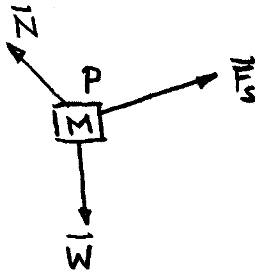
Homework 10 - Solution

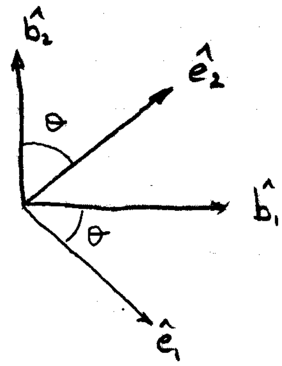
Problem 1

$\Sigma \vec{F} = m \cdot \vec{a}$

FBD



$\Sigma \vec{F} = \vec{N} + \vec{F}_s + \vec{W} = M \cdot \vec{a}$



$\vec{W} = -M \cdot g \, \hat{b}_2$

$\vec{N} = -N c\theta \, \hat{b}_1 + N s\theta \, \hat{b}_2$

$\vec{F}_s = -K(s-\ell)\, \hat{u}_s$

$\vec{R}^{PQ} = (1{,}1 - 0{,}9 c\theta)\, \hat{b}_1 + (0{,}9 s\theta - 0{,}6)\, \hat{b}_2$

$$\hat{u}_s = \frac{(1{,}1-0{,}9c\theta)\hat{b}_1 + (0{,}9s\theta - 0{,}6)\hat{b}_2}{\sqrt{(1{,}1-0{,}9c\theta)^2 + (0{,}9s\theta-0{,}6)^2}} \qquad s = \sqrt{(1{,}1-0{,}9c\theta)^2 + (0{,}9s\theta-0{,}6)^2}$$

$$\Sigma \vec{F} = -N c\theta\, \hat{b}_1 + N s\theta\, \hat{b}_2 - Mg\, \hat{b}_2 - K(s-\ell)\frac{(1{,}1-0{,}9c\theta)\hat{b}_1 + (0{,}9s\theta-0{,}6)\hat{b}_2}{\sqrt{(1{,}1-0{,}9c\theta)^2 + (0{,}9s\theta-0{,}6)^2}}$$

Since we have two unknowns (N, θ) we can either solve for N using fsolve in Matlab or we can rewrite the equation in the $\hat{e}$ system. That way we can get one equation with only unknown θ.

$$\hat{e}_2: \quad -Mg\, c\theta - K(s-\ell)\frac{(1{,}1-0{,}9c\theta)s\theta + (0{,}9s\theta-0{,}6)c\theta}{\sqrt{(1{,}1-0{,}9c\theta)^2 + (0{,}9s\theta-0{,}6)^2}} =$$

$$= -Mg\, c\theta - K(s-\ell)\frac{(1{,}1 s\theta - 0{,}6 c\theta)}{s}$$

Next, we find ${}^{I}\vec{a}^{PQ}$;

$${}^{I}\vec{V}^{PQ} = 0{,}9\,\dot{\theta}\, s\theta\ \hat{b}_1 + 0{,}9\,\dot{\theta}\, c\theta\ \hat{b}_2$$

$${}^{I}\vec{a}^{PQ} = \left(0{,}9\,\ddot{\theta}\, s\theta + 0{,}9\,\dot{\theta}^2 c\theta\right)\hat{b}_1 + \left(0{,}9\,\ddot{\theta}\, c\theta - 0{,}9\,\dot{\theta}^2 s\theta\right)\hat{b}_2$$

transform to the $\hat{e}$ system in order to write $\Sigma\vec{F} = m\cdot\vec{a}$

$${}^{I}\vec{a}^{PQ} = \left(0{,}9\,\ddot{\theta}\, s\theta + 0{,}9\,\dot{\theta}^2 c\theta\right)\left(c\theta\,\hat{e}_1 + s\theta\,\hat{e}_2\right) + \left(0{,}9\,\ddot{\theta}\, c\theta - 0{,}9\,\dot{\theta}^2 s\theta\right)\left(-s\theta\,\hat{e}_1 + c\theta\,\hat{e}_2\right)$$

$$= \left[\cancel{0{,}9\,\ddot{\theta}\, s\theta c\theta} + 0{,}9\,\dot{\theta}^2 c^2\theta - \cancel{0{,}9\,\ddot{\theta}\, s\theta c\theta} + 0{,}9\,\dot{\theta}^2 s^2\theta\right]\hat{e}_1 +$$

$$+\left[0{,}9\,\ddot{\theta}\, s^2\theta + \cancel{0{,}9\,\dot{\theta}^2 s\theta c\theta} + 0{,}9\,\ddot{\theta}\, c^2\theta - \cancel{0{,}9\,\dot{\theta}^2 s\theta c\theta}\right]\hat{e}_2$$

$${}^{I}\vec{a}^{PQ} = 0{,}9\,\dot{\theta}^2\ \hat{e}_1 + 0{,}9\,\ddot{\theta}\ \hat{e}_2$$

$\underline{\Sigma\vec{F} = m\cdot\vec{a}}$

$$\hat{e}_2:\ M\cdot 0{,}9\,\ddot{\theta} = -Mg\,c\theta - K(s-\ell)\,\frac{(1{,}1\,s\theta - 0{,}6\,c\theta)}{s}$$

$$\hat{e}_1:\ M\cdot 0{,}9\,\dot{\theta}^2 = -N c^2\theta - N s^2\theta + Mg\,s\theta - K(s-\ell)\,\frac{(1{,}1 - 0{,}9\,c\theta)c\theta - (0{,}9\,s\theta - 0{,}6)s\theta}{s}$$

$$M\cdot 0{,}9\,\dot{\theta}^2 = -N + Mg\,s\theta - K(s-\ell)\,\frac{1{,}1\,c\theta - 0{,}9 + 0{,}6\,s\theta}{s}$$

