Analytical Solutions of Continuum Equations in Hypersonic Transitional Flow Over Blunt Bodies

Irina G. Brykina*, Boris V. Rogov#, and Grigoriy A. Tirskiy*

*Institute of Mechanics of Moscow State University, Michurinskiy Pr. 1, 119192, Moscow, Russia
#Institute of Mathematical Modeling, Miusskaja Sq. 4a, 125047, Moscow, Russia

Abstract. Two-dimensional hypersonic rarefied flow over blunt bodies is studied. Basic regimes and parameters of hypersonic flow are found by means of asymptotic analysis. Thin viscous shock layer asymptotic solutions at low Reynolds numbers are obtained. Analytical solutions provide pressure, heat transfer and skin friction coefficients that approach their free molecular limits. Viscous shock layer numerical solutions are obtained. Applicability of various continuum solutions for transitional flow is investigated by comparison with direct simulation Monte Carlo results.

INTRODUCTION

Hypersonic rarefied flow over blunt bodies at low Reynolds number *Re,* typical for space vehicles reentry at altitudes higher than 90 km, is investigated. In such transitional to free-molecule flow regime, continuum models – full and parabolic Navier-Stokes (NS), viscous shock layer (VSL) - give heat-transfer and skin-friction coefficients that increase unlimitedly as *Re* decrease. Taking into account slip wall conditions reduces these coefficients and thus extends the range of applicability of continuum models to lower *Re* number, but does not remove their tendency to increase. So in the transitional flow regime the direct simulation Monte Carlo method (DSMC) is widely applied [1-4] and also approximated engineering bridging methods are used [1,5,6]. But restriction on application of continuum models at low *Re* number does not exclude using the continuum approach for prediction some flow parameters.

In [7,8] a thin viscous shock layer (TVSL) model was proposed and it was shown for a stagnation point of axisymmetric blunt body in case that strongly cold surface, linear viscosity-temperature relation and zero pressure gradient, that TVSL solution gives heat-transfer and skin-friction coefficients approaching their free-molecule limits (at a unit accommodation coefficient) as *Re* number decrease. In [9] it was shown that at $Re\to 0$, $k\to 0$, $(k/Re)^{1/2}\to 0$ (k – the ratio of densities before and behind a shock) NS equations, or TVSL equations, are reduced to the “local” Reynolds equations with vanishing inertia and pressure forces, solution of which for skin-friction and heat-transfer coefficients is a free-molecule flow solution. For 3D stagnation point of blunt body in case that cold surface and power viscosity-temperature relation TVSL asymptotic solution was obtained and this solution gives for heat skin-friction and heat-transfer coefficients the correct free-molecule limits as *Re* number decrease [10]. For 3D stagnation point with arbitrary surface temperature applicability of continuum-flow models was investigated in [11] and it was shown, that TVSL can be used for heat-transfer and skin friction prediction in transitional flow regime.

A question can arise, why TVSL is used for hypersonic flow study at low Re number, while TVSL (and also VSL) equations had been derived at high *Re* numbers [7,8]. Analysis of the NS equations in hypersonic shock layer near blunt body shows that in non-dimensional NS equations parameter $\chi = (\mu_s/((\rho_s Re))^{1/2}$ arises; $Re=\rho_\infty V_\infty R_0/\mu(T_0)$, ρ_∞ and V_∞ - freestream density and velocity, $\mu(T_0)$ – viscosity at freestream stagnation temperature T_0, R_0 – nose radius. Parameter χ is basic in the NS equations at low *Re* number. The other basic parameter is $\tau = (Re/(\mu_s \rho_s))^{1/2}$. Estimation of all terms of the NS equations had been carried out and it had been shown [11], that VSL and TVSL equations are derived from the NS equations at low Re number on the assumption of small parameter χ.

In present work TVSL asymptotic solutions are obtained for two-dimensional transitional flows; they give correct free-molecule limits for pressure, heat-transfer and skin-friction coefficients as $Re \to 0$. The basic similarity parameters of hypersonic rarefied flow are founded for various flow regimes. Applicability of continuum-flow models for transitional flow is studied by comparing analytical and numerical solutions with DSMC results.

