Buckling and Initial Post-Buckling of Generally Stiffened Conical Shells

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The sensitivity of stiffened conical shells to imperfection is considered, via the initial post-buckling analysis. Unlike stiffened cylindrical shells, in the case of generally stiffened conical shells the stiffeners inclination and the distance between the stiffeners vary with the shell coordinates, which complicates the problem considerably. The main objective of the study is to investigate the influence of the stiffeners on the buckling load and on the imperfection sensitivity. Thus, by finding the parameters that influence the shell's imperfection sensitivity, it is possible to improve the behavior of the whole structure.

I. Introduction

HELL structures are widely used in aeronautic, marine and civil engineering structures, they belong to the thin-wall structure family, which are very sensitive to imperfection, and their sensitivity depends on the geometry of the shell and its mechanical properties. One of the main goals, in this field, is to find the various parameters that influence the shell’s sensitivity, thus to improve the behavior of the whole structure. One of the ways to increase the buckling load and to decrease the sensitivity to imperfection as well is to add stiffeners to the shell structure. However, adding stiffeners to the shell structures must be done with strict attention, because adding stiffeners incorrectly would, in some cases, not change the behavior of the shell at all.

Conical shells are usually used as a connection between two cylindrical shells with different diameters. Unlike stiffened cylindrical shells, in the case of conical shells with stiffeners in a general position the distance between the stiffeners and the angle of inclination of the stiffeners vary with the shell coordinates, which ultimately results in coordinate dependence of the contribution to the stiffness coefficients of the A, B and D matrices. This effect complicates the problem considerably. The first level of complexity is attributed to the need to find an analytical representation of those functions. An exhaustive study of the stiffness functions and their dependence on these factors has been performed in Ref. 1. The second level of complexity is associated with the introduction of coordinate dependent stiffness matrices into the mathematical model and the solution of the system of nonlinear governing partial differential equations with variable coefficients. In earlier investigations2-4 the stiffeners were considered only as stringers (positioned in the axial direction, the distance between the stringers vary linearly) or as rings (positioned in circumferential direction, equally distributed). Other investigators5-6 used finite elements commercial computer code to calculate the buckling and post-buckling states of stringer-stiffened conical shells. As far as the author knows, the buckling and the initial post-buckling behavior and the imperfection sensitivity of generally stiffened conical shells taking into account the variation of the material properties with the shell’s coordinates has not been investigated, as yet.

This work presents a quantitative study of the imperfection-sensitivity of eccentrically stiffened conical shells in a general direction, by consideration of the variation of the stiffness coefficients and their dependence on the shell’s coordinate. Investigation is made within the framework of Koiter’s7 general theory of post-buckling behavior. The calculations provide a measure of the extent to which the shells are sensitive or insensitive to imperfections in their shape and thus indicate to what extent the classical buckling results can be reliable.

The non-linear equilibrium differential equations are derived from the basis of their kinematic approach, using the displacement components (axial (u), circumferential (v) and normal (w)) as the unknown dependent variables. The asymptotic technique is used to convert the non-linear equations into three linear sets. These equations are solved through expansion of the dependent variables in Fourier series in the circumferential direction and in finite differences in the axial direction. Afterwards the Galerkin procedure is used to minimize the error due to the truncated form of the series. A special computer code was developed and used for a wide range of parametric study of buckling and sensitivity behavior of generally stiffened conical shells.

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II. Governing Equation

This solution procedure has been used before\textsuperscript{8-10}, however, for the case of stiffened conical shells some additional complications occur.

