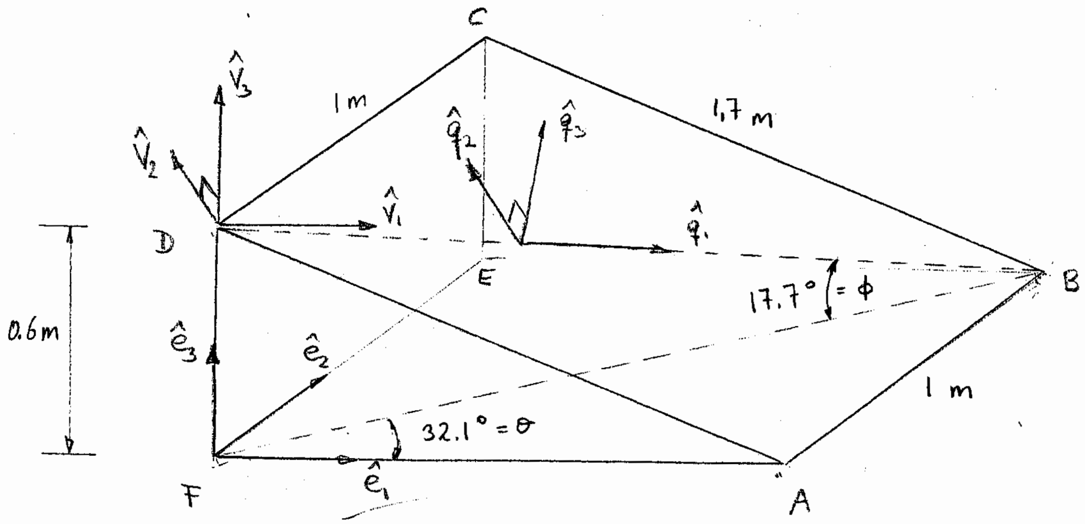
Problem 1



Define the intermediate set of unit vectors as follows:

$\hat{v}_3 = \hat{e}_3$, $\hat{v}_2 = \hat{q}_2$, and $\hat{v}_1$ completes the right handed set.

First, express $\hat{e}$'s in terms of $\hat{v}$'s. Take some vector $\vec{V}$:

$$\vec{V} = x_1 \hat{e}_1 + x_2 \hat{e}_2 + x_3 \hat{e}_3 = y_1 \hat{v}_1 + y_2 \hat{v}_2 + y_3 \hat{v}_3$$

$$y_1 = \hat{v}_1 \cdot \vec{V} = x_1 (\hat{v}_1 \cdot \hat{e}_1) + x_2 (\hat{v}_1 \cdot \hat{e}_2) + x_3 (\hat{v}_1 \cdot \hat{e}_3)$$

$$y_2 = \hat{v}_2 \cdot \vec{V} = x_1 (\hat{v}_2 \cdot \hat{e}_1) + x_2 (\hat{v}_2 \cdot \hat{e}_2) + x_3 (\hat{v}_2 \cdot \hat{e}_3)$$

$$y_3 = \hat{v}_3 \cdot \vec{V} = x_1 (\hat{v}_3 \cdot \hat{e}_1) + x_2 (\hat{v}_3 \cdot \hat{e}_2) + x_3 (\hat{v}_3 \cdot \hat{e}_3)$$

<u>In matrix form:</u>

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} (\hat{v}_1 \cdot \hat{e}_1) & (\hat{v}_1 \cdot \hat{e}_2) & (\hat{v}_1 \cdot \hat{e}_3) \\ (\hat{v}_2 \cdot \hat{e}_1) & (\hat{v}_2 \cdot \hat{e}_2) & (\hat{v}_2 \cdot \hat{e}_3) \\ (\hat{v}_3 \cdot \hat{e}_1) & (\hat{v}_3 \cdot \hat{e}_2) & (\hat{v}_3 \cdot \hat{e}_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

We know that the dot-product of two perpendicular vectors is zero! Also, $\hat{v}_3 \cdot \hat{e}_3 = 1$

