Stability of Rarefied Gas Flow in a Channel for Ray-Diffuse Scattering Function

I.A. Khalidov, E.A. Shalaginov

St.-Petersburg State University, Department of Mathematics and Mechanics,
198504, Universitetskiy pr., 28, Petrodvorets, St.-Petersburg, Russia

Abstract. Stability of free-molecular flows in long flat or cylindrical channels is investigated numerically applying obtained in [1] analytical limit solutions, which are valid when the number of gas atom collisions with the walls turns to infinity. Scattering function on the surface of channel walls is assumed ray-diffuse, i.e. ray model of reflection (unique determined velocity of reflected particles as a function of the velocity of incident gas atoms) is combined with diffuse scattering. Calculated by Monte-Carlo simulation distributions of the number of gas atoms and of the angles of inclination of gas atom trajectories along the channel demonstrate that the flow becomes instable if the values of gas-surface interaction parameters are close to the points where analytical limit solution has singularity. Instability is comprehended as large difference between calculated characteristics of two flows with very close to each other (it means negligible small difference) corresponding parameters of the gas and of the surface.

INTRODUCTION

Assuming large Knudsen number (free-molecular flow, Kn→∞), rarefied gas flow in a channel is entirely determined by the scattering function $V(\vec{u}_0,\vec{u})$ of gas atoms on the surface, where $\vec{u}_0$ and $\vec{u}$ are the velocities of incident upon the channel wall and reflected gas atoms. Previous analytical and numerical investigations showed that, if flat or cylindrical channel is long enough and scattering function V is described by means of ray model, then there exist transition parameter values of V that lead to the instability of the flow. Consequently, nonlinear dynamic system describing successive gas atoms reflections from channel walls gets many different attractors and the cascade of bifurcations appears [1]. Correspondingly close to these values of the parameters of scattering function V the aerodynamic characteristics of the flow change abruptly.

In the present paper more general ray-diffuse model is considered, i.e. the scattering function V takes a form [2]

$$V(\vec{u}_0,\vec{u}) = (1-\sigma)\delta(\vec{u}-\vec{u}_*(\vec{u}_0)) + \sigma\frac{2h_d^2}{\pi}u_n e^{-h_d u^2}, \tag{1}$$

where σ and h_d are constant parameters, σ changing from 0 to 1. According to this model one part of the molecules (σ) follows diffuse scattering, and another part $(1-\sigma)$ reflects in compliance with ray model. Second term on the right side is diffuse addition to ray model of scattering, it describes the effects of adsorption and contamination of the surface of channel walls. Generally speaking it causes randomization and changes fundamentally the limit behavior of the dynamic system.

The purpose of present investigation is to study the influence of diffuse addition to scattering function V on the instability of the flow.

STATEMENT OF THE PROBLEM

If the scattering function V an the geometrical shape of the channel are given, nonlinear relationship between the angles θ_m, θ'_m, θ_{m+1} etc. can be determined describing the trajectory of gas atoms in different points of the collisions with channel walls (fig.1).

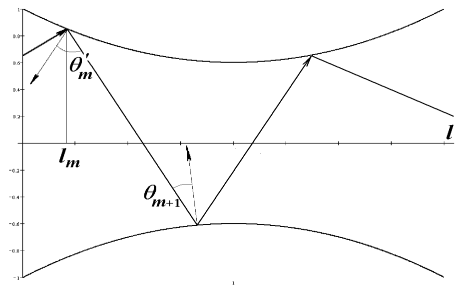

FIGURE 1. Iterative scheme in a channel.

For free-molecular flow in flat or cylindrical channel we apply the equation proposed in [3]

$$x_{m+1} = \frac{x_m}{a\sqrt{1+x_m^2} - b}, \tag{2}$$

where $x_m = \tan\theta_m$, $x_{m+1} = \tan\theta_{m+1}$; θ_m and θ_{m+1} are the angles of incidence in m-th and $(m+1)$-th collisions of gas atom with the surface, and the variables a and b are constant parameters connected with the momentum exchange coefficients p and τ by the equalities

$$p(\theta) = p_1\cos\theta + p_2\cos^2\theta, \quad \tau(\theta) = \tau_0\sin\theta\cos\theta, \quad a = \frac{p_1}{2-\tau_0}, \ b = \frac{2-p_2}{2-\tau_0}, \tag{3}$$

considered in our papers [1], [4].

