ABSTRACT

Ruan, Wei. Ph.D., Purdue University, August 2012. Modeling Vertical U-tube Ground Heat Exchangers for Ground-Coupled Heat Pump Applications. Major Professor: W. Travis Horton.

Buildings were responsible for 40% of the primary energy use in the United States. The energy consumed in the building section was broken down by end-users that 21% and 13% of primary energy was used for space heating and space cooling respectively. Ground-couple heat pump (GCHP) systems use stored energy in the ground at a relatively constant temperature as a heat source (in the winter) and a heat sink (in the summer) so that they offer higher energy efficiency than the conventional heating and cooling system. However, drilling of deep boreholes comes with a high initial cost, which hinders the application of the GCHP systems. To effectively use a GCHP, it is necessary in the design phase to accurately predict the thermal performance of GCHP systems throughout their entire life spans.

This research aims to develop the heat conduction model and the hybrid moisture and groundwater model for vertical U-tube GHEs to evaluate its performance within their life spans. Based on these models, the sensitivity analysis will be conducted to study the influence of parameters on the thermal performance of vertical U-tube GHEs. In order to achieve this goal, it can be broken down to three sub-objectives:

- **Objective 1:** Develop the two-dimensional heat conduction model for constant volume and variable volume vertical U-tube GHEs in cylindrical coordinate system.
- **Objective 2:** Develop hybrid moisture and groundwater model in the three-dimensional unstructured grid to consider the influence of the moisture transfer and groundwater movement on the thermal performance.
- **Objective 3:** Conduct the sensitivity analysis of the parameters on the performance of vertical U-tube GHEs. The screening design will select the relatively important parameters and the sensitivity analysis will rank these screened parameters. The result of sensitivity analysis will provide the guidance for further optimization of the model of vertical U-tube GHEs.

Based on the local geological surveys or the data from testing boreholes, the models developed in this research would have great significance for the designers to predict the thermal performance of GCHP systems. The appropriate design for the GCHP systems will be achieved to trade off the initial cost and the operating cost in their life spans. In the future research, the models of vertical U-tube GHEs could be integrated with the models of cooling towers or solar collectors to simulate the hybrid GCHP systems, which provide much higher efficiency for the thermally imbalanced buildings.

CHAPTER 1 INTRODUCTION

1.1 Background

According to Building Energy Data Book (DOE 2011), buildings in the United States were responsible for 41% of the primary energy use. The energy consumed in the building section was broken down by end-users that 22.5% and 14.8% of primary energy was used for space heating and space cooling respectively in the United States, which was about 9.56×10^{18} Joules (9.07×10^{15} Btu) and 6.31×10^{18} Joules (5.98×10^{15} Btu).

The high demand of alternative low-cost energy resources has given rise to the development of geothermal heat pump systems for residential and commercial heating and cooling applications. Geothermal heat pump systems use stored energy in the ground at a relatively constant temperature as a heat source (in the winter) and a heat sink (in the summer) so that they offer higher energy efficiency than air-source systems. The average heating and cooling efficiency for existing GCHPs in the United States were that coefficient of performance (COP) was 3.4 in heating season and energy efficiency ratio (EER) was 16 in cooling season; whereas, the average cooling and heating efficiency of existing air-source heat pump were that COP was 2.3 and EER was 13 (DOE 2011). Therefore, the efficiency of GCHP systems would increase by 47.8% in heating season and 23.0% in cooling season than that of air-source heat pump systems. The performance criteria of geothermal heat pumps for EnergyStar$^{®}$ certification are required by EPA (EnergyStar 2009) for three levels from Tier 1 to Tier 3. The test procedures (ISO-13256 1998) for close-loop and open-loop geothermal heat pumps should be followed to rate the performance of geothermal heat pumps.

Limited by the local geographic limitations, surface water is not frequently available for geothermal heat pump systems as a heat source and a heat sink for space heating and space cooling. Ground-coupled heat pumps (GCHPs) are typical heating, ventilating, air-conditioning & refrigeration (HVAC&R) equipment to in geothermal heat pump systems. GCHPs have the closed-loop ground heat exchangers (GHEs) that can effectively avoid fouling in the system and contaminating the groundwater, which reduce the environmental effect of the operation of geothermal heat pumps. There are two kinds of closed-loop GHE: horizontal and vertical. The advantages of the vertical GHEs include their efficient utilization of available land for installation, and a reduced influence of ground temperature fluctuations on the performance of vertical GHEs.