Therefore we can simplify the matrix:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} (\hat{v}_1 \cdot \hat{e}_1) & (\hat{v}_1 \cdot \hat{e}_2) & 0 \\ (\hat{v}_2 \cdot \hat{e}_1) & (\hat{v}_2 \cdot \hat{e}_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{transforms any vector in the } \hat{e}\text{-system to a vector in the } \hat{v}\text{-system.}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Next, do the same thing to find a transformation matrix from the $\hat{v}$'s to the q's.

$$\vec{V} = y_1 \hat{v}_1 + y_2 \hat{v}_2 + y_3 \hat{v}_3 = z_1 \hat{q}_1 + z_2 \hat{q}_2 + z_3 \hat{q}_3$$

$$z_1 = \hat{q}_1 \cdot \vec{V} = y_1 (\hat{q}_1 \cdot \hat{v}_1) + y_2 (\hat{q}_1 \cdot \hat{v}_2) + y_3 (\hat{q}_1 \cdot \hat{v}_3)$$

$$z_2 = \hat{q}_2 \cdot \vec{V} = y_1 (\hat{q}_2 \cdot \hat{v}_1) + y_2 (\hat{q}_2 \cdot \hat{v}_2) + y_3 (\hat{q}_2 \cdot \hat{v}_3)$$

$$z_3 = \hat{q}_3 \cdot \vec{V} = y_1 (\hat{q}_3 \cdot \hat{v}_1) + y_2 (\hat{q}_3 \cdot \hat{v}_2) + y_3 (\hat{q}_3 \cdot \hat{v}_3)$$

In matrix form

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} (\hat{q}_1 \cdot \hat{v}_1) & (\hat{q}_1 \cdot \hat{v}_2) & (\hat{q}_1 \cdot \hat{v}_3) \\ (\hat{q}_2 \cdot \hat{v}_1) & (\hat{q}_2 \cdot \hat{v}_2) & (\hat{q}_2 \cdot \hat{v}_3) \\ (\hat{q}_3 \cdot \hat{v}_1) & (\hat{q}_3 \cdot \hat{v}_2) & (\hat{q}_3 \cdot \hat{v}_3) \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \longrightarrow \text{simplify}$$

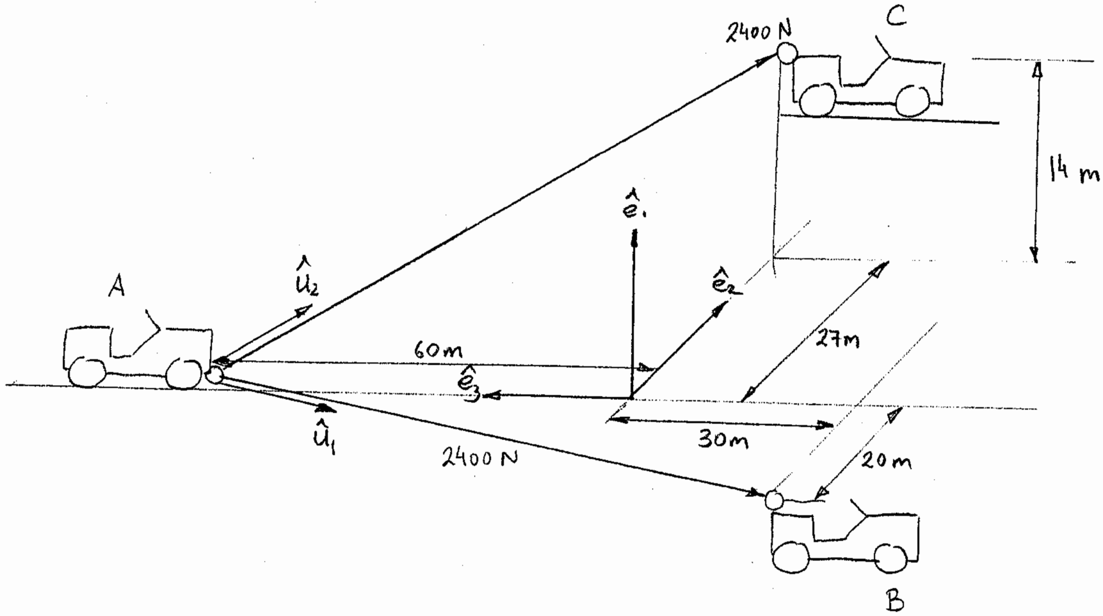
$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \underbrace{\begin{bmatrix} (\hat{q}_1 \cdot \hat{v}_1) & 0 & (\hat{q}_1 \cdot \hat{v}_3) \\ 0 & 1 & 0 \\ (\hat{q}_3 \cdot \hat{v}_1) & 0 & (\hat{q}_3 \cdot \hat{v}_3) \end{bmatrix}}_{\text{transforms a vector in the } \hat{v} \text{ system to a vector in the } \hat{q} \text{ system.}} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Finally, we multiply both matices together to obtain the transformation matrix from $\hat{e}$ to $\hat{q}$.

