Homework #9 – solution

Problem 1

$$\vec{R}^{QP} = x\,\hat{q}_1 + y\,\hat{q}_2 + z\,\hat{q}_3$$

$$^{A}\vec{V}^{QP} = u\,\hat{q}_1 + v\,\hat{q}_2 + w\,\hat{q}_3$$

Want to find inertial acceleration of the shuttle, $^{I}\vec{a}^{OP}$

$$^{I}\vec{a}^{OP} = {}^{I}\vec{a}^{OQ} + {}^{I}\vec{a}^{QP}$$

First, lets find $^{I}\vec{a}^{OQ}$:

$$\vec{R}^{OQ} = R_o\,\hat{b}_2 + L\,\hat{b}_1$$

$$^{S}\vec{V}^{OQ} = 0$$

$$^{S}\vec{a}^{OQ} = 0$$

$$^{I}\vec{a}^{OQ} = {}^{S}\vec{a}^{OQ} + \underbrace{2\,{}^{I}\vec{\omega}^{S} \times {}^{S}\vec{V}^{OQ}}_{=0} + {}^{I}\vec{\omega}^{S} \times \left({}^{I}\vec{\omega}^{S} \times \vec{R}^{OQ}\right) + \frac{^{S}d\,{}^{I}\vec{\omega}^{S}}{dt} \times \vec{R}^{OQ}$$

$$^{I}\vec{\omega}^{S} = -\Omega\,\hat{b}_3 \qquad \frac{^{S}d\,{}^{I}\vec{\omega}^{S}}{dt} = 0$$

$$^{I}\vec{a}^{OQ} = -\Omega\,\hat{b}_3 \times \left(-\Omega\,\hat{b}_3 \times \left(R_o\,\hat{b}_2 + L\,\hat{b}_1\right)\right) =$$

$$= \Omega^2\,\hat{b}_3 \times \left(-R_o\,\hat{b}_1 + L\,\hat{b}_2\right) =$$

$$\boxed{^{I}\vec{a}^{OQ} = -R_o\Omega^2\,\hat{b}_2 - L\,\Omega^2\,\hat{b}_1}$$

Next, we will calculate $^{I}\vec{a}^{QP}$:

$$^{I}\vec{a}^{QP} = {}^{A}\vec{a}^{QP} + 2\,{}^{I}\vec{\omega}^{A} \times {}^{A}\vec{V}^{QP} + {}^{I}\vec{\omega}^{A} \times \left({}^{I}\vec{\omega}^{A} \times \vec{R}^{QP}\right) + \frac{^{A}d\,{}^{I}\vec{\omega}^{A}}{dt} \times \vec{R}^{QP}$$

Define a new set of unit vectors $\hat{a}$ so that $\hat{a}_1 = \hat{b}_1$ and $\hat{a}_2, \hat{a}_3$ rotate about $\hat{a}_1 = \hat{b}_1$ at the constant rate $\Lambda = \dot{\phi}$.

$$\hat{a}_1 = \hat{q}_1 \, c40^\circ - \hat{q}_2 \, s40^\circ$$

$$\hat{a}_2 = \hat{q}_1 \, s40^\circ + \hat{q}_2 \, c40^\circ$$

$$\hat{a}_3 = \hat{q}_3$$

Write $\vec{R}^{op}$ and ${}^A\vec{V}^{ap}$ in terms of $\hat{a}$'s :

$$\vec{R}^{ap} = x' \hat{a}_1 + y' \hat{a}_2 + z' \hat{a}_3$$

$${}^A\vec{V}^{ap} = u' \hat{a}_1 + v' \hat{a}_2 + w' \hat{a}_3$$

Calculate all terms for ${}^I\vec{a}^{ap}$:

$${}^A\vec{a}^{ap} = \dot{u}' \hat{a}_1 + \dot{v}' \hat{a}_2 + \dot{w}' \hat{a}_3$$

$${}^I\vec{\omega}^A = {}^I\vec{\omega}^S + {}^S\vec{\omega}^A = -\Omega \hat{b}_3 + \Lambda \hat{a}_1 \qquad \hat{b}_3 = \hat{a}_2 c\phi - \hat{a}_3 s\phi$$

$$= -\Omega c\phi \, \hat{a}_2 + \Omega s\phi \, \hat{a}_3 + \Lambda \hat{a}_1$$

$$\frac{d\,{}^I\vec{\omega}^A}{dt} = \Omega \Lambda s\phi \, \hat{a}_2 + \Omega \Lambda c\phi \, \hat{a}_3 = \Lambda \Omega \left[s\phi \, a_2 + c\phi \, a_3 \right]$$