However, drilling of deep boreholes comes with a high initial cost, which hinders the application of GCHP systems. According to California's Consumer Energy Center (CEC 2011), the cost of vertical GHEs for GCHP is about 50% - 70% of the total initial cost of the whole GCHP system depending on the terrain and other local factors. Another factor that prevents GCHP being widely used is the gradual degradation of the GCHP's performance. Because of the imbalance of building load, the ground temperatures around GHEs gradually increase or decrease with the operation of GCHPs, which will correspondingly change the fluid temperatures existing from ground loops to degrade the GCHP's performance.

Cane and Forgas (Cane & Forgas, 1991) estimated that the length of GHEs was oversized by about 10% - 30% in the North American market. To effectively utilize GCHPs, it is necessary in the design phase to accurately predict the heat extraction and injection rates, and the ground thermal response of the GHE throughout its entire life span. With the development of computing technique, the validated numerical model for vertical U-tube GHEs will be used to achieve this goal in this thesis. A sensitivity analysis of this numerical model will evaluate the contributions of the factors to the annual electricity consumption in the GCHP systems.

1.2 Research motivation

Currently, with the development of numerical techniques and computation hardware, many numerical models of vertical U-tube GHEs are developed to predict the thermal performance in their life spans for space cooling and space heating. There is a trade-off between the accuracy and computation effort to widely use the numerical models of vertical U-tube GHE. Therefore, some important factors should be included into the numerical models to capture their influence on the performance of vertical U-tube GHEs.

Limited by the available land for installing vertical U-tube GHE, the distance between boreholes is typically about 4-6 m (13-20 ft), which cannot avoid thermal interference between boreholes in the life spans of vertical U-tube GHEs. A corrected capacitance resistance model, called CaRM, was developed (De Carli et al., 2010) to consider the thermal interference between boreholes, which had good agreement with the simulation results of HEAT2 (Blomberg, 1999). In this thesis, a three-dimensional capacitance resistance model will be used in the entire domain of vertical U-tube GHE to calibrate the thermal interference between boreholes in the life span.

Driven by heat flux of boreholes, moisture movement is coupled with heat conduction in the region around boreholes. The thermal conductivity of the unsaturated soil is significantly impacted by the water content in soil. Côté and Konrad (Côté & Konrad, 2005) obtained the correlations of thermal conductivity and water content for different types of soils by regression of literature data. Therefore, the coupled heat and mass transfer in the vicinity of boreholes should be included into the numerical model in this thesis.

Due to the existence of hydraulic head in some locations, groundwater flows in horizontal direction through the entire domain of vertical U-tube GHE with a constant velocity, which depends on the hydraulic head and hydraulic conductivity. This advection flow term should be integrated into the heat conduction equations to investigate the impact of groundwater movement on the thermal performance of vertical U-tube GHE.

There are many factors in the numerical model of vertical U-tube GHEs, which will influence on their performance to different extent. Screening designs rank the order of important parameters and the sensitivity analysis evaluate how much the parameters contribute to the annual electricity consumption of GCHP systems. The result of sensitivity analysis will be used to improve the model by investigating the most important parameters in detail and simplifying the least important parameters, which would trade off between the accuracy and computation effort.

1.3 Research objectives

This research aims to develop the numerical models of vertical U-tube GHEs to predict the thermal performance within the life span. Based on this model, the sensitivity analysis will be made to study the influence of parameters on the performance. In order to achieve this goal, it can be broken down to four sub-objectives:

Objective 1: Develop the two-dimensional heat conduction model for constant volume and variable volume vertical U-tube GHEs in cylindrical coordinate system. Thermal interference between boreholes is considered by the correction coefficients based on the relative locations of individual boreholes in the borefield.

Objective 2: Develop hybrid moisture and groundwater model in the three-dimensional unstructured grid to consider the influence of the moisture transfer and groundwater movement on the thermal performance. The governing equations for unsaturated and saturated soil are applied along different types of boreholes, which make this model suitable for the specific geographic locations with high water content in soil and with high advection velocity of groundwater.

Objective 3: Conduct the sensitivity analysis of the parameters on the performance of vertical U-tube GHEs. The screening design will select the relatively important parameters and the sensitivity analysis will rank these screened parameters. The result of sensitivity analysis will provide the guidance for further optimization of the model of vertical U-tube GHEs, which will better balance between the accuracy and the computation effort.

