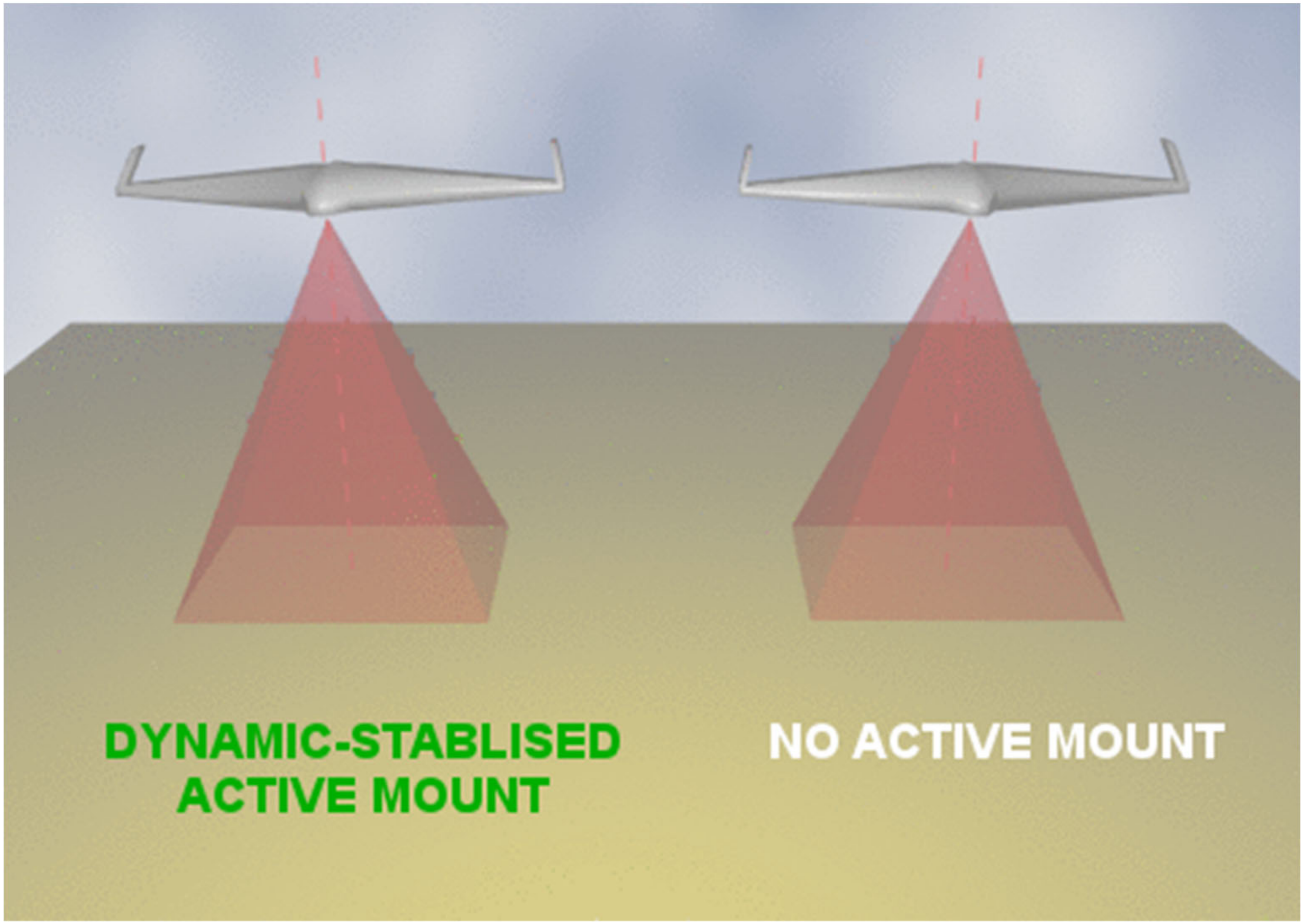
Incorporating GNSS/INS Position

- To incorporate the GNSS/INS-integrated position, we need to consider:
  - The spatial offset between the IMU body frame and the image coordinate system (lever arm)

$$r_b^m(t) = r_c^m(t) + R_c^m(t)\,\underbrace{r_b^c}_{\text{Lever arm}} + e_b^m(t)$$

- Problem: Camera stabilization device
  - The camera is rotated within the aircraft to have the optical axis as close as possible to the plumb line.

Camera Stabilization Device



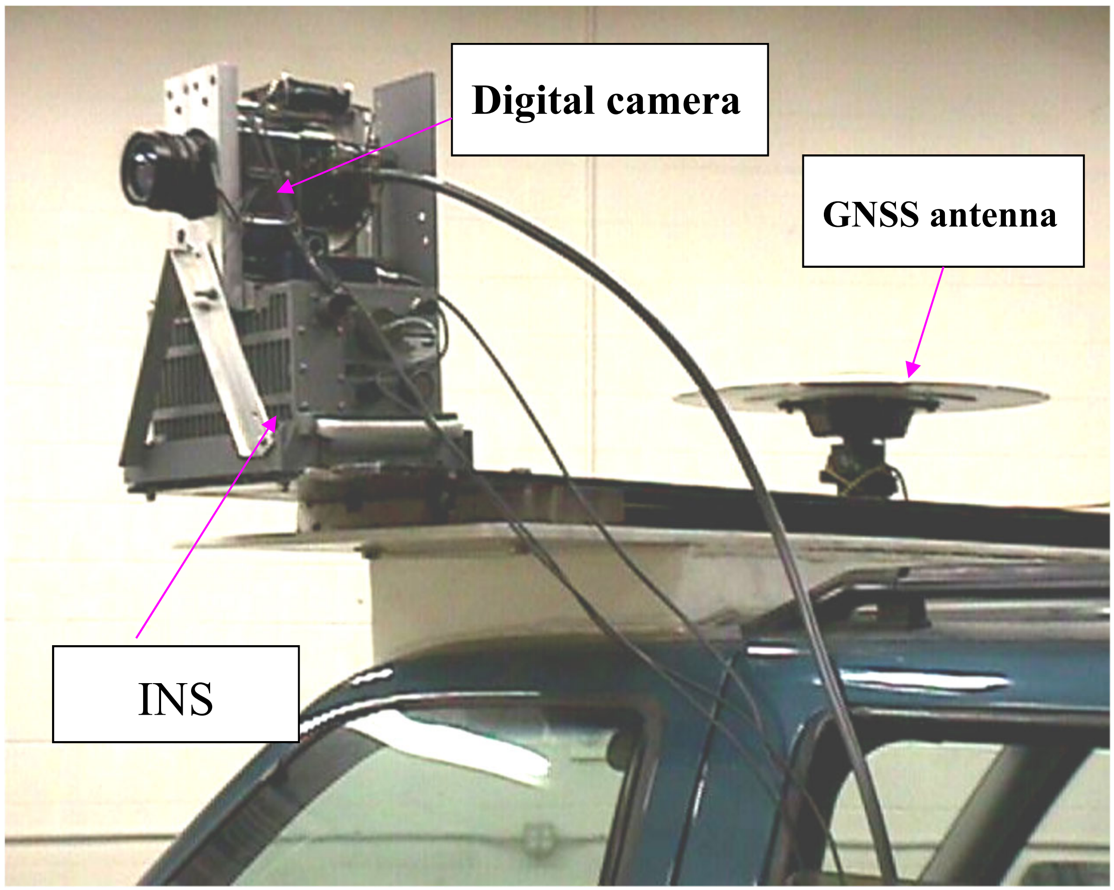
Incorporating GNSS/INS Attitude

- To incorporate the GNSS/INS-integrated attitude, we need to consider:
  - The rotational offset between the IMU body frame and the image coordinate system (boresight matrix)

$$R_b^m(t) = R_c^m(t)\ \underbrace{R_b^c}_{\text{Boresight matrix}}$$

- Problem: Camera stabilization device
  - The camera is rotated within the aircraft to have the optical axis as close as possible to the plumb line.

Incorporating GNSS/INS Position & Attitude



Incorporating GNSS/INS Position & Attitude

GNSS/INS Position: Mathematical Model

- The GNSS/INS-integrated position can be incorporated into the bundle adjustment according to the following model:

$$r_b^m(t) = r_c^m(t) + R_c^m(t)\, r_b^c + e_b^m(t)$$

$$\begin{bmatrix} X^t_{GNSS/INS} \\ Y^t_{GNSS/INS} \\ Z^t_{GNSS/INS} \end{bmatrix} = \begin{bmatrix} X^t_o \\ Y^t_o \\ Z^t_o \end{bmatrix} + R(\omega_t, \varphi_t, \kappa_t) \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} + \begin{bmatrix} e_{x_{GNSS/INS}} \\ e_{y_{GNSS/INS}} \\ e_{z_{GNSS/INS}} \end{bmatrix}$$

$$\begin{bmatrix} e_{x_{GNSS/INS}} \\ e_{y_{GNSS/INS}} \\ e_{z_{GNSS/INS}} \end{bmatrix} \sim (\underline{0}, \Sigma_{GNSS/INS})$$

  - Used as additional observations in the bundle adjustment procedure

GNSS/INS Attitude: Mathematical Model

- The GNSS/INS-integrated attitude can be incorporated into the bundle adjustment according to the following model:

$$R_b^m(t) = R_c^m(t)\ R_b^c$$

- 9 Equations
- Should be reduced to 3 independent equations

$$R_b^m{}_{(1,2)}(t) = (R_c^m(t) R_b^c)_{(1,2)} + e_{R_b^m{}_{(1,2)}(t)}$$

$$R_b^m{}_{(1,3)}(t) = (R_c^m(t) R_b^c)_{(1,3)} + e_{R_b^m{}_{(1,3)}(t)}$$

$$R_b^m{}_{(2,3)}(t) = (R_c^m(t) R_b^c)_{(2,3)} + e_{R_b^m{}_{(2,3)}(t)}$$

- Used as additional observations in the bundle adjustment procedure

GNSS/INS Attitude: Mathematical Model

- If the GNSS/INS-attitude angles have been reduced to the camera coordinate system, we can use the following model:

$$\begin{bmatrix} \omega_{t_{GNSS/INS}} \\ \varphi_{t_{GNSS/INS}} \\ \kappa_{t_{GNSS/INS}} \end{bmatrix} = \begin{bmatrix} \omega_t \\ \varphi_t \\ \kappa_t \end{bmatrix} + \begin{bmatrix} e_\omega \\ e_\varphi \\ e_\kappa \end{bmatrix}$$

$$\begin{bmatrix} e_\omega \\ e_\varphi \\ e_\kappa \end{bmatrix} \sim (\underline{0}\ , \Sigma_{GNSS/INS})$$

  - Used as additional observations in the bundle adjustment procedure

GNSS/INS-Controlled Aerial Triangulation

- Questions:
  - Do we need additional control in a GNSS/INS-controlled aerial triangulation?
    - Image block?
    - Single flight line?
  - For object reconstruction, do we need to perform a triangulation procedure?
    - Can we simply use intersection for object space reconstruction?
      - Direct georeferencing
- Answers:
  - Refer to the next section