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}_{\hat{q}} = \begin{bmatrix} (\hat{q}_1 \cdot \hat{v}_1) & 0 & (\hat{q}_1 \cdot \hat{v}_3) \\ 0 & 1 & 0 \\ (\hat{q}_3 \cdot \hat{v}_1) & 0 & (\hat{q}_3 \cdot \hat{v}_3) \end{bmatrix} \begin{bmatrix} (\hat{v}_1 \cdot \hat{e}_1) & (\hat{v}_1 \cdot \hat{e}_2) & 0 \\ (\hat{v}_2 \cdot \hat{e}_1) & (\hat{v}_2 \cdot \hat{e}_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{\hat{e}}$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}_{\hat{q}} = \begin{bmatrix} (\hat{q}_1 \cdot \hat{v}_1)(\hat{v}_1 \cdot \hat{e}_1) & (\hat{q}_1 \cdot \hat{v}_1)(\hat{v}_1 \cdot \hat{e}_2) & (\hat{q}_1 \cdot \hat{v}_3) \\ (\hat{v}_2 \cdot \hat{e}_1) & (\hat{v}_2 \cdot \hat{e}_2) & 0 \\ (\hat{q}_3 \cdot \hat{v}_1)(\hat{v}_1 \cdot \hat{e}_1) & (\hat{q}_3 \cdot \hat{v}_1)(\hat{v}_1 \cdot \hat{e}_2) & (\hat{q}_3 \cdot \hat{v}_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{\hat{e}}$$

Problem 2



Define unit vectors $\hat{u}_1$ and $\hat{u}_2$ along the cables.

$$\vec{AB} = 0\,\hat{e}_1 - 20\,\hat{e}_2 - 90\,\hat{e}_3 \quad m$$

$$|\vec{AB}| = 92 \quad m$$

$$\hat{u}_1 = \frac{\vec{AB}}{|\vec{AB}|} = -\,0{,}22\,\hat{e}_2 - 0{,}98\,\hat{e}_3$$

$$\vec{AC} = 14\,\hat{e}_1 + 27\,\hat{e}_2 - 60\,\hat{e}_3 \quad m$$

$$|\vec{AC}| = 67{,}3 \quad m$$

$$\hat{u}_2 = 0{,}21\,\hat{e}_1 + 0{,}40\,\hat{e}_2 - 0{,}89\,\hat{e}_3$$

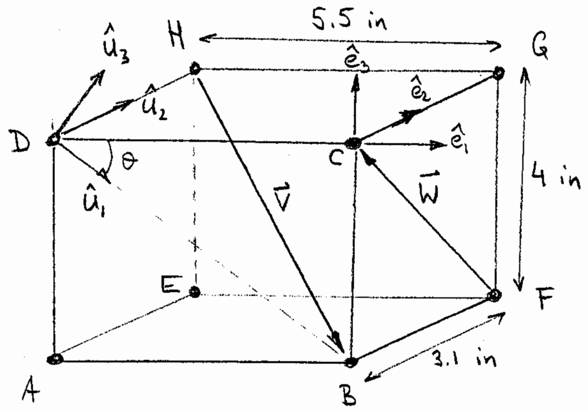
$$\vec{F}_{AB} = 2400\text{ N} \cdot \hat{u}_1 = -\,53 \cdot 10\,\hat{e}_2 - 24 \cdot 10^2\,\hat{e}_3 \quad N$$

$$\vec{F}_{AC} = 2400\text{ N} \cdot \hat{u}_2 = 50 \cdot 10\,\hat{e}_1 + 96 \cdot 10\,\hat{e}_2 - 21 \cdot 10^2\,\hat{e}_3 \quad N$$