The flow described by the iterative equation (2) becomes unstable for certain values of the parameters a and b [1]. It means that the macroscopic parameters of the flow can vary greatly while the modification of gas-surface interaction parameters a and b is negligible. However the regions in which the flow becomes unstable are very narrow, therefore it is difficult to find them numerically or experimentally. To find the values a and b where the instability takes place we apply known analytical results [1] concerning analytical limit solution of nonlinear iterative equation (2).

NUMERICAL RESULTS

In the Monte-Carlo simulations in the initial section of the channel is set uniform flow consisting of N gas atoms having identical velocities with the angle of inclination θ_0. The length l of the channel relative to its width is assumed quite large, in calculations it varies from 20 to 100 (and N changes from 1000 to 5000). Other parameters (σ, a, b and θ_0) vary in accordance with known limit analytical solution of equation (2).

To illustrate obtained results we present the comparative graphs of velocity distribution in different directions and of the number of gas atoms in sections of a channel for ray-diffuse model (1) with identical value $\sigma = 0.1$. To demonstrate various points of instability different parameters are changed at many values. For instance, variable a changes near 1.7 (figs. 2 and 3), 1.2 (figs. 4 and 5) and 0.81 (fig. 6).

The results of numerical calculations show that for relative small values of σ the effect of significant change of flow parameters by small modification of gas-surface interaction coefficients a and b stays qualitative the same as for ray scattering ($\sigma = 0$) and by the same parameter values of a and b.

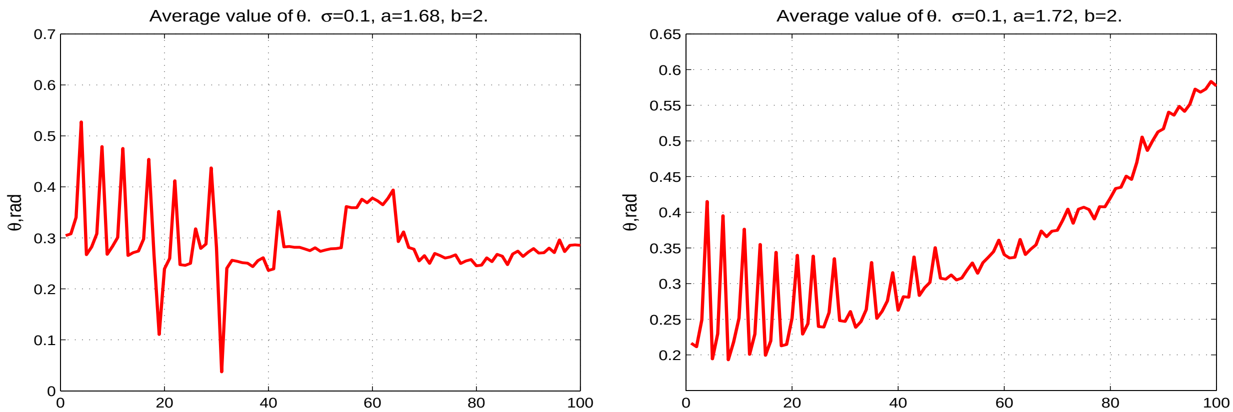

FIGURE 2. The change of average scattering angle θ'_m along the channel by the modification of the parameter from $a = 1.68$ (left graph) to $a = 1.72$ (right graph) by constant $b = 2$, ray-diffuse scattering function, $\sigma = 0.1$.

