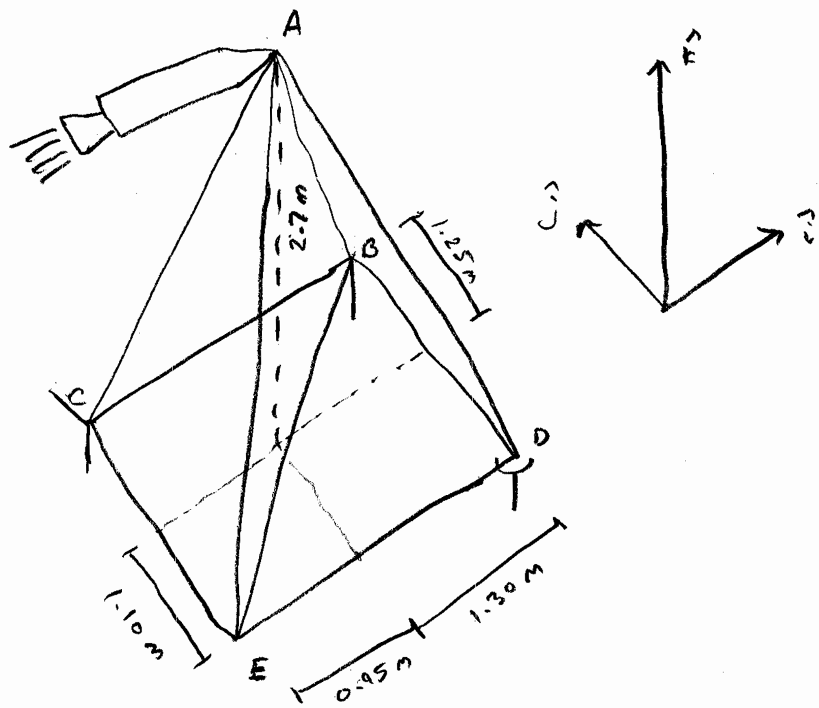
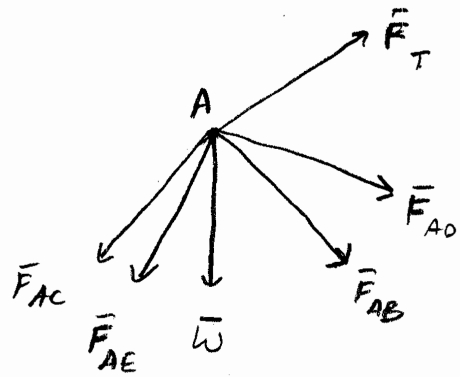
Philip Haberlen | Homework #5 Solution | 10/18/04



First, analyze point A with a free-body diagram.



Assuming Tension

Define unit vectors for each 2-force member. Expressions for $\bar{F}_T$ and $\bar{W}$ can be written directly.

$$\hat{u}_{AC} = \frac{-.95\hat{i} + 1.25\hat{j} - 2.7\hat{k}}{\sqrt{(.95)^2 + (1.25)^2 + (2.7)^2}} = -.3042\hat{i} + .4002\hat{j} - .8645\hat{k}$$

$$\hat{u}_{AE} = \frac{-.95\hat{i} - 1.10\hat{j} - 2.7\hat{k}}{\sqrt{(-.95)^2 + (1.1)^2 + (2.7)^2}} = -.3098\hat{i} - .3587\hat{j} - .8805\hat{k}$$

$$\hat{u}_{AB} = \frac{1.3\hat{i} + 1.25\hat{j} - 2.7\hat{k}}{\sqrt{(1.3)^2 + (1.1)^2 + (2.7)^2}} = .4072\hat{i} + .3916\hat{j} - .8458\hat{k}$$

$$\hat{u}_{AO} = \frac{1.3\hat{i} - 1.1\hat{j} - 2.7\hat{k}}{\sqrt{(1.3)^2 + (1.1)^2 + (2.7)^2}} = .4072\hat{i} - .3446\hat{j} - .8458\hat{k}$$

$$\bar{W} = -mg\hat{k} = -(125\ kg)(9.806\ \tfrac{m}{s^2})\hat{k} = -1225.75\hat{k}\ N$$