•) <u>EOM</u>:

$$0{,}9\,M\ddot{\theta} = -Mg\,c\theta - K(s-\ell)\,\frac{(1{,}1\,s\theta - 0{,}6\,c\theta)}{s}$$

•) Order = 2nd order

•) 1 DoF

•) state variable form

$$S_1 = \theta$$
$$S_2 = \dot{\theta}$$

$$\dot{S}_1 = S_2$$
$$\dot{S}_2 = -\frac{g}{0,9}\, c\theta - \frac{K}{0,9M}\,(s-\ell)\,\frac{(1,1\, s\theta - 0,6\, c\theta)}{s}$$

•) States = 2

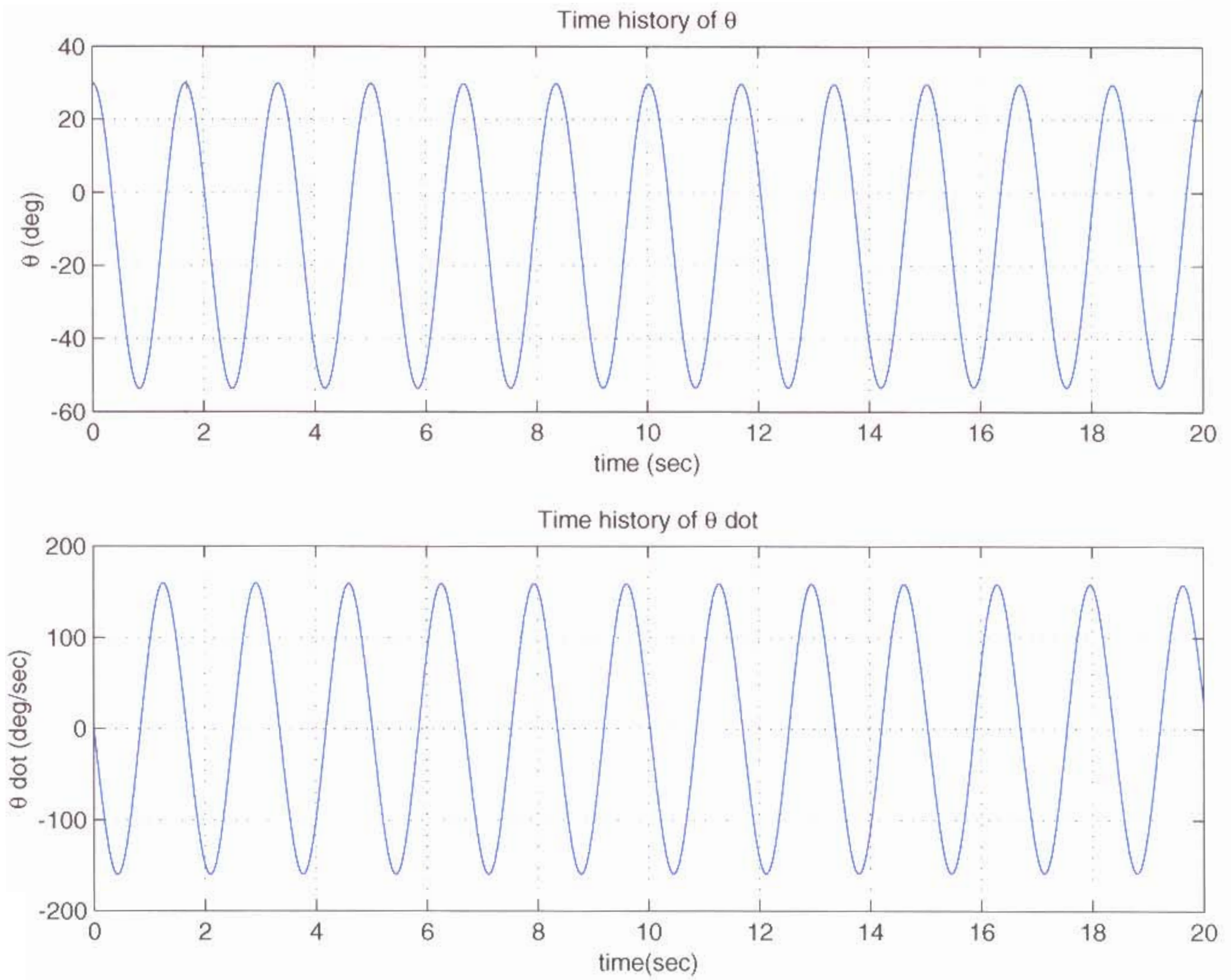
Problem 2

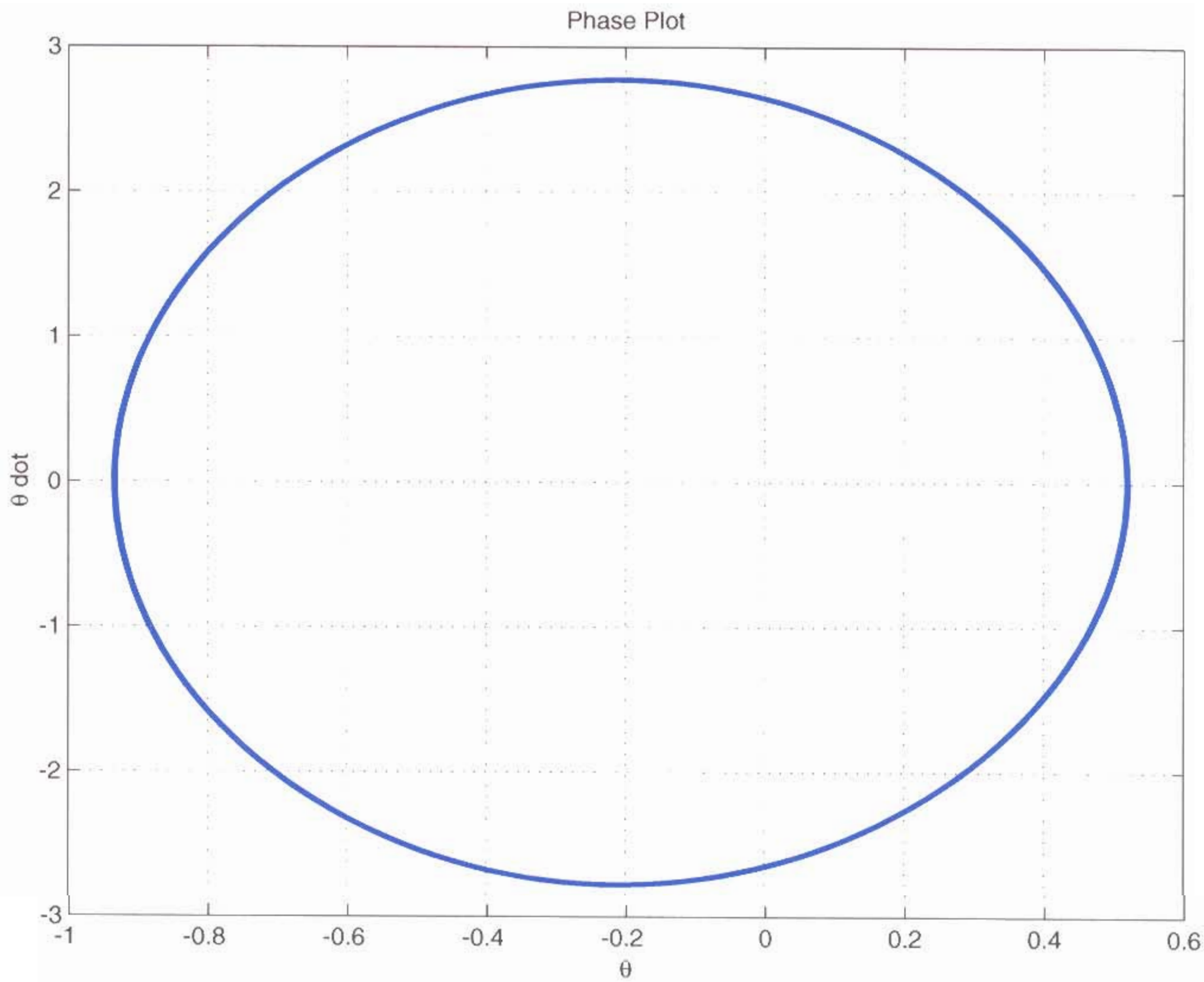
$K = 150 \ \frac{N}{m}$ IC's: $\theta(0) = 30°$ time = 20 seconds

$\dot{\theta}(0) = 0$ deg/sec

$\ell = 0,05$ m

$$\text{Period} = \frac{1}{\text{frequency}} \approx 1,7 \text{ sec} \Rightarrow \text{frequency} = \boxed{0,588 \text{ Hz}}$$


Time history of θ
θ (deg)
time (sec)
40
20
0
-20
-40
-60
0
2
4
6
8
10
12
14
16
18
20
Time history of θ dot
θ dot (deg/sec)
time(sec)
200
100
0
-100
-200



From this plot we can tell that the motion is periodic since the curve is closed.

Problem 3

$\sum \vec{F} = m\cdot\vec{a}$