Total Force $\vec{F} = \vec{F}_{AB} + \vec{F}_{AC}$

$$\boxed{\vec{F} = 50 \cdot 10\,\hat{e}_1 + 43 \cdot 10\,\hat{e}_2 - 45 \cdot 10^2\,\hat{e}_3 \quad N}$$

Problem 3



$\theta = 36°$

1) $\boxed{\vec{V} = 6.8\,\hat{u}_1 - 3.1\,\hat{u}_2 \ \text{(in)}}$

Introduce intermediate set of unit vectors $\hat{e}$. $\hat{e}_1$ is in the direction DC, $\hat{e}_2 = \hat{u}_2$, and $\hat{e}_3$ completes the right handed set.

Write $\vec{W}$ in terms of $\hat{e}$ - vectors:

$$\vec{W} = 0\,\hat{e}_1 - 3.1\,\hat{e}_2 + 4\,\hat{e}_3 \quad \text{(in)}$$

$$\vec{W} = x_1\,\hat{u}_1 + x_2\,\hat{u}_2 + x_3\,\hat{u}_3$$

$$x_1 = \hat{u}_1 \cdot \vec{W} = 0\,(\hat{u}_1 \cdot \hat{e}_1) - 3.1\,(\hat{u}_1 \cdot \hat{e}_2) + 4\,(\hat{u}_1 \cdot \hat{e}_3)$$

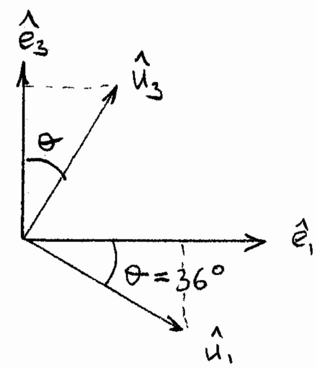
$$x_2 = \hat{u}_2 \cdot \vec{W} = 0\,(\hat{u}_2 \cdot \hat{e}_1) - 3.1\,(\hat{u}_2 \cdot \hat{e}_2) + 4\,(\hat{u}_2 \cdot \hat{e}_3)$$

$$x_3 = \hat{u}_3 \cdot \vec{W} = 0\,(\hat{u}_3 \cdot \hat{e}_1) - 3.1\,(\hat{u}_3 \cdot \hat{e}_2) + 4\,(\hat{u}_3 \cdot \hat{e}_3)$$

We can get the transformation matrix:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} (\hat{u}_1 \cdot \hat{e}_1) & 0 & (\hat{u}_1 \cdot \hat{e}_3) \\ 0 & 1 & 0 \\ (\hat{u}_3 \cdot \hat{e}_1) & 0 & (\hat{u}_3 \cdot \hat{e}_3) \end{bmatrix} \begin{bmatrix} 0 \\ -3.1 \\ 4 \end{bmatrix}$$

$$\hat{u}_1 \cdot \hat{e}_1 = \cos\theta = 0.809$$
$$\hat{u}_1 \cdot \hat{e}_3 = -\sin\theta = -0.588$$
$$\hat{u}_3 \cdot \hat{e}_1 = \sin\theta = 0.588$$
$$\hat{u}_3 \cdot \hat{e}_3 = \cos\theta = 0.809$$



Now we can get $\vec{W}$ in the $\hat{u}$ set.

$$\boxed{\vec{W} = -2.4\,\hat{u}_1 - 3.1\,\hat{u}_2 + 3.2\,\hat{u}_3} \quad (\text{in})$$

2) $|\vec{v}| = \sqrt{6.8^2 + 3.1^2} = 7.5$ in

$|\vec{w}| = \sqrt{2.4^2 + 3.1^2 + 3.2^2} = 5.1$ in

$$\boxed{\begin{aligned} \hat{u}_v &= \frac{\vec{V}}{|\vec{V}|} = 0.91\,\hat{u}_1 - 0.41\,\hat{u}_2 \\ \hat{u}_w &= \frac{\vec{W}}{|\vec{W}|} = -0.47\,\hat{u}_1 - 0.61\,\hat{u}_2 + 0.63\,\hat{u}_3 \end{aligned}}$$