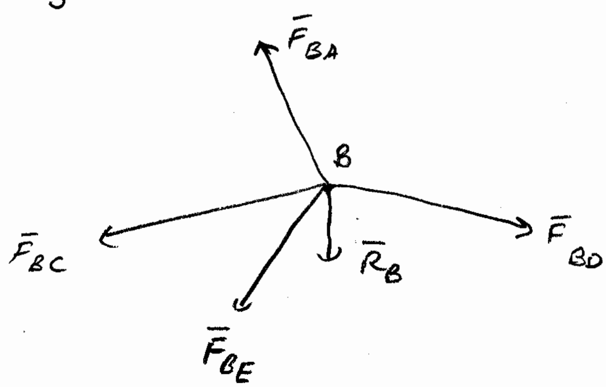
$$\bar{F}_T = 22000\hat{i}\ N$$

Point A must be in static equilibrium, so we have

$$\bar{F}_{AC} + \bar{F}_{AE} + \bar{F}_{AB} + \bar{F}_{AO} + \bar{W} + \bar{F}_T = 0$$

I will collect all of these equations and explicitly substitute in the appropriate expressions at the end.

Point B is analyzed below



$$\hat{u}_{BA} = \frac{-1.3\hat{i} - 1.25\hat{j} + 2.7\hat{k}}{\sqrt{(1.3)^2 + (1.25)^2 + (2.7)^2}} = -.4004\hat{i} - .3850\hat{j} + .8316\hat{k}$$

$$\hat{u}_{BC} = -\hat{i}$$

$$\hat{u}_{BE} = \frac{-2.25\hat{i} - 2.35\hat{j}}{\sqrt{(2.25)^2 + (2.35)^2}} = -.6916\hat{i} - .7223\hat{j}$$

$$\hat{u}_{BO} = -\hat{j}$$

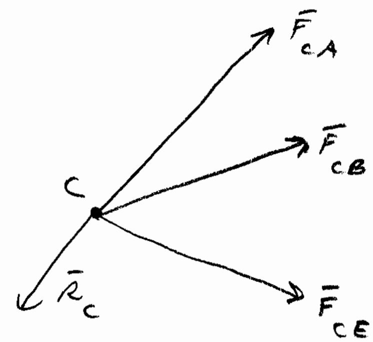
$\bar{R}_B = R_{Bz}\hat{k}$ ← Short link in $\hat{k}$ direction

$\bar{R}_{Bz}$ is the force that the short link exerts on point B.

As usual, point B must be in static equilibrium, so

$$\bar{F}_{BA} + \bar{F}_{BC} + \bar{F}_{BE} + \bar{R}_{Bz} + \bar{F}_{BO} = 0$$

Now analyze point C



$\hat{u}_{CA} = -\hat{u}_{AC} = .3042\hat{i} - .4002\hat{j} + .8645\hat{k}$

$\hat{u}_{CB} = \hat{i}$

$\hat{u}_{CE} = -\hat{j}$

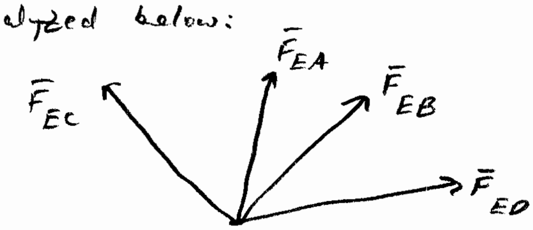
$\bar{R}_c = R_{cy}\hat{j} + R_{cz}\hat{k}$

Note that 2 short links are drawn to point C, so there are 2 orthogonal components to $\bar{R}_c$.