A. Kinematic Relation – Donnell’s theory

The strain-displacement relation can be written as:

$$\{ \varepsilon \} = \{ \varepsilon^0 \} + z \{ \chi \}$$

(1)

where $\{ \varepsilon^0 \}$ and $\{ \chi \}$ are, respectively, the strain and change-of-curvature vectors of the reference surface, composed as follows, see Ref. 9:

$$\varepsilon^0_\alpha = u_\alpha + \frac{1}{2} w^2_\alpha$$
$$\varepsilon^0_{\alpha\theta} = \frac{w_\theta}{r(x)} \cos(\alpha) + \frac{w_\alpha}{r(x)} \sin(\alpha) + \frac{w^2_\theta}{2r(x)^2}$$
$$\gamma^0_{\alpha\theta} = \frac{w_\theta}{r(x)} v_\alpha + \frac{v}{r(x)} \sin(\alpha) + \frac{w_\alpha w_\theta}{r(x)}$$

(2-3)

where $( )_\alpha$ and $( )_\theta$ denote the derivatives with respect to the axial $(x)$ and circumferential $(\theta)$ coordinate, respectively; $r(x)$ is the radius at the $x$–coordinate, and $\alpha$ is the cone semi-vertex angle (see Fig. 1).

B. Constitutive equations – Stiffened Conical Shells

The present work allows for both internally and externally eccentric stiffener in a general position. The shell is assumed to be closely stiffened. Therefore, there is no possibility of local buckling between stiffeners and it is possible to “smear” the stiffeners\textsuperscript{11}, so the stress resultants and moment per unit length are given by:

$$N_\alpha = N_{\alpha}^{\text{shell}} + N_{\alpha}^{\text{eff}}$$
$$M_\alpha = M_{\alpha}^{\text{shell}} + M_{\alpha}^{\text{eff}}$$

(4)

where $N_{\alpha}^{\text{shell}}$ and $M_{\alpha}^{\text{shell}}$ are the stress resultants and couples per unit length of the shells’ sheet. $N_{\alpha}^{\text{eff}}$ and $M_{\alpha}^{\text{eff}}$ are the stiffeners contributions to the load carrying capacity of the shell (per unit length). These contributions are based on the following assumptions regarding the constitutive relations\textsuperscript{1,11}:

1) The stiffeners are made of isotropic material with linear elastic behavior.
2) The stiffeners can be positioned in any direction and are distributed mathematically, for calculation purposes, over the whole surface of the shell.
3) The stiffener-laminate connection is monolithic, hence the normal strains $(\varepsilon_{zz})$ vary linearly in the stiffener as well as in the sheet and equal at their point of contact.
4) The stiffeners do not transmit shear perpendicular to their axes.
5) The stiffener resists torsion due to its torsional rigidity.

As a result of these assumptions the internal forces and moments about the surface of reference, due to the stiffeners only, are obtained as functions of the strains and the curvatures of the reference surface in the direction of the stiffeners (1, 2).
\[
\begin{align*}
N_i &= a_i e_i^a + b_i X_i \\
M_i &= b_i e_i^a + d_{i1} X_i \\
M_{i2} &= d_{66} X_{i2}
\end{align*}
\]

where

\[
\begin{align*}
a_i &= \frac{E_s A_{s_0}}{b(x)} \\
d_{i1} &= \frac{E_s (I_{s_0} + A_s e^2)}{b(x)} \\
b_i &= \frac{E_s A_s e}{b(x)} \\
d_{66} &= \frac{1}{4} \frac{G_s I_{s_0}}{b(x)}
\end{align*}
\]

(5-6)

\(E_s\) is the modulus of elasticity of the stiffener, \(G_s\) is the shear modulus of elasticity of the stiffener, \(A_s\) is the cross-sectional area of the stiffener, \(e\) is the distance of the centroid of the stiffener cross-section from the surface of reference, \(I_{s_0}\) is the moment of inertia of the stiffener cross-section about itself, \(I_{s_0}\) is the torsion constant of the stiffener cross-section. \(b\) is the distance between the stiffeners and for a conical shell it depends on the shell’s coordinates, \(b(x, \theta)\). For optimization purpose the geometrical characteristics of the cross-section of the stiffeners can be chosen to be also some function of \(x\) and \(\theta\). Therefore, \(a_{i1}, b_{i1}, d_{i1}\) and \(d_{66}\) are strong functions of the shell’s coordinate.