$$2\,{}^I\vec{\omega}^A \times {}^A\vec{V}^{ap} = 2 \Big[\Omega u' c\phi \, \hat{a}_3 + \Omega u' s\phi \, \hat{a}_2 - \Omega v' s\phi \, \hat{a}_1 + \Lambda v' \hat{a}_3 - \Omega w' c\phi \, \hat{a}_1 - $$
$$- \Lambda w' \hat{a}_2 \Big]$$

$${}^I\vec{\omega}^A \times ({}^I\vec{\omega}^A \times \vec{R}^{ap}) = {}^I\vec{\omega}^A \times \Big[\Omega x' c\phi \, \hat{a}_3 + \Omega x' s\phi \, \hat{a}_2 - \Omega y' s\phi \, \hat{a}_1 + \Lambda y' \hat{a}_3 - $$
$$- \Omega z' c\phi \, \hat{a}_1 - \Lambda z' \hat{a}_2 \Big] =$$

$$= -\Omega^2 x' c^2\phi \, \hat{a}_1 - \Omega^2 y' s\phi c\phi \, \hat{a}_3 - \Omega \Lambda y' c\phi \, \hat{a}_1 - \Omega^2 z' c^2\phi \, \hat{a}_3 - $$
$$- \Omega^2 x' s^2\phi \, \hat{a}_1 - \Omega^2 y' s^2\phi \, \hat{a}_2 - \Omega^2 z' s\phi c\phi \, \hat{a}_2 + \Omega \Lambda z' s\phi \, \hat{a}_1 - $$

$$- \Omega\Lambda x' c\phi\, \hat{a}_2 + \Omega\Lambda x' s\phi\, \hat{a}_3 - \Lambda^2 y' \hat{a}_2 - \Lambda^2 z' \hat{a}_3$$

$${}^{I}\vec{\omega}^{A} \times \left({}^{I}\vec{\omega}^{A} \times \vec{R}^{ap}\right) = \left[-\Omega^2 x' - \Omega\Lambda\left(y' c\phi - z' s\phi\right)\right] \hat{a}_1 +$$

$$+ \left[-\Omega^2 y' s^2\phi - \Omega^2 z' s\phi\, c\phi - \Omega\Lambda x' c\phi - \Lambda^2 y'\right] \hat{a}_2 +$$

$$+ \left[-\Omega^2 y' s\phi\, c\phi - \Omega^2 z' c^2\phi + \Omega\Lambda x' s\phi - \Lambda^2 z'\right] \hat{a}_3$$

$$\frac{d\, {}^{I}\vec{\omega}^{A}}{dt} \times \vec{R}^{ap} = - \Omega\Lambda s\phi\, x' \hat{a}_3 + \Omega\Lambda s\phi\, z' \hat{a}_1 + \Omega\Lambda c\phi\, x' \hat{a}_2 -$$

$$- \Omega\Lambda c\phi\, y' \hat{a}_1 =$$

$$= \Omega\Lambda\left[z' s\phi - y' c\phi\right] \hat{a}_1 + \Omega\Lambda x' c\phi\, \hat{a}_2 - \Omega\Lambda x' s\phi\, \hat{a}_3$$

Put all the terms together:

$${}^{I}\vec{a}^{ap} = \left[\ddot{u}' - 2\Omega v' s\phi - 2\Omega w' c\phi - \Omega^2 x' - \Omega\Lambda\left(y' c\phi - z' s\phi\right) + \Omega\Lambda\left(z' s\phi - y' c\phi\right)\right] \hat{a}_1$$

$$+ \left[\ddot{v}' + 2\Omega u' s\phi - 2\Lambda w' - \Omega^2 y' s^2\phi - \Omega^2 z' s\phi\, c\phi - \Omega\Lambda x' c\phi - \Lambda^2 y' + \right.$$

$$\left. + \Omega\Lambda x' c\phi\right] \hat{a}_2 +$$

$$+ \left[\ddot{w}' + 2\Omega u' c\phi + 2\Lambda v' - \Omega^2 y' s\phi\, c\phi - \Omega^2 z' c^2\phi + \Omega\Lambda x' s\phi - \Lambda^2 z' - \right.$$