FBD

$\sum \vec{F} = \vec{W} + \vec{N} = m\cdot\vec{a}$

$\vec{W} = -m\cdot g\cdot \hat{e}_2 = -mg\left[c\gamma\ \hat{b}_2 - s\gamma\ \hat{b}_1\right] = mg\,s\gamma\ \hat{b}_1 - mg\,c\gamma\ \hat{b}_2$

$\vec{N} = N\ \hat{b}_2$

$\sum \vec{F} = mg\,s\gamma\ \hat{b}_1 + \left[N - mg\,c\gamma\right]\hat{b}_2 = m\vec{a}$

$\vec{R}^{OP} = -x\ \hat{b}_1 + y\ \hat{b}_3 = -x\left(c\gamma\ \hat{b}_1' - s\gamma\ \hat{b}_2'\right) + y\ \hat{b}_3'$

The $\hat{b}'$ system rotates at Ω wrt. the $\hat{e}$ system.

$${}^{I}\vec{V}^{OP} = {}^{D}\vec{V}^{OP} + {}^{I}\vec{\omega}^{D} \times \vec{R}^{OP} = -\dot{x}\,c\gamma\ \hat{b}_1' + \dot{x}\,s\gamma\ \hat{b}_2' + \dot{y}\ \hat{b}_3' +$$

$$+ \Omega\ \hat{b}_2' \times \left(-x\,c\gamma\ \hat{b}_1' + x\,s\gamma\ \hat{b}_2' + y\ \hat{b}_3'\right) =$$

$${}^{I}\vec{V}^{OP} = -\dot{x}\,c\gamma\ \hat{b}_1' + \dot{x}\,s\gamma\ \hat{b}_2' + \dot{y}\ \hat{b}_3' + \Omega x\,c\gamma\ \hat{b}_3' + \Omega y\ \hat{b}_3'$$