3) $\hat{u}_v \cdot \hat{u}_w = \underbrace{|\hat{u}_v|}_{=1}\,\underbrace{|\hat{u}_w|}_{=1} \cdot \cos\theta$

$\cos\theta = (0.91\,\hat{u}_1 - 0.41\,\hat{u}_2)\cdot(-0.47\,\hat{u}_1 - 0.61\,\hat{u}_2 + 0.63\,\hat{u}_3) = -0.178$

$$\boxed{\theta = \cos^{-1}(0.178) = 100°}$$

4) To find $\hat{u}_\perp$ that is $\perp$ to $\vec{v}$ and $\vec{w}$ $\rightarrow$ take $\vec{u}_v \times \vec{u}_w$

$$\vec{u}_v \times \vec{u}_w = \begin{vmatrix} \hat{u}_1 & \hat{u}_2 & \hat{u}_3 \\ 0.91 & -0.41 & 0 \\ -0.47 & -0.61 & 0.63 \end{vmatrix} = -0.26\,\hat{u}_1 - 0.57\,\hat{u}_2 - 0.75\,\hat{u}_3$$

$$\boxed{\hat{u}_\perp = -0.26\,\hat{u}_1 - 0.57\,\hat{u}_2 - 0.75\,\hat{u}_3}$$

5) $\vec{Q} = 2.1\,\hat{u}_1 + 1.8\,\hat{u}_2 + 3\,u_\perp$

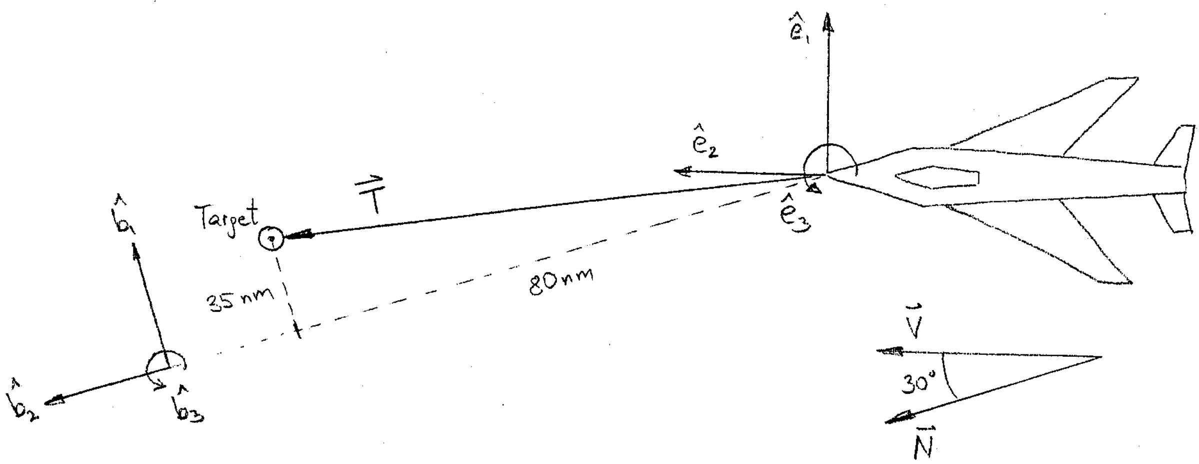
$\vec{Q} = 2.1\,\hat{u}_1 + 1.8\,\hat{u}_2 + 3(-0.26\,\hat{u}_1 - 0.57\,\hat{u}_2 - 0.75\,\hat{u}_3) =$

$= 1.3\,\hat{u}_1 + 0.09\,\hat{u}_2 - 2.3\,\hat{u}_3$

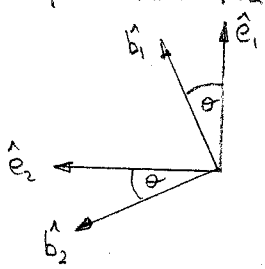
$|\vec{Q}| = \sqrt{1.3^2 + 0.09^2 + 2.3^2}$

$$\boxed{|\vec{Q}| = 2.6 \text{ in}}$$

Problem 4



First define intermediate coordinate system $\hat{b}_1, \hat{b}_2, \hat{b}_3$ and find transformation matrix from $\hat{b} \to \hat{e}$. (rotation about $\hat{b}_3 = \hat{e}_3$)