Direct Georeferencing

Simple Intersection Procedure

Photogrammetric Mapping

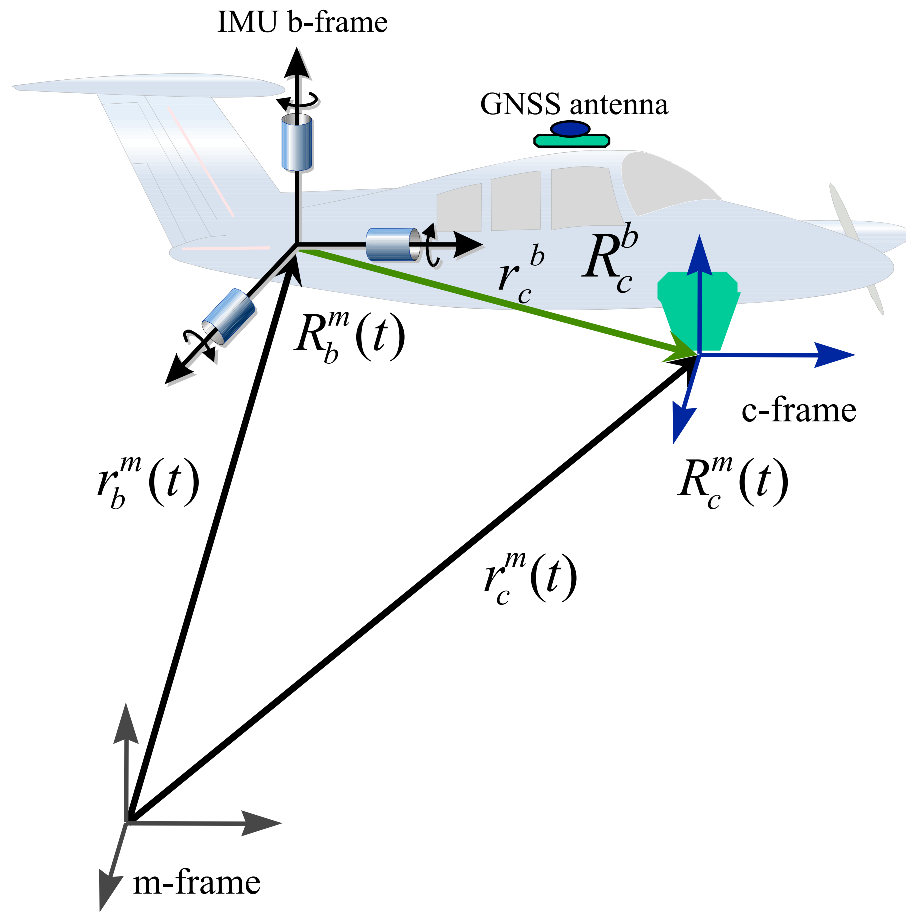
Direct Georeferencing

$$r_c^m(t) = r_b^m(t) + R_b^m(t)\, r_c^b$$

Camera position — GNSS/INS position — GNSS/INS attitude — Calibration

$$R_c^m(t) = R_b^m(t)\; R_c^b$$

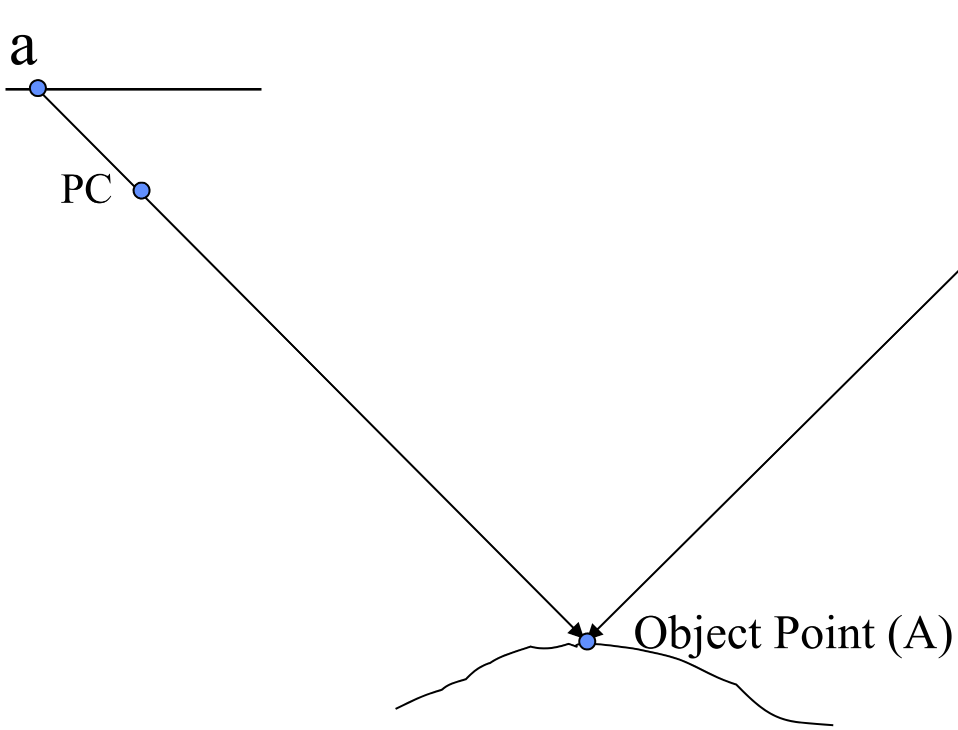
Camera attitude — GNSS/INS attitude — Calibration



Direct Georeferencing & Intersection

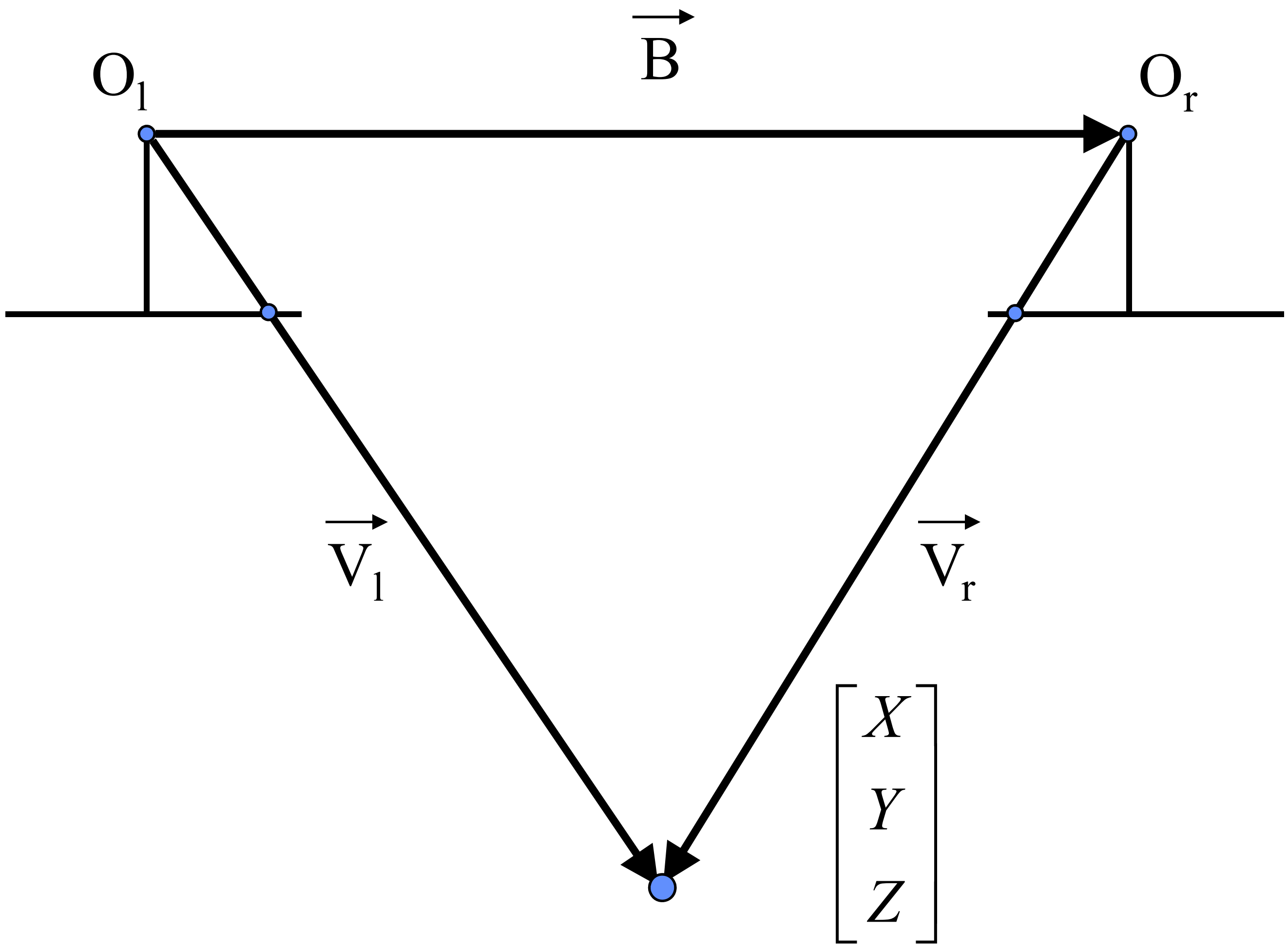
- The EOPs of the images are directly derived from the integrated GNSS/INS-position and attitude information.
  - The lever arm and the boresight matrix relating the camera and IMU coordinate systems are available from a system calibration procedure.
- The IOPs of the involved camera(s) are also available.
- We want to estimate the ground coordinates of points in the overlap area among the involved images.
- For each tie point, we have:
  - 2* n Observation equations (n is the number of images where the tie point has been observed)
  - 3 Unknowns
- Non-linear model: approximations are needed.

Direct Georeferencing & Intersection



Special Case: Stereo-pair

Intersection: Linear Model



Special Case: Stereo-pair

Intersection: Linear Model

$$\vec{B} = \begin{bmatrix} X_{o_r} - X_{o_l} \\ Y_{o_r} - Y_{o_l} \\ Z_{o_r} - Z_{o_l} \end{bmatrix}$$

- These vectors are given w.r.t. the ground coordinate system.

$$\vec{V}_l = \lambda \; R_{(\omega_l,\, \phi_l,\, \kappa_l)} \begin{bmatrix} x_l - x_p - dist_x \\ y_l - y_p - dist_y \\ -c \end{bmatrix}$$

$$\vec{V}_r = \mu \; R_{(\omega_r,\, \phi_r,\, \kappa_r)} \begin{bmatrix} x_r - x_p - dist_x \\ y_r - y_p - dist_y \\ -c \end{bmatrix}$$

Intersection: Linear Model

$$\vec{V}_l = \vec{B} + \vec{V}_r$$

$$\begin{bmatrix} X_{o_r} - X_{o_l} \\ Y_{o_r} - Y_{o_l} \\ Z_{o_r} - Z_{o_l} \end{bmatrix} = \lambda \, R_{(\omega_l,\, \phi_l,\, \kappa_l)} \begin{bmatrix} x_l - x_p - dist_x \\ y_l - y_p - dist_y \\ -c \end{bmatrix} - \mu \, R_{(\omega_r,\, \phi_r,\, \kappa_r)} \begin{bmatrix} x_r - x_p - dist_x \\ y_r - y_p - dist_y \\ -c \end{bmatrix}$$

- Three equations in two unknowns (λ, μ).
- They are linear equations.

Intersection: Linear Model

$$\begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}_l = \begin{bmatrix} X_{o_l} \\ Y_{o_l} \\ Z_{o_l} \end{bmatrix} + \hat{\lambda}\, R_{(\omega_l, \phi_l, \kappa_l)} \begin{bmatrix} x_l - x_p - dist_x \\ y_l - y_p - dist_y \\ -c \end{bmatrix},$$

$$\begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}_r = \begin{bmatrix} X_{o_r} \\ Y_{o_r} \\ Z_{o_r} \end{bmatrix} + \hat{\mu}\, R_{(\omega_r, \phi_r, \kappa_r)} \begin{bmatrix} x_r - x_p - dist_x \\ y_r - y_p - dist_y \\ -c \end{bmatrix}, \textbf{ or}$$

, weighted average of the above two estimates

Intersection: Multi-Light Ray Intersection

$$\begin{bmatrix} x_i^j - x_p - dist_x \\ y_i^j - y_p - dist_y \\ -c \end{bmatrix} = \lambda\, R_m^{c^j} \begin{bmatrix} X_I - X_o^j \\ Y_I - Y_o^j \\ Z_I - Z_o^j \end{bmatrix}$$

$$\lambda \begin{bmatrix} X_I - X_o^j \\ Y_I - Y_o^j \\ Z_I - Z_o^j \end{bmatrix} = R_{c^j}^m \begin{bmatrix} x_i^j - x_p - dist_x \\ y_i^j - y_p - dist_y \\ -c \end{bmatrix} = \begin{bmatrix} u_i^j \\ v_i^j \\ w_i^j \end{bmatrix}$$

$$\frac{X_I - X_o^j}{Z_I - Z_o^j} = \frac{u_i^j}{w_i^j}$$

$$\frac{Y_I - Y_o^j}{Z_I - Z_o^j} = \frac{v_i^j}{w_i^j}$$

i: point index

j: image index

Intersection: Multi-Light Ray Intersection

$$\frac{X_I - X_o^j}{Z_I - Z_o^j} = \frac{u_i^j}{w_i^j} \rightarrow u_i^j\,(Z_I - Z_o^j) = w_i^j(X_I - X_o^j)$$

$$\frac{Y_I - Y_o^j}{Z_I - Z_o^j} = \frac{v_i^j}{w_i^j} \rightarrow v_i^j\,(Z_I - Z_o^j) = w_i^j(Y_I - Y_o^j)$$

$$w_i^j X_I - u_i^j Z_I = w_i^j X_o^j - u_i^j Z_o^j$$
$$w_i^j Y_I - v_i^j Z_I = w_i^j Y_o^j - v_i^j Z_o^j$$

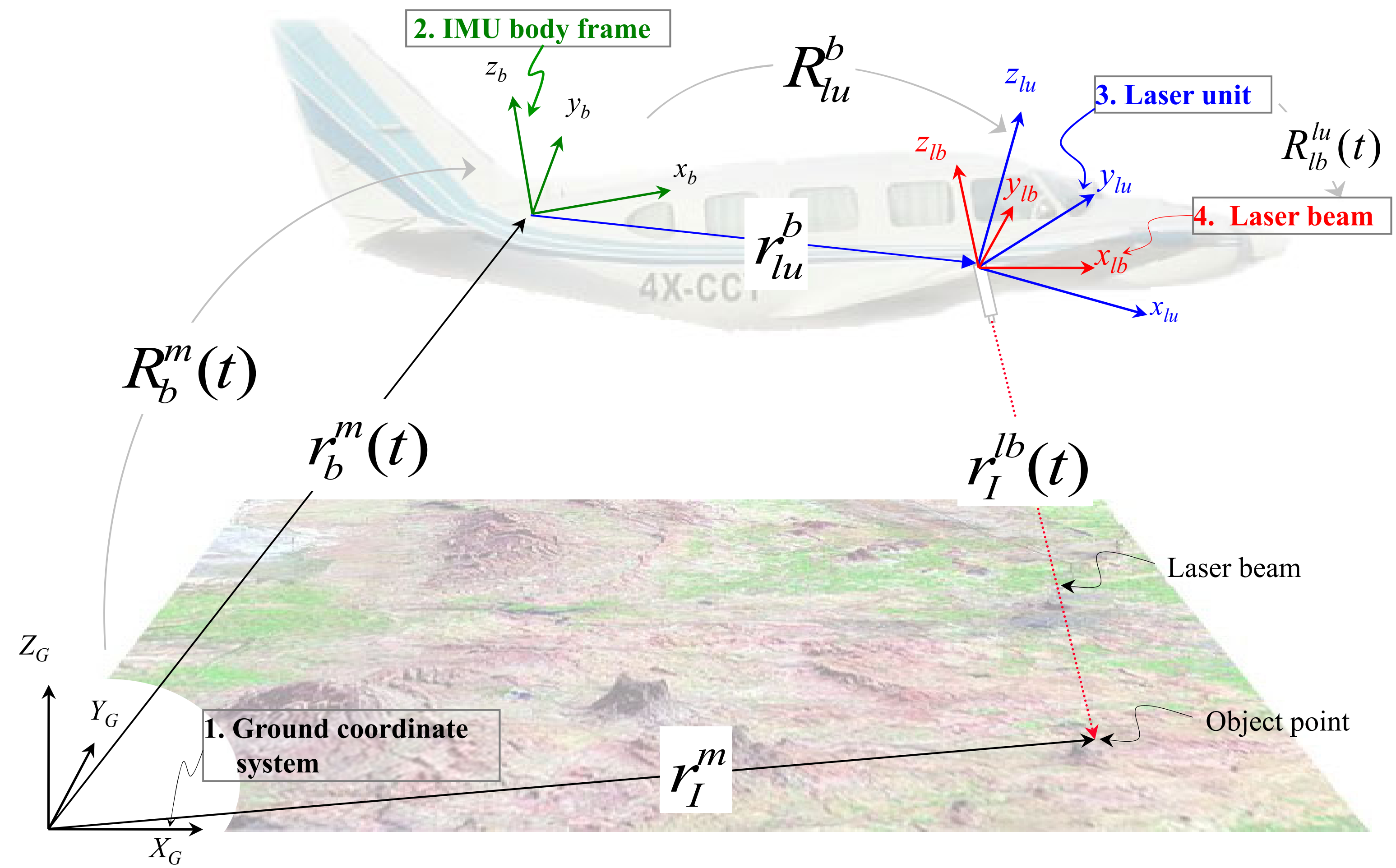
i: point index
j: image index

n images → 2n equations in 3 unknowns

Direct Georeferencing

LiDAR Mapping

LiDAR Equation & Coordinate Systems



- LiDAR equation is a vector summation procedure.