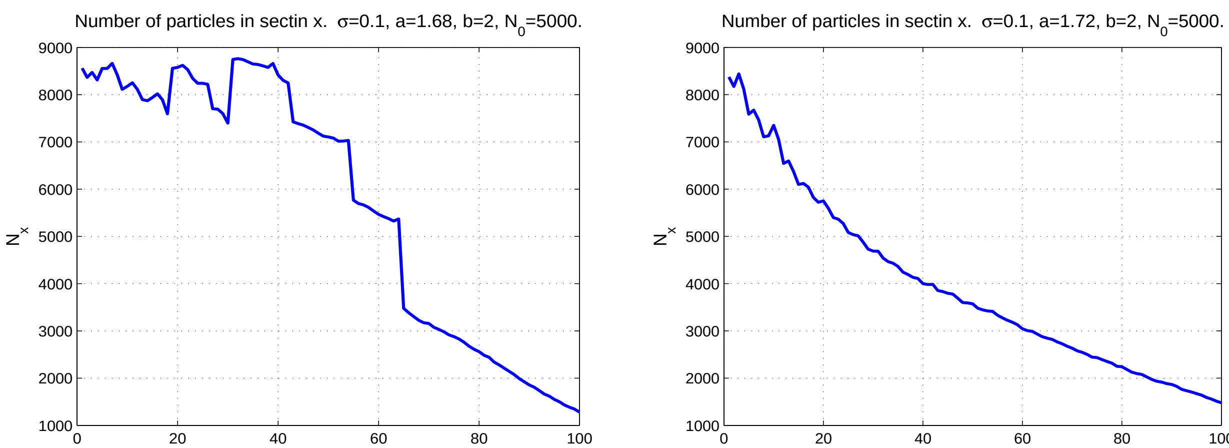

FIGURE 3. The change of the number N of gas atoms along the channel by the modification of the parameter from $a = 1.68$ (left graph) to $a = 1.72$ (right graph) by constant $b = 2$, ray-diffuse scattering function, $\sigma = 0.1$.

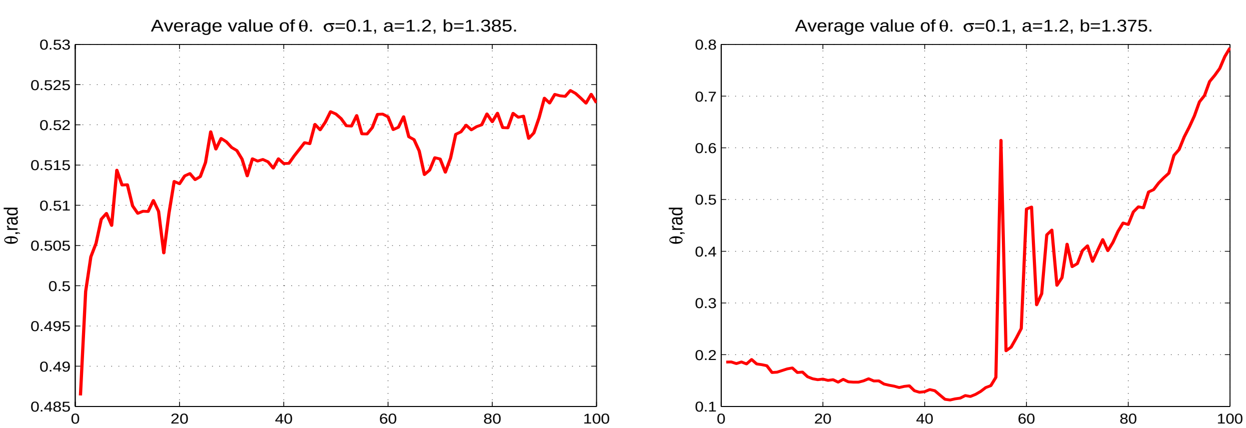

FIGURE 4. The change of average scattering angle θ'_m along the channel by the modification of the parameter from $b = 1.385$ (left graph) to $b = 1.375$ (right graph) by constant $a = 1.2$, ray-diffuse scattering function, $\sigma = 0.1$.

