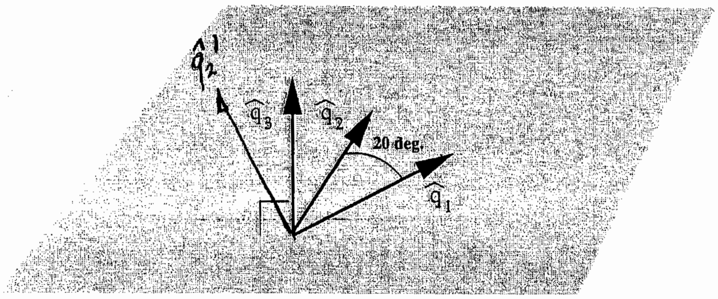
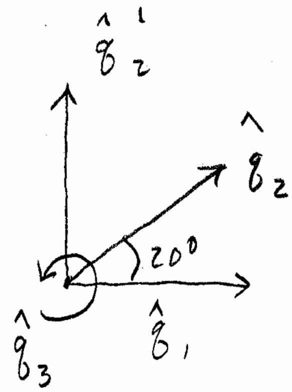
1. (10 percent) The set of unit vectors $\hat{q}_1, \hat{q}_2, \hat{q}_3$ are defined in the drawing below, in which $\hat{q}_1$ and $\hat{q}_2$ are in the plane, and $\hat{q}_3$ is orthogonal to the plane.





The velocity vector, $\vec{V}$, is given below:

Observe: System is <u>Not</u> <u>Orthogonal</u> !!!

$$\vec{V} = 3.15\hat{q}_1 - 2.11\hat{q}_2 + 0.95\hat{q}_3 \text{ m/sec}$$

- What is the magnitude of $\vec{V}$ (what is the speed ?)
- What is the cross product $\hat{q}_1 \times \hat{q}_2$?

•) introduce unit vector $\hat{q}_2'$ $\perp$ $\hat{q}_1$ and $\hat{q}_3$ to complete an orthogonal set. $\hat{q}_2'$ is in the plane.

$$\hat{q}_2 = \cos 20\, \hat{q}_1 + \sin 20\, \hat{q}_2'$$

$$\therefore \vec{V} = 3.15\, \hat{q}_1 - 2.11\left[\cos 20\, \hat{q}_1 + \sin 20\, \hat{q}_2'\right] + 0.95\, \hat{q}_3 \quad \frac{m}{sec}$$

$$\vec{V} = 1.16\, \hat{q}_1 - 0.721\, \hat{q}_2' + 0.95\, \hat{q}_3 \quad \frac{m}{sec}$$

Now find the magnitude of $\vec{V} = |\vec{V}|$ – since $\hat{q}_1$, $\hat{q}_2'$, and $\hat{q}_3$ are orthogonal