LiDAR Equation (Mobile Systems)

$$r_I^m = r_b^m(t) + R_b^m(t)\; r_{lu}^b + R_b^m(t)\, R_{lu}^b\, R_{lb}^{lu}(t)\, r_I^{lb}(t)$$

r_I^m	ground coordinates of the object point under consideration
$r_b^m(t)$	ground coordinates of the origin of the IMU coordinate system
$R_b^m(t)$	rotation matrix relating the ground and IMU coordinate systems
r_{lu}^b	offset between the laser unit and IMU coordinate systems (lever arm offset)
R_{lu}^b	rotation matrix relating the IMU and laser unit coordinate systems (boresight matrix)
$R_{lb}^{lu}(t)$	rotation matrix relating the laser unit and laser beam coordinate systems
$r_I^{lb}(t)$	coordinates of the object point relative to the laser beam coordinate system

- Note: There is no redundancy in the surface reconstruction process.

Terrestrial Mobile Mapping Systems

Operational Example

Photogrammetric Mapping

Mobile Mapping Systems: Introduction

- Definition
  - Mobile Mapping Systems (MMS) can be defined as moving platforms upon which multiple sensors / measurement systems have been integrated to provide three-dimensional near-continuous positioning of both the platform's path in space and simultaneously collected geospatial data.
- Includes therefore
  - Planes, trains, automobiles

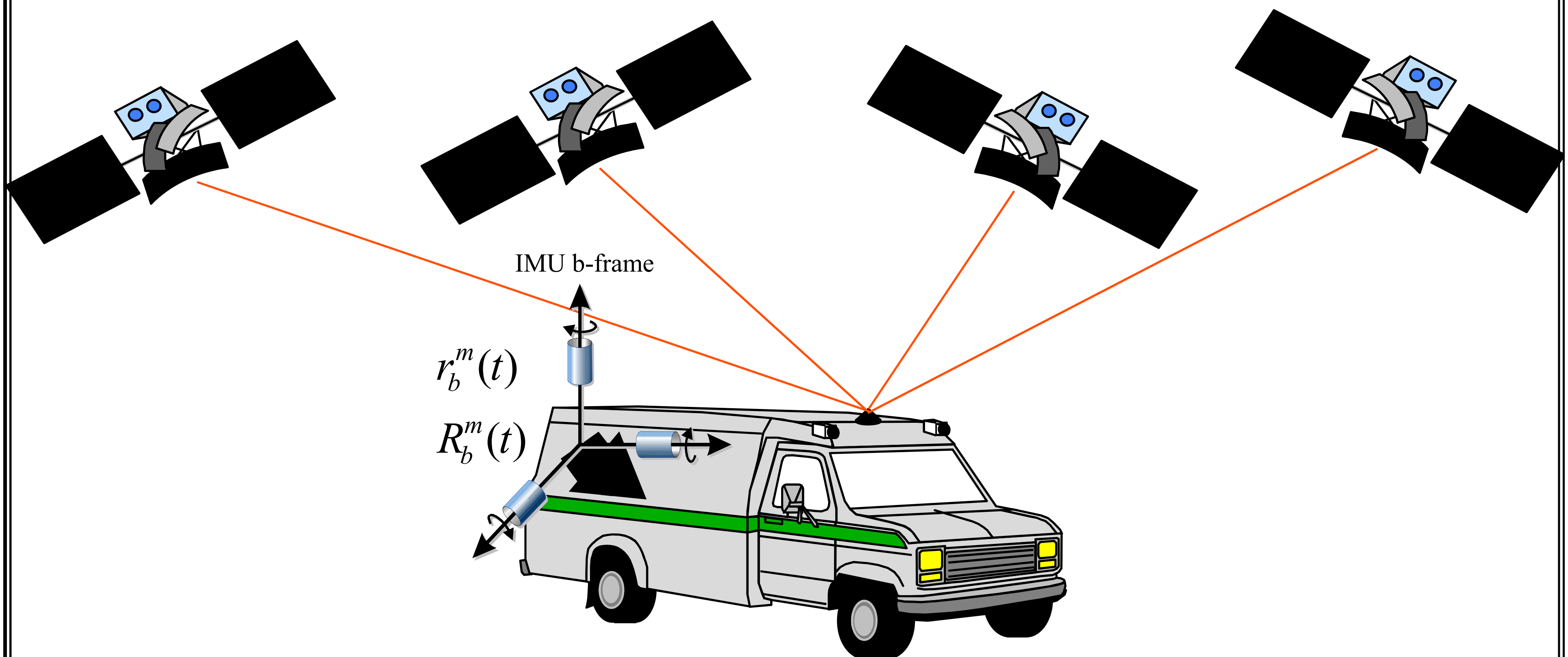
Terrestrial MMS: Motivation

- Increasing need for digital land-related information, (GIS)
- Road network data is of special interest.
- Road network data can be collected via:
  - Digitizing existing maps (inherit existing errors), or
  - Site surveying
- Mobile Mapping Systems (MMS) are fast, accurate, economic, and current data collection devices.

Mobile Mapping Systems

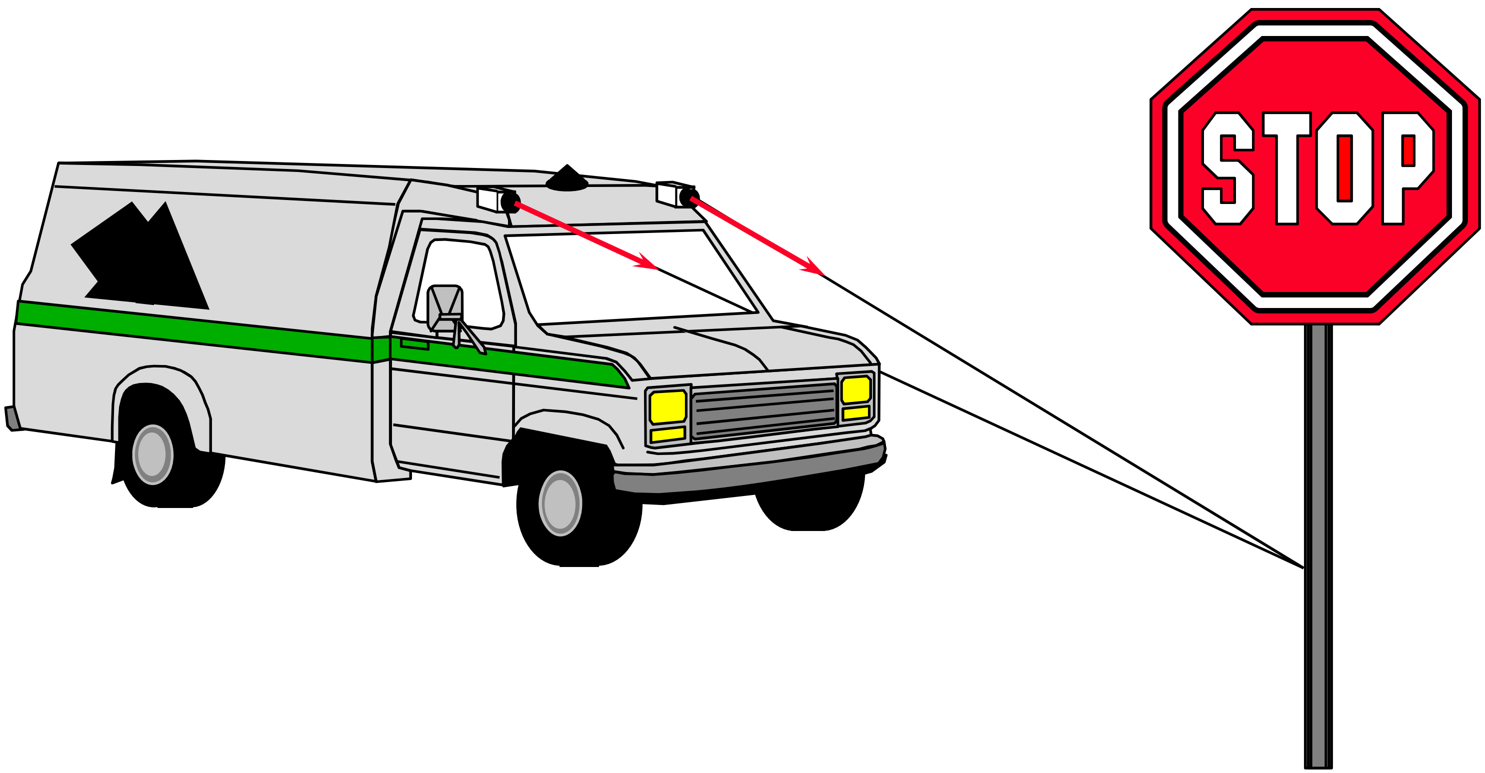
- Basic requirements:
  - Positioning capabilities
    - GNSS and INS
  - Knowledge about the surrounding environment
    - Radar,
    - Laser, and/or
    - Optical camera(s)
- The involved operational example includes:
  - GNSS receiver,
  - Inertial Navigation System (INS), and
  - Stereo-vision system.

GNSS/INS Component



Stereo-Vision System

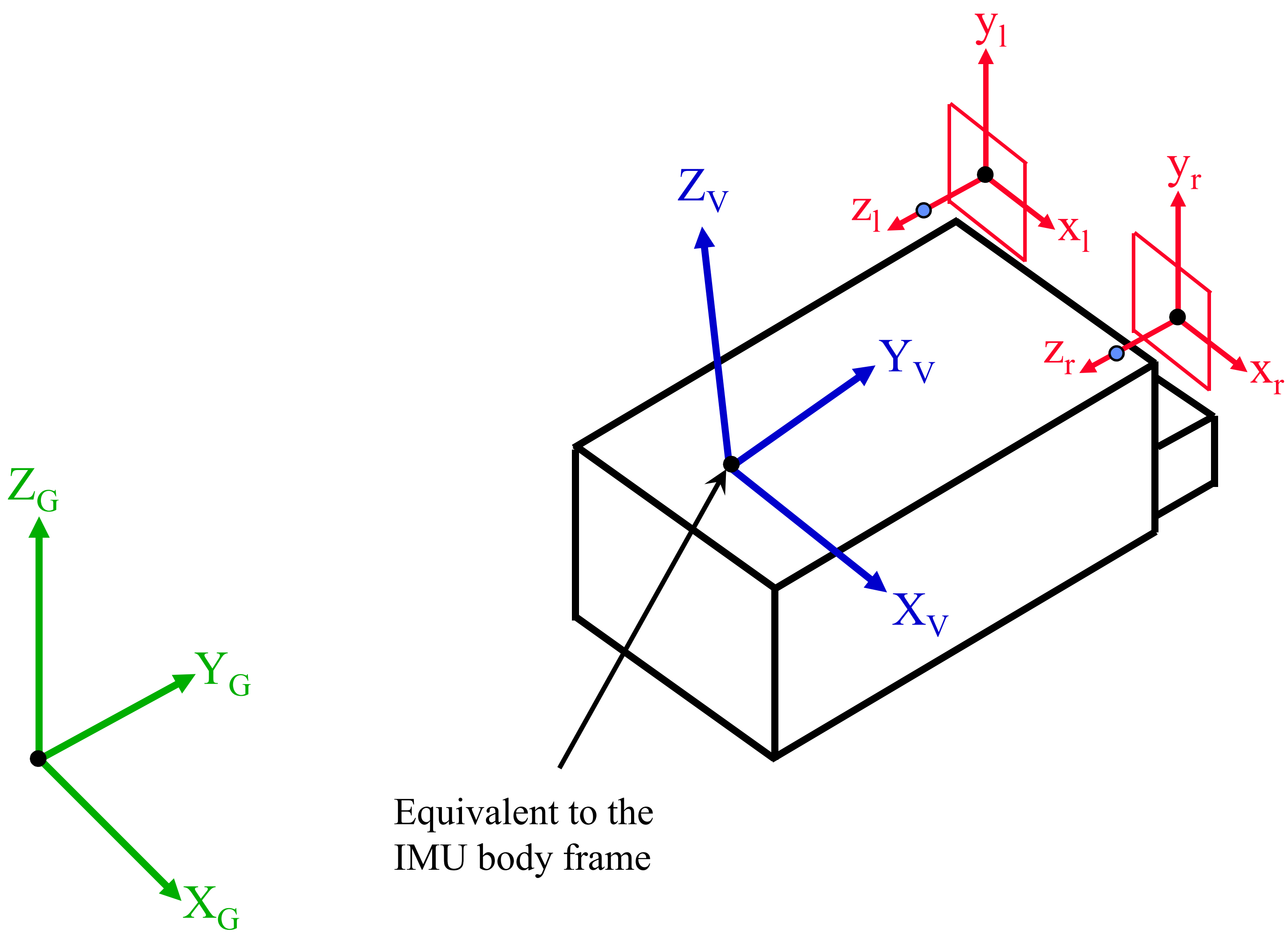
$$\left.\begin{array}{l} r_{c_i}^{m}(t) \\ R_{c_i}^{m}(t) \end{array}\right\} i=1:n \text{ (n is the number of cameras)}$$



Terrestrial MMS: Operational Example

- **Rack:** up to 4 cameras with 4 possible combinations
- **GNSS Receiver**
- **Inertial Navigation System**