$\theta = 30°$

$$\hat{b}_1 = \cos\theta \; \hat{e}_1 + \sin\theta \; \hat{e}_2$$
$$\hat{b}_2 = -\sin\theta \; \hat{e}_1 + \cos\theta \; \hat{e}_2$$
$$\hat{b}_3 = \hat{e}_3$$

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$$

Write $\vec{T}$ in terms of $\hat{b}$'s (don't forget that the plane is flying at 45000 ft !)

$$45000 \text{ ft} \cdot \frac{1 \text{ nm}}{6076 \text{ ft}} = 7.406 \text{ nm}$$

$$\vec{T} = 35 \, \hat{b}_1 + 80 \, \hat{b}_2 - 7.4 \, \hat{b}_3 \; (\text{nm})$$

Find the unit vector $\hat{T} = \frac{\vec{T}}{|\vec{T}|}$

$$|\vec{T}| = \sqrt{35^2 + 80^2 + 7.406^2} = 87.63 \text{ nm}$$

$$\hat{T} = 0.40\ \hat{b}_1 + 0.91\ \hat{b}_2 - 0.08\ \hat{b}_3$$

Use transformation matrix to convert to the $\hat{e}$ system.

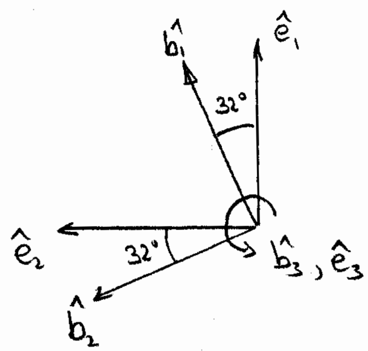
$$\begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 0.40 \\ 0.91 \\ -0.08 \end{bmatrix}$$

$$\boxed{\hat{T} = -0.11\ \hat{e}_1 + 0.99\ \hat{e}_2 - 0.08\ \hat{e}_3}$$

$$\vec{T} = \hat{T} \cdot |\vec{T}| = -9.6\ \hat{e}_1 + 87\ \hat{e}_2 - 7.0\ \hat{e}_3 \quad (\text{nm})$$

To find the error if the plane is actually flying at 32° we simply take $\vec{T}$ and rotate it back to the $\hat{b}$ system.

Note: we need to calculate a $\boxed{\text{new}}$ transformation matrix since we are now rotating 32° instead of 30°



$$\hat{b}_1 = \cos\theta\ \hat{e}_1 + \sin\theta\ \hat{e}_2$$
$$\hat{b}_2 = -\sin\theta\ \hat{e}_1 + \cos\theta\ \hat{e}_2$$
$$\hat{b}_3 = \hat{e}_3$$

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \begin{bmatrix} 0.848 & 0.529 & 0 \\ -0.529 & 0.848 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$$

Our new $\vec{T}$ becomes:

$$\vec{T}_{new} = x_1\ \hat{b}_1 + x_2\ \hat{b}_2 + x_3\ \hat{b}_3$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{\hat{b}} = \begin{bmatrix} 0{,}848 & 0{,}529 & 0 \\ -0{,}529 & 0{,}848 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -9{,}6 \\ 87 \\ -7{,}0 \end{bmatrix}_{\hat{e}} = \begin{bmatrix} 38 \\ 79 \\ 7.0 \end{bmatrix}_{\hat{b}}$$

$$\vec{T}_{new} = 38\ \hat{b}_1 + 79\ \hat{b}_2 - 7.0\ \hat{b}_3 \quad (nm)$$

To find the targeting error, find $|\vec{T} - \vec{T}_{new}|$

$$\vec{T} - \vec{T}_{new} = (35-38)\ \hat{b}_1 + (80-79)\ \hat{b}_2 + (-7{,}4+7.0)\ \hat{b}_3$$

$$\vec{T} - \vec{T}_{new} = -3\ \hat{b}_1 + 1\ \hat{b}_2 - 0{,}4\ \hat{b}_3$$

$$\text{Error} = |\vec{T} - \vec{T}_{new}| = \sqrt{3^2 + 1^2 + 0{,}4^2}$$

$$\boxed{\text{Error} = 3.2\ nm}$$