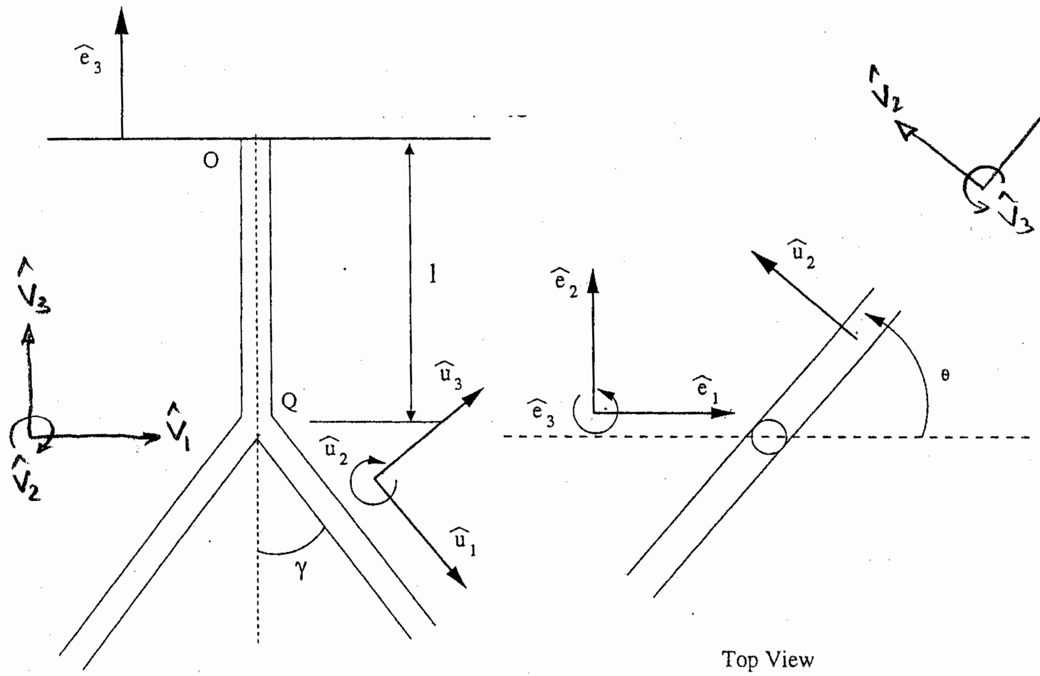
$$V = \sqrt{1.16^2 + 0.721^2 + 0.95^2} = 1.66 \ \frac{m}{s}$$

$$\boxed{V = 1.66 \ \frac{m}{s}}$$

•) $\hat{q}_1 \times \hat{q}_2 = |\hat{q}_1||\hat{q}_2| \sin 20°\, \hat{q}_3 = 0.342\, \hat{q}_3$

↳ Cross product produces a <u>Vector</u> !

2. (30 percent) In the Y-shaped mechanism drawn in the two figures below, the $\hat{u}_1, \hat{u}_2, \hat{u}_3$ system of unit vectors are attached to the rotating mechanism. Another system of unit vectors, $\hat{e}_1, \hat{e}_2, \hat{e}_3$ are attached to the ceiling. The Y-mechanism can rotate to the angle θ.

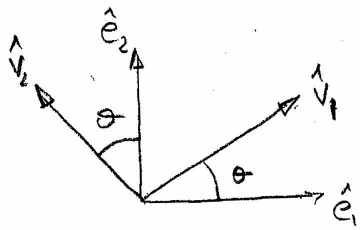


Top View

Find the transformation matrix from components in the $\hat{e}_1, \hat{e}_2, \hat{e}_3$ system to components in the $\hat{u}_1, \hat{u}_2, \hat{u}_3$ system.

Introduce intermediate coordinate system $\hat{V}$ (orthogonal) attached to the Y-mechanism

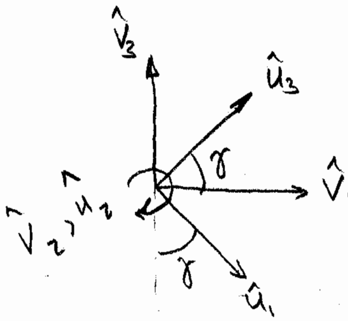
First rotation about $\hat{e}_3 = \hat{V}_3$



$$\begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$$

/ (1)

Second rotation about $\hat{v}_2 = \hat{u}_2$ by angle γ



$$\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \end{bmatrix} = \begin{bmatrix} \sin\gamma & 0 & -\cos\gamma \\ 0 & 1 & 0 \\ \cos\gamma & 0 & \sin\gamma \end{bmatrix} \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \hat{v}_3 \end{bmatrix} \quad \checkmark \quad (2)$$