$${}^{I}\vec{a}^{OP} = {}^{D}\vec{a}^{OP} + 2\,{}^{I}\vec{\omega}^{D} \times {}^{D}\vec{V}^{OP} + {}^{I}\vec{\omega}^{D} \times \left({}^{I}\vec{\omega}^{D} \times \vec{R}^{OP}\right) + \cancel{\frac{{}^{D}d\,{}^{I}\vec{\omega}^{D}}{dt}}^{\,0} \times \vec{R}^{OP}$$

$${}^{D}\vec{a}^{OP} = -\ddot{x}\,c\gamma\ \hat{b}_1' + \ddot{x}\,s\gamma\ \hat{b}_2' + \ddot{y}\ \hat{b}_3'$$

$$2\,{}^{I}\vec{\omega}^{D} \times {}^{D}\vec{V}^{OP} = 2\left[\Omega\dot{x}\,c\gamma\ \hat{b}_3' + \Omega\dot{y}\ \hat{b}_1'\right]$$

$${}^{I}\vec{\omega}^{D} \times \left({}^{I}\vec{\omega}^{D} \times \vec{R}^{OP}\right) = \Omega\ \hat{b}_2' \times \left[\Omega x\,c\gamma\ \hat{b}_3' + \Omega y\ \hat{b}_1'\right] =$$

$$= \Omega^2 x\,c\gamma\ \hat{b}_1' - \Omega^2 y\ \hat{b}_3'$$

$$ {}^I\vec{a}^{OP} = \left[-\ddot{x}\, c\gamma + 2\Omega\dot{y} + \Omega^2 x\, c\gamma\right]\hat{b}_1' + $$
$$ + \left[\ddot{x}\, s\gamma\right]\hat{b}_2' + \left[\ddot{y} + 2\Omega\dot{x}\, c\gamma - \Omega^2 y\right]\hat{b}_3' $$

$$ \hat{b}_1' = c\gamma\, \hat{b}_1 + s\gamma\, \hat{b}_2 $$

$$ \hat{b}_2' = -s\gamma\, \hat{b}_1 + c\gamma\, \hat{b}_2 $$

$$ \hat{b}_3' = \hat{b}_3 $$

$$ {}^I\vec{a}^{OP} = \left[-\ddot{x}\, c^2\gamma + 2\Omega\dot{y}\, c\gamma + \Omega^2 x\, c^2\gamma - \ddot{x}\, s^2\gamma\right]\hat{b}_1 + $$
$$ + \left[-\ddot{x}\, s\gamma\, c\gamma + 2\Omega\dot{y}\, s\gamma + \Omega^2 x\, s\gamma\, c\gamma - \ddot{x}\, s\gamma\, c\gamma\right]\hat{b}_2 + $$
$$ + \left[\ddot{y} + 2\Omega\dot{x}\, c\gamma - \Omega^2 y\right]\hat{b}_3 $$

$$ {}^I\vec{a}^{OP} = \left[-\ddot{x} + 2\Omega\dot{y}\, c\gamma + \Omega^2 x\, c^2\gamma\right]\hat{b}_1 + \left[-2\ddot{x}\, s\gamma\, c\gamma + 2\Omega\dot{y}\, s\gamma + \Omega^2 x\, s\gamma\, c\gamma\right]\hat{b}_2 + $$
$$ + \left[\ddot{y} + 2\Omega\dot{x}\, c\gamma - \Omega^2 y\right]\hat{b}_3 $$

$\underline{\sum \vec{F} = m\cdot\vec{a}}$

$$ \boxed{\hat{b}_1:\quad \cancel{m} g\, s\gamma = \cancel{m}\cdot\left[-\ddot{x} + 2\Omega\dot{y}\, c\gamma + \Omega^2 x\, c^2\gamma\right]} $$

$$ \hat{b}_2:\quad N - mg\, c\gamma = m\left[-2\ddot{x}\, s\gamma\, c\gamma + 2\Omega\dot{y}\, s\gamma + \Omega^2 x\, s\gamma\, c\gamma\right] $$

$$ \boxed{\hat{b}_3:\quad 0 = \left[\ddot{y} + 2\Omega\dot{x}\, c\gamma - \Omega y\right]} $$

<u>EOM's</u>

- 4 th order system
- DoF = 2

•) <u>State variable form</u> :

$$\begin{aligned}
s_1 &= x & \dot{s}_1 &= s_3 \\
s_2 &= y & \dot{s}_2 &= s_4 \\
s_3 &= \dot{x} & \dot{s}_3 &= 2\Omega s_4 c\gamma + \Omega^2 s_1 c^2\gamma - g s\gamma \\
s_4 &= \dot{y} & \dot{s}_4 &= \Omega^2 s_2 - 2\Omega s_3 c\gamma
\end{aligned}$$