Coordinate Systems



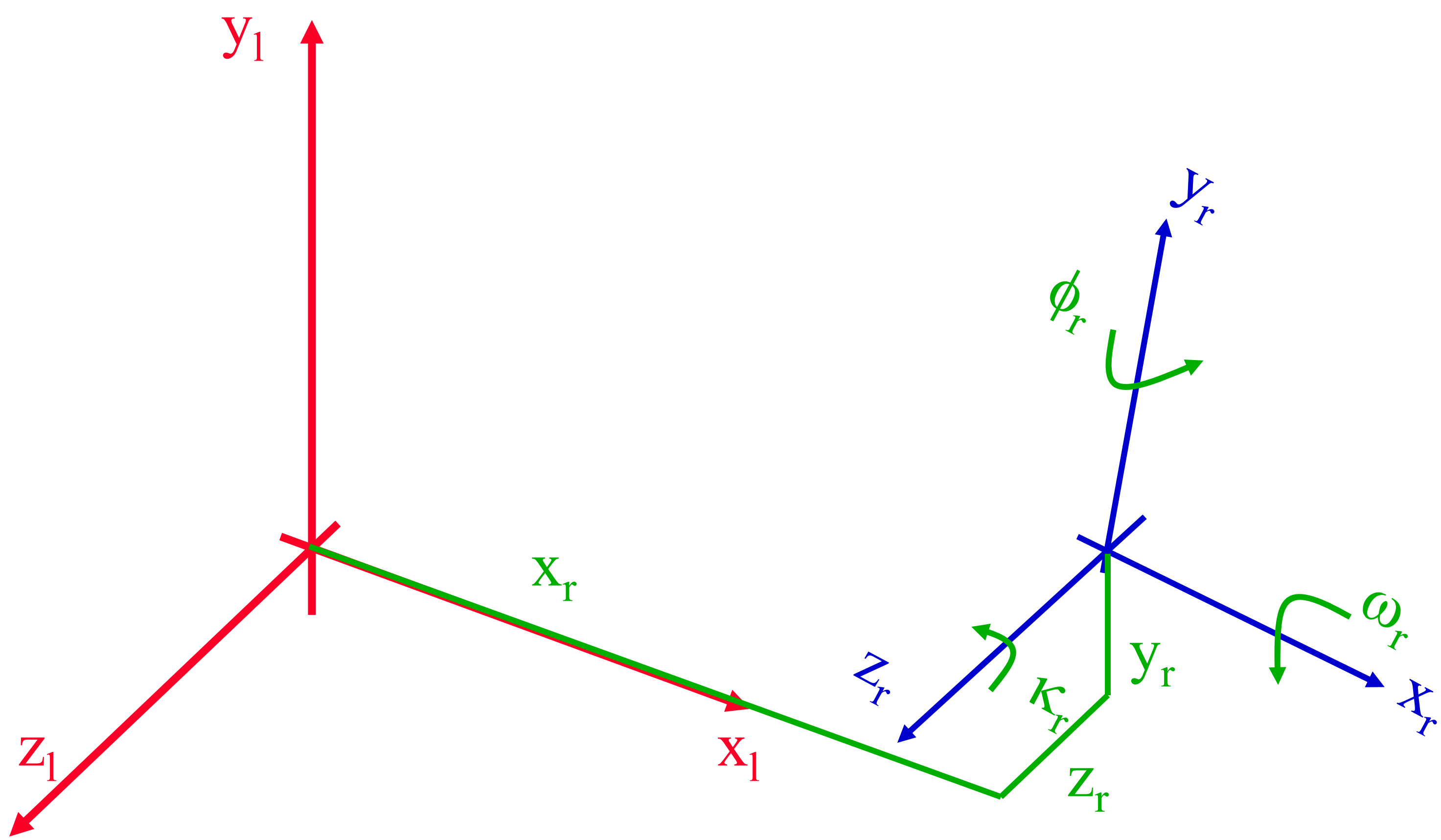
Coordinate Systems

- (x_l, y_l, z_l) Image coordinate system for the left camera station
- (x_r, y_r, z_r) Image coordinate system for the right camera station
- (X_V, Y_V, Z_V) Van coordinate system:
  - Origin at the IMU body frame
  - Y_V coincides with the driving direction
  - Z_V is pointing upward
  - The van coordinate system is parallel to the IMU body frame coordinate system.
- (X_G, Y_G, Z_G) Ground coordinate system

System Calibration

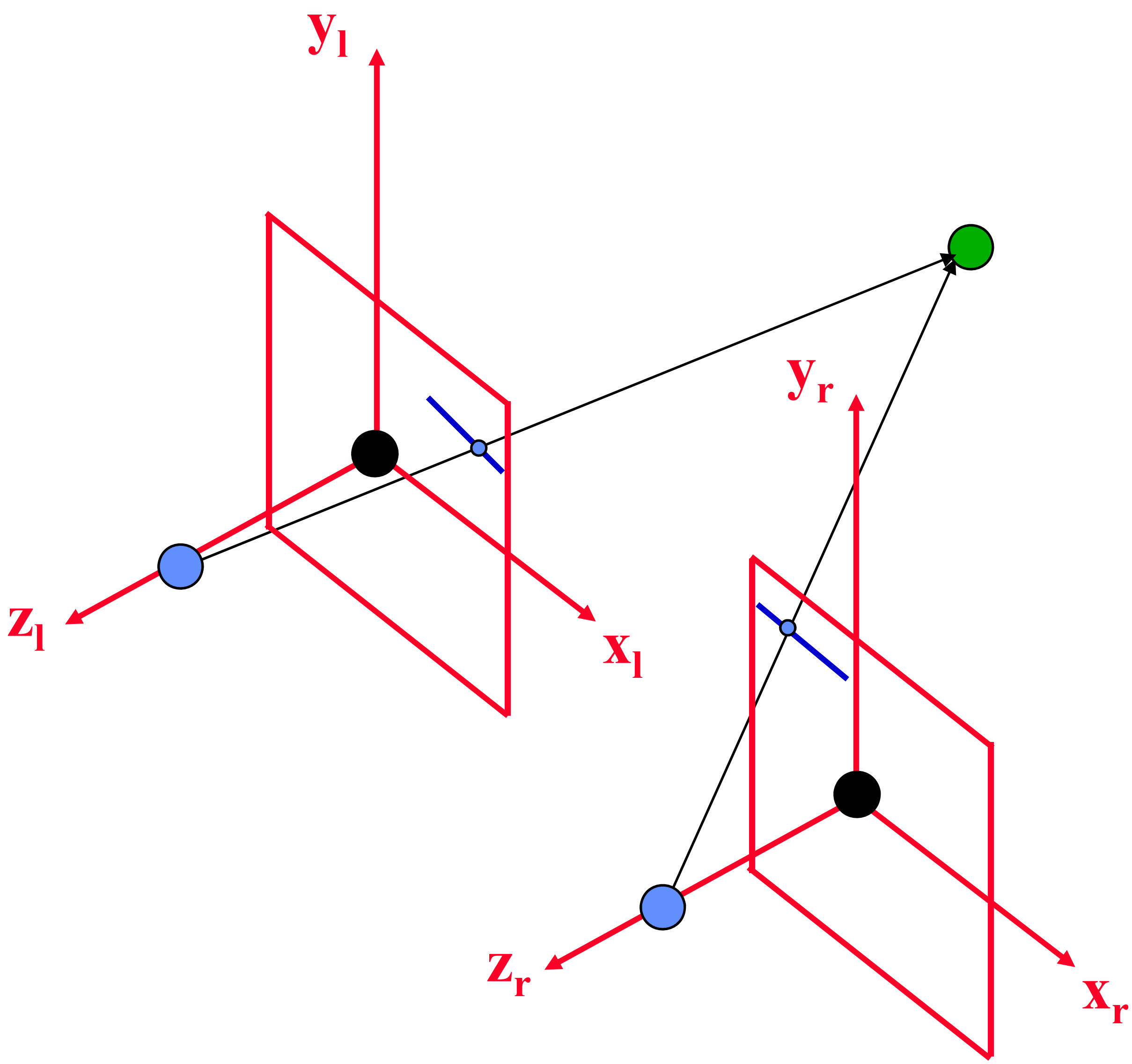
- The interior orientation parameters of the used cameras
  - The coordinates of the principal point,
  - The focal length, and
  - Distortion parameters
- The spatial and rotational offsets between the right and the left camera stations.
  - X_r Y_r Z_r ω_r ϕ_r κ_r
  - Those offsets can be determined through:
    - Bundle adjustment using some tie points and distance measurements in the object space
- The spatial and rotational offsets between the left camera and the IMU body frame.

Relationship Between the Two Camera Stations



- The left camera coordinate system defines the model coordinate system.

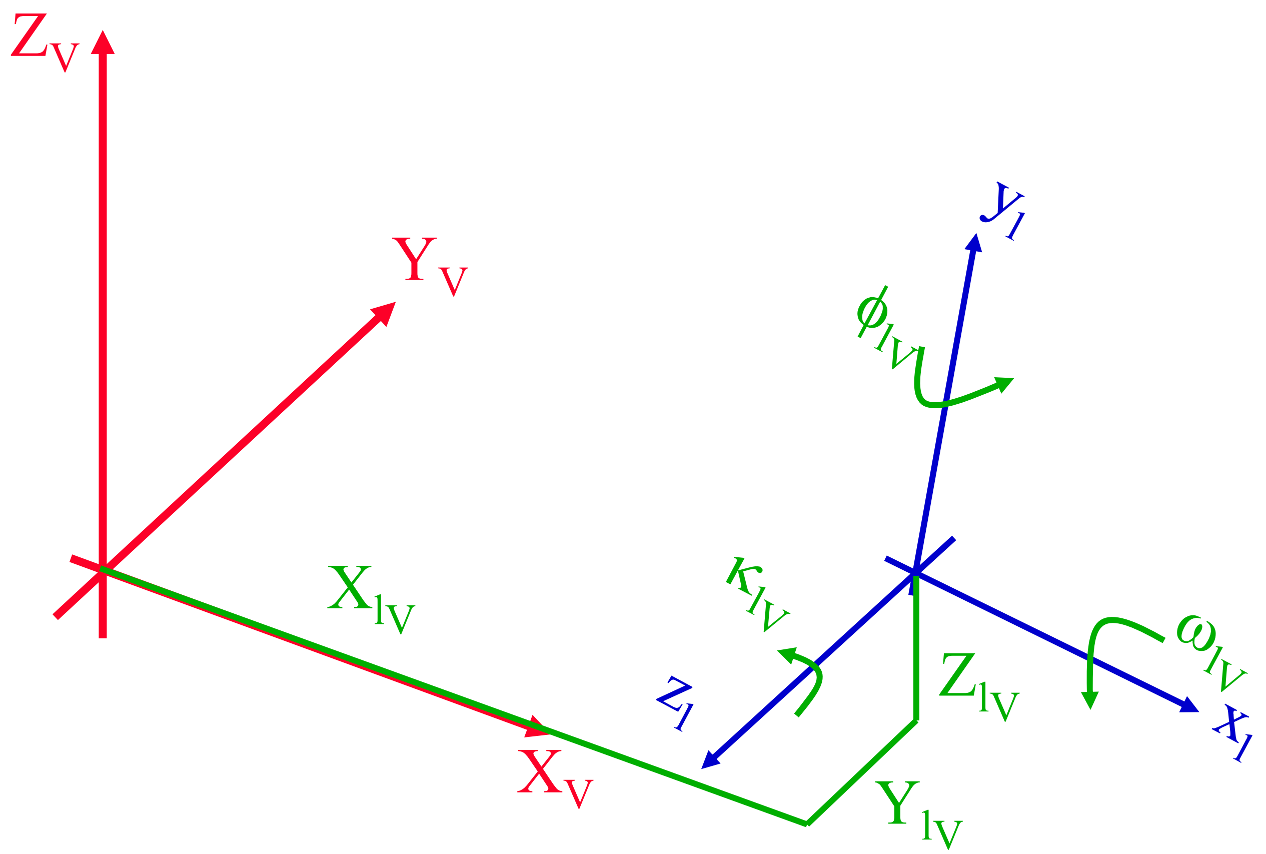
Stereo Positioning: Model Coordinates



Model-to-Van Coordinate Transformation

- The spatial and rotational offsets between the left camera station and the van coordinate system
  - X_{lV} Y_{lV} Z_{lV} ω_{lV} ϕ_{lV} κ_{lV}
  - The components of the spatial and rotational offsets can be determined through a system calibration procedure.

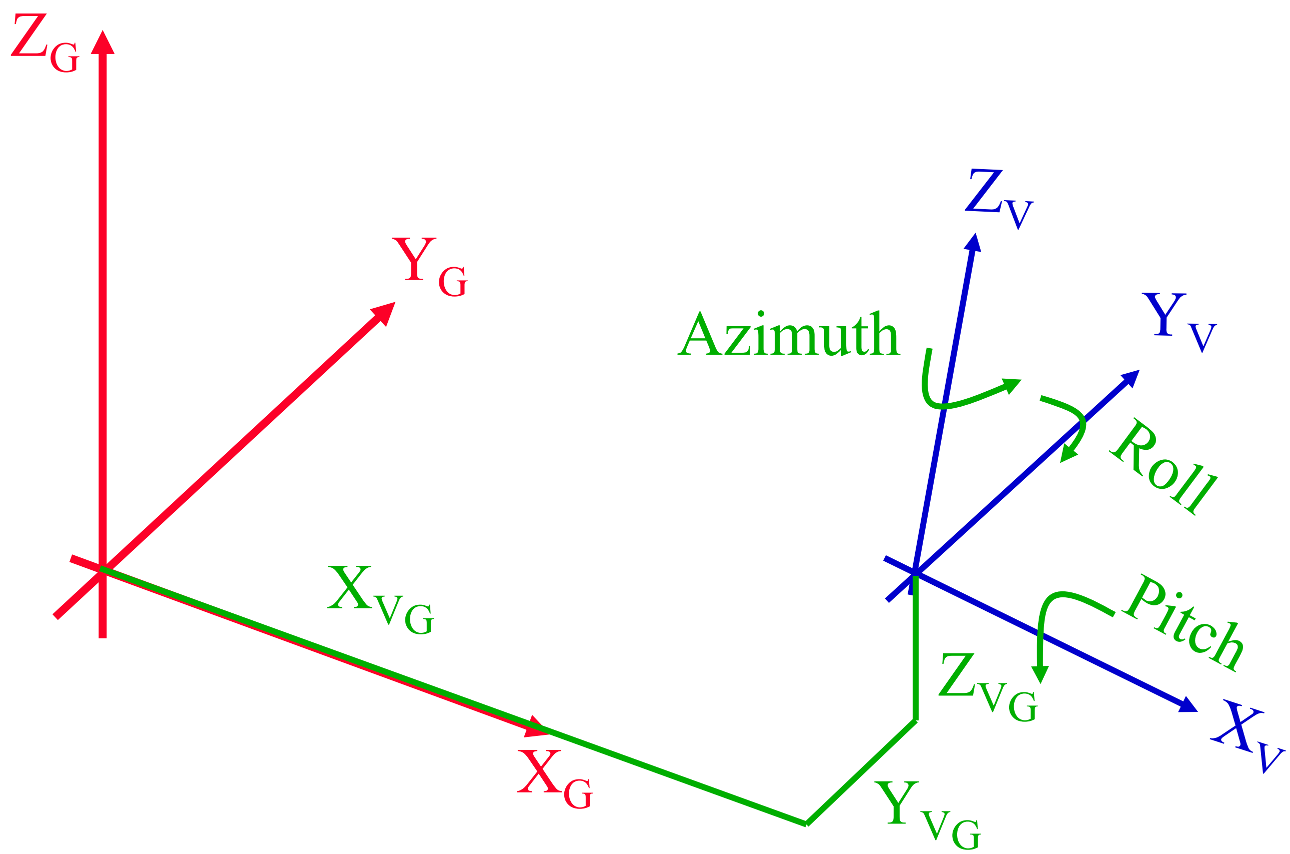
Model-to-Van Coordinate Transformation



Van-to-Ground Coordinate Transformation

- The spatial and rotational offsets between the van and ground coordinate systems
  - X_{V_G} Y_{V_G} Z_{V_G} Azimuth Pitch Roll
- Those offsets are determined from the onboard GNSS/INS unit (GNSS/INS-integration process).