1.4 Organization of dissertation

In Chapter 2, the GCHP system and its major components are firstly introduced, followed by a state-of-the-art literature review on the researches that have been done on the properties of soils, modeling of vertical U-tube GHEs, and sensitivity analysis.

The main body of the thesis is presented in Chapter 3 – 6. In Chapter 3, the two-dimensional heat conduction model of vertical U-tube GHEs is developed for the constant volume and variable volume systems. The results of dynamic case are validated by the published experimental data with good agreement.

In Chapter 4, two cases are studied for the constant volume and variable volume system respectively. A theoretical office building located in Chicago, Illinois, is selected as the object building. The heat transfer rate profiles, ground temperature distribution, and the fluid outlet temperature are calculated for the long-term operation of the GCHP system.

In Chapter 5, the moisture transfer and groundwater advection model are developed for unsaturated and saturated soil. These two models are integrated as the hybrid model because of their similar format of governing equations. A case study is conducted to investigate the influence of moisture transfer and groundwater advection on the thermal performance.

In Chapter 6, the sensitivity analysis is conducted for the variable volume heat conduction model and the hybrid moisture and groundwater model to rank the importance of the screened parameters to the electricity consumption.

In Chapter 7, the summary and conclusions of this thesis are presented. And the recommendations for the future work are listed.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction of Ground-Coupled Heat Pump Systems

GCHP systems, also called ground-source heat pumps, are a kind of geothermal heat pump systems, which use stored energy in the ground at a relatively constant temperature as a heat source (in the winter) and a heat sink (in the summer) to provide space heating and space cooling for residential and commercial buildings. The schematic diagram of a typical GCHP is shown in Figure 2.1 and Figure 2.2 for heating mode and cooling mode. The major components include GHE, source heat exchanger (outdoor heat exchanger), load heat exchanger (indoor heat exchanger), compressor, throttling valve, reverse valve, and ground loop pump.

