Drag and heat transfer coefficients in free molecular flows

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Abstract. Analytical calculations of drag and heat transfer coefficients are carried out for free molecular flows. Such calculations are useful in aerospace research for the thermal protection of the re-entry vehicles or for the trajectory control of Earth satellites. The gas/surface interaction is described here by means of a new scattering kernel model, modifying and generalizing the well known Maxwell scattering kernel. The structure of the new kernel allows us to handle easily and to develop, in a rigorous theoretical frame, concepts previously introduced in a phenomenological way by Schaaf and Chambré for aerodynamic coefficient calculations, namely the fluxes of normal and tangential momentum components and their differentiated accommodation processes. The theoretical drag coefficient, heat transfer coefficient, and recovered temperature predicted by the new model are compared with available experimental data.

1. INTRODUCTION

Predictions of aerodynamic forces and heat transfer between gases and solid body surfaces in the transitional and the free molecular gas flow regimes are important in aerospace research. Despite the great interest devoted to this topic during the past [1, 2], it remains an open question where the main difficulty is the theoretical description of the gas/surface interaction [3, 4].

Using a new model of gas/surface interaction derived in the framework of scattering kernel formalism [5], we present here calculations of the drag coefficient, the heat coefficient, and the recovered temperature on a flat plate surface under free molecular flow conditions. The new model of gas/surface interaction involves a set of accommodation coefficients characterizing the various molecular freedom degrees.

The theoretical aero-thermodynamic parameters have been compared, first, with experimental results obtained by Bellomo et al. [6] for helium gas and a gold surface. Then, our results have been compared to more recent experimental results in nearly free molecular flows for helium gas on a clean lithium fluoride (Lif001) crystal surface [7].

2. THE PROBLEM UNDER INVESTIGATION

We consider a flow of rarefied gas around a flat plate surface. The normal unit vector to the surface, $\overrightarrow{N}$, is oriented from the surface into the flow. The incoming flow speed is denoted by $\overrightarrow{U_e}$. The angle of incidence θ_e is the geometrical angle between the flow direction and the normal to the surface, $\theta_e = \widehat{(\overrightarrow{N}, U_e)}$, and $\overrightarrow{N} \cdot \overrightarrow{U_e} = -U_e \cos\theta_e$. If $\overrightarrow{d_S}$ is a vector element of surface area on the plate, oriented in the direction of $-\overrightarrow{N}$, the projected area in the flow direction called the cross section area is given here by, $\overrightarrow{d_S} \cdot \overrightarrow{N} = -d_s \cos\theta_e$.

The reflection of the gaseous particles at the solid surface is described by the following scattering kernel [5]:

$$B(\xi', \xi) = [(1-\alpha_x)\delta(\xi'_x + \xi_x) + \alpha_x \frac{2\xi_x}{C_w^2} e^{-\frac{\xi_x^2}{C_w^2}}][(1-\alpha_y)\delta(\xi'_y - \xi_y) +$$

$$\alpha_y \frac{1}{C_w\sqrt{\pi}} e^{-\frac{\xi_y^2}{C_w^2}}][(1-\alpha_z)\delta(\xi_z'-\xi_z)+\alpha_z \frac{1}{C_w\sqrt{\pi}} e^{-\frac{\xi_z^2}{C_w^2}}] \; , \tag{1}$$

where $C_w^2 = \frac{2kT_w}{m}$; T_w is the surface temperature, k is the Boltzmann constant, and m is the gaseous particle molecular mass. The velocity of the impinging gaseous particle referred to the wall is, $\xi' = (\xi_x', \xi_y', \xi_z') \in \{\mathbf{\Omega}' = \mathbf{R}_- \times \mathbf{R} \times \mathbf{R}\}$, and ξ is the velocity of the reflected one also referred to the wall, $\xi = (\xi_x, \xi_y, \xi_z) \in \{\mathbf{\Omega} = \mathbf{R}_+ \times \mathbf{R} \times \mathbf{R}\}$, where x , y and z are the three space coordinates with x in $\overrightarrow{N}$ direction.

The three coefficients α_x, α_y and α_z correspond to the accommodation coefficients of the three momentum components [5]. Accordingly, the reflection law (1) allows a more flexible treatment of the momentum exchange by the particles with the surface according to the different directions. On isotropic surfaces, both the tangential accommodation coefficients will be equal, $\alpha_y = \alpha_z = \alpha_\tau$.