Van-to-Ground Coordinate Transformation



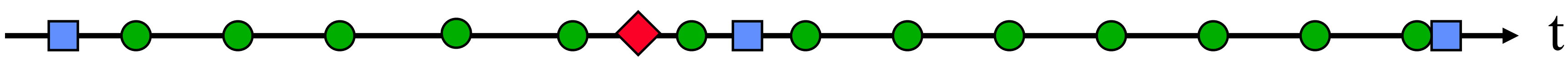
Sample Calibration File (*.cop)

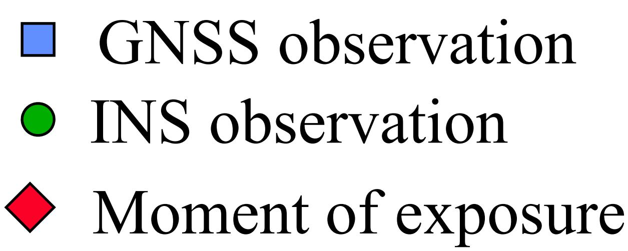
- Relationship between the two camera stations:
  - **0.000 0.000 0.000 0.000 0.000 0.000**
  - **2.130 -0.009 -0.208 -1.0366 9.8562 0.6427**
- IOPs for the left camera station:
  - **720 400 0.0120 0.0136 0.2099 -0.4865 6.6731**
  - **-0.0043 0.0000 0.0007 -0.0013 -0.0197738 0.0033121**
- IOPs for the right camera station:
  - **720 400 0.0120 0.0136 -0.0946 -0.4106 6.7160**
  - **-0.0046 0.0000 -0.0004 -0.0007 -0.0189568 0.0022773**
- Relationship between the left camera station and the van coordinate system:
  - **0.000000 90.000000 0.000000**
  - **-1.1389 3.0211 2.523000 -2.549210 -8.476720 -1.191110**

Van Orientation Parameters (*.vop)

- The relationship between the van and ground coordinate systems
  - X_{V_G} Y_{V_G} Z_{V_G} Azimuth Pitch Roll
- Spatial offset:
  - **587 321753.97150 4449805.51690 252.99000**
- Rotational offset (Azimuth, Pitch and Roll):
  - **93.5870400 -0.2518300 0.0000000**
- Those offsets are computed after GNSS/INS-integration at the moment of exposure for a specific stereo-pair (stereo-pair # 587 in this case).