$$\left. - \Omega\Lambda x' s\phi\right] \hat{a}_3$$

Convert all u', v', etc. back to $u, v, \ldots$

$\ddot{u}' = \dot{u}\, c40 - \dot{v}\, s40$ $\qquad$ $x' = x\, c40 - y\, s40$

$\ddot{v}' = \dot{u}\, s40 + \dot{v}\, c40$ $\qquad$ $y' = x\, s40 + y\, c40$

$\ddot{w}' = \dot{w}$ $\qquad$ $z' = z$

$$
\begin{aligned}
{}^{I}\vec{a}^{\,QP} = \Big[& \dot{u}\,c40 - \dot{v}\,s40 - 2\Omega s\phi\,(u\,s40 + v\,c40) - 2\Omega c\phi\,(w) - \Omega^2(x\,c40 - y\,s40) - \\
& - \underline{\Omega\Lambda c\phi\,(x\,s40 + y\,c40)} + \underline{\Omega\Lambda s\phi\,(z)} + \underline{\Omega\Lambda s\phi\,(z)} - \underline{\Omega\Lambda c\phi\,(x\,s40 + y\,c40)}\Big]\hat{a}_1 \\
& + \Big[\dot{u}\,s40 + \dot{v}\,c40 + 2\Omega s\phi\,(u\,c40 - v\,s40) - 2\Lambda w - \Omega^2 s^2\phi\,(x\,s40 + y\,c40) - \\
& - \Omega^2 s\phi\,c\phi\,(z) - \cancel{\Omega\Lambda c\phi\,(x\,c40 - y\,s40)} - \Lambda^2(x\,s40 + y\,c40) + \\
& + \cancel{\Omega\Lambda c\phi\,(x\,c40 - y\,s40)}\Big]\hat{a}_2 + \\
& + \Big[\dot{w} + 2\Omega c\phi\,(u\,c40 - v\,s40) + 2\Lambda\,(u\,s40 + v\,c40) - \Omega^2 s\phi\,c\phi\,(x\,s40 + y\,c40) \\
& - \Omega^2 c^2\phi\,(z) + \cancel{\Omega\Lambda s\phi\,(x\,c40 - y\,s40)} - \Lambda^2 z - \cancel{\Omega\Lambda s\phi\,(x\,c40 - y\,s40)}\Big]\hat{a}_3
\end{aligned}
$$

$$
\begin{aligned}
{}^{I}\vec{a}^{\,QP} = \Big[& \dot{u}\,c40 - \dot{v}\,s40 - 2\Omega s\phi\,(u\,s40 + v\,c40) - 2\Omega c\phi\,w - \Omega^2(x\,c40 - y\,s40) - \\
& - 2\Omega\Lambda c\phi\,(x\,s40 + y\,c40) + 2\Omega\Lambda s\phi\,z\Big]\hat{a}_1 + \\
& + \Big[\dot{u}\,s40 + \dot{v}\,c40 + 2\Omega s\phi\,(u\,c40 - v\,s40) - 2\Lambda w - \Omega^2 s^2\phi\,(x\,s40 + y\,c40) - \\
& - \Omega^2 s\phi\,c\phi\,z - \Lambda^2(x\,s40 + y\,c40)\Big]\hat{a}_2 + \\
& + \Big[\dot{w} + 2\Omega c\phi\,(u\,c40 - v\,s40) + 2\Lambda(u\,s40 + v\,c40) - \Omega^2 s\phi\,c\phi\,(x\,s40 + y\,c40) - \\
& - \Omega^2 c^2\phi\,z - \Lambda^2 z\Big]\hat{a}_3
\end{aligned}
$$

Convert to $\hat{b}$'s:

$$\hat{b}_1 = \hat{a}_1 \qquad\qquad \hat{a}_1 = \hat{b}_1$$

$$\hat{b}_2 = c\phi\,\hat{a}_2 - s\phi\,\hat{a}_3 \qquad\qquad \hat{a}_2 = c\phi\,\hat{b}_2 + s\phi\,\hat{b}_3$$

$$\hat{b}_3 = s\phi\,\hat{a}_2 + c\phi\,\hat{a}_3 \qquad\qquad \hat{a}_3 = -s\phi\,\hat{b}_2 + c\phi\,\hat{b}_3$$