Problem 4

$\Omega = 0{,}5 \cdot 2\pi \ \frac{rad}{sec}$

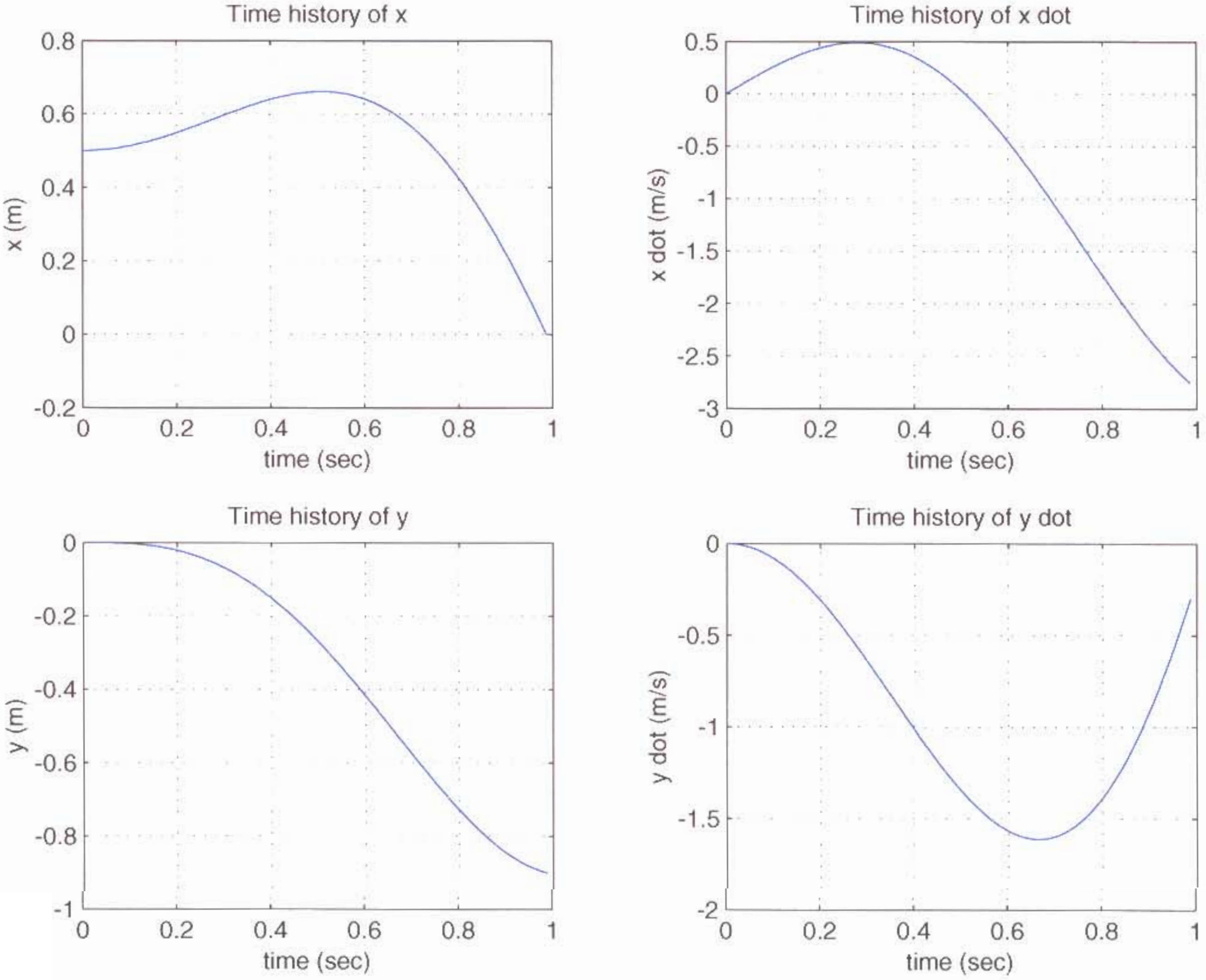
$\gamma = 12^\circ$

The mass will fall off the plane when $x \leq 0$.

IC's:

$x = 0.5$ m

$y = 0$ m

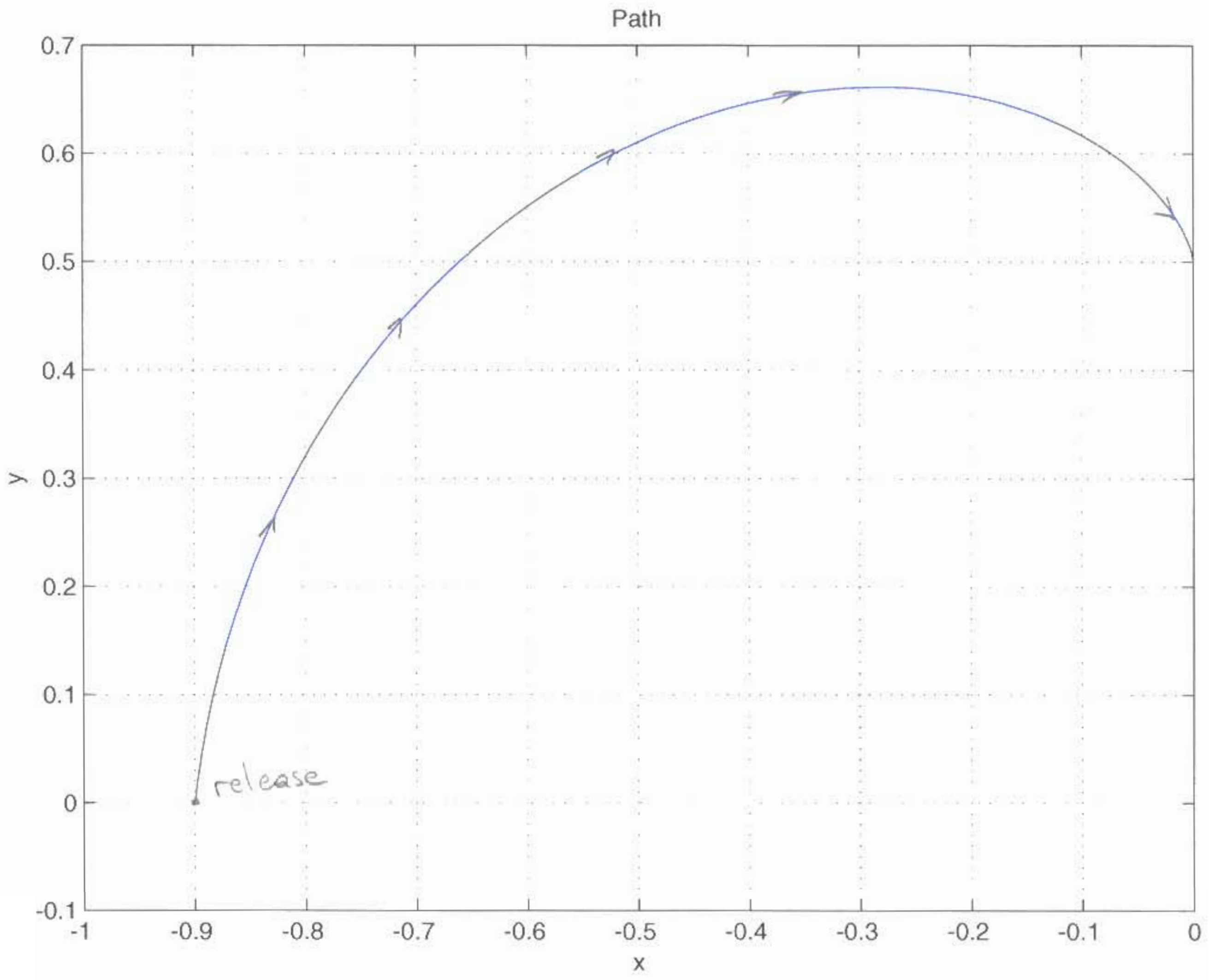


The mass falls off the plane at $\approx$ 1 sec

IC's:

X = 0.5 m

Y = 0 m

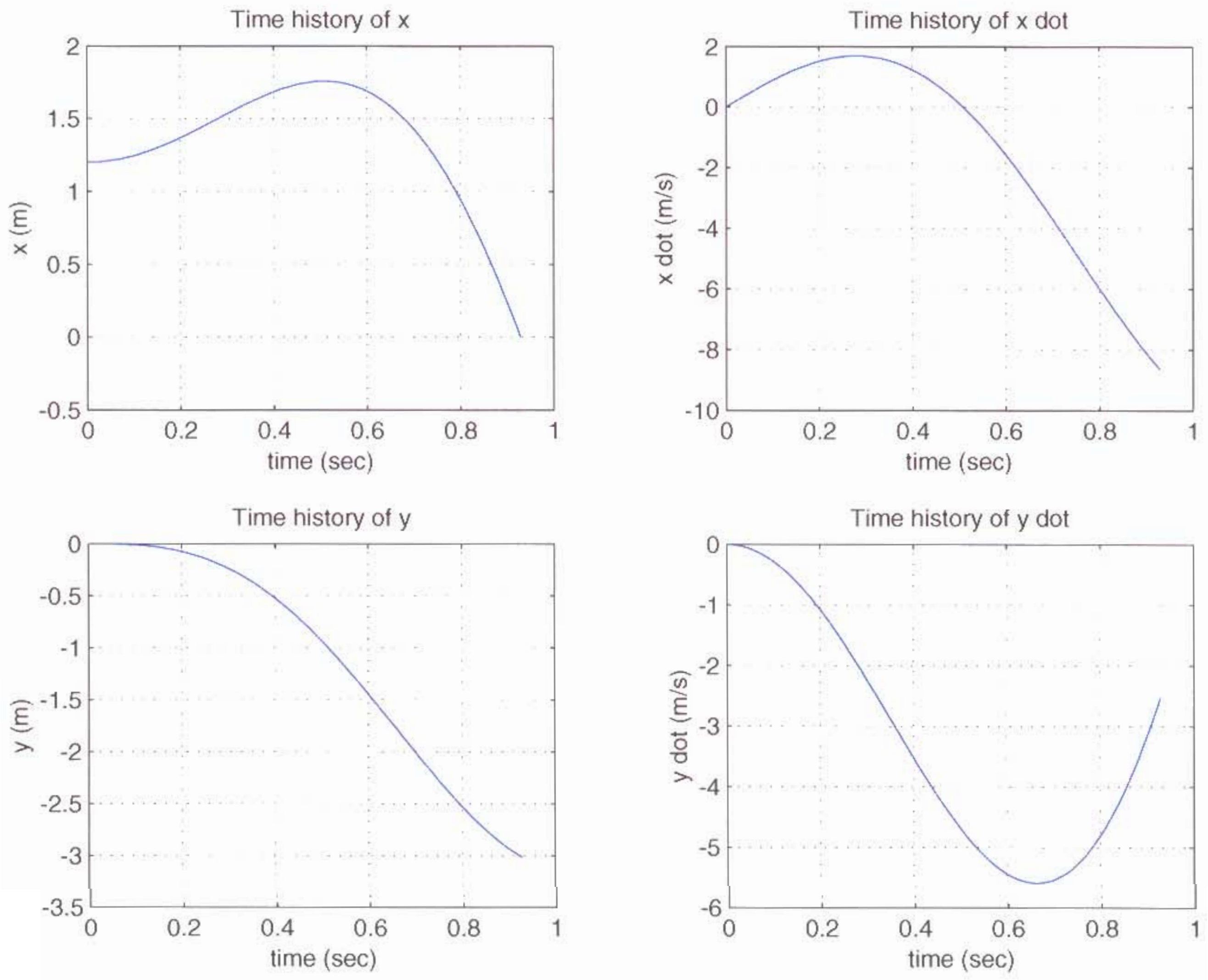


Path
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
y
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
x
release

IC's:

$x = 1.2$ m

$y = 0$ m

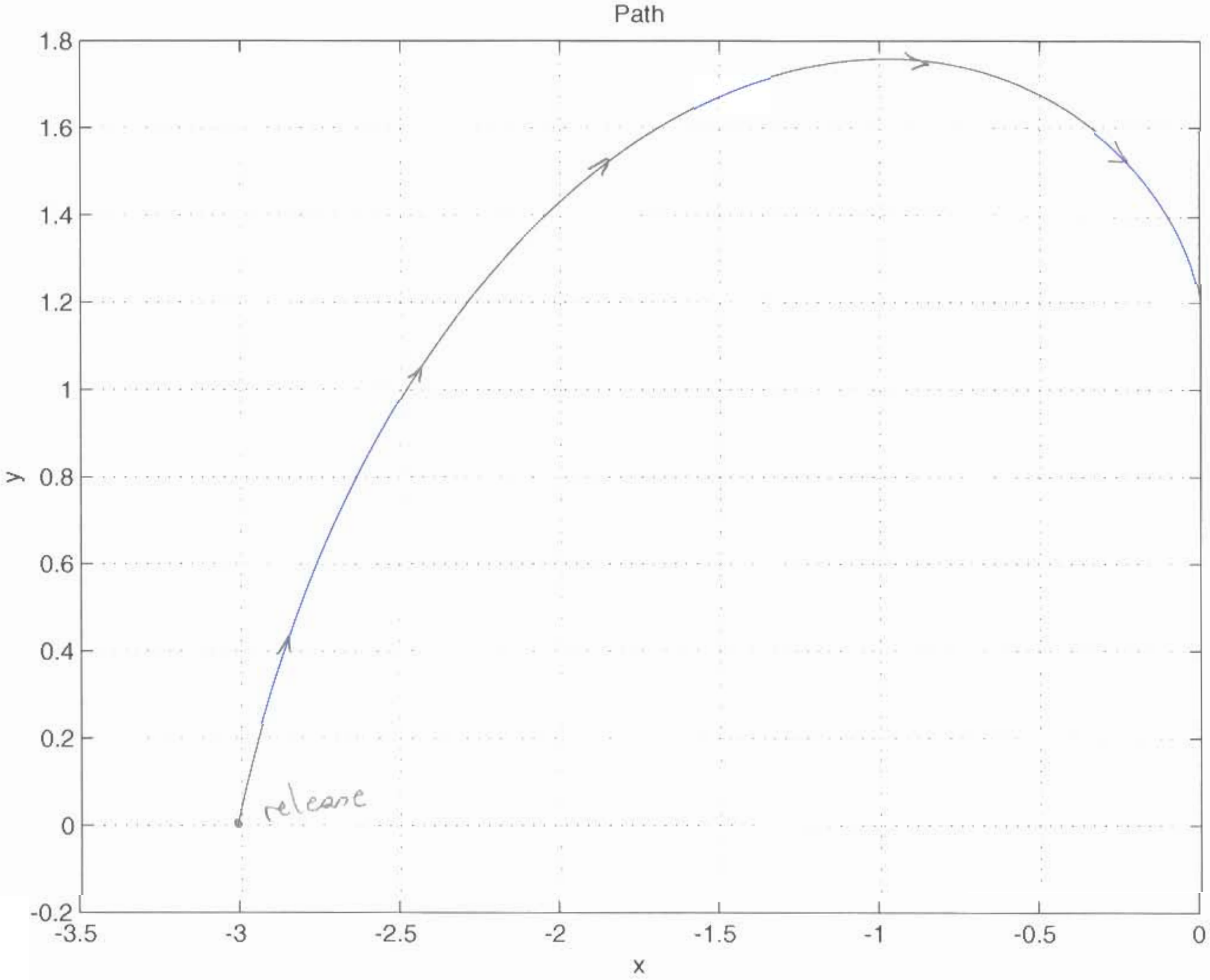


The mass falls off the plane at $\approx$ 0.93 sec

IC's:

x = 1.2 m

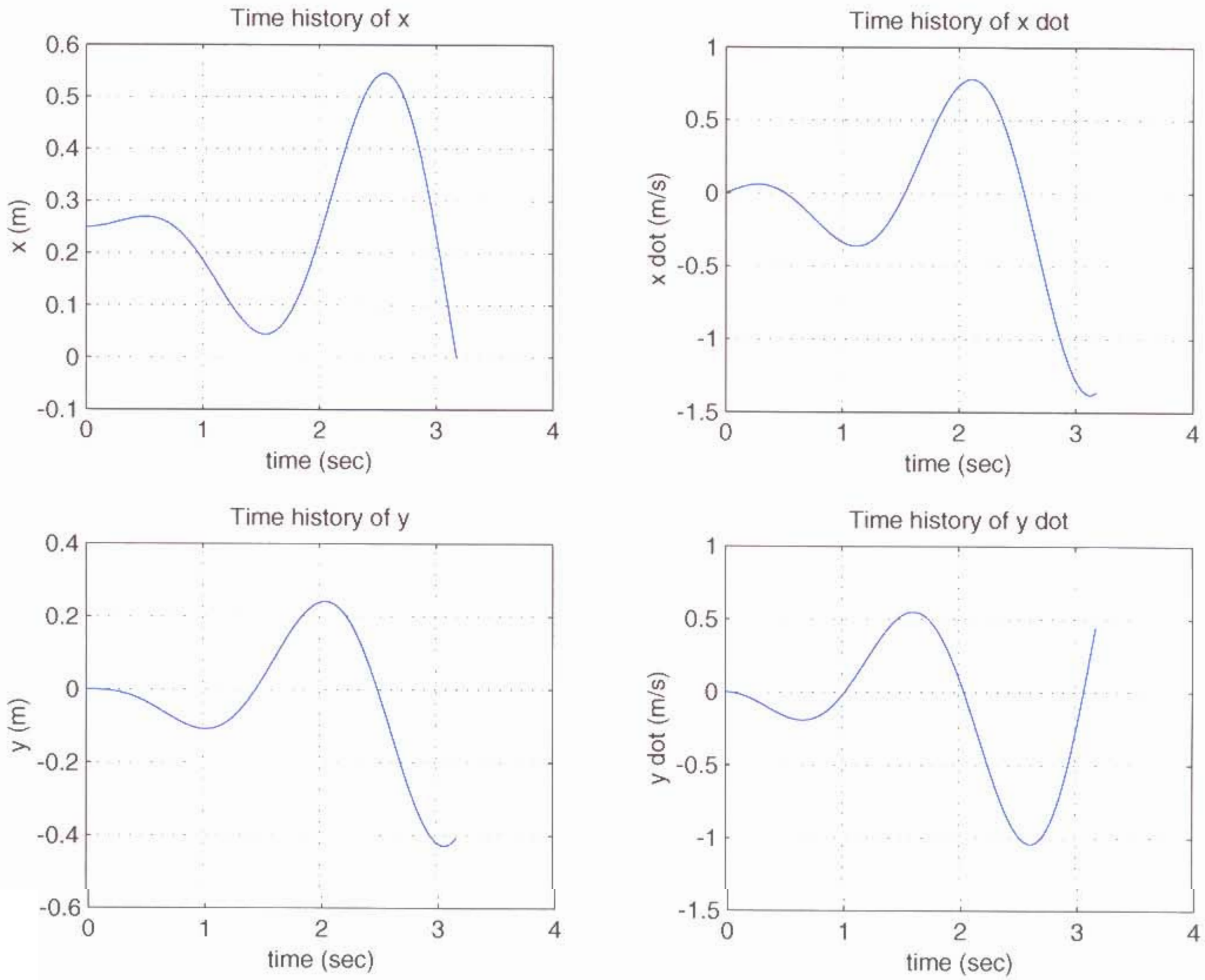
y = 0 m



I C's:

$x = 0.25\,m$

$y = 0\,m$

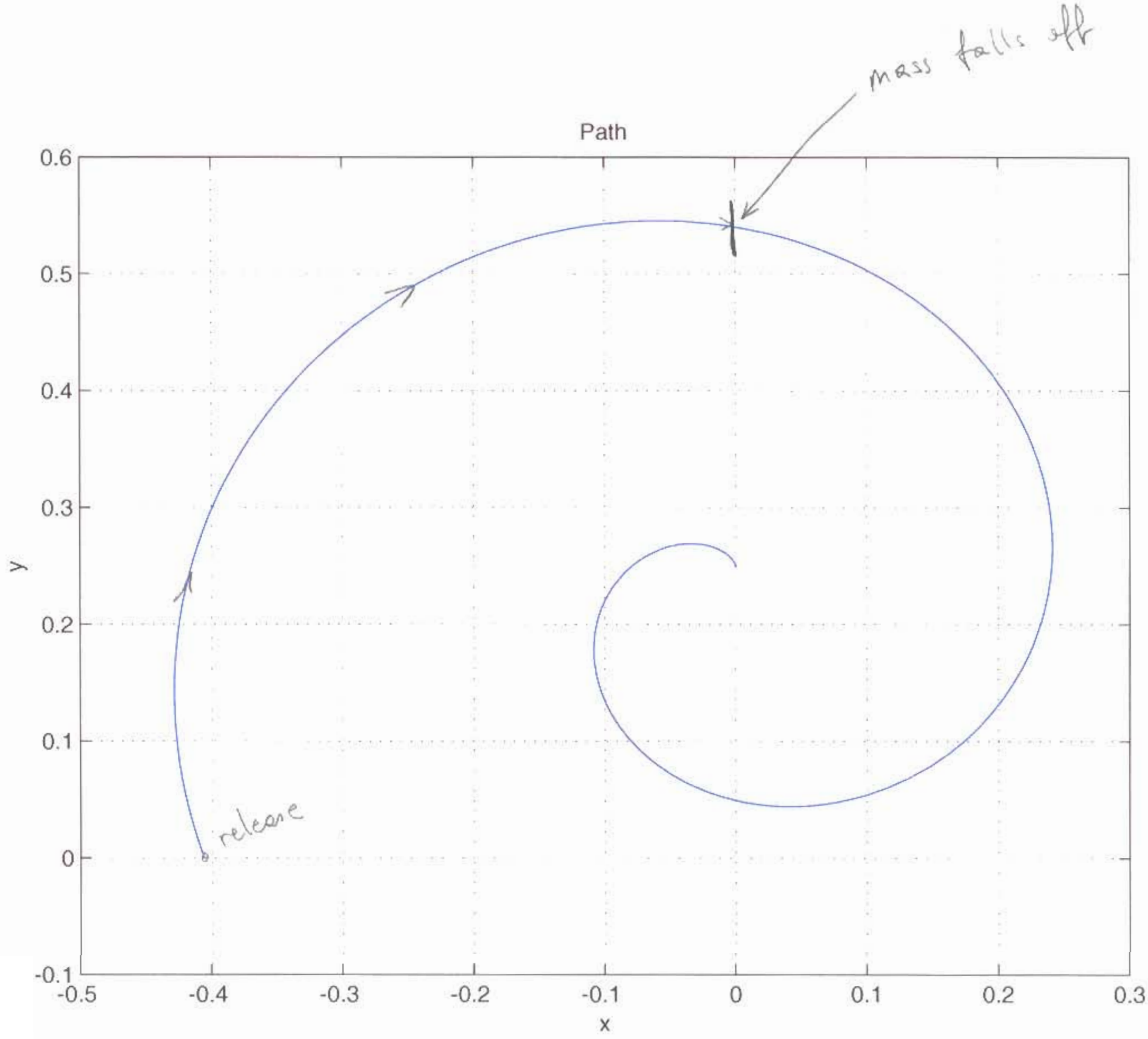


The mass falls off the plane at $\approx$ 3.2 sec.

IC's:

$x = 1.2\,m$

$y = 0\,m$



mass falls off
Path
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
y
release
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
x