Point C is in static equilibrium, so therefore

$$\bar{F}_{CA} + \bar{F}_{CB} + \bar{F}_{CE} + \bar{R}_c = 0$$

Point E is analyzed below:



$\hat{u}_{EC} = -\hat{j}$

$\hat{u}_{EA} = -\hat{u}_{AE} = .3098\hat{i} + .3587\hat{j} + .8805\hat{k}$

$\hat{u}_{EB} = -\hat{u}_{BE} = .6916\hat{i} + .7223\hat{j}$

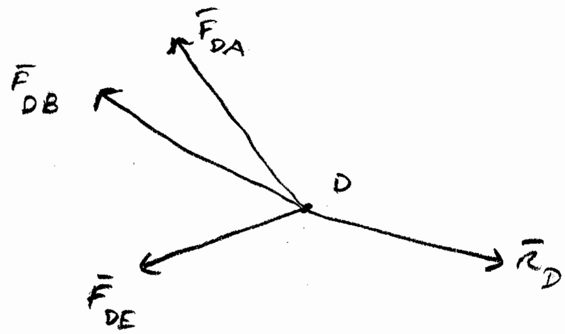
$\hat{u}_{ED} = \hat{i}$

Point E is in static equilibrium

$$\bar{F}_{EC} + \bar{F}_{EA} + \bar{F}_{EB} + \bar{F}_{ED} = 0$$

Note that there are no reaction forces at point E.

Point D is analyzed below:



$$\hat{u}_{DA} = -u_{AD} = -.4072\,\hat{i} + .3446\,\hat{j} + .8458\,\hat{k}$$

$$\hat{u}_{DB} = \hat{j}$$

$$\hat{u}_{DE} = -\hat{i}$$

$$\bar{R}_D = R_{Dx}\,\hat{i} + R_{Dy}\,\hat{j} + R_{Dz}\,\hat{k}$$

A ball and socket joint can exert a reaction force with components in all 3 orthogonal directions.

Static equilibrium at point D

$$\bar{F}_{DA} + \bar{F}_{DB} + \bar{F}_{DE} + \bar{R}_D = 0$$

In summary, we have a total of 5 vector equations from static equilibrium:

$$\bar{F}_{AC} + \bar{F}_{AE} + \bar{F}_{AB} + \bar{F}_{AD} + \bar{w} + \bar{F}_T = 0$$

$$\bar{F}_{BA} + \bar{F}_{BC} + \bar{F}_{BE} + \bar{R}_{Bz} + \bar{F}_{BD} = 0$$

$$\bar{F}_{CA} + \bar{F}_{CB} + \bar{F}_{CE} + \bar{R}_C = 0$$

$$\bar{F}_{EC} + \bar{F}_{EA} + \bar{F}_{EB} + \bar{F}_{ED} = 0$$

$$\bar{F}_{DA} + \bar{F}_{DB} + \bar{F}_{DE} + \bar{R}_D = 0$$

I can rewrite these five equations using the unit vectors:

$$F_{AC}\hat{u}_{AC} + F_{AE}\hat{u}_{AE} + F_{AB}\hat{u}_{AB} + F_{AD}\hat{u}_{AD} - 1225.75\hat{k} + 22000\hat{i} = 0$$

$$F_{BA}\hat{u}_{BA} + F_{BC}\hat{u}_{BC} + F_{BE}u_{BE} + R_{BZ}\hat{k} + F_{BD}\hat{u}_{BD} = 0$$

$$F_{CA}\hat{u}_{CA} + F_{CB}u_{CB} + F_{CE}\hat{u}_{CE} + R_{Cy}\hat{j} + R_{Cz}\hat{k} = 0$$

$$F_{EC}\hat{u}_{EC} + F_{EA}\hat{u}_{EA} + F_{EB}\hat{u}_{EB} + F_{ED}\hat{u}_{ED} = 0$$

$$F_{DA}\hat{u}_{DA} + F_{DB}\hat{u}_{DB} + F_{DE}\hat{u}_{DE} + R_{Dx}\hat{i} + R_{Dy}\hat{j} + R_{Dz}\hat{k} = 0$$