Under the classical laminate theory, the contribution of the stiffeners to stiffness matrices expressed in the basic coordinates system of the shell \((x, \theta)\) is given by:

\[
\begin{align*}
A^{eff} &= a_i J_i \\
B^{eff} &= b_i J_i \\
D^{eff} &= d_{i1} J_i + d_{66} L_i
\end{align*}
\]

Where

\[
J_i = \begin{bmatrix}
C^4 & C^2 S^2 & C^3 S \\
C^2 S^2 & S^4 & C S^3 \\
C^3 S & C S^3 & C^2 S^2
\end{bmatrix}
\quad
L_i = \begin{bmatrix}
4 C^2 S^2 & -4 C^2 S^2 & -2 C S (C^2 - S^2) \\
-4 C^2 S^2 & 4 C^2 S^2 & 2 C S (C^2 - S^2) \\
-2 C S (C^2 - S^2) & 2 C S (C^2 - S^2) & (C^2 - S^2)^2
\end{bmatrix}
\]

(8-9)

where \(C = \cos(\beta(x, \theta))\) and \(S = \sin(\beta(x, \theta))\), \(\beta\) is the angle of inclination of the stiffeners and is a function of \(x\) and \(\theta\) which depends on the chosen position of the stiffeners.

In this work a geodesic path is chosen to represents the variation of the stiffeners in the shell’s coordinate, therefore the stiffeners inclination \(\beta\) and the distance between the stiffeners \(b\) change only in the axial direction and are constant in the circumferential direction. The stiffeners inclination \(\beta(x)\) is given by:

\[
\beta(x) = \arcsin \left( \frac{R_x \sin \alpha}{R_x \sin \alpha - \sin \beta_1} \right)
\]

(10)

where \(\beta_1\) is the stiffener inclination at the small end of the shell, \(x\) is the longitudinal coordinate in the cone surface, see Fig. 1.

The distance between the stiffeners \(b(x)\) is given by:

\[
b(x) = b_1 \frac{(x + R_x \sin \alpha \cos \beta)}{R_x \sin \alpha \cos \beta_1}
\]

(11)

where \(b_1\) is the distance between the fibers at the narrower edge of the cone.

Therefore, the stiffness matrices \(A^{eff}, B^{eff}\) and \(D^{eff}\), are strong functions of the longitudinal coordinate and have to be

![Figure 2: Variation of the stiffeners inclination and the distance between the stiffeners along the cone slant length for conical shell \(\alpha=45^\circ\).](image-url)
added to the stiffness matrices given by the shell’s sheet. Note that for $\beta=0$ one obtains the case of stringers (contribution to the stiffness coefficients $A_{11}$, $B_{11}$, and $D_{11}$, which are vary linearly along the axial direction). In the case of a stiffener orientation of $\beta=\pi/2$ the assumption of a geodesic path is not valid. The stiffener inclination and the distance between stiffeners remain constant along the axial direction and the buckling load and the initial post buckling behavior can be calculated on the basis of constant stiffness coefficients. In this case ($\beta=\pi/2$), one obtains the case of ring stiffeners (constant contribution to the stiffness coefficients $A_{22}$, $B_{22}$ and $D_{22}$).

In Fig. 2 the change of the stiffener inclination and the distance between the stiffeners are plotted vs. the longitudinal coordinate for conical shell (with cone semi-vertex angle of $\alpha=45^\circ$, a slant length of $L=2.54$ m and a shorter radius of the truncated cone $R_1=1.27$m) with various initial stiffeners inclination ($\beta_1$). The stiffener inclination decreases along the axial direction and the distance between stiffeners increases along the axial direction. The higher the initial stiffener inclination the rapid the change along the axial direction.