THIN VISCOUS SHOCK LAYER

TVSL model was proposed [7,8] for high *Re* number. At low *Re* number TVSL equations were derived from the NS equations on the assumption of small parameter χ [11]: with neglecting terms $O(\chi^2)$ and $O(\chi)$ except tangential pressure gradient term ~ χ, which is taken into account to extend a range of applicability of TVSL to higher *Re* numbers. The correct TVSL equations at low *Re* number are equations (1) without pressure gradient term in the first momentum equation. Two-dimensional TVSL equations in Dorodnitsyn-Lees variables for plane ($\nu=0$) and axisymmetric ($\nu=1$) flows are:

$$\frac{\xi\,\mathrm{tg}\,\alpha}{R}u^2+\xi u\frac{\partial u}{\partial\xi}-\left(\beta' f+\xi\frac{\partial f}{\partial\xi}\right)\frac{\partial u}{\partial\zeta}=-\frac{\xi}{\rho\cos^2\alpha}\frac{\partial p}{\partial\xi}+\frac{\partial}{\partial\zeta}\left(\frac{\mu\rho\xi}{\mathrm{Re}\cos\alpha\,\Delta^2}\frac{\partial u}{\partial\zeta}\right),\quad \frac{\partial f}{\partial\zeta}=u$$
$$\xi u\frac{\partial g}{\partial\xi}-\left(\beta' f+\xi\frac{\partial f}{\partial\xi}\right)\frac{\partial g}{\partial\zeta}=\frac{\partial}{\partial\zeta}\left(\frac{\mu\rho\xi}{\sigma\,\mathrm{Re}\cos\alpha\,\Delta^2}\frac{\partial}{\partial\zeta}\left(g-\frac{(1-\sigma)\cos^2\alpha}{1-T_w}u^2\right)\right),\quad \frac{\partial p}{\partial\zeta}=\frac{\Delta\cos^2\alpha}{R}u^2 \tag{1}$$
$$\frac{1}{\rho}=\frac{\varepsilon T}{p},\quad \mu=T^{\omega},\quad g=\frac{H-H_w}{H_\infty-H_w},\quad T=g(1-T_w)+T_w,\quad \beta'=\frac{d\ln\Delta}{d\ln\xi}+\xi\left(\frac{\xi\,\mathrm{tg}\,\alpha}{R}+\frac{\nu\sin\alpha}{r_w}\right)$$
$$\xi=\frac{x}{R_0},\quad \zeta=\frac{1}{\Delta R_0}\int_0^y\rho\,dy,\quad \Delta=\frac{1}{R_0}\int_0^{y_s}\rho\,dy,\quad y_s=\Delta R_0\int_0^1\frac{1}{\rho}d\eta,\quad \varepsilon=\frac{\gamma-1}{2\gamma},\quad \mathrm{Re}=\frac{\rho_\infty V_\infty R_0}{\mu(T_0)}$$

Here $H=c_pT_0T+V_\infty^2\cos^2\alpha\,u^2/2$ – total enthalpy, $V_\infty\cos\alpha\,u$ – tangential velocity component, $\rho\rho_\infty$ - density, $\mu\mu(T_0)$ – viscosity, T_0T – temperature, T_wT_0 – wall temperature, $\rho_\infty V_\infty^2 p$ - pressure; RR_0 – radius of curvature, α – an angle between a tangent to surface contour and freestream velocity $\mathbf{V}_\infty$, r_wR_0 – a distance from a surface to an axis of symmetry, γ - specific heats ratio, σ - Prandtl number, T_0 - freestream stagnation temperature, R_0 – nose radius; x – the length of arc from a stagnation point along a surface, y – the normal coordinate.