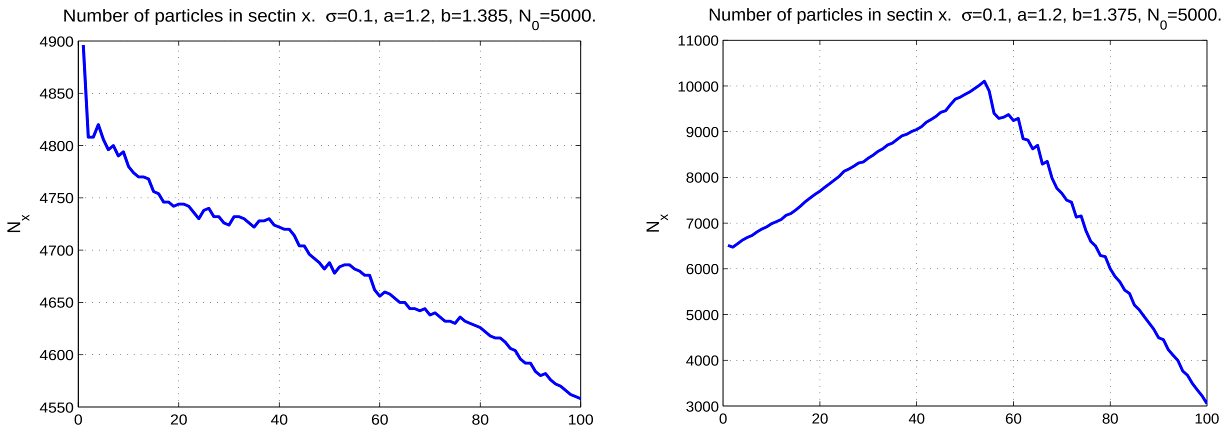

FIGURE 5. The change of the number N of gas atoms along the channel by the modification of the parameter from $b = 1.385$ (left graph) to $b = 1.375$ (right graph) by constant $a = 1.2$, ray-diffuse scattering function, $\sigma = 0.1$.

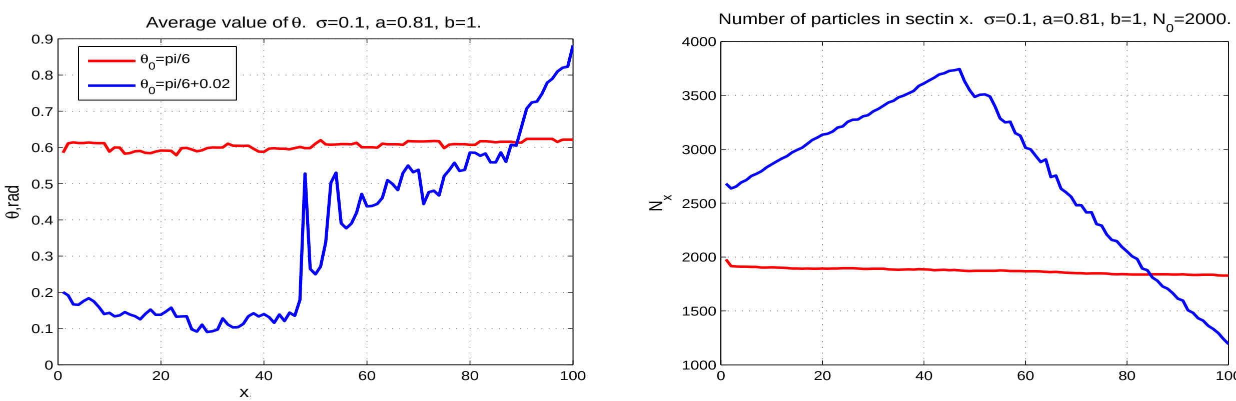

FIGURE 6. The change of average scattering angle θ'_m and of the number N of gas atoms along the channel by the modification of the initial angle θ_0 from $\theta_0 = \pi/6$ (left graph) to $\theta_0 = \pi/6 + 0.02$ (right graph) by constant $a = 0.81$, to $b = 1$, ray-diffuse scattering function, $\sigma = 0.1$.

REFERENCES

1. Aksenova, O.A., and Khalidov, I.A., in *Proc. of the 24th Int. Symp. on Rarefied Gas Dynamics*, Ed. Mario Capitelli, Melville, New York, 2005, pp. 232-235.
2. Barantsev, R.G., Interaction of Rarefied Gases with Streamline Surfaces, Mir Publishers, Moscow, 1975 (in Russian).
3. Aksenova, O.A., and Khalidov, I.A., Surface Roughness in Rarefied Gas Aerodynamics, St.-Petersburg University Publishers, St.-Petersburg, 2004 (in Russian).
4. Khalidov, I. A., and Miroshin, R.N., Local Methods in Continuum Mechanics, St.-Petersburg University Publishers, St.-Petersburg, 2002 (in Russian).