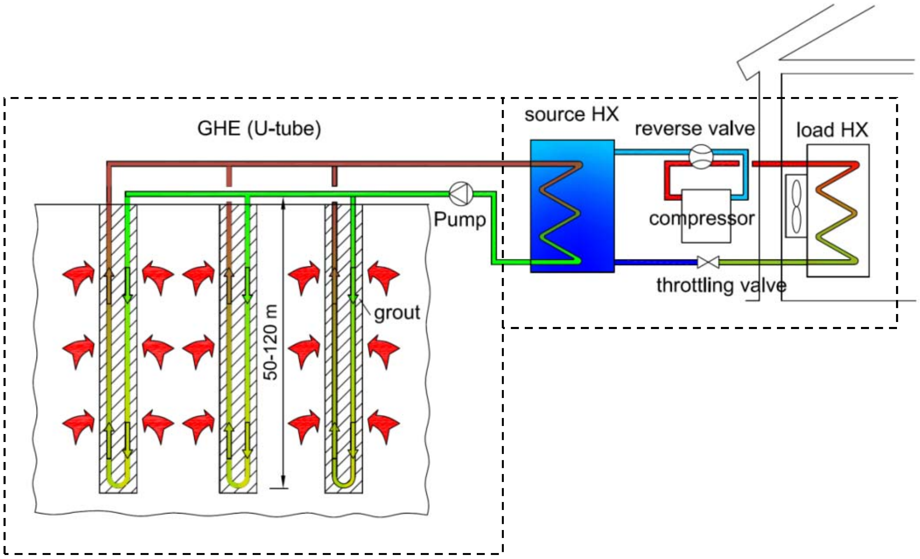

Figure 2.1 Schematic Diagram of a GCHP System in Heating Mode

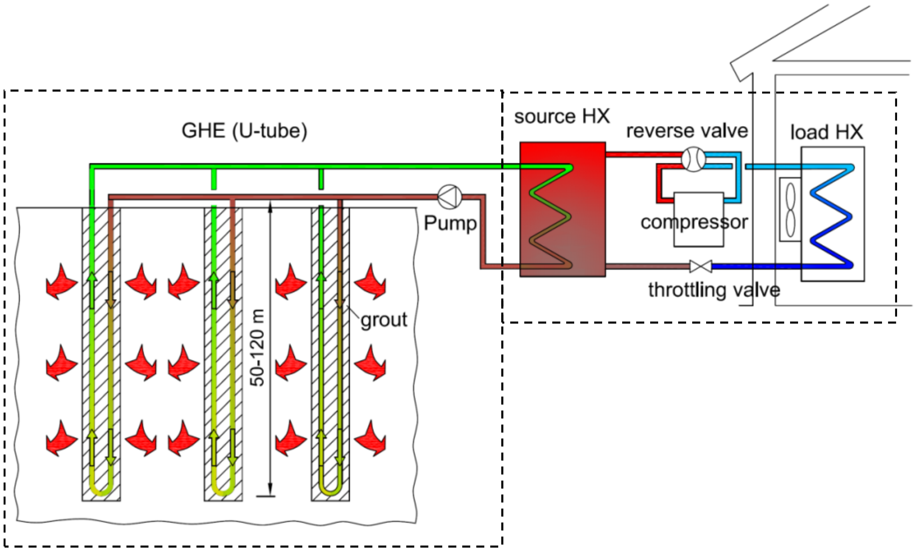

Figure 2.2 Schematic Diagram of a GCHP System in Cooling Mode

GCHP systems transfer heat from a low-temperature medium to a high-temperature one at the cost of mechanical work input. Therefore, the electricity is supplied to the motor connected to the compressor to input mechanical work into the GCHP systems. Through vapor compression cycle (Cengel & Boles, 2001; Moran and Shapiro, 2003), heat is transferred from a low-temperature medium to a high-temperature one with refrigerant circulating in the heat pump loop.

According to the second law of thermodynamics, the COP and EER of GCHP systems are highly related to the evaporating temperature and condensing temperature during operation. Because the ground has relatively constant temperature than air and there is no frosting problem for ground loops in heating mode, it is a perfect medium to use as a heat source and a heat sink for heat pump systems. To achieve the higher efficiency, ground loops are integrated with heat pumps to replace air to be used as a heat source and a heat sink in GCHP systems.

2.1.1 Ground Heat Exchangers

There are two kinds of GHEs for GCHP systems: one is open-loop and the other is close-loop. There are one discharge well and one recharge well in the open-loop GHE, as shown in Figure 2.3. This type of GHE uses discharge well or surface body water as the heat exchange fluid that circulates directly through the GCHP system. Once it has circulated through the system, the water returns to the ground through the recharge well, or surface discharge. Open-loop GHEs would significantly improve heat transfer efficiency with the advection flow of groundwater through the ground. However, this option is obviously practical only where there is an adequate supply of relatively clean water, and all local codes and regulations (MassDEP, 2009; NHDES, 2009; NREPA, 1994) regarding groundwater discharge are met. If the hardness of groundwater is high, it is easy to scale and foul on the heat transfer area of source heat exchanger, water softening device has to be used to reduce hard water’s adverse effects.