For a general angle of inclination $\beta$, the stiffening of the stiffeners will be "unbalanced". To have balanced stiffening one must add the same stiffeners with an angle of inclination -$\beta$. For this case one obtains:

$$A^{\text{eff}} = 2a_{11}J_2$$
$$B^{\text{eff}} = 2b_{11}J_2$$
$$D^{\text{eff}} = 2d_{11}J_2 + 2d_{22}L_2$$

where the balanced matrices $J_2$ and $L_2$ are given by:

$$J_2 = \begin{bmatrix} C^4 & C^2S^2 & 0 \\ C^2S^2 & S^4 & 0 \\ 0 & 0 & C^2S^2 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 4C^2S^2 & -4C^2S^2 & 0 \\ -4C^2S^2 & 4C^2S^2 & 0 \\ 0 & 0 & (C^2-S^2)^2 \end{bmatrix}$$

It must be emphasized that for cylindrical shell, the stiffness coefficients, due to the stiffeners with constant geometrical characteristics, will become automatically constants.

C. Equilibrium equations

The nonlinear equilibrium equations and the appropriate boundary conditions are derived on the basis of the stationary potential energy criterion. The following three nonlinear equilibrium equations are obtained as:

$$N_{xx} + \frac{N_{w\theta}}{r(x)} + \frac{\sin(\alpha)}{r(x)} [N_{w\theta} - N_{w\theta}] + q_x = 0$$

$$N_{x\theta} + \frac{N_{w\theta}}{r(x)} + 2\frac{\sin(\alpha)}{r(x)} N_{x\theta} + q_r = 0$$

$$M_{xx} + \frac{M_{w\theta\theta}}{r(x)} + \frac{2M_{w\theta\theta}}{r(x)} - \frac{N_{w\theta}}{r(x)} \cos(\alpha), \frac{\sin(\alpha)}{r(x)} \left[ 2M_{xx} - M_{w\theta} + 2M_{w\theta\theta} \right]$$

$$+ \frac{1}{r(x)} r(x) N_{w\theta} + N_{w\theta} w_x \frac{1}{r(x)} + \frac{1}{r(x)} r(x) N_{w\theta} + N_{w\theta} w_y \frac{1}{r(x)} + q_x = 0$$

with the following boundary conditions:

$$N_{xx} = \overline{N}_{xx} \quad \text{or} \quad u = \overline{u}$$

$$N_{x\theta} = \overline{N}_{x\theta} \quad \text{or} \quad \nu = \overline{\nu}$$

$$M_{xx} + \frac{2M_{w\theta\theta}}{r(x)} + N_{w\theta} w_x + \frac{N_{w\theta} w_y}{r(x)} + \frac{\sin(\alpha)}{r(x)} (M_{xx} - M_{w\theta}) = \overline{M} \quad \text{or} \quad w = \overline{w}$$

$$M_{xx} = \overline{M}_{xx} \quad \text{or} \quad w = \overline{w}$$

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Using Eqs. (2-3) and the constitutive relations the equilibrium equations and the boundary conditions can be obtained in terms of the displacement function.

### D. Imperfection sensitivity

In Koiter’s general theory of elastic stability the imperfection sensitivity of a structure is closely related to its initial postbuckling behavior and the theory is exact in the asymptotic sense. This determines whether the load initially increases or decreases after buckling. The classical buckling load of the perfect structure is denoted by \( \lambda_c \), and in all cases considered here it is the load at which a nonaxisymmetric bifurcation from the prebuckling state occurs. Assuming that the eigenvalue problem for the buckling load \( \lambda_c \) will yield a unique buckling mode \( \mathbf{u}_\text{b} \), a solution to be valid in the initial postbuckling regime, is sought in the form of the following asymptotic expansion.

\[
\frac{\lambda}{\lambda_c} = 1 + a\xi + b\xi^2 + \cdots
\]

\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix}
= \begin{bmatrix}
  u^{(0)} \\
  v^{(0)} \\
  w^{(0)}
\end{bmatrix}
+ \xi \begin{bmatrix}
  u^{(1)} \\
  v^{(1)} \\
  w^{(1)}
\end{bmatrix}
+ \xi^2 \begin{bmatrix}
  u^{(2)} \\
  v^{(2)} \\
  w^{(2)}
\end{bmatrix}
+ \cdots
\]

(17)

Where \( \lambda \) is the load parameter deviating from the bifurcation buckling load \( \lambda_c \), and \( \xi \) is the amplitude of the buckling mode. The superscripts \((0), (1)\) and \((2)\) denote the prebuckling, buckling and postbuckling state, respectively.