The flow is assumed to be sufficiently rarefied, $K_n \geq 10$, so that the incoming free stream is represented by a Maxwellian distribution function:

$$f^-(\xi') = \frac{n_e}{(\sqrt{\frac{2kT_e}{m}\pi})^3} e^{-\frac{m(\xi'-U_e)^2}{2kT_e}} \; , \tag{2}$$

where n_e and T_e are respectively the incident flow number-density and kinetic temperature. We denote $C_e = \sqrt{2kT_e/m}$. The reflected particle distribution function, $f^+(\xi)$, is known through the reflection law (1) and the following relation:

$$\xi_x f^+(\xi) = \int_{\Omega'} \mid \xi_x' \mid f^-(\xi') B(\xi', \xi) d_{\xi'} \; . \tag{3}$$

3. THE DRAG, THE LIFT AND THE HEAT TRANSFER COEFFICIENTS

3.1. Calculation of drag and lift coefficient

To investigate the aerodynamic forces, we have to determine the flux of tangential momentum component, P_τ, and the flux of normal momentum component, P_n, exchanged by the gaseous particles with the surface. These fluxes are expressed by,

$$P_n = \Phi^+(|\xi_x|) + \Phi^-(|\xi_x'|) \quad , \quad P_\tau = \Phi^+(\xi_y) - \Phi^-(\xi_y') \; , \tag{4}$$

where $\Phi^-(M_\chi)$ and $\Phi^+(M_\chi)$ are respectively the incident and the reflected fluxes of a quantity M_χ. They are expressed through,

$$\Phi^-(M_\chi) = \int_{\Omega'} m|\xi_x'| M_\chi f^-(\xi') d_{\xi'} \quad , \quad \Phi^+(M_\chi) = \int_{\Omega} m|\xi_x| M_\chi f^+(\xi) d_\xi \; . \tag{5}$$

The drag and the lift coefficients are defined using the momentum fluxes through,

$$C_D = \frac{P_n\cos\theta_e + P_\tau\sin\theta_e}{\frac{1}{2}mn_eU_e^2\cos\theta_e} \quad , \quad C_L = \frac{P_n\sin\theta_e - P_\tau\cos\theta_e}{\frac{1}{2}mn_eU_e^2\cos\theta_e} \; . \tag{6}$$

The calculation of the drag coefficient using the reflection law (1) yields:

$$C_D = \frac{C_e^2}{U_e^2}\left[\frac{\alpha_x}{2}\sqrt{\frac{T_w}{T_e}} - (2-\alpha_x)\frac{U_{e_x}}{\sqrt{\pi}C_e} - \alpha_y\frac{\sin\theta_e}{\cos\theta_e}\frac{U_{e_y}}{\sqrt{\pi}C_e}\right] e^{-\frac{U_{e_x}^2}{C_e^2}} +$$
$$\frac{C_e^2}{U_e^2}\left[(2-\alpha_x)\left(\frac{1}{2}+\frac{U_{e_x}^2}{C_e^2}\right) - \frac{\alpha_x\sqrt{\pi}}{2}\frac{U_{e_x}}{C_e}\sqrt{\frac{T_w}{T_e}} + \alpha_y\frac{U_{e_x}U_{e_y}}{C_e^2}\frac{\sin\theta_e}{\cos\theta_e}\right]\left(1 - erf(\frac{U_{e_x}}{C_e})\right) \; .$$

For comparison of the aerodynamic coefficients with experiment, it is convenient to define the classical parameters: the speed ratio, $S_\infty = \sqrt{U_e^2/C_e^2}$, and the stagnation temperature, $T_s = T_e(1 + 2S_\infty^2/5)$. Therefore, as in our choice of reference frame we have, $\vec{U_e} = (-U_e \cos\theta_e, -U_e \sin\theta_e)$, the drag coefficient is also expressed:

$$C_D = \left[\frac{\alpha_x\sqrt{\pi}}{2S_\infty} \cos\theta_e \sqrt{\frac{T_w(1+\frac{2S_\infty^2}{5})}{T_s}} + (2-\alpha_x)\left(\frac{1}{2S_\infty^2} + \cos^2\theta_e\right) + \alpha_y \sin^2\theta_e \right] \times$$
$$(1+erf(S_\infty\cos\theta_e)) + \left[\frac{\alpha_x}{2S_\infty^2} \sqrt{\frac{T_w(1+\frac{2S_\infty^2}{5})}{T_s}} + (2-\alpha_x)\frac{\cos\theta}{S_\infty\sqrt{\pi}} + \frac{\alpha_y}{S_\infty\sqrt{\pi}} \frac{\sin^2\theta_e}{\cos\theta_e} \right] e^{-S_\infty^2\cos^2\theta_e} \; . \tag{7}$$

In most applications to earth satellites, for example at altitudes between 150 km and 700 km, it is observed that the ratio of the satellite speed to the flow thermal speed is high: $S_\infty > 5$ [8]. Therefore in these situations (hyperthermal flow conditions), the expression (7) can be approximated, by making $S_\infty \to +\infty$,

$$C_D = \alpha_x \cos\theta_e \sqrt{\frac{2\pi T_w}{5T_s}} + 2\left[\alpha_y + (2-\alpha_x-\alpha_y)\cos^2\theta_e\right] \; . \tag{8}$$

The calculation of the lift coefficient gives:

$$C_L = \frac{\sin\theta_e}{\cos\theta_e} \left[\frac{\alpha_x\sqrt{\pi}}{2S_\infty} \cos\theta_e \sqrt{\frac{T_w(1+\frac{2S_\infty^2}{5})}{T_s}} + \frac{(2-\alpha_x)}{2S_\infty^2} + (2-\alpha_x-\alpha_y)\cos^2\theta_e \right] \times \tag{9}$$
$$(1+erf(S_\infty\cos\theta_e)) + \frac{\sin\theta_e}{\cos\theta_e} \left[\frac{\alpha_x}{2S_\infty^2} \sqrt{\frac{T_w(1+\frac{2S_\infty^2}{5})}{T_s}} + (2-\alpha_x-\alpha_y)\frac{\cos\theta}{S_\infty\sqrt{\pi}} \right] e^{-S_\infty^2\cos^2\theta_e} \; .$$

which, again, in hyperthermal flow conditions, is approximated by:

$$C_L = \alpha_x \sin\theta_e \sqrt{\frac{2\pi T_w}{5T_s}} + 2\,(2-\alpha_x-\alpha_y)\sin\theta_e\cos\theta_e \; . \tag{10}$$

3.2. Calculation of the heat transfer coefficient and the recovered temperature

Another important aerothermodynamic parameter to be calculated is the heat transfer coefficient, defined by,

$$C_H = \frac{\Phi^+(\frac{1}{2}\xi^2) - \Phi^-(\frac{1}{2}\xi^2)}{\frac{1}{2} mnU_e^3 \cos\theta_e} \; , \tag{11}$$

where the fluxes are expressed as in relation (5). In relation (11) the heat coefficient is normalised using a surface area projected in the flow direction. We will denote $C_H' = C_H \cos\theta_e$.

Using definition (11), the calculation of the coefficient C_H based on the reflection law (1) yields:

$$C_H = \frac{C_e^3}{2\sqrt{\pi}U_e^3\cos\theta_e} e^{-\frac{U_{e_x}^2}{C_e^2}} \left[\sigma_0(1-\frac{C_w^2}{C_e^2}) + \alpha_x\frac{U_{e_x}^2}{C_e^2} + \alpha_y\frac{U_{e_y}^2}{C_e^2} + \alpha_z\frac{U_{e_z}^2}{C_e^2} \right] -$$
$$\frac{C_e^2 U_{e_x}}{2U_e^3\cos\theta_e}\left(1 - erf(\frac{U_{e_x}}{C_e})\right) \left[\sigma_0(\frac{\sigma_1}{2\sigma_0} - \frac{C_w^2}{C_e^2}) + \alpha_x\frac{U_{e_x}^2}{C_e^2} + \alpha_y\frac{U_{e_y}^2}{C_e^2} + \alpha_z\frac{U_{e_z}^2}{C_e^2} \right] \; , \tag{12}$$