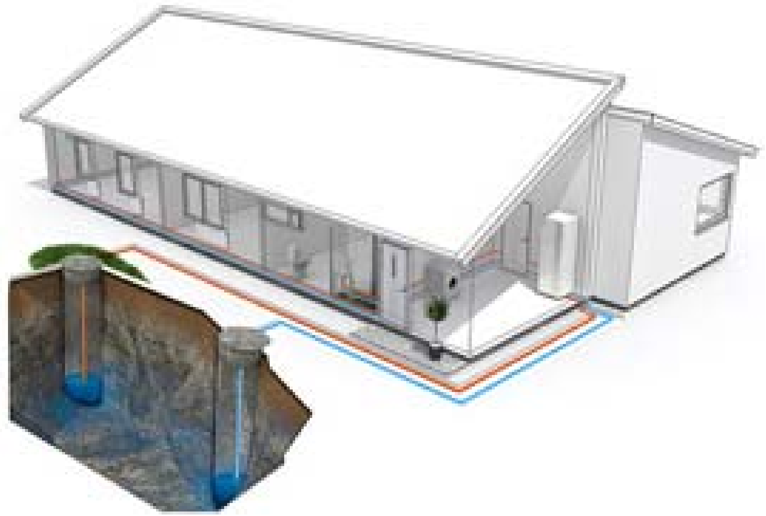

Figure 2.3 Diagram of an Open-loop GHE

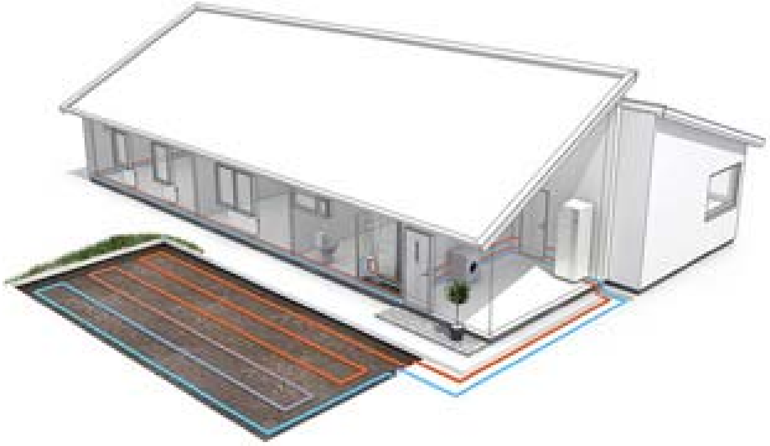

Figure 2.4 Diagram of a Horizontal Close-loop GHE

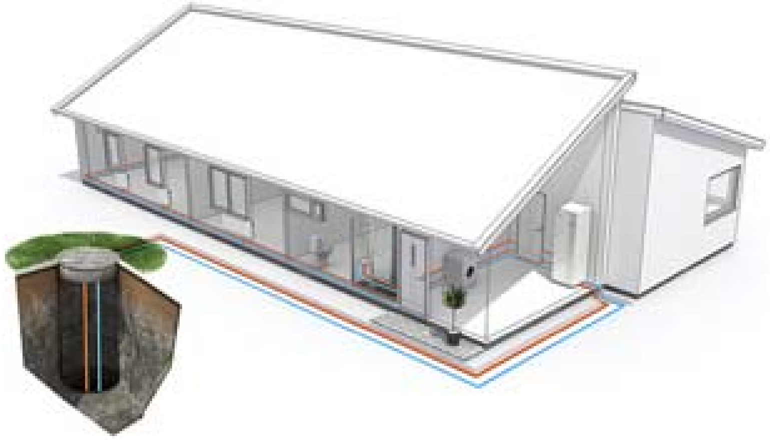

Figure 2.5 Diagram of a Vertical Close-loop GHE

To reduce the negative environmental impacts, polypropylene pipes can be buried under ground to form close-loop GHEs in the either of two ways: vertical boreholes or horizontal trenches. The diagrams of horizontal close-loop GHEs and vertical close-loop GHEs are shown in Figure 2.4 and Figure 2.5 respectively.

For horizontal close-loop GHEs, the pipes are buried in trench with a depth of about 1.8 meters (6 feet) and the typical distance between pipes is 0.6 meters (2 feet) to reduce thermal interference. The biggest advantage of this horizontal GHE is the low initial cost and the fast pace to install. However, the disadvantages are the adverse impact of ground temperature fluctuations on the performance and the high demand of unused land for digging trenches to accommodate pipes.

For vertical close-loop GHEs, U-tube polypropylene pipes are inserted into 0.1-0.15 meters (4-6 inches) diameter boreholes drilled to a depth of 60-100 meters (200-330 feet). Bentonite based grout is typically pumped into boreholes to fill all space not occupied by U-tube pipes, which will enhance heat transfer between borehole wall and the U-tube pipes and prevent water movement along the borehole between different aquifers (Carda, 2006). To increase the conductivity of grout to reduce the thermal resistance between the borehole wall and U-tube pipes, some additives, such as silica sand (Park et al, 2011) and graphite powder (Lee et al 2010, Delaleux et al 2012) are investigated for the innovative grout for vertical U-tube GHEs. Although the high initial cost of drilling boreholes is the main challenge for vertical U-tube GHEs, it is attractive to apply the vertical U-tube GHEs for GCHP systems to efficiently utilize available land for installation and to reduce influence of ground temperature fluctuations on the performance.

2.1.2 Heat Pump Machines

A heat pump is a machine or device that transfers thermal energy from one location, called the “heat source," which is at a lower temperature, to another location called the "heat sink", which is at a higher temperature. Thus, heat pumps move thermal energy opposite to the direction that it normally flows at the cost of mechanical work input. From the view of heat source and heat sink, there are air source heat pumps, water source heat pumps, ground source heat pumps and solar-assisted heat pumps for HVAC&R applications (ASHARE, 2000).

The schematic diagrams of GCHP systems are shown in Figure 2.1and Figure 2.2 for heating and cooling mode. The switchover of heating mode and cooling mode is accomplished by adding a reversing valve to the vapor compression cycle. The compressor in the GCHP system is a kind of positive displacement compressor. The performance rating of positive displacement compressor can be evaluated by a polynomial equation with the suction dew point temperature and discharge dew point temperature as parameters. This polynomial equation is obtained by regression from manufacturer-tested tabular data under testing conditions (AHRI, standard-540 2004). Source heat exchangers in GCHP system are liquid-to-refrigerant heat exchangers. In ground loops of vertical U-tube GHEs, anti-frozen solutions (glycol solution) are used as working fluids to transfer heat between ground loops to source heat exchangers. For load heat exchangers, air-to-refrigerant heat exchangers or liquid-to-refrigerant heat exchangers may be utilized depending on the heating and cooling distribution systems in residential and commercial buildings.