A formal substitution of this expansion into the nonlinear governing equations (Eqs. 15) generates a sequence of equations for the functions appearing in the expansions:

The zero-th order terms yield the partial differential equations of the prebuckling state:

\[
L^{[e]}[S^{(0)}] + LL^{[e]}[S^{(0)}, T^{(0)}] + LLL^{[e]}[S^{(0)}, T^{(0)}, S^{(0)}] = P^{[e]} \quad e=1,2,3
\]

(18)

The first order terms yield the partial differential equations of the buckling state:

\[
L^{[e]}[S^{(0)}] + LL^{[e]}[S^{(0)}, T^{(0)}] + LLL^{[e]}[S^{(0)}, T^{(0)}, S^{(0)}] + [LLL^{[e]}[S^{(0)}, T^{(0)}, S^{(0)}] + LLL^{[e]}[S^{(0)}, T^{(0)}, S^{(0)}] = 0 \quad e=1,2,3
\]

(19)

The second order terms yield the partial differential equations of the postbuckling state:

\[
L^{[e]}[S^{(0)}] + LL^{[e]}[S^{(0)}, T^{(0)}] + LLL^{[e]}[S^{(0)}, T^{(0)}, S^{(0)}] + LLL^{[e]}[S^{(0)}, T^{(0)}, S^{(0)}] + LLL^{[e]}[S^{(0)}, T^{(0)}, S^{(0)}] = 0 \quad e=1,2,3
\]

(20)

\( L^{[e]}, LL^{[e]} \) and \( LLL^{[e]}, [e]=1, 2, 3 \), are respectively linear, quadratic and cubic differential operators having variable coefficients. They are given by Ref. 9.

Notice that the sum of the indices of the prebuckling operators is zero, those of the buckling operators is one, and that of the postbuckling operators is two.

The non-linear solution of the prebuckling state, Eq. (18), should be performed by an adequate numerical procedure which ultimately yields a limit-point. In order to simplify the problem the solution procedure of the prebuckling state is performed by ignoring the non-linear terms and solving only the linear part of Eq. (18), that is \( L^{[e]}[S^{(0)}] = P^{[e]} \). The applied loading \( P^{[e]} \) consists of axial compression, internal or external pressure and clockwise or counter-clockwise torque. It is assumed to have a uniform spatial distribution and is divided into a fixed part and a variable part. The magnitude of the variable part is allowed to vary in proportion to a load parameter \( \lambda \). This leads to an eigenvalue problem for the critical load \( \lambda_c \).

These equations admit separable solutions, in the circumferential direction by using Fourier series and the \( \theta \)-dependence is eliminated by applying Galerkin’s procedure. A central finite difference scheme is used in the axial direction to reduce the ordinary differential equations to the following algebraic ones:
For the prebuckling state,

$$\begin{array}{c}
\begin{bmatrix} K \end{bmatrix} \begin{bmatrix} Z^{(0)} \end{bmatrix} = \begin{bmatrix} P \end{bmatrix} \\
\end{array}$$

(21)

and for the buckling state,

$$\begin{array}{c}
\begin{bmatrix} K \end{bmatrix} + \lambda \begin{bmatrix} G \end{bmatrix} = 0 \\
\end{array}$$

(22)

where \( K \) and \( G \) are the stiffness and geometry matrices respectively, \( Z^{(0)} \) and \( Z^{(1)} \) are unknown vectors consisting of \( u, v, w, u_{xx}, v_{xx} \) and \( w_{xx} \) for the prebuckling state and the buckling state, respectively. Eq. (22) is an eigenvalue problem in which \( \lambda \) represents the buckling load parameters and \( Z \) the buckling mode.