where $\sigma_0 = \alpha_x + 1/2(\alpha_y + \alpha_z)$ and $\sigma_1 = 3\alpha_x + \alpha_y + \alpha_z$. On an isotropic surface, a two-dimensional flow configuration could be assumed and these coefficients become, $\sigma_0 = \alpha_x + \alpha_\tau$ and $\sigma_1 = 3\alpha_x + 2\alpha_\tau$. Furthermore, the expression (12) may be written using the stagnation temperature and the speed ratio as follows:

$$C_H = \frac{1}{2\sqrt{\pi}S_\infty^3\cos\theta_e} e^{-S_\infty^2\cos^2\theta_e} \left[\sigma_0\left(1 - (1+\frac{2S_\infty^2}{5})\frac{T_w}{T_s}\right) + S_\infty^2\left(\alpha_x\cos^2\theta_e + \alpha_\tau\sin^2\theta_e\right) \right] +$$
$$\frac{1}{2S_\infty^2}\left(1+erf(S_\infty\cos\theta_e)\right) \left[\sigma_0\left(\frac{\sigma_1}{2\sigma_0} - (1+\frac{2S_\infty^2}{5})\frac{T_w}{T_s}\right) + S_\infty^2\left(\alpha_x\cos^2\theta_e + \alpha_\tau\sin^2\theta_e\right) \right] \; . \tag{13}$$

The recovered temperature, T_r, is the surface temperature at which no heat is transferred between the surface and the gas: $C_H = 0$. Moreover, in application domains where S_∞ has rather large values, expression (13) is mainly driven by the term on the second line of relation (13). Then, from the heat coefficient (13), the following expression is deduced for the recovered temperature:

$$\frac{T_r}{T_s} = \frac{1}{\sigma_0(\frac{1}{S_\infty^2} + \frac{2}{5})} \left(\alpha_x \cos^2\theta_e + \alpha_\tau \sin^2\theta_e\right) + \frac{\sigma_1}{2\sigma_0(1+\frac{2S_\infty^2}{5})} \ . \tag{14}$$

4. COMPARISON WITH EXPERIMENT

It can be shown that the normal and the tangential reflected fluxes given by reflection law (1) are the same as those postulated by Schaaf and Chambré [2], who introduced empirically separate accommodation coefficients for normal and tangential momentum fluxes in aerodynamic coefficient calculations. Then, the expression obtained for the drag coefficient (relations (7) and (8)), and for the lift coefficient (relations (9) and (10)), are not completely new expressions but appeared as approximated forms in the literature [2, 9]. In contrast, the expression obtained for the heat coefficient in relation (13), is a complete new expression.

In figures 1 and 2 the drag coefficients given by relation (7) are compared with experimental values published by Bellomo et. al in 1984 [6, 4]. The experiments concerned a helium gas impinging on a gold surface at Mach 14. The comparisons show good agreement, and, also show the variation of the couple of coefficients (α_x, α_τ) with the angle of incidence. In fact, theoretical drag coefficient predictions based on normal and tangential accommodation coefficient phenomenological descriptions were known to give rather good results in earth satellite trajectory predictions [9]. Thus, the most interesting parameters which could be investigated, among the aerothermodynamic parameters predicted by the new reflection law (1), may be the heat coefficient and the recovered temperature.

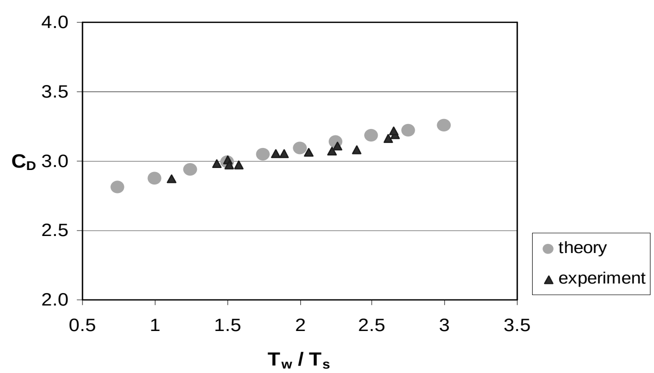

FIGURE 1. Comparison of C_D, relation (7), with experiment from [6]. $\theta_e = 45$. Experimental conditions: gold surface, $T_s = 296K$, $S_\infty = 13$, $K_n = 10$. The best fit is obtained with $\alpha_x = 0.65$ and $\alpha_y = 1$