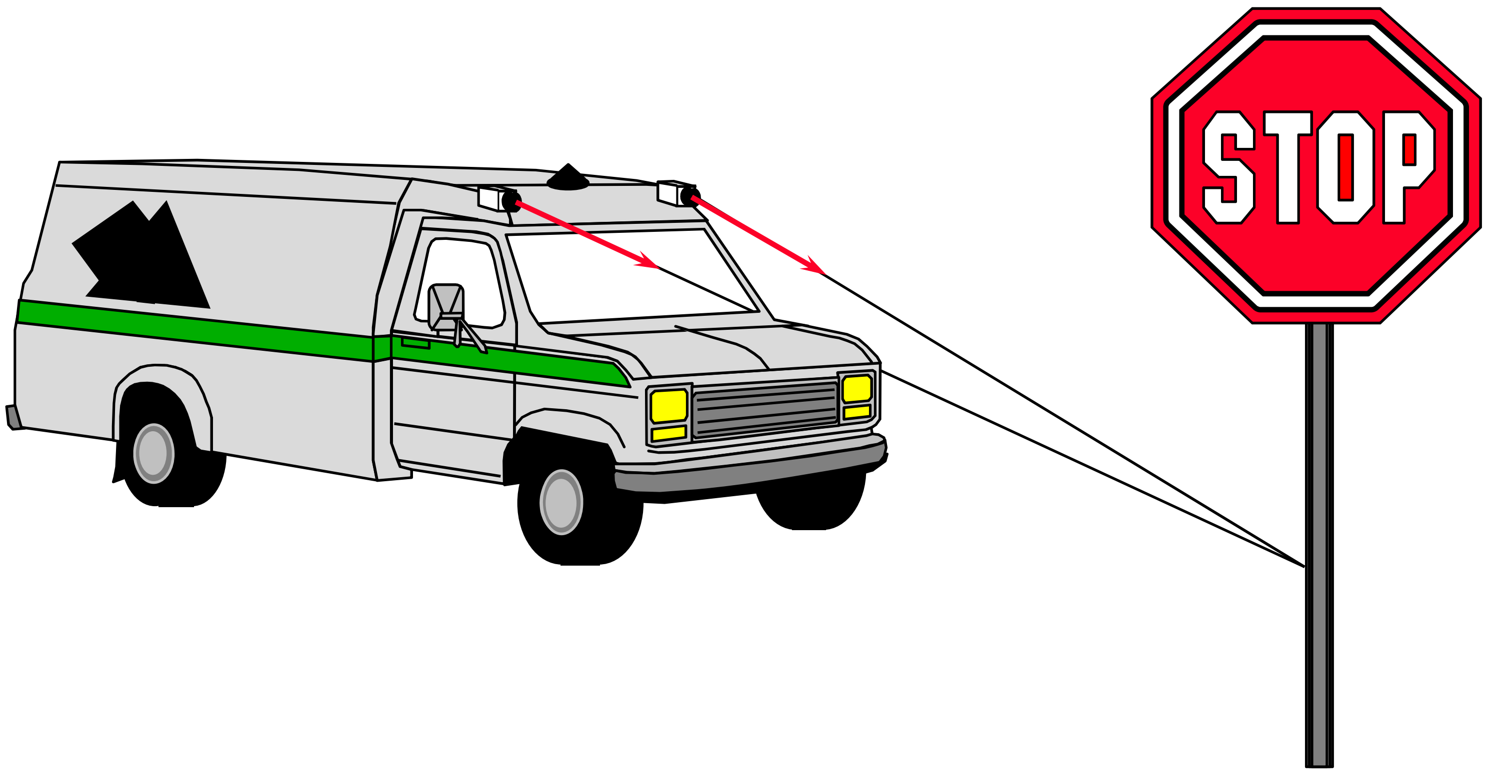
Time Synchronization





Terrestrial MMS: 3-D Positioning

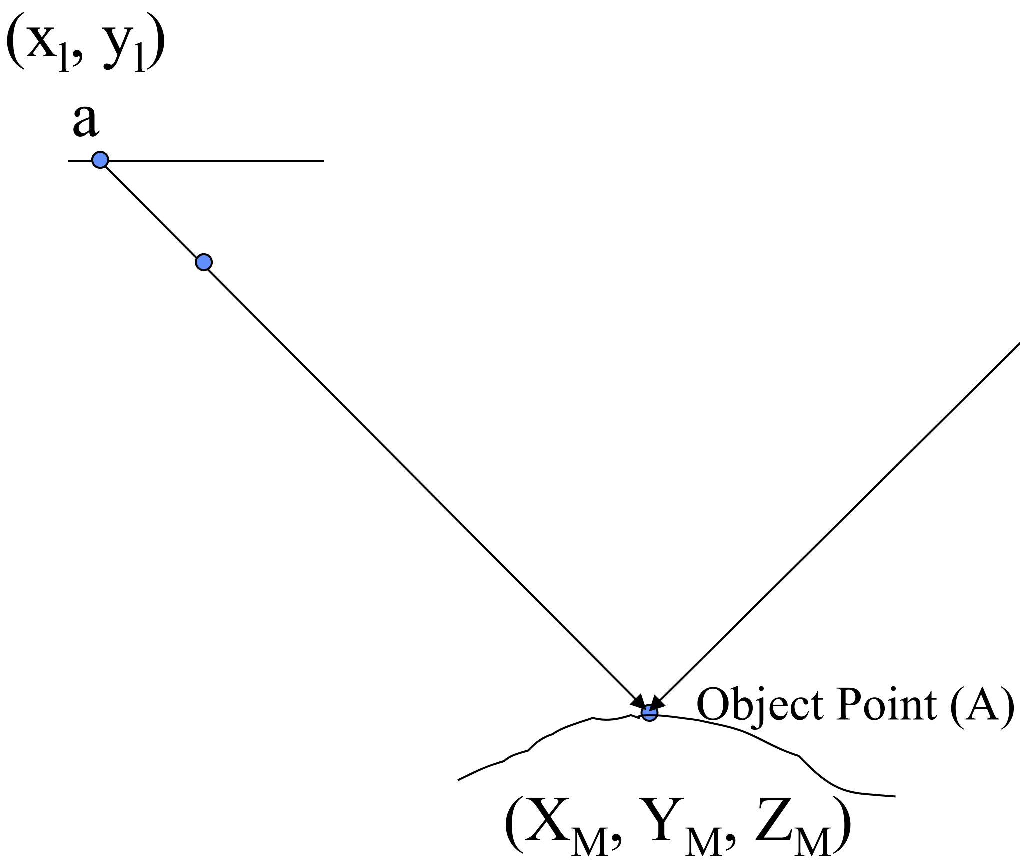
Step 1: Stereo Measurement



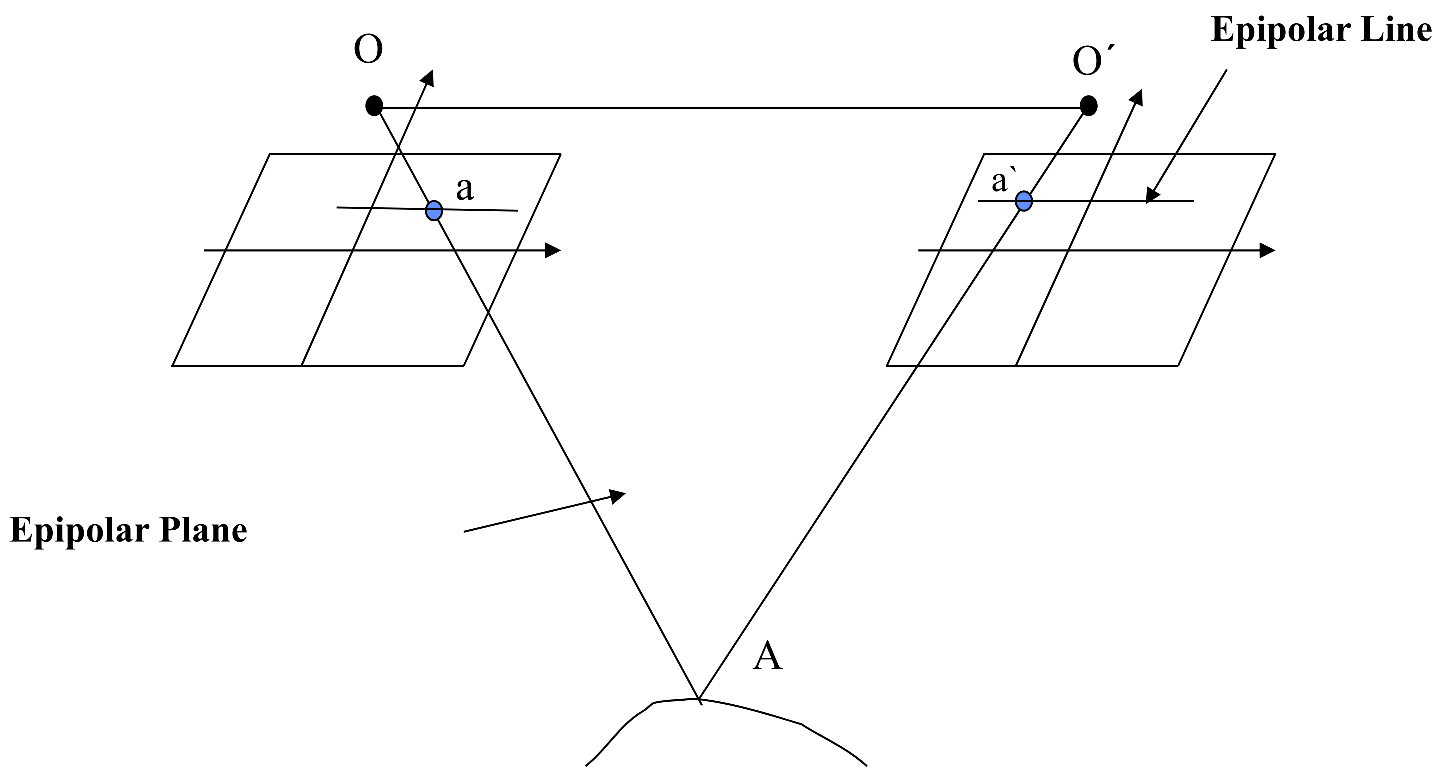
Step 1: Stereo Measurement

- Output: (x_l, y_l) & (x_r, y_r)
- Red lines ≡ epipolar lines

Step 2: Intersection



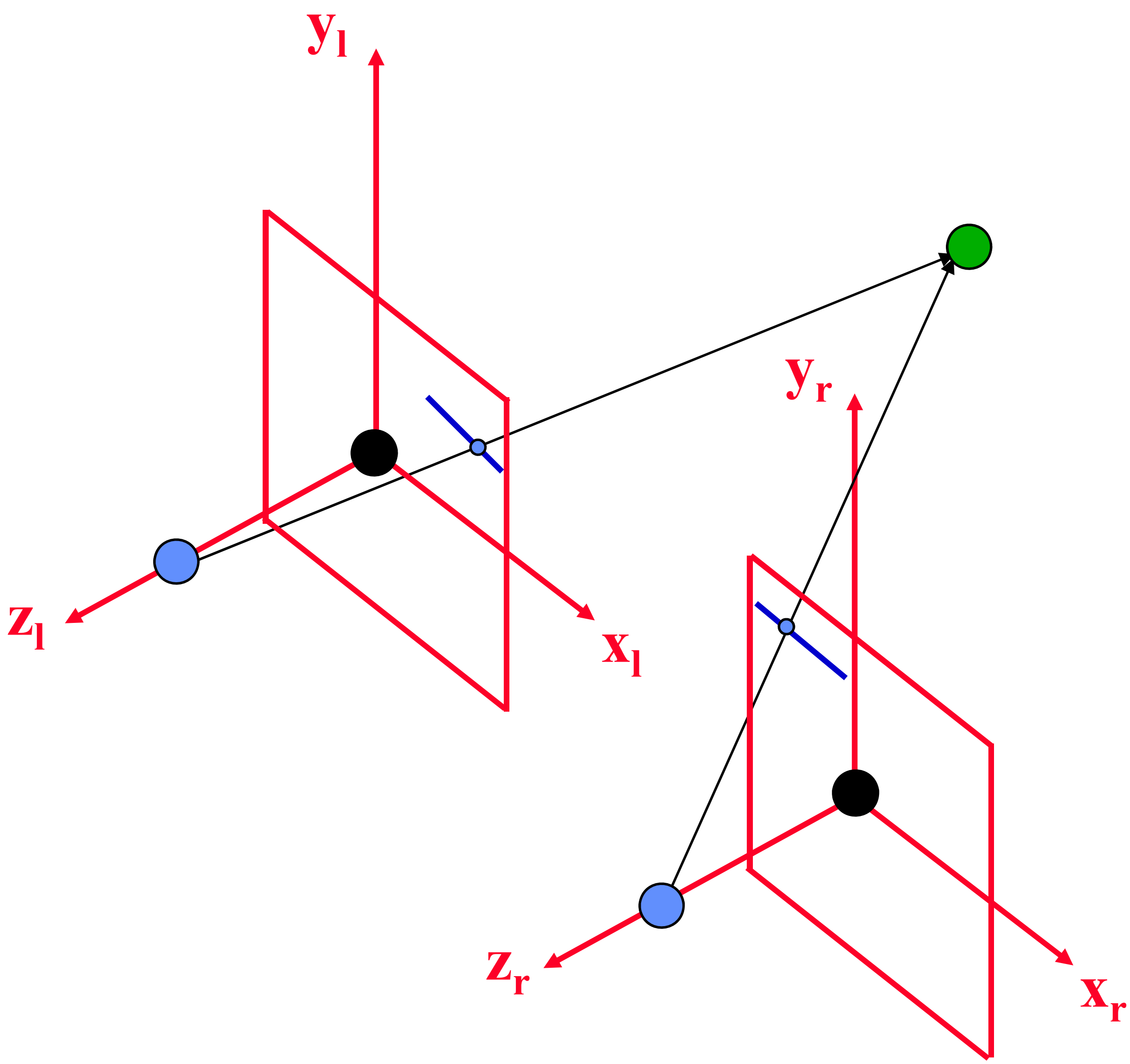
Epipolar Geometry



Epipolar Geometry (Remarks)

- The epipolar plane can be defined once we have:
  - The Relative Orientation Parameters (ROPs) relating the two images of a stereo-pair, and
  - Image coordinate measurements in either the left or right image.
- Conjugate points are located along conjugate epipolar lines.

Step 2: Intersection



Step 2: Intersection

- Given:
  - Left and right image coordinates of a selected feature in one stereo-pair,
  - The IOPs of the left and right cameras, and
  - The spatial and rotational offsets between the left and right camera stations
- Output:
  - (X_M, Y_M, Z_M) model coordinates of the selected feature relative to the left camera coordinate system

Step 3: Model-to-Global Coordinate Transformation

- Input:
  - (X_M, Y_M, Z_M) model coordinates of the selected feature relative to the left camera coordinate system,
  - The spatial and rotational offsets between the left camera station and the van coordinate systems, and
  - The spatial and rotational offsets between the van and ground coordinate systems
- Output:
  - (X_G, Y_G, Z_G) ground coordinates of the selected feature

Step 3-a: Model-to-Van Coordinate Transformation

$$\begin{bmatrix} X_V \\ Y_V \\ Z_V \end{bmatrix} = \begin{bmatrix} X_{l_V} \\ Y_{l_V} \\ Z_{l_V} \end{bmatrix} + R(\omega_{l_V}, \varphi_{l_V}, \kappa_{l_V}) \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix}$$

$r_I^V(t)$ $\quad r_l^V$ $\quad R_l^V$ $\quad r_I^l(t)$

Step 3-b: Van-to-Ground Coordinate Transformation

$$\begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} = \begin{bmatrix} X_{V_G} \\ Y_{V_G} \\ Z_{V_G} \end{bmatrix} + R(Azimuth,\ Pitch,\ Roll) \begin{bmatrix} X_V \\ Y_V \\ Z_V \end{bmatrix}$$

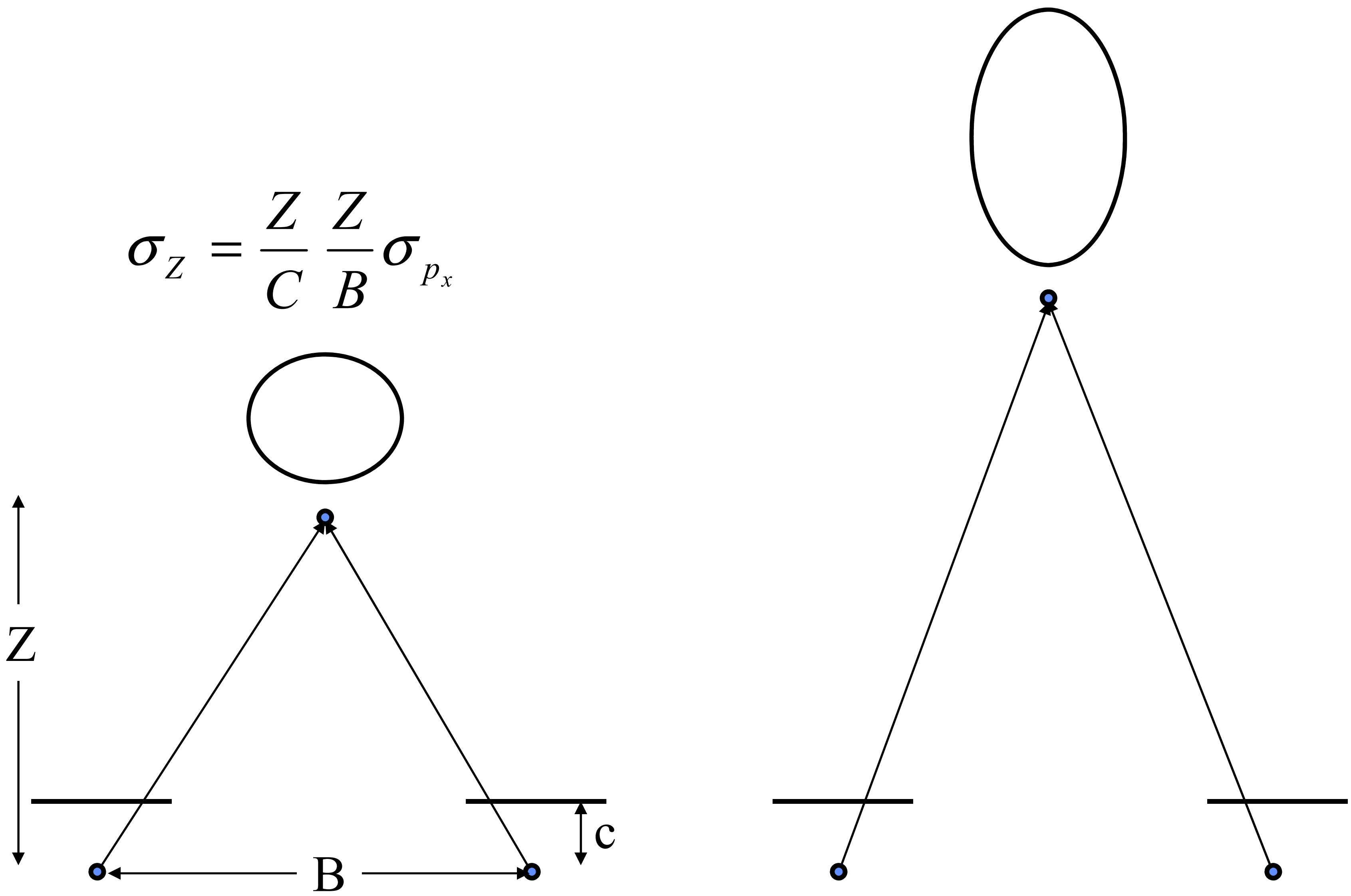
$$r_I^m \qquad r_V^m(t) \qquad R_V^m(t) \qquad r_I^V(t)$$

Error Sources

- Measurement errors
- Interior Orientation Parameters (IOPs)
- Relative relationship between the two camera stations
- Offset between the left camera station and the van coordinate system
- GNSS/INS errors
  - GNSS blockage - foliage, bridges
  - Base stations
- Distance from cameras

Measurement Errors & Object Distance

$$\sigma_Z = \frac{Z}{C}\frac{Z}{B}\sigma_{p_x}$$

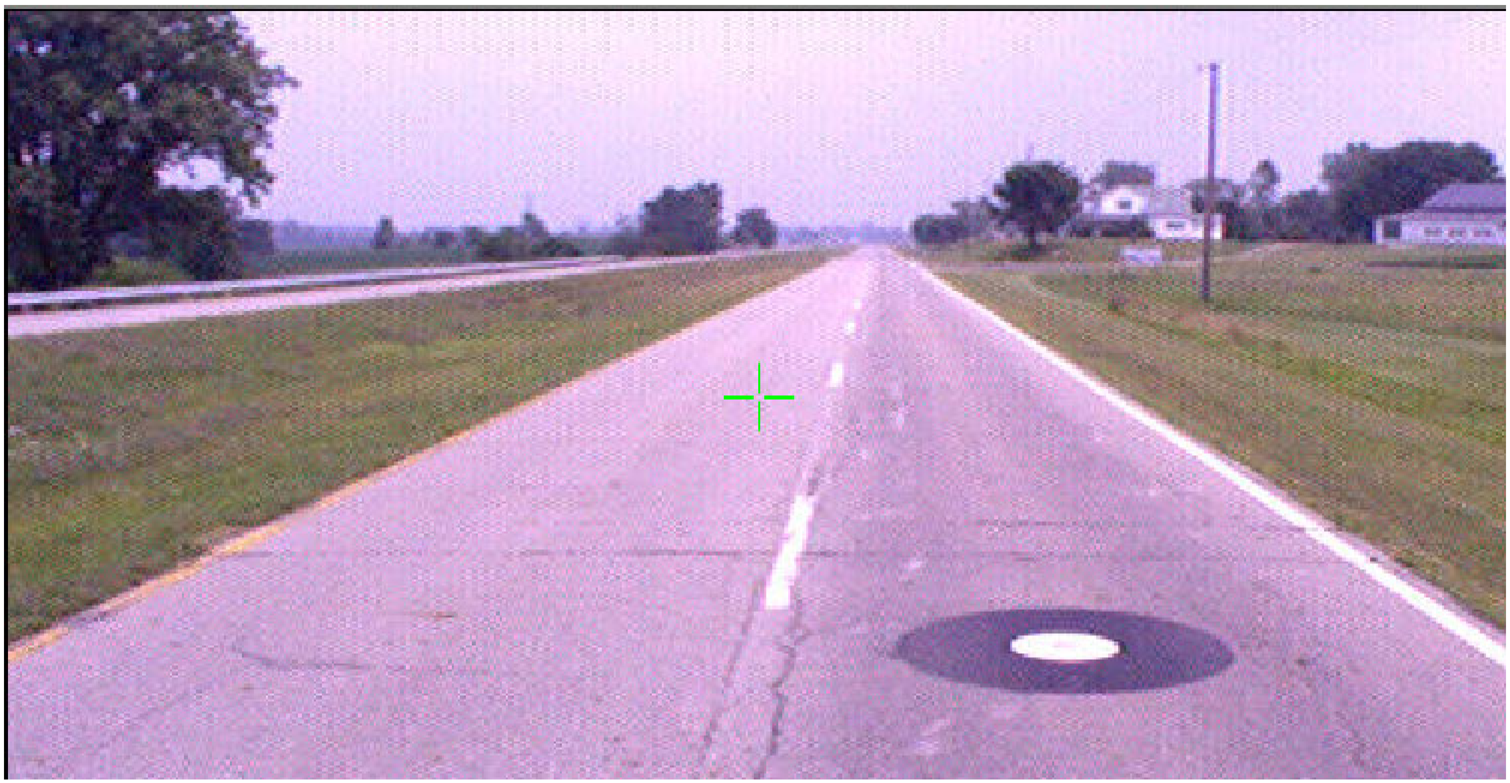


Field Procedure

- Drive along all roads
- Two GNSS base stations
  - Quality control
  - Datum, map projections, heights
- Check points
  - Independent check of system accuracy

Quality Control Points: Check Points

- $(XYZ)_1$: Derived from the MMS
- $(XYZ)_2$: Derived from direct geodetic measurements (e.g., GNSS)



Data Processing

- GNSS post-processing
- Integration of INS and GNSS
- Image storage - JPEG archives
- Camera calibration
- Output:
  - XYZ coordinates of objects in the stereo-vision system field of view
  - Additional attributes (e.g., feature type and some notes)

MMS Application: Traffic Signs Inventory

sign type

height above pavement

offset from road edge

coordinate locations

size of sign

MMS Application: Asset Management

- **Collecting inventories**
- **Database integration**
- **On-going maintenance**

Direct Versus Indirect Georeferencing

Accuracy Analysis

Photogrammetric Mapping

Overview

- Objectives
- Performance criterion and analysis environment
- Experimental results:
  - Aerial Triangulation
    - Integrated sensor orientation, and
    - Indirect georeferencing
  - Intersection
    - Intersection (direct georeferencing) versus aerial triangulation
- Conclusions

Objective

- The main objective of this work is to investigate several issues associated with direct and indirect georeferencing:
  - Accuracy
  - Configuration requirements
  - Sensitivity against problems in the IOPs
  - Triangulation versus intersection
- We implemented synthetic/simulated data for the experiments to restrict the error analysis to the assumed error sources.