Using a linear prebuckling analysis the equations governing the post-buckling state become

$$\begin{array}{c}
L^{(3)} \left( S^{(2)} \right) + \lambda \left[ L^{(2)} \left( S^{(0)}, T^{(2)} \right) + L^{(3)} \left( S^{(0)}, T^{(0)} \right) \right] + \\
+ \lambda^2 \left[ L^{(1)} \left( S^{(0)}, T^{(0)} \right) + L^{(2)} \left( S^{(0)}, T^{(0)}, T^{(0)} \right) + L^{(3)} \left( S^{(0)}, T^{(0)}, S^{(0)} \right) \right] = \\
e^{1,2,3} \\
\end{array}$$

(23)

The postbuckling-state (2nd-order terms) \( w^{(2)}, v^{(2)} \) and \( u^{(2)} \) are obtained from the solution of the set of three inhomogeneous linear partial differential equation linear with the associated boundary conditions.

Since the right hand side of Eq. (23) represents the quadratic level of the known buckling mode, the post-buckling state obviously depends on \( c^2 \), \( c \) being the scalar by which the buckling mode is normalized. Solving the post-buckling state using the same separable solutions, the following algebraic equations obtained for the post-buckling state:

$$\begin{array}{c}
\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} Z^{(2)} \end{bmatrix} = \begin{bmatrix} F \end{bmatrix} \\
\end{array}$$

(24)

where \( A \) and \( F \) are the post-buckling matrix and the right hand side vector, respectively. \( Z^{(2)} \) is unknown vectors consisting of \( u^{(2)}, v^{(2)}, w^{(2)}, u_{xx}^{(2)}, v_{xx}^{(2)} \) and \( w_{xx}^{(2)} \) for the post-buckling state.

For perfect shells one is interested in the variation of \( \lambda(\xi) \) with \( \xi \) in the vicinity of \( \lambda=\lambda_c \) near the bifurcation point \( \lambda_c \), the asymptotic expansion given in Eq. (17) is valid. Due to the periodicity of the buckling mode in the circumferential direction the first postbuckling coefficient "\( c \)" vanishes.

The well-known formula for the case of a membrane prebuckling state derived in Ref. 12, for the case of linear prebuckling the following formula obtained:

$$b = \frac{\sigma_s \cdot L_s(u_1) + 2\sigma_s \cdot L_{00}(u_1, u_1)}{\lambda_c \left[ 2\sigma_s \cdot L_{01}(u_1, u_1) + \sigma_s \cdot L_{21}(u_1) \right]}$$

(25)

When \( b \) has a positive value the structure can develop considerable postbuckling strength, and loss of stability of the primary path does not result in structural collapse. However, when \( b \) has a negative value then in most cases buckling will occur violently and the magnitude of the critical load is subject to the degrading influence of initial imperfections.

For conical shell, with the variables \( u, v, w \) the operator will be:

$$\sigma_s \cdot L_{\alpha\gamma}\left( u_{ij}, u_{ij} \right) = \int_0^{\pi/\alpha} \left[ \begin{bmatrix} N_{00}(u_j^{(k)}) w_{x}^{(k)} + N_{00}(u_j^{(k)}) \frac{w_{x}^{(k)}}{r(x)} \end{bmatrix} + 2N_{00}(w_{x}^{(k)} + \frac{w_{x}^{(k)}}{2r(x)}) \right] d\theta$$

(26)

Here, the superscripts \( (i), (j) \) and \( (k) \) denote the appropriate state ((0)-prebuckling, (1)-buckling and (2)- initial postbuckling).
III. Results and Discussion