$$
\begin{aligned}
{}^{I}\vec{a}^{QP} = {} & \left[\dot u\, c40 - \dot v\, s40 - 2\Omega s\phi\,(u\, s40 + v\, c40) - 2\,\Omega c\phi\, w - \Omega^2 (x\, c40 - y\, s40) - \right.\\
& \left. - 2\,\Omega \Lambda c\phi\,(x\, s40 + y\, c40) + 2\,\Omega\Lambda s\phi\, z\right] \hat b_1 \; + \\
& + \left[\dot u\, c\phi\, s40 + \dot v\, c\phi\, c40 + 2\,\Omega\, s\phi\, \cancel{c\phi (u\, c40 - v\, s40)} - 2\Lambda c\phi\, w - \right.\\
& - \Omega^2 s^2\phi\, \cancel{c\phi (x\, s40 + y\, c40)} - \Omega^2 \cancel{s\phi}\, c^2\phi\, z - \Lambda^2 c\phi\,(x\, s40 + y\, c40) - \\
& - \dot w\, s\phi - 2\,\Omega\, s\phi\, \cancel{c\phi\, (u\, c40 - v\, s40)} - 2\,\Lambda s\phi\,(u\, s40 + v\, c40) + \\
& \left. + \Omega^2 s^2\phi\, \cancel{c\phi (x\, s40 + y\, c40)} + \Omega^2 \cancel{c^2\phi\, s\phi}\, z + \Lambda^2 s\phi\, z\right] \hat b_2 \; + \\
& + \left[\dot u\, s\phi\, s40 + \dot v\, s\phi\, c40 + \underline{2\,\Omega\, s^2\phi\,(u\, c40 - v\, s40)} - 2\,\Lambda s\phi\, w - \right.\\
& - \underline{\Omega^2 s^2\phi\, s\phi\,(x\, s40 + y\, c40)} - \underline{\Omega^2 s^2\phi\, c\phi\, z} - \underline{\Lambda^2 s\phi (x\, s40 + y\, c40)} + \\
& + \dot w\, c\phi + \underline{2\,\Omega\, c^2\phi\,(u\, c40 - v\, s40)} + 2\Lambda\, c\phi\,(u\, s40 + v\, c40) - \\
& \left. - \underline{\Omega^2 s\phi\, c^2\phi\,(x\, s40 + y\, c40)} - \underline{\Omega^2 c^2\phi\, c\phi\, z} - \Lambda^2 c\phi\, z\right] \hat b_3
\end{aligned}
$$

$$
\begin{aligned}
{}^{I}\vec{a}^{OP} = {} & \left[- L\Omega^2 + \dot u\, c40 - \dot v\, s40 - 2\,\Omega s\phi\,(u\, s40 + v\, c40) - 2\,\Omega\, c\phi\, w - \Omega^2 (x\, c40 - y\, s40) - \right.\\
& \left. - 2\,\Omega\Lambda\, c\phi\,(x\, s40 + y\, c40) + 2\,\Omega\Lambda s\phi\, z\right] \hat b_1 \; + \\
& + \left[- R_0 \Omega^2 + \dot u\, c\phi\, s40 + \dot v\, c\phi\, c40 - 2\,\Lambda c\phi\, w - \Lambda^2 c\phi\,(x\, s40 + y\, c40) - \right.\\
& \left. - \dot w\, s\phi - 2\,\Lambda s\phi\,(u\, s40 + v\, c40) + \Lambda^2 s\phi\, z\right] \hat b_2 \; + \\
& + \left[\dot u\, s\phi\, s40 + \dot v\, s\phi\, c40 + 2\,\Omega\,(u\, c40 - v\, s40) - 2\,\Lambda\, s\phi\, w - \right.\\
& - \Omega^2 s\phi\,(x\, s40 + y\, c40) - \Omega^2 c\phi\, z - \Lambda^2 s\phi\,(x\, s40 + y\, c40) + \dot w\, c\phi + \\
& \left. + 2\,\Lambda c\phi\,(u\, s40 + v\, c40) - \Lambda^2 c\phi\, z\right] \hat b_3
\end{aligned}
$$

Problem 2

Given $^{I}\vec{a}^{OP} = -g\,\hat{g}$ we want to find $^{F}\vec{a}^{OP}$, where F is the observer on the ladder (= fire fighter)

$$\hat{g} = -c\theta_1\,\hat{u}_3 + s\theta_1\,\hat{u}_2$$

BKE (Method III)

$$^{F}\vec{a}^{OP} = {}^{I}\vec{a}^{OP} + 2\,{}^{F}\vec{\omega}^{I} \times {}^{I}\vec{V}^{OP} + {}^{F}\vec{\omega}^{I} \times \left({}^{F}\vec{\omega}^{I} \times \vec{R}^{OP}\right) + \frac{{}^{I}d\,{}^{F}\vec{\omega}^{I}}{dt} \times \vec{R}^{OP}$$

$$^{I}\vec{a}^{OP} = -g\,s\theta_1\,\hat{u}_2 + g\,c\theta_1\,\hat{u}_3$$

$$^{F}\vec{\omega}^{I} = -\omega_1\,\hat{u}_1 + \omega_2\,\hat{g} = -\omega_1\,\hat{u}_1 + \omega_2\left(-c\theta_1\,\hat{u}_3 + s\theta_1\,\hat{u}_2\right) =$$

$$= -\omega_1\,\hat{u}_1 + \omega_2\,s\theta_1\,\hat{u}_2 - \omega_2\,c\theta_1\,\hat{u}_3$$

$$\vec{R}^{OP} = (L+x)\,\hat{u}_2$$

$$^{I}\vec{V}^{OP} = {}^{F}\vec{V}^{OP} + {}^{I}\vec{\omega}^{F} \times \vec{R}^{OP} = v\,\hat{u}_2 - (L+x)\,\omega_1\,\hat{u}_3 + (L+x)\,\omega_2\,c\theta_1\,\hat{u}_1$$

