Analog Accelerators for Computational Solid Mechanics

by Osama Abdelaleim

Abstract:

The solution of large systems of linear equations represents one of the primary computational bottlenecks in finite element analysis (FEM) used in computational solid mechanics (CSM). Digital algorithms have been the main solvers for linear systems of equations, but their computational cost continues to scale with system size, motivating exploration of alternative computing paradigms. Analog compute-in-memory (CIM) architectures have recently gained attention for accelerating matrix-vector multiplication and solution of linear systems of equations.

Solving full linear systems with feedback-based analog solvers is limited by convergence dynamics. Instead, this dissertation investigates the behavior of a class of analog accelerators, referred to as the Equivalent System Solver (ESS), which is inspired by the analogy between 1D spring systems and resistive networks. Instead of simulating the step-by-step numerical process of solving the system, as traditional digital solvers do, this architecture maps the computational problem to a physical system whose nodal voltages correspond to the solution vector. This enables it to solve symmetric positive definite (SPD) systems at a speed that is independent of system size.

This research details the principles behind the ESS design, optimizations of the system architecture, and numerical results that demonstrate the robustness of the design for general linear systems and FEM stiffness matrices. Theoretical analysis shows that, for symmetric diagonally dominant systems, the ESS maps the system to a strictly passive resistive network with effectively zero settling time. More generally, for SPD systems, the convergence dynamics of the ESS is governed by matrix properties, particularly the smallest eigenvalue and deviation from diagonal dominance, rather than by the number of unknowns. The circuit dynamics in this case is shown to correspond to an overdamped second-order system that implicitly solves a preconditioned problem, improving numerical conditioning. The complexity studies of the variations of the ESS solver show that the negative resistance design and the choice of system transformation define the speed, accuracy, occupied physical space, and power consumption of the solver.

The applicability of the ESS to FEM is evaluated through benchmark studies ranging from one-dimensional bar problems to a two-dimensional plane stress elliptic membrane problem. 1D bar tests demonstrate that, when systems are scaled consistently with finite element principles, the ESS maintains nearly mesh-independent convergence time and stable accuracy under refinement. In contrast, feedback-based analog solvers exhibit quadratically increasing error and solution time as stiffness matrices evolve with mesh refinement. The two-dimensional benchmark further demonstrated that analog solutions can be incorporated into conventional finite element workflows without loss in accuracy.