We also know that the forces in each 2-force member, a scalar quantity, are related in the following way:

$F_{AC} = F_{CA}$

$F_{AE} = F_{EA}$

$F_{AD} = F_{DA}$

$F_{AB} = F_{BA}$

$F_{CB} = F_{BC}$

$F_{CE} = F_{EC}$

$F_{DE} = F_{ED}$

$F_{DB} = F_{BD}$

$F_{BE} = F_{EB}$

After substituting these relationships in as well as the expressions for each unit vector, we have

$$F_{AC}(-.3042\hat{i} + .4002\hat{j} - .8645\hat{k}) + F_{AE}(-.3098\hat{i} - .3587\hat{j} - .8805\hat{k})$$
$$+ F_{AB}(.4072\hat{i} + .3916\hat{j} - .8458\hat{k}) + F_{AD}(.4072\hat{i} - .3446\hat{j} - .8458\hat{k})$$
$$= 1225.75\hat{k} - 22000\hat{i}$$

$$F_{AB}(-.4004\hat{i} - .3850\hat{j} + .8316\hat{k}) + F_{BC}(-\hat{i}) + F_{BE}(-.6916\hat{i} - .7223\hat{j})$$
$$+ R_{BZ}\hat{k} + F_{BD}(-\hat{j}) = 0$$

$$F_{AC}(.3042\hat{i} - .4002\hat{j} + .8645\hat{k}) + F_{BC}(\hat{i}) + F_{CE}(-\hat{j}) + R_{cy}(\hat{j})$$
$$+ R_{cz}(\hat{k}) = 0$$

$$F_{CE}(\hat{j}) + F_{AE}(.3098\hat{i} + .3587\hat{j} + .8805\hat{k}) + F_{BE}(.6916\hat{i} + .7273\hat{j})$$
$$+ F_{ED}(\hat{i}) = 0$$

$$F_{AD}(-.4072\hat{i} + .3446\hat{j} + .8458\hat{k}) + F_{BD}(\hat{j}) + F_{ED}(-\hat{i})$$
$$+ R_{Dx}\hat{i} + R_{Dy}\hat{j} + R_{Dz}\hat{k} = 0$$

Each vector equation contains 3 scalar equations (one for the $\hat{i}$, $\hat{j}$, and $\hat{k}$ directions) which are linearly independent. Since there are 5 vector equations, this yields a total of 15 scalar equations, which are written below:

$$-.3042F_{AC} - .3098F_{AE} + .4072F_{AB} + .4072F_{AD} = -22000$$
$$.4002F_{AC} - .3587F_{AE} + .3916F_{AB} - .3446F_{AD} = 0$$
$$-.8645F_{AC} - .8805F_{AE} - .8458F_{AB} - .8458F_{AD} = 1225.75$$
$$-.4004F_{AB} - F_{BC} - .6916F_{BE} = 0$$
$$-.3850F_{AB} - .7223F_{BE} - F_{BD} = 0$$
$$.8316F_{AB} + R_{Bz} = 0$$
$$.3042F_{AC} + F_{BC} = 0$$
$$-.4002F_{AC} - F_{CE} + R_{cy} = 0$$
$$.8645F_{AC} + R_{cz} = 0$$
$$.3098F_{AE} + .6916F_{BE} + F_{ED} = 0$$
$$.3587F_{AE} + .7223F_{BE} + F_{CE} = 0$$
$$.8805F_{AE} = 0$$
$$-.4072F_{AD} - F_{ED} + R_{Dx} = 0$$
$$.3446F_{AD} + F_{BD} + R_{Dy} = 0$$
$$.8458F_{AD} + R_{Dz} = 0$$

All of the variables in the above equations are listed below

F_{AC} F_{AE} F_{AB} F_{AD} F_{BC} F_{BE} R_{Bz} F_{BD} F_{CE} R_{Cy} R_{Cz} F_{ED}
R_{Dx} R_{Dy} R_{Dz}

There are 15 unknowns and 15 equations, so this system is solvable. It only remains to arrange the equations in $Ax=B$ form. These equations have been put into this form in an Excel spreadsheet for clarity. However, the equations were actually solved in MATLAB. The spreadsheet, code, and results appear next.