For the procedure outlined above simply-supported (SS3-SS4) stiffened conical shells under axial compression are examined. The geometric properties are: slant length \( L=2.54\,\text{m} \), the radius at the small end \( R_1=1.27\,\text{m} \), the thickness of the shell’s sheet \( t=0.0127\,\text{m} \) (\( R_1/t=100 \)). The geometric properties of the stiffeners are: the distance between the stiffeners at the small end of the shell \( b_1=2\pi R_1/30 \) (30 stiffeners in the circumferential direction assure that no local buckling will occur), the cross-sectional area of the stiffener \( A_s=0.75 b_1 t \), the moment of inertia of the stiffener cross-section about itself \( I_{st}=50 t^3 b_1/12(1-\nu^2) \), the torsional rigidity of the stiffener cross-section is neglected \( (I_{ts}=0) \), the distance of the centroid of the stiffener cross-section from the surface of reference \( e=\pm4.66666 t \). For balanced stiffeners configuration \( A_{st}^{\text{balanced}}=A_s/2 \) and \( I_{st}^{\text{balanced}}=I_{st}/2 \), thus balanced and unbalanced configurations have the same amount of material. The sheet and the stiffeners are made of isotropic material with material properties as follow: modulus of elasticity \( E=1.404\times10^{11} \,\text{N/m}^2 \), Poisson’s ratio \( \nu=0.2 \).

In Fig. 3 the buckling load and the Koiter-b parameter are plotted against the stiffeners inclination at the small end of the shell \( (\beta_1) \) for conical shell with \( \alpha=45^\circ \) having different stiffeners configurations. As was expected balanced stiffener configurations give higher buckling load than unbalanced stiffener configurations, however usually conical shell with balanced stiffener configuration are more sensitive to imperfection than unbalanced configuration. The position of the stiffeners has a significant influence on the buckling load and on the imperfection sensitivity; external stiffeners give higher buckling load but are more sensitive to imperfection than internal stiffeners. Internal stiffeners give relatively lower buckling load but usually assure insensitive shell. Centric stiffeners have no significant advantage compare to the other configuration, especially in the case of balanced configuration (it gives lower buckling load and also very sensitive to imperfection) since it has kind of orthotropic configuration. For optimization purpose, it seems that the stiffeners configuration of \( \beta_1=45^\circ \) for balanced stiffeners and \( \beta_1=0^\circ \) for unbalanced configuration give the highest buckling load but are sensitive to imperfection.

Figure 3: Axial compressive buckling load and Koiter b parameter versus stiffener inclination at the small end of the shell for conical shell with \( \alpha=45^\circ \).

In Fig. 4 the buckling load and the Koiter-b parameter are plotted against the cone semi-vertex angle for conical shells with stiffeners having \( \beta_1=30^\circ \) at the small end of the shell. Here again, the unbalanced stiffener configurations give lower buckling load but insensitive shells. External stiffeners give higher buckling load but more sensitive shell. Centric stiffeners is the worst configuration for balanced stiffeners, it gives lower buckling load and very sensitive shell. For both examples the circumferential wave numbers of the balanced configurations are lower than those of the unbalanced configuration.
IV. Conclusion

In this study, the buckling load and the imperfection sensitivity of a stiffened conical shell were investigated under the assumption that stiffeners orientation changes as a geodesic path. The improvement in this research is achieved by the adoption of a suitable analytical representation to describe the coordinate dependent stiffness and, especially, by the study of the influence of the variation of the stiffness coefficients on the imperfection sensitivity of the stiffened conical shells. The main conclusion from this investigation is that the position of the stiffeners has a significant influence on the buckling load and on the imperfection sensitivity. External stiffeners give higher buckling load but are more sensitive to imperfection than internal stiffeners compare to internal stiffeners. Centric stiffeners have no significant improvement for the shells behavior. Balanced stiffeners configurations give higher buckling load but more sensitivity to imperfection.

References