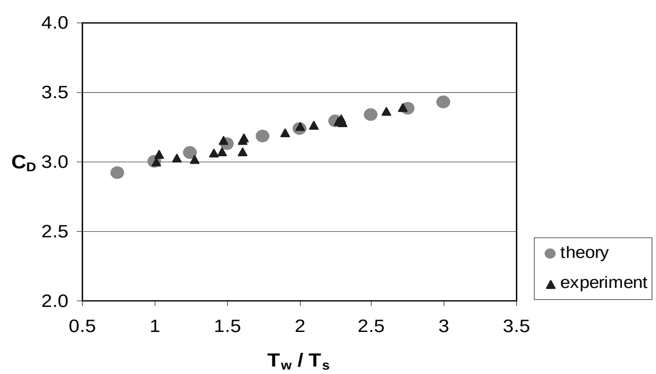

FIGURE 2. Comparison of C_D, relation (7), with experiment of [6] [4]: $\theta_e = 30$. Experimental conditions: gold surface, $T_s = 294.65K$, $S_\infty = 12.78$, $Kn = 10$. The best fit is obtained with $\alpha_x = 0.6$ and $\alpha_y = 0.6$

The most usual theoretical expression for the heat coefficient and the recovered temperature in the free molecular regime are based on a single energy accommodation coefficient representation which led to $T_r = 5/2T_s$. Thereby this representation provides a recovered temperature which is independent of the angle of incidence of the flow and independent of any accommodation coefficient. This does not give an excellent prediction of the recovered temperature as experiments show a variation of the recovered temperature with the angle of incidence [7]. In our new relation (14) the recovered temperature depends precisely on the angle of incidence. It can also be noted that the normal and the tangential accommodation coefficient involved in the scattering kernel (1) are theoretically allowed to depend on the flow macroscopic parameters such as the angle of incidence. Then, at a given angle of incidence, this recovered temperature should be expressed using the corresponding accommodation coefficients. The heat coefficients given by relation (13) are compared first to experimental data given in reference [6], and second to the results given in reference [7]. The experimental data from reference [7] are for helium gas on a clean single crystal Lif(001) surface. The Knudsen number in the second experiments is about 4.3, which is within the transition flow regime. But the comparison with the present theoretical calculation makes sense because the experiment consists of a nearly free monoenergetic beam that is consistent with a Maxwellian representation for the incident distribution function (as in expression (2)). We obtained in both comparisons a good agreement between the theoretical expression (13) and the experimental data. In addition, in the experiments concerning the gold surface (figures 3 and 4), the accommodation coefficients for which the theoretical curve best fits the experiments are slightly higher than in the second experiment concerning the clean crystal surface (figures 6 and 5). In all the curves representing the heat coefficients, the recovered temperatures, which correspond to the zero intercept of the curves, are also in good agreement with the experiments.

However, we should note that the values of the accommodation coefficients giving respectively the best fit, for the drag coefficient (figures 1 and 2) and for the heat transfer coefficients (figures 3 and 4), are different. This remains an unresolved question because these measurement are performed in the same conditions.

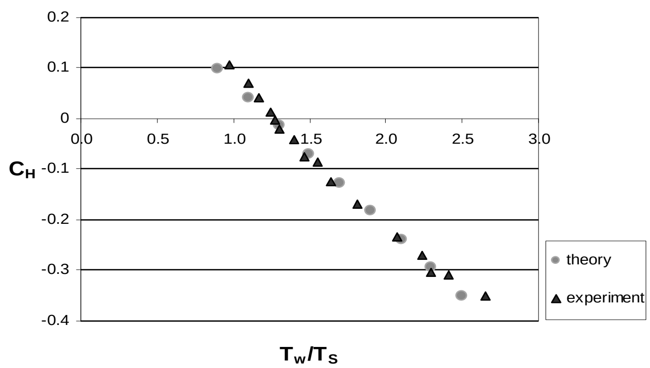

FIGURE 3. Comparison of C_H, relation (13), with experiment [6] [4]: $\theta_e = 45$. Experimental parameters: gold surface, $T_s = 296K$, $S_\infty = 13$, $Kn = 10$. The best fit is obtained using $\alpha_x = 0.34$ and $\alpha_y = 0.35$