Performance Criterion

- The performance of different scenarios is evaluated through Root Mean Square Error (RMSE) analysis:
  – Compares the adjusted ground coordinates from the triangulation or intersection procedures with the true values used for the simulation
- This criterion is very important since it addresses the quality of the reconstructed object space (the ultimate objective of photogrammetric mapping).

Analysis Environment

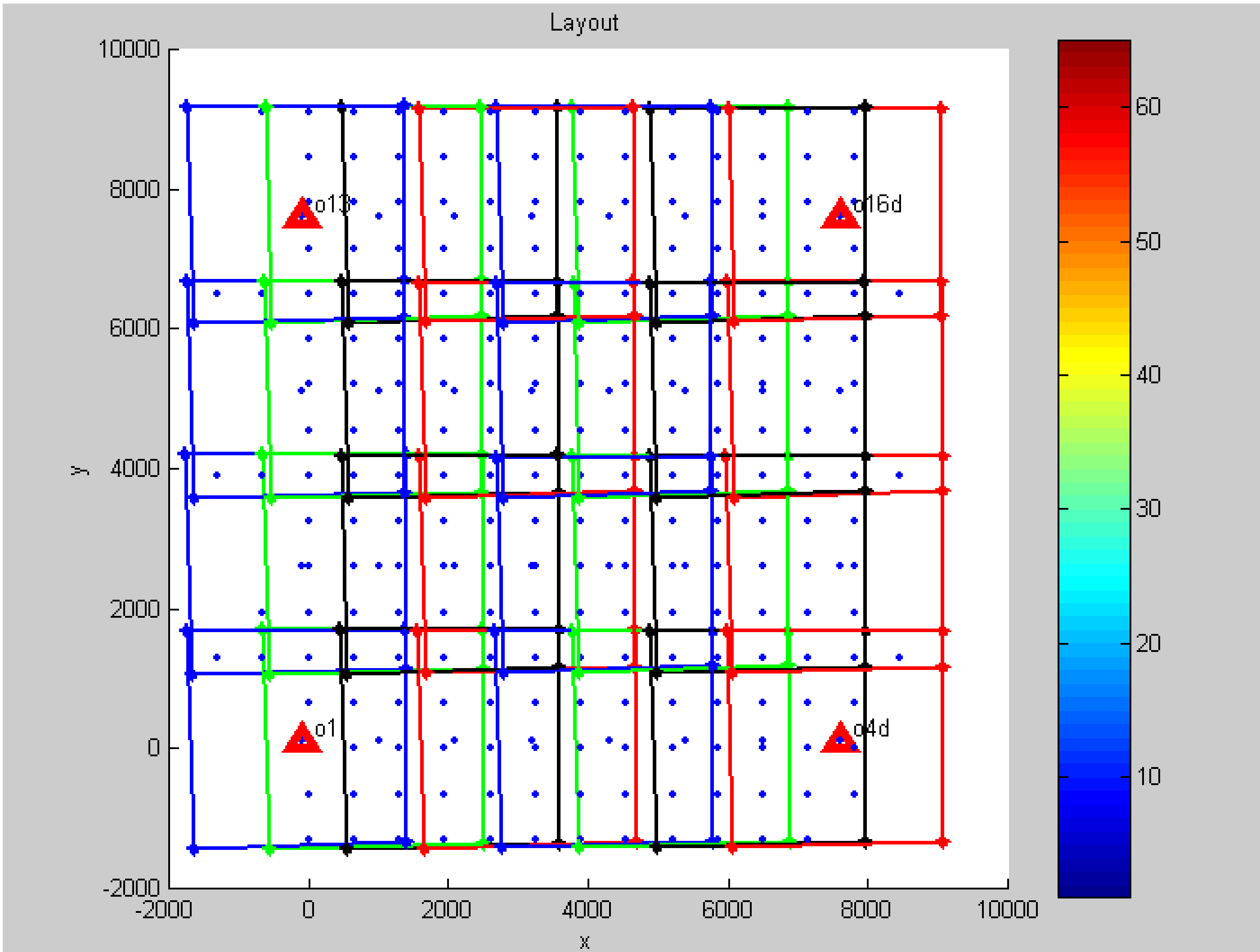
- Bundle adjustment software is used to conduct the experiments.
- This software can incorporate the following prior information:
  - Stochastic ground coordinates of the control points,
  - Stochastic IOPs, and
  - Stochastic GNSS/INS-position/orientation at the perspective centers

Test Data & Configurations

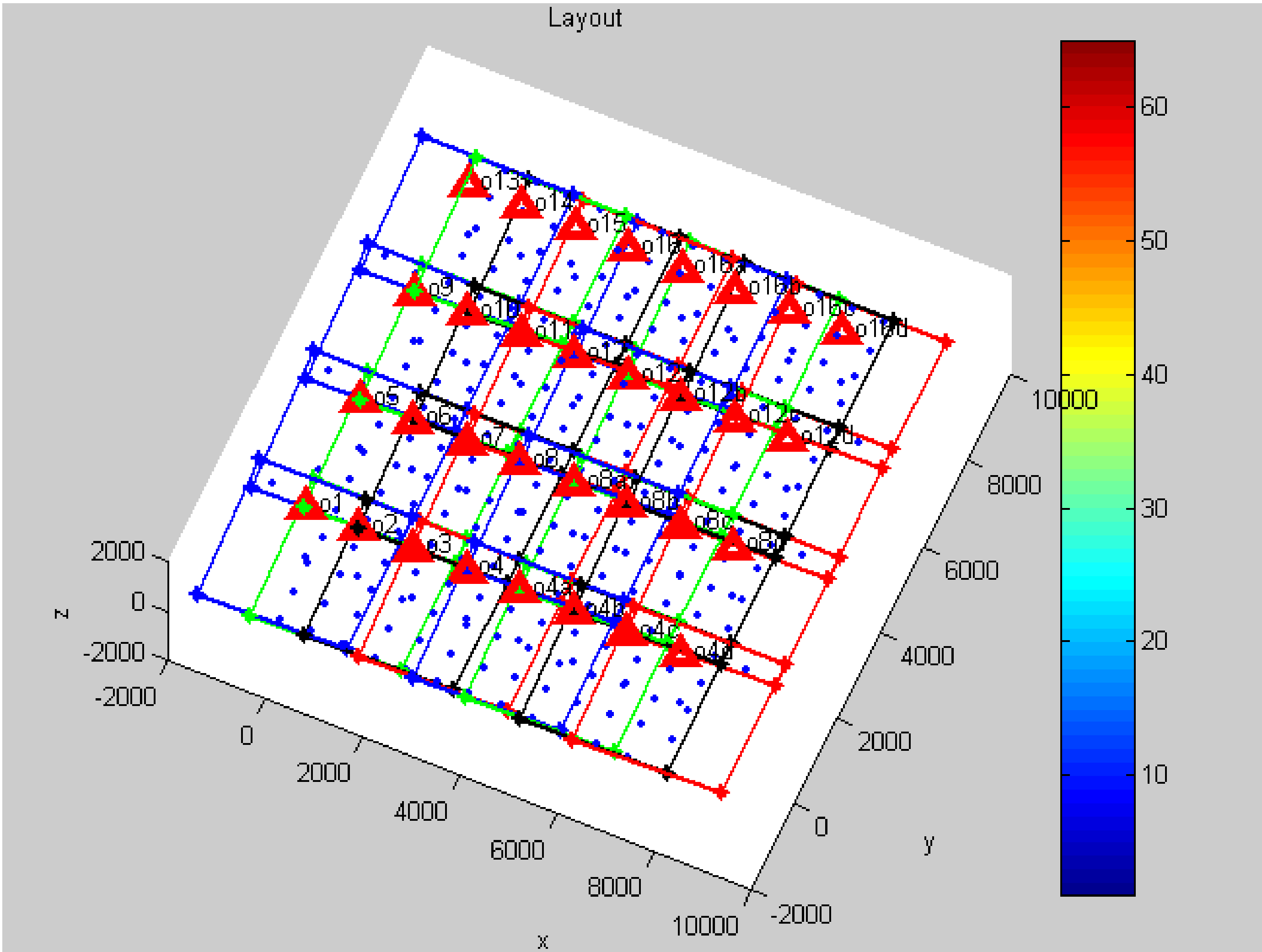
Configuration

- Flying height = 2000.0m
- Focal length = 150mm
- Thirty-two images in four strips
- 60% overlap
- (20 and 60)% side-lap
- Four/ten ground control points at the corners/edges of the block (±10cm)
- Image coordinate measurement accuracy (±5µm)
- IOPs (±5µm): 50 µm Bias
- GNSS/INS-position information at the perspective centers (±10cm): 10cm Lever Arm Bias
- GNSS/INS-attitude information (±10sec): 0.05° Boresight Bias