$$\frac{{}^{I}d\,{}^{F}\vec{\omega}^{I}}{dt} = \omega_1\omega_2\,c\theta_1\,\hat{u}_2 + \omega_1\omega_2\,s\theta_1\,\hat{u}_3$$

$$^{F}\vec{a}^{OP} = -g\,s\theta_1\,\hat{u}_2 + g\,c\theta_1\,\hat{u}_3 + 2\left[-v\omega_1\,\hat{u}_3 - \omega_1^2(L+x)\,\hat{u}_2 - (L+x)\,\omega_1\omega_2\,s\theta_1\,\hat{u}_1 - \right.$$
$$\left. - (L+x)\,\omega_2^2\,s\theta_1 c\theta_1\,\hat{u}_3 + v\,\omega_2\,c\theta_1\,\hat{u}_1 - (L+x)\,\omega_2^2\,c^2\theta_1\,\hat{u}_2\right] +$$
$$- (L+x)\,\omega_1^2\,\hat{u}_2 - (L+x)\,\omega_1\omega_2\,s\theta_1\,\hat{u}_1 - (L+x)\,\omega_2^2\,s\theta_1 c\theta_1\,\hat{u}_3 -$$
$$- (L+x)\,\omega_2^2\,c^2\theta_1\,\hat{u}_2 + \frac{{}^{I}d\,{}^{F}\vec{\omega}^{I}}{dt} \times \vec{R}^{OP}$$

$$^{F}\vec{a}^{OP} = \left[-2(L+x)\,\omega_1\omega_2\,s\theta_1 + 2v\,\omega_2\,c\theta_1 - (L+x)\,\omega_1\omega_2\,s\theta_1\right]\hat{u}_1 +$$
$$+ \left[-g\,s\theta_1 - 2(L+x)\,\omega_1^2 - 2(L+x)\,\omega_2^2\,c^2\theta_1 - (L+x)\,\omega_1^2 - (L+x)\,\omega_2^2\,c^2\theta_1\right]\hat{u}_2$$
$$+ \left[g\,c\theta_1 - 2v\,\omega_1 - 2(L+x)\,\omega_2^2\,s\theta_1 c\theta_1 - (L+x)\,\omega_2^2\,s\theta_1 c\theta_1\right]\hat{u}_3 -$$

$$- (L+x)\,\omega_1 \omega_2 s\theta_1 \;\hat{u}_1$$

$$F_{\vec{a}}^{op} = \left[2 v \omega_2 c\theta_1 - 4 (L+x)\,\omega_1 \omega_2 s\theta_1\right] \hat{u}_1$$

$$- \left[g\, s\theta_1 + 3 (L+x)\,\omega_1^2 + 3 (L+x)\,\omega_2^2 c^2\theta_1\right] \hat{u}_2$$

$$+ \left[g\, c\theta_1 - 2 v \omega_1 - 3 (L+x)\,\omega_2^2 s\theta_1 c\theta_1\right] \hat{u}_3$$

Problem 3

Refer to HW #3 for figure.

Basepoint O on the gimbal, basepoint S on the satellite.

Observer A on the antenna.

$$\vec{R}^{OS} = R\,\hat{b}_1$$

BKE (Method III)

$${}^I\vec{a}^{OS} = {}^A\vec{a}^{OS} + 2\,{}^I\vec{\omega}^A \times {}^A\vec{V}^{OS} + {}^I\vec{\omega}^A \times \left({}^I\vec{\omega}^A \times \vec{R}^{OS}\right) + \frac{{}^A d\,{}^I\vec{\omega}^A}{dt} \times \vec{R}^{OS}$$

$${}^A\vec{V}^{OS} = \dot{R}\,\hat{b}_1$$

$${}^A\vec{a}^{OS} = \ddot{R}\,\hat{b}_1$$

$${}^I\vec{\omega}^A = -\Omega\, c\phi\, \hat{b}_1 - \Omega\, s\phi\, \hat{b}_2 - \Gamma\, \hat{b}_3$$

$$\frac{{}^A d\,{}^I\vec{\omega}^A}{dt} = \Omega\Gamma\, s\phi\, \hat{b}_1 - \Omega\Gamma\, c\phi\, \hat{b}_2$$

$$2\,{}^I\vec{\omega}^A \times {}^A\vec{V}^{OS} = 2\left[\dot{R}\Omega\, s\phi\, \hat{b}_3 - \dot{R}\Gamma\, \hat{b}_2\right]$$

$${}^I\vec{\omega}^A \times \left({}^I\vec{\omega}^A \times \vec{R}^{OS}\right) = {}^I\vec{\omega}^A \times \left[R\Omega\, s\phi\, \hat{b}_3 - R\Gamma\, \hat{b}_2\right] =$$