2.1.3 Heating and Cooling Distribution Systems

Heating and cooling distribution systems use air or water as heat transfer medium to provide space heating and space cooling to maintain thermal comfort in buildings.

For air distribution systems, they provide complete sensible and latent cooling, heating, and dehumidification/humidification capacity in the supply air that is boosted by the supply fans. The major components in the air distribution systems are fans, air ducts, terminal air units, and thermostat controllers. The pre-processed air is delivered by the fans through air ducts to the terminal air units. In the terminal air units, there is a kind of device that is connected to the thermostats in rooms to maintain the thermal comfort by feedback control.

For hydronic distribution systems, water is typically used as heat transfer medium to be transmitted by pumps through pipes to radiant panels, baseboard, and finned-tube units. These hydronic terminal units provide space heating and space cooling through a combination of radiation and convection. It is an energy efficiency measure to integrate hydronic distribution systems with dedicated outdoor air systems. The hydronic distribution systems supply sensible heating and cooling and the dedicated outdoor air systems meet the requirement of ventilation and latent loads in residential and commercial buildings. Compared with air distribution systems, hydronic distribution systems can deliver space heating and space cooling more efficiently (ASHRAE, 2000). Therefore, the sizes of pipes in hydronic distribution systems are much smaller than those of air ducts in air distribution systems. Furthermore, the water temperatures in hydronic distribution systems are relatively lower in heating mode and relatively higher in cooling mode, which will significantly increase the COP of heating and the EER in cooling for heat pump machines.

2.2 Literature Review

2.2.1. Properties of Soil

Properties of soil are very vital to the thermal performance of GCHP systems because the energy stored in soil can provide more constant temperatures as a heat source or a heat sink for space heating and space cooling than air-source heat pump systems, which will significantly increase the COP in heating mode and the EER in cooling mode. The thermal conductivity, the density, the specific heat, the porosity, the hydraulic conductivity should be investigated to study the thermal performance of vertical U-tube GHEs for GCHP systems.

Soil is a natural body consisting of layers (soil horizons) of primarily mineral constituents (Birkeland, 1999), which is made up in part of finely ground rock particles. According to the size of particles, soil is grouped as sand, silt and clay. Each size plays a significantly different role. For example, the largest particles, sand, determine aeration and drainage characteristics, while the tiniest, sub-microscopic clay particles, are chemically active and are easy to bind with water and plant nutrients. The ratio of these sizes determines soil type: clay, loam, clay-loam, silt-loam, and so on. There are twelve soil types determined by the soil texture triangle (USDA, 2000), as shown in Figure 2.6. This soil texture triangle is generated by feel analysis (Thien, 1979). Soil textures are classified by the fractions of each soil separate (sand, silt, and clay) present in a soil. Classifications are typically named for the primary constituent particle size or a combination of the most abundant particles sizes, e.g. "sandy-clay" or "silty-clay." A fourth term, loam, is used to describe a roughly equal concentration of sand, silt, and clay.

