ABSTRACT

Uncertainty quantification techniques based on the spectral approach have been studied extensively in the literature to characterize and quantify, at low computational cost, the impact that uncertainties may have on large-scale engineering problems. One such technique is the *generalized polynomial chaos* (gPC) which utilizes a time-independent orthogonal basis to expand a stochastic process in the space of random functions. The method uses a specific Askey-chaos system (that is concordant with the measure defined in the probability space) in order to ensure exponential convergence to the solution. For nearly two decades, this technique has been used widely by several researchers in the area of uncertainty quantification to solve stochastic problems using the spectral approach. However, a major drawback of the gPC method is that it cannot be used in the resolution of problems that feature strong nonlinear dependencies over the probability space as time progresses. Such downside arises due to the time-independent nature of the random basis, which has the undesirable property to lose unavoidably its optimality as soon as the probability distribution of the system's state starts to evolve dynamically in time.

Another technique is the *time-dependent generalized polynomial chaos* (TD-gPC) which utilizes a time-dependent orthogonal basis to better represent the stochastic part of the solution space (aka random function space or RFS) in time. The development of this technique was motivated by the fact that the probability distribution of the solution changes with time, which in turn requires that the random basis is frequently updated during the simulation to ensure that the mean-square error is kept orthogonal to the discretized RFS. Though this technique works well for problems that feature strong nonlinear dependencies over the probability space, the TD-gPC method possesses a serious issue: it suffers from the curse of dimensionality at the RFS level. This is because in all gPC-based methods the RFS is constructed using a tensor product of vector spaces with each of these vector spaces representing a single RFS over one of the dimensions of the probability space. As a result, the higher the dimensionality of the probability space, the more vector spaces needed in the construction of a suitable RFS. To reduce the dimensionality of the RFS—and thus the associated computational cost—, gPC-based methods require the use of versatile sparse tensor products within their numerical

schemes to alleviate to some extent the curse of dimensionality at the RFS level. Therefore, this curse of dimensionality in the TD-gPC method alludes to the need of developing a more compelling spectral method that can quantify uncertainties in long-time response of dynamical systems at much lower computational cost.

In this work, a novel numerical method based on the spectral approach is proposed to resolve the curse-of-dimensionality issue mentioned above. The method has been called the *flow-driven spectral chaos* (FSC) because it uses a novel concept called *enriched stochastic flow maps* to track the evolution of a finite-dimensional RFS efficiently in time. The enriched stochastic flow map does not only push the system's state forward in time (as would a traditional stochastic flow map) but also its first few time derivatives. The push is performed this way to allow the random basis to be constructed using the system's enriched state as a germ during the simulation and so as to guarantee exponential convergence to the solution. It is worth noting that this exponential convergence is achieved in the FSC method by using only a few number of random basis vectors, even when the dimensionality of the probability space is considerably high. This is due to the fact that (1) the cardinality of the random basis does not depend upon the dimensionality of the probability space and that (2) the cardinality is bounded from above by $M + n + 1$, where M is the order of the stochastic flow map and n is the order of the governing stochastic ODE. The boundedness of the random basis from above is what makes the FSC method be curse-of-dimensionality free at the RFS level. For instance, for a dynamical system that is governed by a second-order stochastic ODE ($n = 2$) and driven by a stochastic flow map of fourth-order ($M = 4$), the maximum number of random basis vectors to consider within the FSC scheme is just 7, independent whether the dimensionality of the probability space is as low as 1 or as high as 10 000.

With the aim of reducing the complexity of the presentation, this dissertation includes three levels of abstraction for the FSC method, namely: a *specialized version* of the FSC method for dealing with structural dynamical systems subjected to uncertainties (Chapter 2), a *generalized version* of the FSC method for dealing with dynamical systems governed by (nonlinear) stochastic ODEs of arbitrary order (Chapter 3), and a *multi-element version* of the FSC method for dealing with dynamical systems that exhibit discontinuities over the probability space (Chapter 4). This dissertation also includes an implementation of the FSC method to address the dynamics of large-scale structural systems more effectively (Chapter 5). The implementation is done via a modal decomposition of the spatial function space (aka modal analysis in structural dynamics) in order to reduce the number of degrees of freedom in the system substantially, and thus, save computational runtime.