$$= R\Omega^2 s\phi\, c\phi\, \hat{b}_2 + R\Gamma\Omega\, c\phi\, \hat{b}_3 - R\Omega^2 s^2\phi\, \hat{b}_1 - R\Gamma^2\, \hat{b}_1$$

$$= R\left[-\Omega^2 s^2\phi - \Gamma^2\right]\hat{b}_1 + \left[R\Omega^2 s\phi\, c\phi\right]\hat{b}_2 + \left[R\Gamma\Omega\, c\phi\right]\hat{b}_3$$

$$\frac{{}^A d\,{}^I\vec{\omega}^A}{dt} \times \vec{R}^{OS} = R\Omega\Gamma\, c\phi\, \hat{b}_3$$

$$
{}^{I}\bar{a}^{os} = \left[\ddot{R} - R\Omega^2 s^2\phi - R\Gamma^2\right]\hat{b}_1 + \left[-2\dot{R}\Gamma + R\Omega^2 s\phi\, c\phi\right]\hat{b}_2 + \left[2\dot{R}\Omega s\phi + R\Gamma\Omega c\phi + R\Omega\Gamma c\phi\right]\hat{b}_3
$$

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$$
{}^{I}\bar{a}^{os} = \left[\ddot{R} - R\Omega^2 s^2\phi - R\Gamma^2\right]\hat{b}_1 + \left[R\Omega^2 s\phi\, c\phi - 2\dot{R}\Gamma\right]\hat{b}_2 + \left[2\dot{R}\Omega s\phi + 2R\Gamma\Omega c\phi\right]\hat{b}_3
$$

Problem 4

Call the inertial observer I and observer on earth E.

We want to find ${}^{I}\vec{a}^{OS}$ where O is now the center of the earth. Call the basepoint on the gimbal Q.

$${}^{I}\vec{a}^{OS} = {}^{I}\vec{a}^{OQ} + {}^{I}\vec{a}^{QS}$$

First, write the acceleration from problem #3 in the $\hat{E}, \hat{N}, \hat{u}$ frame.

$${}^{E}\vec{a}^{QS} = [\ddot{R} - R\Omega^2 s^2\phi - R\Gamma^2]\hat{b}_1 - [R\Omega^2 s\phi c\phi - 2\dot{R}\Gamma]\hat{b}_2 + $$
$$+ [2\dot{R}\Omega s\phi + 2R\Gamma\Omega c\phi]\hat{b}_3$$

$$\hat{b}_1 = c\phi\,\hat{u} + s\phi[s\theta\,\hat{N} - c\theta\,\hat{E}] = -s\phi c\theta\,\hat{E} + s\phi s\theta\,\hat{N} + c\phi\,\hat{u}$$

$$\hat{b}_2 = s\phi\,\hat{u} + c\phi[-s\theta\,\hat{N} + c\theta\,\hat{E}] = c\phi c\theta\,\hat{E} - c\phi s\theta\,\hat{N} + s\phi\,\hat{u}$$

$$\hat{b}_3 = c\theta\,\hat{N} + s\theta\,\hat{E}$$

$${}^{E}\vec{a}^{QS} = [\ddot{R} - R\Omega^2 s^2\phi - R\Gamma^2][-s\phi c\theta\,\hat{E} + s\phi s\theta\,\hat{N} + c\phi\,\hat{u}] -$$
$$- [R\Omega^2 s\phi c\phi - 2\dot{R}\Gamma][c\phi c\theta\,\hat{E} - c\phi s\theta\,\hat{N} + s\phi\,\hat{u}] +$$
$$+ [2\dot{R}\Omega s\phi + 2R\Gamma\Omega c\phi][c\theta\,\hat{N} + s\theta\,\hat{E}] =$$
$$= [-\ddot{R}s\phi c\theta + R\Omega^2 s^3\phi c\theta + R\Gamma^2 s\phi c\theta - R\Omega^2 s\phi c^2\phi c\theta + 2\dot{R}\Gamma c\phi c\theta +$$
$$+ 2\dot{R}\Omega s\phi s\theta + 2R\Gamma\Omega c\phi s\theta]\hat{E} +$$
$$+ [\ddot{R}s\phi s\theta - R\Omega^2 s^3\phi s\theta - R\Gamma^2 s\phi s\theta + R\Omega^2 s\phi c^2\phi s\theta - 2\dot{R}\Gamma c\phi s\theta +$$
$$+ 2\dot{R}\Omega s\phi c\theta + 2R\Gamma\Omega c\phi c\theta]\hat{N} +$$
$$+ [\ddot{R}c\phi - R\Omega^2 s^2\phi c\phi - R\Gamma^2 c\phi - R\Omega^2 s^2\phi c\phi + 2\dot{R}\Gamma s\phi]\hat{u}$$