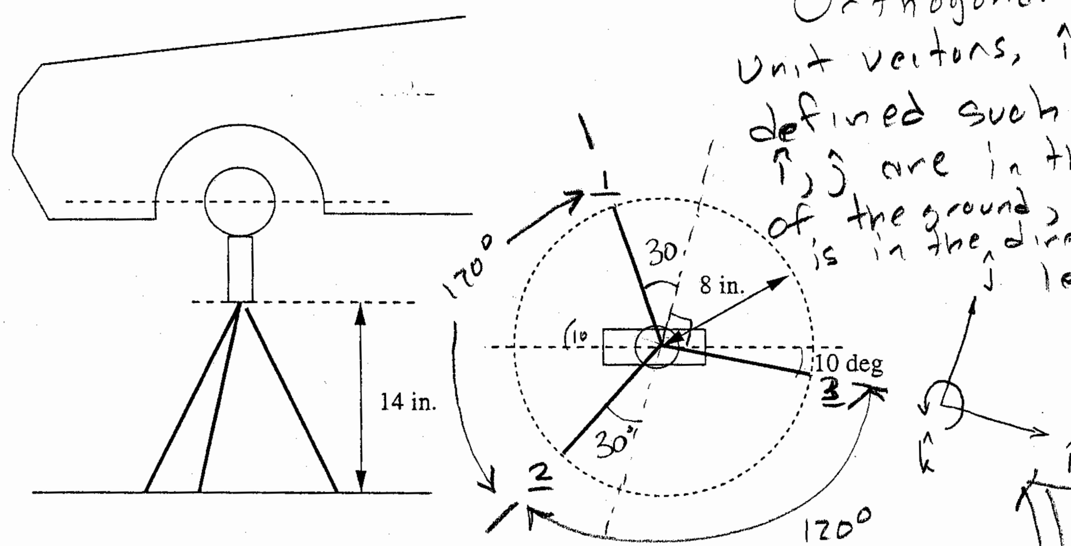
Now multiply both rotations together to obtain transformation matrix from $\hat{e}$'s to $\hat{u}$'s, by substituting (1) into (2) and eliminating

$$\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \end{bmatrix} = \begin{bmatrix} \sin\gamma & 0 & -\cos\gamma \\ 0 & 1 & 0 \\ \cos\gamma & 0 & \sin\gamma \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$$

$$\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta\sin\gamma & \sin\theta\sin\gamma & -\cos\gamma \\ -\sin\theta & \cos\theta & 0 \\ \cos\theta\cos\gamma & \sin\theta\cos\gamma & \sin\gamma \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$$

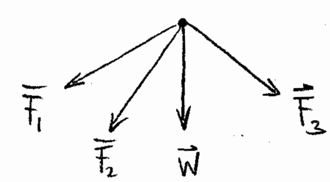
3. (30 percent) One of the four hydraulic jacks used to support a car that is being repaired is shown below. This jack, by itself, supports a weight of 950 lbs (one fourth of the total weight of the car). The legs of the jack are evenly distributed around a circle which has a radius 8 in on the ground. What are the forces in each of the three legs of the jack ?

Orthogonal set of unit vectors, $\hat{\imath}, \hat{\jmath}, \hat{k}$ defined such that $\hat{\imath}, \hat{\jmath}$ are in the plane of the ground, and $\hat{\imath}$ is in the direction of leg $\underline{3}$.



(Hint: Think about how to define the unit vectors: some ways will make the problem easier !)

FBD



$W = -950\,\hat{k}$ lb

assume tension for all forces.

$$\Sigma \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{W} = 0$$

$$\hat{u}_1 = \frac{-8\sin 30\,\hat{\imath} + 8\cos 30\,\hat{\jmath} - 14\hat{k}}{\sqrt{4^2 + 6{,}928^2 + 14^2}} = -0{,}248\,\hat{\imath} + 0{,}429\,\hat{\jmath} - 0{,}868\,\hat{k} \checkmark$$

$$\hat{u}_2 = \frac{-8\sin 30\hat{\imath} - 8\cos 30\hat{\jmath} - 14\hat{k}}{\sqrt{4^2 + 6{,}928^2 + 14^2}} = -0{,}248\,\hat{\imath} - 0{,}429\,\hat{\jmath} - 0{,}868\,\hat{k} \checkmark$$

$$\hat{u}_3 = \frac{8\hat{\imath} - 14\hat{k}}{\sqrt{8^2 + 14^2}} = 0{,}496\,\hat{\imath} - 0{,}868\,\hat{k}$$