Linear equation matrix setup

F_{AC}	F_{AE}	F_{AB}	F_{AD}	F_{BC}	F_{BE}	R_{BZ}	F_{BD}	F_{CE}	R_{CY}	R_{CZ}	F_{ED}	R_{DX}	R_{DY}	R_{DZ}		B
-0.3042	-0.3098	0.4072	0.4072	0	0	0	0	0	0	0	0	0	0	0		-22000
0.4002	-0.3587	0.3916	-0.3446	0	0	0	0	0	0	0	0	0	0	0		0
-0.8645	-0.8805	-0.8458	-0.8458	0	0	0	0	0	0	0	0	0	0	0		1225.75
0	0	-0.4004	0	-1	-0.6916	0	0	0	0	0	0	0	0	0		0
0	0	-0.385	0	0	-0.7223	0	-1	0	0	0	0	0	0	0		0
0	0	0.8316	0	0	0	1	0	0	0	0	0	0	0	0		0
0.3042	0	0	0	1	0	0	0	0	0	0	0	0	0	0		0
-0.4002	0	0	0	0	0	0	0	-1	1	0	0	0	0	0		0
0.8645	0	0	0	0	0	0	0	0	0	1	0	0	0	0		0
0	0.3098	0	0	0	0.6916	0	0	0	0	0	1	0	0	0		0
0	0.3587	0	0	0	0.7223	0	0	1	0	0	0	0	0	0		0
0	0.8805	0	0	0	0	0	0	0	0	0	0	0	0	0		0
0	0	0	-0.4072	0	0	0	0	0	0	0	-1	1	0	0		0
0	0	0	0.3446	0	0	0	1	0	0	0	0	0	1	0		0
0	0	0	0.8458	0	0	0	0	0	0	0	0	0	0	1		0

```
clear
format long
A = [-.3042, -.3098, .4072, .4072, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
    .4002, -.3587, .3916, -.3416, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
    -.8645, -.8805, -.8458, -.8458, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
    0, 0, -.4004, 0, -1, -.6919, 0, 0, 0, 0, 0, 0, 0, 0, 0;
    0, 0, -.3850, 0, 0, -.7223, 0, -1, 0, 0, 0, 0, 0, 0, 0;
    0, 0, .8316, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0;
    .3042, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
    -.4002, 0, 0, 0, 0, 0, 0, 0, -1, 1, 0, 0, 0, 0, 0;
    .8645, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0;
    0, .3098, 0, 0, 0, .6916, 0, 0, 0, 0, 0, 1, 0, 0, 0;
    0, .3587, 0, 0, 0, .7223, 0, 0, 1, 0, 0, 0, 0, 0, 0;
    0, .8805, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
    0, 0, 0, -.4072, 0, 0, 0, 0, 0, 0, 0, -1, 1, 0, 0;
    0, 0, 0, .3446, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0;
    0, 0, 0, .8458, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1];
B = [-22000; 0; 1225.75; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];
X = inv(A)*B
```

X =

F_{AC} = 29719.3115121271
F_{AE} = 0
F_{AB} = -31049.2285393514
F_{AD} = -00776.3742062550
F_{BC} = -09040.6145619891
F_{BE} = 31034.4351338999
R_{BZ} = 25820.5384533246
F_{BD} = -10462.2195095656
F_{CE} = -22416.1724972159
R_{Cy} = -10522.5040300626
R_{Cz} = -25692.3448022339
F_{ED} = -21463.4153386052
R_{Dx} = ~~[illegible]~~ -21779.95549153922
R_{Dy} = 10729.7580610411
R_{Dz} = 00656.6573036505