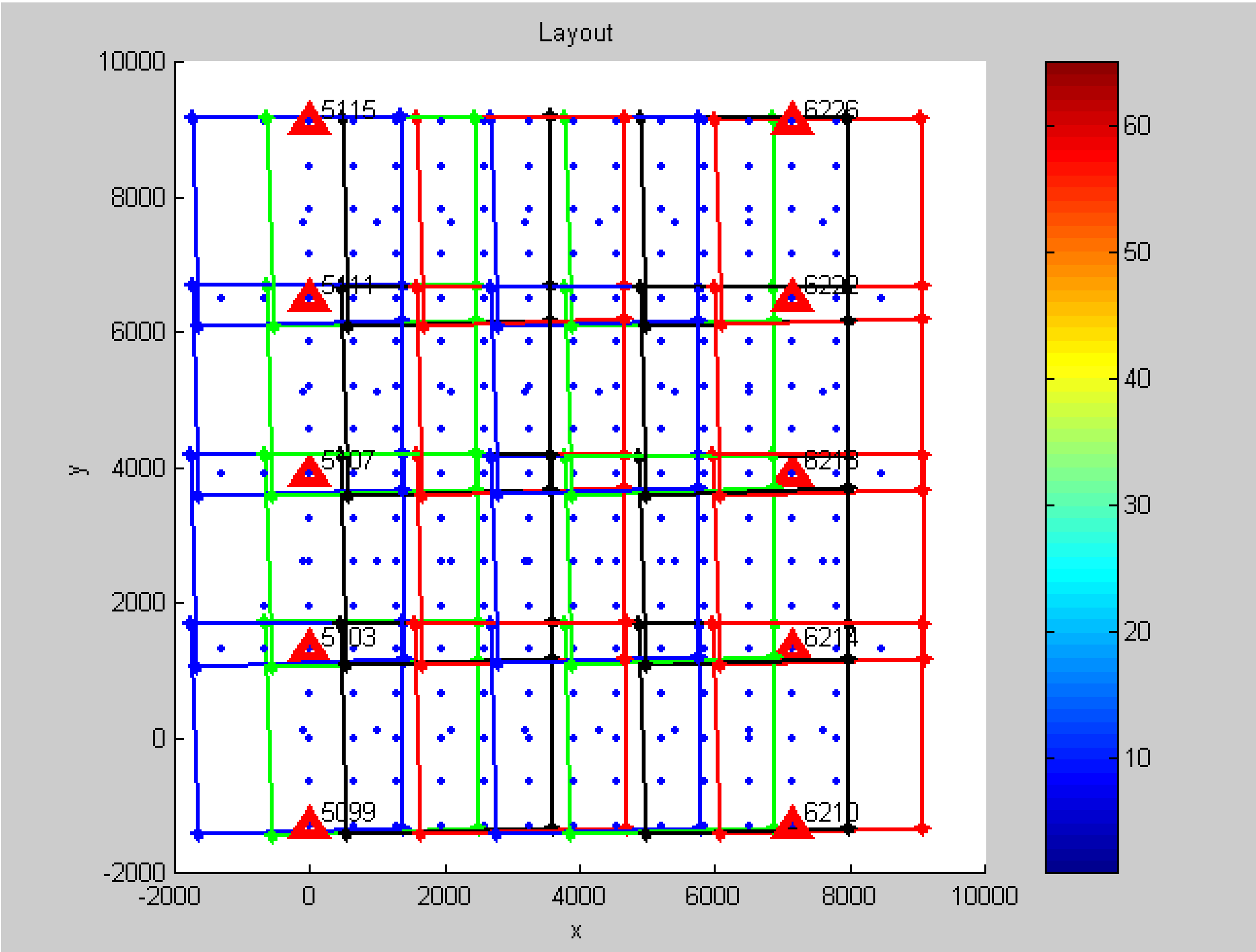
Experiment I



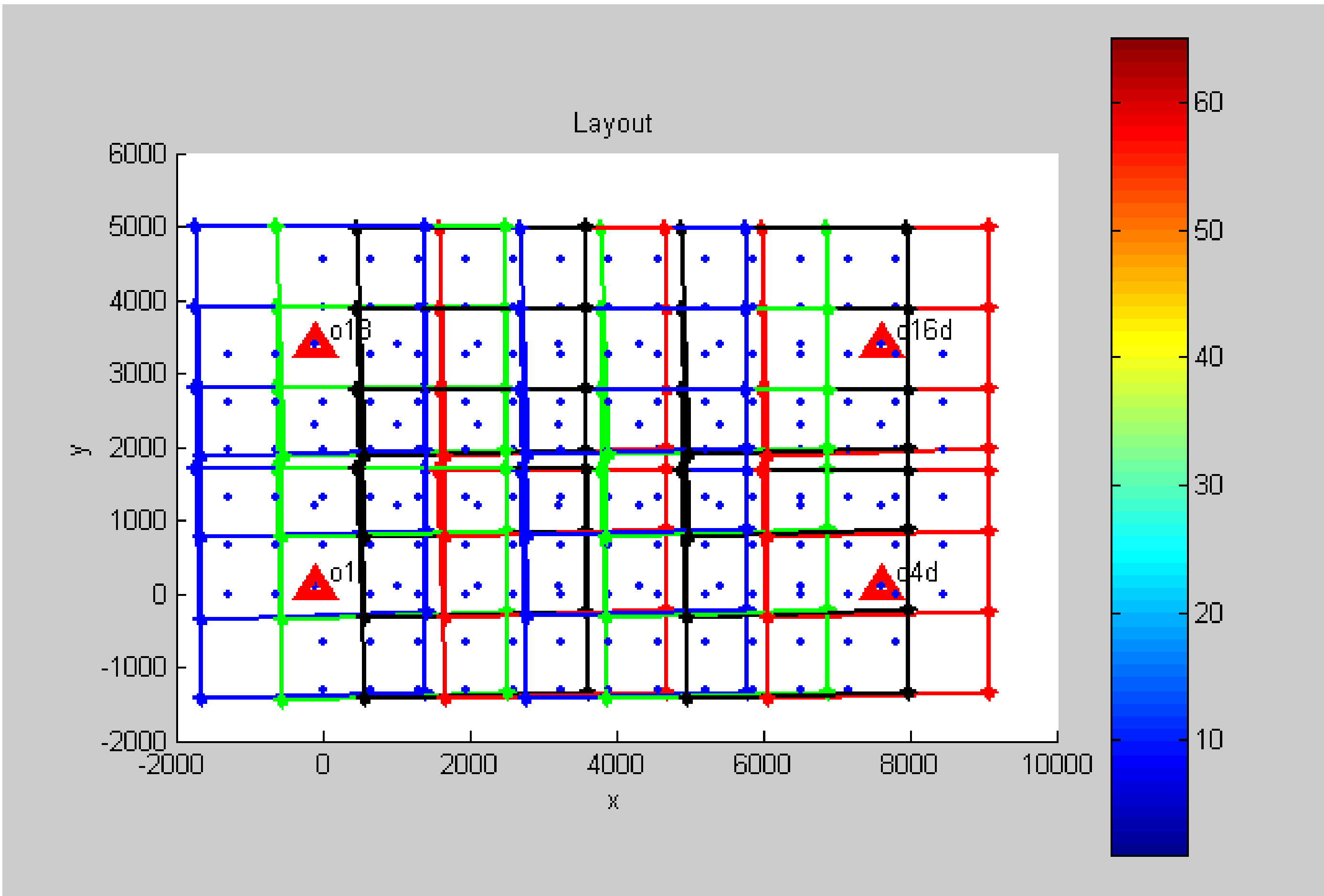
Experiment II



Experiment III



Experiment IV

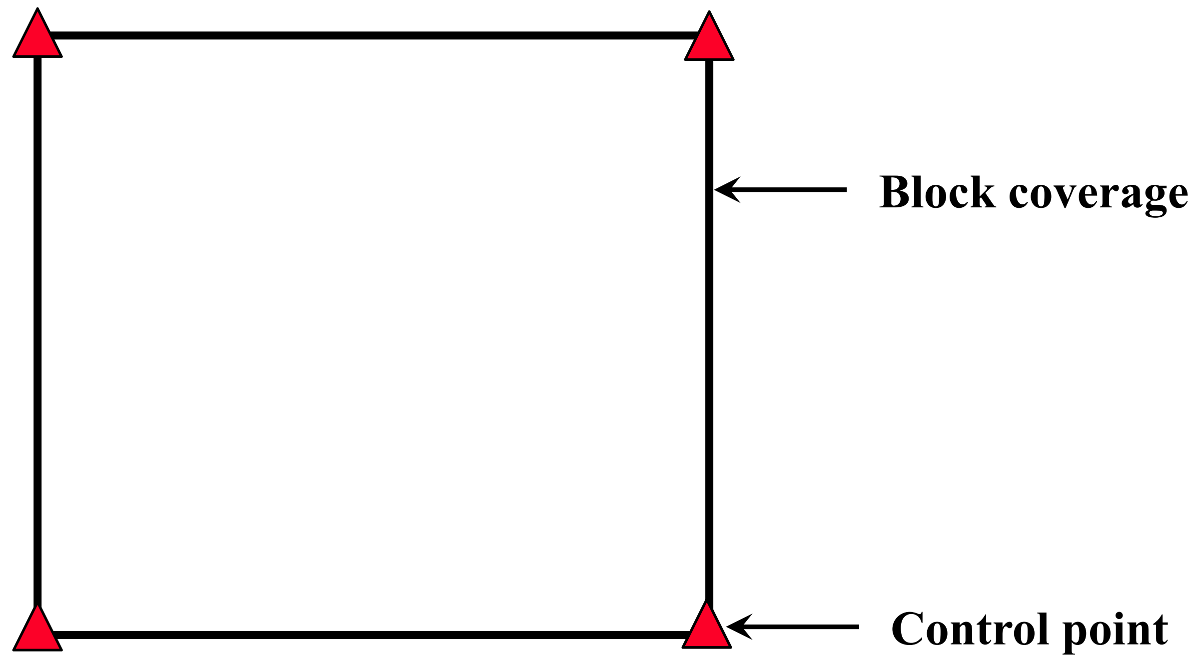


RMSE Results: No Biases

ISO

	GCP-4 (I)	GNSS/INS Pos. (II)	GNSS/INS Pos./Attitude (II)	GCP_10 (III)	GCP_4 60%SL (IV)
X (m)	0.11	0.08	0.06	0.07	0.05
Y (m)	0.14	0.13	0.11	0.10	0.07
Z (m)	1.74	0.17	0.14	0.20	0.13

Indirect Georeferencing



- The vertical accuracy within a block, which has control only at its corners, is worse at the center of the block.
- The vertical accuracy will deteriorate as the size of the block increases.
- Incorporating the GNSS or GNSS/INS observations at the exposure stations in the bundle adjustment procedure (ISO) would improve the vertical accuracy within the block.

Remarks

- Using GNSS/INS – Pos._{pc} or GCPs almost yields equivalent horizontal accuracy.
- GNSS/INS – Pos. observations at the perspective centers help in de-coupling ω and Y_o, which significantly improves the vertical accuracy.
- Adding GNSS/INS – attitude information at the perspective centers has a minor effect on improving the results (as far as the object space is concerned).

Experiment I (GCP-4)
EOP Variance-Correlation Matrix

$\omega(sec^2)$	$\phi(sec^2)$	$\kappa(sec^2)$	$X_o(m^2)$	$Y_o(m^2)$	$Z_o(m^2)$
6772.340	0.048	0.126	0.013	**-0.990**	-0.390
0.048	185.124	0.008	**0.830**	-0.043	-0.077
0.126	0.008	25.789	-0.031	-0.151	0.158
0.013	**0.830**	-0.031	0.036	-0.011	-0.0106
-0.990	-0.043	-0.151	-0.011	**0.655**	0.397
-0.390	0.077	0.158	-0.106	0.397	**1.569**

Experiment II (GNSS/INS – Position) EOP Variance-Correlation Matrix

$\omega(sec^2)$	$\phi(sec^2)$	$\kappa(sec^2)$	$X_o(m^2)$	$Y_o(m^2)$	$Z_o(m^2)$
38.889	-0.009	-0.015	-0.009	**-0.813**	-0.100
-0.009	38.712	0.060	**0.850**	0.011	0.048
-0.015	0.060	10.654	0.026	-0.025	-0.005
-0.009	**0.850**	0.026	0.005	-0.005	0.063
-0.813	0.011	-0.025	-0.005	**0.005**	0.010
-0.100	0.048	-0.005	0.063	0.010	**0.002**

Experiment II (GNSS/INS – Position/Attitude) EOP Variance-Correlation Matrix

$\omega(sec^2)$	$\phi(sec^2)$	$\kappa(sec^2)$	$X_o(m^2)$	$Y_o(m^2)$	$Z_o(m^2)$
25.610	0.007	-0.084	-0.004	**-0.802**	-0.076
0.007	27.578	0.021	**0.826**	0.003	-0.071
-0.084	0.021	9.633	-0.004	-0.016	0.002
-0.004	**0.826**	-0.004	0.004	0.007	-0.053
-0.802	0.003	-0.016	0.007	**0.003**	0.049
-0.076	-0.071	0.002	-0.053	0.049	**0.002**

Experiment III (GCP – 10)
EOP Variance-Correlation Matrix

$\omega(sec^2)$	$\phi(sec^2)$	$\kappa(sec^2)$	$X_o(m^2)$	$Y_o(m^2)$	$Z_o(m^2)$
186.584	-0.006	0.119	-0.001	**-0.865**	-0.044
-0.006	133.875	0.030	**0.826**	0.021	-0.454
0.119	0.030	14.008	-0.007	-0.129	-0.032
-0.001	**0.826**	-0.007	0.022	0.010	-0.364
-0.865	0.021	-0.129	0.010	**0.029**	0.026
-0.044	-0.454	-0.032	-0.364	0.026	**0.021**

Experiment IV (60% Side Lap) EOP Variance-Correlation Matrix

$\omega(sec^2)$	$\phi(sec^2)$	$\kappa(sec^2)$	$X_o(m^2)$	$Y_o(m^2)$	$Z_o(m^2)$
82.215	0.019	-0.162	0.028	**-0.825**	0.055
0.019	98.240	-0.030	**0.816**	-0.010	-0.495
-0.162	-0.030	9.071	-0.085	0.171	-0.009
0.028	**0.816**	-0.085	0.017	-0.020	-0.339
-0.825	-0.010	0.171	-0.020	**0.017**	-0.057
0.055	-0.495	-0.009	-0.339	-0.057	**0.017**

RMSE Results: IOP Biases

- Bias in the IOPs (50µm)
  - x_p, y_p, & f
- Bias in f (50µm)

ISO

	GCP-4 IOP (I)	GNSS/INS Pos. IOP (II)	GNSS/INS Pos. GCP-2 IOP (II)	GNSS/INS Pos./Attit. IOP (II)	GNSS/INS Pos. f (II)
X (m)	0.11	0.63	0.40	0.64	0.09
Y (m)	0.15	0.79	0.53	0.77	0.15
Z (m)	1.73	0.71	0.59	0.69	0.71

RMSE Results

	GCP-10 (III)	GCP-10 IOP (III)	ISO: GNSS/INS Pos. (II)	ISO: GNSS/INS Pos. IOP (II)
X (m)	0.07	0.07	0.08	0.63
Y (m)	0.10	0.11	0.13	0.79
Z (m)	0.20	0.20	0.17	0.71

Experiment II (GNSS/INS)

- GNSS/INS-attitude information with 0.05º bias in the boresight angles
  - Assumed to be accurate up to ±10sec
- RMSE Values (Check Point Analysis):
  - X = 1.16 m
  - Y = 1.54 m
  - Z = 1.14 m

Aerial Triangulation / Intersection

Bias	GNSS/INS (POS.) – AT (m)			Intersection (m)		
No Bias	0.08	0.13	0.17	0.15	0.21	0.37
IOPs	0.63	0.79	0.71	0.68	0.78	0.78
Lever Arm	0.10	0.07	0.18	0.17	0.21	0.39
Boresight	1.16	1.54	1.14	2.00	2.11	1.08

AT - Experiment II (GNSS/INS) EOP Variance-Correlation Matrix

$\omega(sec^2)$	$\phi(sec^2)$	$\kappa(sec^2)$	$X_o(m^2)$	$Y_o(m^2)$	$Z_o(m^2)$
25.610	0.007	-0.084	-0.004	-0.802	-0.076
0.007	**27.578**	0.021	0.826	0.003	-0.071
-0.084	0.021	**9.633**	-0.004	-0.016	0.002
-0.004	0.826	-0.004	**0.004**	0.007	-0.053
-0.802	0.003	-0.016	0.007	**0.003**	0.049
-0.076	-0.071	0.002	-0.053	0.049	**0.002**

- **Input GNSS/INS position accuracy: ±10cm.**
- **Input GNSS/INS attitude accuracy: ±10sec**

AT - Experiment II (GNSS/INS)
IOP Variance-Correlation Matrix

x_p (mm^2)	y_p (mm^2)	c (mm^2)
$\boxed{2.501e\text{-}005}$	1.2351e-008	-8.782e-010
1.235e-008	$\boxed{2.501e\text{-}005}$	9.259e-010
-8.782e-010	9.2589681766e-010	$\boxed{2.505e\text{-}005}$

- **Input accuracy for x_p, y_p, and c: ±5µm.**

Remarks

- In case of a bias in the IOPs, RMSE values obtained from GNSS/INS (position/attitude) – AT and Intersection are almost the same.
- In contrast, GNSS/INS (position/attitude) – AT significantly improves the point precision if either no bias, a bias in the lever arm, or bias in the boresight matrix is present.

Conclusions

- For photogrammetric mapping:
  - The main emphasis should be placed on the quality of the reconstructed object space rather than the quality of the derived EOPs from the onboard GNSS/INS unit.
  - In the absence of systematic errors, integrated sensor orientation and indirect georeferencing yield comparable results.
    - Integrated sensor orientation leads to better results than intersection.
  - In the presence of systematic errors, indirect georeferencing produces best results.
  - Indirect georeferencing:
    - IOPs + ΔIOPs → EOPs + ΔEOPs
    - EOPs + ΔEOPs + IOPs + ΔIOPs → Correct Object Space
  - Direct georeferencing:
    - (IOPs + ΔIOPs) & GNSS/INS → EOPs
    - EOPs + IOPs + ΔIOPs → Wrong Object Space

Direct Georeferencing

Accuracy Analysis: Will be discussed in detail in Chapters 3 & 4

LiDAR Mapping