$\Sigma \vec{F} = 0$

$\hat{i}$: $-0,248\,F_1 - 0,248\,F_2 + 0,496\,F_3 = 0$

$\hat{j}$: $0,429\,F_1 - 0,429\,F_2 = 0$

$\hat{k}$: $-0,868\,F_1 - 0,868\,F_2 - 0,868\,F_3 - 950 = 0$

In matrix form:

$$\begin{bmatrix} -0,248 & -0,248 & 0,496 \\ 0,429 & -0,429 & 0 \\ -0,868 & -0,868 & -0,868 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 950 \end{bmatrix}$$

$$\boxed{\begin{aligned} F_1 &= -365 \text{ lb (c)} \\ F_2 &= -365 \text{ lb (c)} \\ F_3 &= -365 \text{ lb (c)} \end{aligned}}$$

We assumed tension, so the negative answer indicates compression.

This system can be solved easily without a matrix calculator, if required.

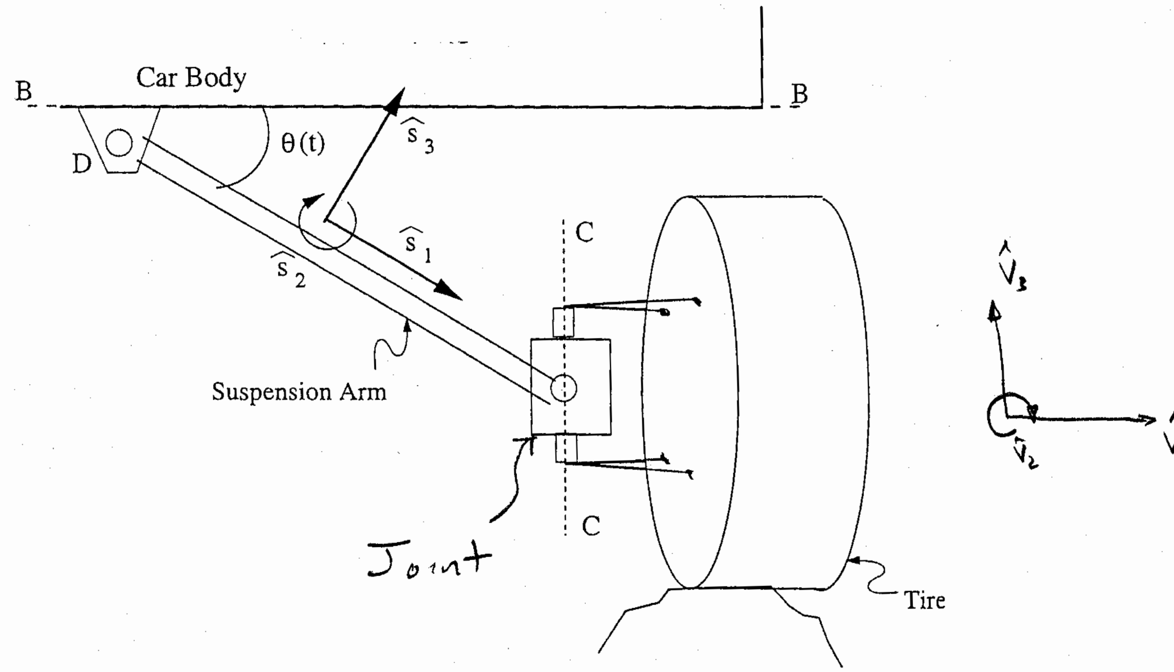
$\hat{j}$: $F_1 = F_2$ substitute into $\hat{i}$ and $\hat{k}$ equations.