Use BKE to get ${}^I\vec{a}^{QS}$:

$${}^I\vec{a}^{QS} = {}^E\vec{a}^{QS} + 2\,{}^I\vec{\omega}^E \times {}^E\vec{V}^{QS} + {}^I\vec{\omega}^E \times \left({}^I\vec{\omega}^E \times \vec{R}^{QS}\right) + \frac{d\,{}^I\vec{\omega}^E}{dt} \times \vec{R}^{QS}$$

$${}^E\vec{V}^{QS} = {}^A\vec{V}^{QS} + {}^E\vec{\omega}^A \times \vec{R}^{QS} \qquad {}^E\vec{\omega}^A = -\Omega c\phi\,\hat{b}_1 - \Omega s\phi\,\hat{b}_2 - \Gamma\,\hat{b}_3$$

$${}^E\vec{V}^{QS} = \dot{R}\,\hat{b}_1 + R\Omega s\phi\,\hat{b}_3 - R\Gamma\,\hat{b}_2 =$$

$$= \left[-\dot{R}s\phi c\theta + R\Omega s\phi s\theta - R\Gamma c\phi c\theta\right]\hat{E} +$$

$$+\left[\dot{R}s\phi s\theta + R\Omega s\phi c\theta + R\Gamma c\phi s\theta\right]\hat{N} +$$

$$+\left[\dot{R}c\phi + R\Gamma s\phi\right]\hat{U}$$

$${}^I\vec{\omega}^E = \omega_E\left(c\gamma\,\hat{N} + s\gamma\,\hat{U}\right)$$

$$2\,{}^I\vec{\omega}^E \times {}^E\vec{V}^{QS} = 2\Big\{\left[\dot{R}s\phi c\theta\,\omega_E c\gamma - R\Omega s\phi s\theta\,\omega_E c\gamma + R\Gamma c\phi c\theta\,\omega_E c\gamma\right]\hat{U} +$$

$$+\left[-\dot{R}s\phi c\theta\,\omega_E s\gamma + R\Omega s\phi s\theta\,\omega_E s\gamma - R\Gamma c\phi c\theta\,\omega_E s\gamma\right]\hat{N} +$$

$$+\left[-\dot{R}s\phi s\theta\,\omega_E s\gamma - R\Omega s\phi c\theta\,\omega_E s\gamma - R\Gamma c\phi s\theta\,\omega_E s\gamma\right]\hat{E} +$$

$$+\left[\dot{R}c\phi\,\omega_E c\gamma - R\Gamma s\phi\,\omega_E c\gamma\right]\hat{E}\Big\} =$$

$$2\,{}^I\vec{\omega}^E \times {}^E\vec{V}^{QS} = 2\left[-\dot{R}s\phi s\theta\,\omega_E s\gamma - R\Omega s\phi c\theta\,\omega_E s\gamma - R\Gamma c\phi s\theta\,\omega_E s\gamma +\right.$$

$$\left.+\dot{R}c\phi\,\omega_E c\gamma - R\Gamma s\phi\,\omega_E c\gamma\right]\hat{E} + 2\left[-\dot{R}s\phi c\theta\,\omega_E s\gamma +\right.$$

$$\left.+R\Omega s\phi s\theta\,\omega_E s\gamma - R\Gamma c\phi c\theta\,\omega_E s\gamma\right]\hat{N} + 2\left[\dot{R}s\phi c\theta\,\omega_E c\gamma -\right.$$

$$\left.- R\Omega s\phi s\theta\,\omega_E c\gamma + R\Gamma c\phi c\theta\,\omega_E c\gamma\right]\hat{U}$$

$$\vec{R}^{QS} = R\,\hat{b}_1 = R\left[-s\phi c\theta\,\hat{E} + s\phi s\theta\,\hat{N} + c\phi\,\hat{U}\right]$$

$$\frac{d\,{}^I\vec{\omega}^E}{dt} = 0$$

$${}^I\vec{\omega}^E \times \left({}^I\vec{\omega}^E \times \vec{R}^{QS}\right) = {}^I\vec{\omega}^E \times \Big[R s\phi c\theta \, \omega_E c\gamma \, \hat{u} + R c\phi \, \omega_E c\gamma \, \hat{E} -$$

$$- R s\phi c\theta \, \omega_E s\gamma \hat{N} - R s\phi s\theta \, \omega_E s\gamma \, \hat{E}\Big] =$$

$$= \Big[R s\phi c\theta \, \omega_E^2 c^2\gamma \, \hat{E} - R c\phi \, \omega_E^2 c^2\gamma \, \hat{u} + R s\phi s\theta \, \omega_E^2 s\gamma c\gamma \, \hat{u} +$$