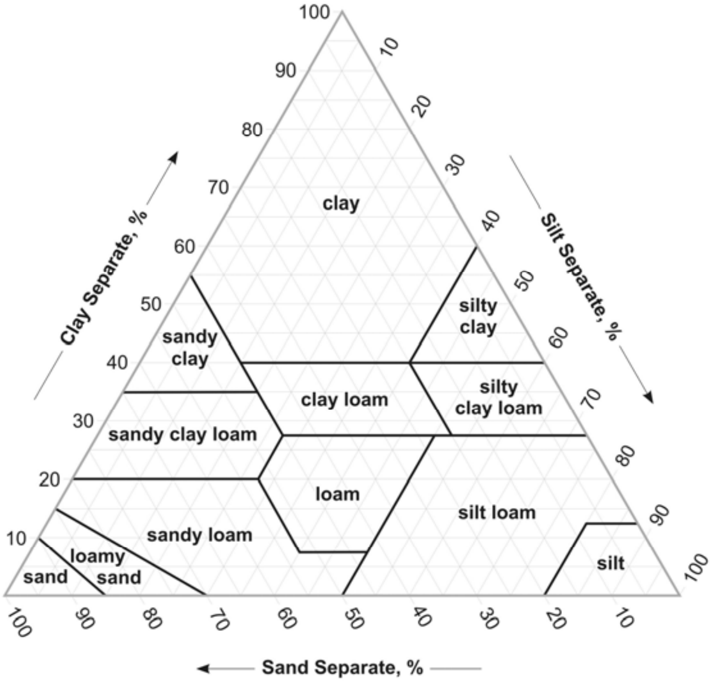

Figure 2.6 Soil Texture Triangle

Soil is porous solid unsolved medium. There is moisture and water existence in pores of soil. Based on the water content in pores, soil is grouped as unsaturated and saturated (Lambe & Whitman, 1969). For the unsaturated soil, water and moisture co-exist in pores of soil. These two paired substances are equalized with the partial pressure of moisture and saturated vapor pressure of water. The moisture movement is driven by temperature difference and moisture partial pressure difference to diffuse in pores. The moisture diffusion coefficients include the thermal moisture diffusivity driven by temperature difference and the isothermal moisture diffusivity driven by partial pressure difference (Piechowski, 1999). The relationship of thermal and isothermal moisture diffusivities with water content was first proposed by Philip and de Vries (Philip & de Vries, 1957) and modified by de Vries (de Vries, 1958).

In saturated soil, it is assumed that groundwater and soil are always thermal equilibrium because the advection velocity of groundwater is very low in the application of vertical U-tube GHEs (Corey, 2003; Kong, 1999). In saturated porous medium, Darcy's law is a simple proportional relationship between the instantaneous discharge rate through a porous medium, the viscosity of the fluid and the pressure drop over a given distance. The mathematical expression is (Darcy, 1856):

$$q_D = \frac{\dot{Q}}{A} = -\frac{\kappa}{\mu}\frac{p_b - p_a}{L} = -\frac{\kappa}{\mu}\nabla p = -K_{sat}\nabla h \tag{2.1}$$

where, κ is a measure of the ability of a porous material to allow fluids to pass through it. K_{sat}, saturated hydraulic conductivity, is used to define the proportionally constant for the flow of groundwater through a porous media. It can be derived from the empirical and experimental approach. Shepherd (Shepherd, 1989) derived an empirical formula for approximating hydraulic conductivity from grain size analyses:

$$K_{sat} = a(D_{10})^b \tag{2.2}$$

where, a and b are empirically derived terms based on the soil type, and D_{10} is the diameter of the 10 percentile grain size of the material. One of the experimental approaches is the constant-head method, which use the Darcy’s law to calculate the saturated hydraulic conductivity. Bear (Bear, 1972) obtained the saturated hydraulic conductivity in the nature in Table 2.1.

Table 2.1 Saturated hydraulic conductivity values found in nature

K_{sat} (cm/s)	10^{2}	10^{1}	1	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}
K_{sat} (ft/day)	10^{5}	10^{4}	10^{3}	100	10	1	0.1	0.01	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
Relative Permeability	Pervious				Semi-Pervious				Impervious				
Aquifer	Good					Poor				None			
Unconsolidated Sand & Gravel	Well Sorted Gravel		Well Sorted Sand or Sand & Gravel			Very Fine Sand, Silt, Loess, Loam							
Unconsolidated Sand & Gravel					Peat		Layered Clay			Fat / Unweathered Clay			
Consolidate Rocks	Highly Fractured Rocks				Oil Reservoir			Fresh Sandstone		Fresh limestone Dolomite		Fresh Granite	

2.2.2 Models of Vertical U-tube Ground Heat Exchangers

The methods used to calculate heat transfer of VGHE have been under development since the 1940’s. There appear to have been three phases in this development. The first phase, which covered a period from the 1940’s to the 1960’s, was focused on the development of theoretical models. The second phase focused on analytical solutions, which covered a timeframe from the 1970’s to the 1990’s. Finally, with the advent of computers came the development of numerical models since the late of 1980’s.