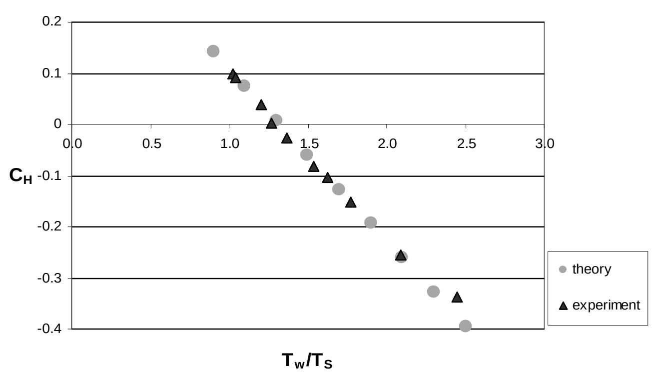

FIGURE 4. Comparison of C_H, relation (13), with experiment [6] [4]: $\theta_e = 0$. Experimental parameters: gold surface, $T_s = 293K$, $S_\infty = 13$, $Kn = 10$. The best fit is obtained using $\alpha_x = 0.44$ and $\alpha_y = 0.39$

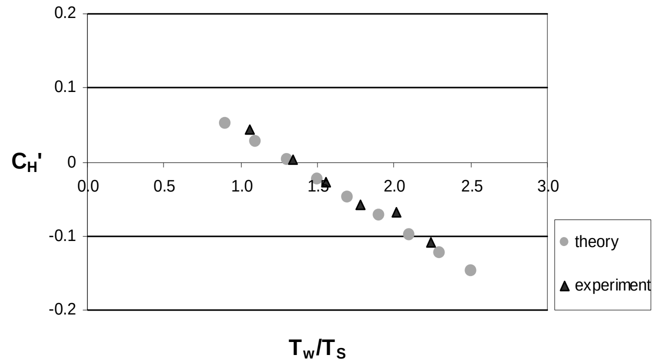

FIGURE 5. Comparison of C'_H, relation (13), with experiment [7]. $\theta_e = 15$. Experimental parameters: clean crystal surface, $T_s = 300K$, $S_\infty = 14.2$, $K_n = 4.3$. The best fit is obtained using $\alpha_x = 0.17$ et $\alpha_y = 0.15$

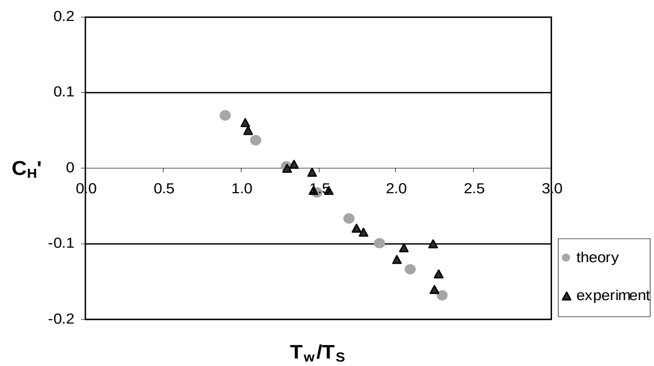

FIGURE 6. Comparison of C'_H, relation (13), with experiment [7]. $\theta_e = 0$. Experimental parameters: clean crystal surface, $T_s = 300K$, $S_\infty = 14.2$, $K_n = 4.3$. The best fit is obtained using $\alpha_x = 0.22$ and $\alpha_y = 0.2$

5. CONCLUSION

In this article we have presented aerodynamic coefficient calculations based on a new model of the scattering kernel. The drag and the lift coefficients are similar to those obtained using the Schaff and Chambré description based on a phenomenological introduction of different accommodations processes for the normal and the tangential momentum fluxes. The heat transfer coefficient and the recovered temperature are new expressions involving the different momentum accommodation coefficients, instead of the classical single energy accommodation coefficient. While the expression obtained for the drag coefficient can give satisfactory results in some trajectory control of Earth satellites [9], a reserve can be cast through the comparisons with the heat transfer coefficient.

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