$$+ R c\phi \, \omega_E^2 s\gamma c\gamma \, \hat{N} + R s\phi c\theta \, \omega_E^2 s^2\gamma \, \hat{E} - R s\phi s\theta \, \omega_E^2 s^2\gamma \, \hat{N}\Big]$$

$${}^I\vec{\omega}^E \times \left({}^I\vec{\omega}^E \times \vec{R}^{QS}\right) = R s\phi c\phi \, \omega_E^2 \, \hat{E} + \Big[R c\phi \, \omega_E^2 s\gamma c\gamma - R s\phi s\theta \, \omega_E^2 s^2\gamma\Big] \hat{N} +$$

$$+ \Big[R s\phi s\theta \, \omega_E^2 s\gamma c\gamma - R c\phi \, \omega_E^2 c^2\gamma\Big] \hat{u}$$

$${}^I\vec{a}^{QS} = \Big[-\ddot{R} s\phi c\theta + R\Omega^2 s^3\phi c\theta + R\Gamma^2 s\phi c\theta - R\Omega^2 s\phi c^2\phi c\theta + 2\dot{R}\Gamma c\phi c\theta +$$

$$+ 2\dot{R}\Omega s\phi s\theta + 2R\Gamma\Omega c\phi s\theta - 2\dot{R} s\phi s\theta \, \omega_E s\gamma - 2R\Omega s\phi c\theta \, \omega_E s\gamma -$$

$$- 2R\Gamma c\phi s\theta \, \omega_E s\gamma + 2\dot{R} c\phi \, \omega_E c\gamma - 2R\Gamma s\phi \, \omega_E c\gamma + R s\phi c\phi \, \omega_E^2\Big] \hat{E} +$$

$$+ \Big[\ddot{R} s\phi s\theta - R\Omega^2 s^3\phi s\theta - R\Gamma^2 s\phi s\theta + R\Omega^2 s\phi c^2\phi s\theta - 2\dot{R}\Gamma c\phi s\theta +$$

$$+ 2\dot{R}\Omega s\phi c\theta + 2R\Gamma\Omega c\phi c\theta - 2\dot{R} s\phi c\theta \, \omega_E s\gamma + 2R\Omega s\phi s\theta \, \omega_E s\gamma -$$

$$- 2R\Gamma c\phi c\theta \, \omega_E s\gamma + R c\phi \, \omega_E^2 s\gamma c\gamma - R s\phi s\theta \, \omega_E^2 s^2\gamma\Big] \hat{N} +$$

$$+ \Big[\ddot{R} c\phi - 2R\Omega^2 s^2\phi c\phi - R\Gamma^2 c\phi + 2\dot{R}\Gamma s\phi + 2\dot{R} s\phi c\theta \, \omega_E c\gamma -$$

$$- 2R\Omega s\phi s\theta \, \omega_E c\gamma + 2R\Gamma c\phi c\theta \, \omega_E c\gamma + R s\phi s\theta \, \omega_E^2 s\gamma c\gamma -$$

$$- R c\phi \, \omega_E^2 c^2\gamma\Big] \hat{u}$$

Find ${}^{I}\vec{a}^{OQ}$:

$$\vec{R}^{OQ} = R_e \, \hat{u}$$

$${}^{I}\vec{V}^{OQ} = \cancel{{}^{E}\vec{V}^{OQ}}^{\,0} + {}^{I}\vec{\omega}^{E} \times \vec{R}^{OQ} = \omega_E \left(c\gamma \, \hat{N} + s\gamma \, \hat{U} \right) \times R_e \, \hat{u} =$$

$$= R_e \, \omega_E \, c\gamma \, \hat{E}$$

$${}^{I}\vec{a}^{OQ} = \cancel{{}^{E}\vec{a}^{OQ}}^{\,0} + 2\,{}^{I}\vec{\omega}^{E} \times \cancel{{}^{E}\vec{V}^{OQ}}^{\,0} + {}^{I}\vec{\omega}^{E} \times \left({}^{I}\vec{\omega}^{E} \times \vec{R}^{OQ} \right) + \cancel{\frac{d\,{}^{I}\vec{\omega}^{E}}{dt}}^{\,0} \times \vec{R}^{OQ}$$

$$= - R_e \, \omega_E^2 \, c^2\gamma \, \hat{u} + R_e \, \omega_E^2 \, s\gamma \, c\gamma \, \hat{N}$$

$${}^{I}\vec{a}^{OS} = {}^{I}\vec{a}^{OQ} + {}^{I}\vec{a}^{QS}$$

$${}^{I}\vec{a}^{OS} = \left[\ldots\ldots \right] \hat{E} + \left[\ldots\ldots + R_E \, \omega_E^2 \, s\gamma \, c\gamma \right] \hat{N} + \left[\ldots\ldots - R_E \, \omega_E^2 \, c^2\gamma \right] \hat{u}$$