2.2.2.1 Theoretical models

Ingersoll et al (Ingersoll et al, 1954) proposed the line source model for the application of VGHE in GSHP systems. In the line source model, heat transfer from a single borehole is treated as a line heat source with constant heat strength in an infinite medium. The medium is assumed to be at a uniform initial temperature and the line heat source starts at time zero. The temperature distribution in the medium is given by:

$$T_s - T_0 = \frac{q'}{2\pi k_s} \int_{\frac{r}{2\sqrt{a_s t}}}^{\infty} \frac{e^{-\beta^2}}{\beta} d\beta = \frac{q'}{2\pi k_s} I\left(\frac{r}{2\sqrt{a_s t}}\right) \tag{2.3}$$

The line source model is assumed to be a one-dimensional (radial) heat transfer with a known constant heat flux. For the application of GCHP, the extracted or ejected load profile varies with time. Ingersoll suggested the integral part in Equation (2.1 be split into parts on a monthly basis. A monthly average heat transfer rate is used to replace the constant heat transfer rate in Equation (2.3.

Carslaw and Jaeger (Carslaw and Jaeger, 1959) developed the cylindrical source model based on one-dimensional heat conduction in a cylindrical coordinate system. The expression is:

$$T_s = \frac{2F_0 kt}{Ka} + \frac{F_0 a}{K} \left\{ \frac{r^2}{2a^2} - \frac{1}{4} - 2 \sum_{n=1}^{\infty} e^{-\frac{k\alpha_n^2 t}{a^2}} \frac{J_0\left(\frac{r\alpha_n}{a}\right)}{\alpha_n^2 J_0(\alpha_n)} \right\} \tag{2.4}$$

Hart and Couvillion (Hart & Couvillion, 1986) defined the far-field radius to determine the range of the ground that would be influenced by a single borehole. The change of ground temperature beyond the far-field radius is negligible. The line source equation is expressed as follows, which is as same as Ingersoll's model.

$$T_s - T_0 = \frac{q'}{4\pi k_s}\int_{\frac{r^2}{4a_s t}}^{\infty}\frac{e^{-\lambda}}{\lambda}d\lambda = \frac{q'}{4\pi k_s}E1\left(\frac{r^2}{4a_s t}\right) \tag{2.5}$$

The integral part in Equation (2.5 is simplified as the sum of a power series of N terms. That is:

$$T_s - T_0 = \frac{q'}{2\pi k_s}\left[ln\frac{r_{far}}{r} - 0.98 + \frac{4r^2}{2r_{far}^2} - \frac{1}{4\times 2!}\left(\frac{4r^2}{2r_{far}^2}\right)^2 + \cdots + \frac{(-1)^{N+1}}{2N\times N!}\left(\frac{4r^2}{2r_{far}^2}\right)^N\right] \tag{2.6}$$

Some terms in Equation (2.6 could be truncated off if the absolute error is within a desired tolerance.

2.2.2.2 Analytical models

IGSHPA (IGSHPA, 1991) defined the ground formation resistance of a single borehole based on the line source model, as expressed in Equation (2.3. An approximation for the integral part in Equation (2.3 is given when η is in the range of $(0, 1]$ and $(1, \infty)$. The superposition method is used to consider the influence of adjacent boreholes on the borehole that is studied. IGSHPA also provided a design method to estimate the length of cooling and heating as follows:

For heating,

$$H_h = \frac{Capacity_h(COP_h - 1)(R_p + R_s * RunFraction_h)}{COP_h(T_{s,min,a} - T_{f,min})} \tag{2.7}$$

For cooling,

$$H_c = \frac{Capacity_c(COP_c + 1)(R_p + R_s * RunFraction_c)}{COP_c(T_{f,max} - T_{s,max,a})} \tag{2.8}$$

The larger of these two lengths will be used as the design length of the borehole. Then, a monthly performance analysis can be performed based on the monthly bin method. The procedure is: first, the $T_{f,min}$ and $T_{f,max}$ in each month are assumed; second, the heat pump run fractions are calculated using the bin method; third, the design length of the borehole is used to calculate the temperature differences $(T_{s,min,a} - T_{f,min})$ and $(T_{f,max} - T_{s,max,a})$ from Equation (2.7 and (2.8; fourth, the calculated and assumed $T_{f,min}$ and $T_{f,max}$ are compared, if necessary, a new set of $T_{f,min}$ and $T_{f,max}$ should be assumed until the convergent solution is obtained.

Kavanaugh (Kavanaugh, 1985) further developed a model based Carslaw's cylinder source model. He used a correction term to consider the non-uniform heat flow rate at the zones close to the pipes and the number of U-pipes inside the borehole. The average water temperature is shown as below:

$$\bar{T}_f - T_0 = \frac{q'}{k_s} G(z,p) + \frac{q'}{CN_t 2\pi r_{po} h_{eq}} \tag{2.9}$$