$\hat{i}$: $-0.496\,F_1 + 0.496\,F_3 = 0 \Rightarrow F_1 = F_3$

or: $F_1 = F_2 = F_3$, which when substituted into $\hat{k}$ gives:

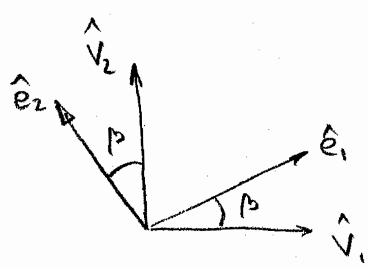
$-2.604\,F_1 = 950 \quad F_1 = -365 \text{ lb} \Rightarrow$ $F_1 = F_2 = F_3 = 365 \text{ lb (c)}$

4. (30 percent) An automobile suspension is drawn in the two figures below. The suspension arm can rotate, relative to the car body, through the angle θ, and the wheel bearing assembly can rotate through an angle β. The system of orthogonal unit vectors $\hat{s}_1, \hat{s}_2, \hat{s}_3$ are attached to the suspension arm, and the system $\hat{e}_1, \hat{e}_2, \hat{e}_3$ are attached to the wheel bearing assembly. Derive the transformation matrix from components in $\hat{e}_1, \hat{e}_2, \hat{e}_3$ to components in $\hat{s}_1, \hat{s}_2, \hat{s}_3$. (Look at the problem carefully!)

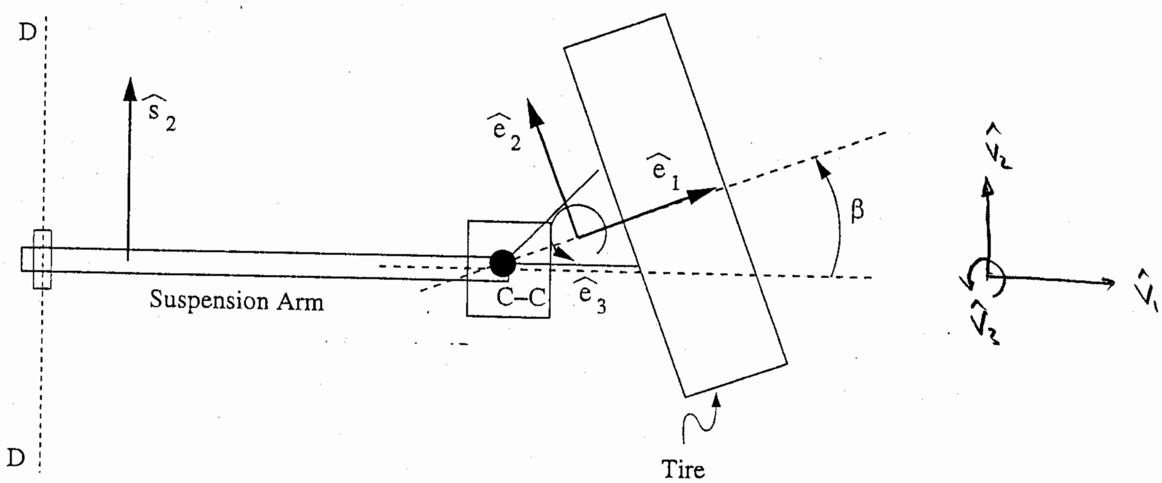


Repeat: **Look at the problem carefully!**

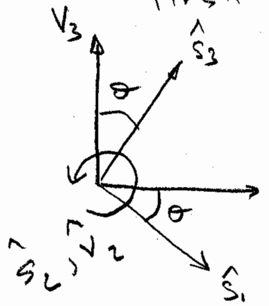
Introduce intermediate orthogonal vector set $\hat{v}$ attached to the joint, as indicated.
First rotation about $\hat{e}_3 = \hat{v}_3$ by angle β (see drawing on next page)



$$\begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \hat{v}_3 \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} \quad (1)$$



Second rotation about $\hat{v}_2 = \hat{s}_2$ by angle θ (see drawing on first page)



$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \hat{v}_3 \end{bmatrix} \quad (2)$$

Substitute (1) into (2), to eliminate components in $\hat{v}$'s

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$$

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\beta & -\cos\theta\sin\beta & -\sin\theta \\ \sin\beta & \cos\beta & 0 \\ \sin\theta\cos\beta & -\sin\theta\sin\beta & \cos\theta \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$$