Boundary conditions for equations (1) at a wall and at a shock (modified Rankine- Hugoniot relations) are:

$$\zeta=0:\quad u=0,\quad g=0,\quad f=0$$
$$\zeta=1:\quad u=1-\frac{\mu\rho}{\mathrm{Re}\,\Delta\sin\alpha}\frac{\partial u}{\partial\zeta},\quad p=\sin^2\alpha,\quad f=\frac{r_w}{(1+\nu)\Delta\cos\alpha} \tag{2}$$
$$g=1-\frac{\mu\rho}{\sigma\,\mathrm{Re}\,\Delta\sin\alpha}\frac{\partial}{\partial\zeta}\left(g-\frac{1-\sigma}{1-T_w}\cos^2\alpha u^2\right)$$

Skin friction and heat-transfer coefficients are defined as

$$c_f=\frac{2\tau}{\rho_\infty V_\infty^2}=\frac{2\cos\alpha\mu\rho}{\Delta\,\mathrm{Re}}\left.\frac{\partial u}{\partial\zeta}\right|_w,\quad c_H=\frac{q}{\rho_\infty V_\infty(H_\infty-H_w)}=\frac{\mu\rho}{\sigma\Delta\,\mathrm{Re}}\left.\frac{\partial g}{\partial\zeta}\right|_w \tag{3}$$

BASIC PARAMETERS AND REGIMES OF 2D HYPERSONIC RAREFIED FLOW

TVSL asymptotic analysis (equations and boundary conditions) shows that three flow regimes can be distinguished:

$$\text{I: } Re\varepsilon >> T_w^{(1+\omega)}/\beta^* \qquad \text{II: } Re\varepsilon = O(T_w^{(1+\omega)}/\beta^*) \qquad \text{III: } Re\varepsilon << T_w^{(1+\omega)}/\beta^*,\qquad \beta^*=\frac{2r_w}{(1+\nu)\sin\alpha\cos\alpha} \tag{4}$$

An analysis of NS and TVSL equations in hypersonic shock layer shows that there are two basic parameters of hypersonic rarefied flow over blunt body. The first one is $\chi=O((\mu_s/(\rho_s Re))^{1/2})$, a shock layer thickness $y_s=O(\chi\beta^*)$. The other parameter is $\tau=O((Re/(\mu_s\rho_s))^{1/2})$; non-dimensional tangential velocity and enthalpy $u,\ g=O(\tau)$. For each regime parameters χ, τ are derived in dependence on Re, ε, ω, T_w, $\sin\alpha$, β^*:

At regimes I, II: $\chi=O(\varepsilon(\beta^*/\sin^2\alpha)^{1/2})$, $\quad\tau=O((\varepsilon Re\beta^*)^{1/(1+\omega)})$.

At regime III: $\chi = O((\varepsilon T_w^{1+\omega}/(Re\sin^2\alpha))^{1/2})$, $\tau = O((\varepsilon Re T_w^{1-\omega}\beta^*)^{1/2})$.

Parameter τ characterizes flow rarefaction; it's part depended on flow conditions defines flow rarefaction near stagnation point, geometric parameter β^* increases τ at a distance from stagnation point. Parameter χ determinates applicability of continuum models in transitional flow. At regimes I and II χ is small in hypersonic flow in stagnation region ($\chi \sim \varepsilon$). At regime III χ depends on Re, ε and non-dimensional wall temperature T_w.

ASYMPTOTIC SOLUTION OF TVSL EQUATIONS

TVSL equations (1) together with boundary conditions (2) were solved by the integral method of successive approximations [12] and approximate analytical solution was obtained. By using series expansion procedure on the assumption of small parameter τ the asymptotic solution of TVSL equations at low Re number is obtained for axisymmetric and plane flows for regimes I - III. Solution for heat transfer coefficient c_H , skin-friction coefficient c_f and wall pressure p_w in dependence on flow parameters Re, ε, σ, ω, T_w and geometric parameters α, r_w, R, ν is

$$c_H = \sin\alpha\left[1-\left(\frac{1+\nu}{\beta+\nu}\phi-\frac{1}{3}\right)\sigma\tau\right]+O(\tau^2),\quad \beta=\frac{r_w}{R\cos\alpha} \tag{5}$$

$$c_f = 2\sin\alpha\cos\alpha\left[1-\left(\frac{1+\nu}{\beta+\nu}\phi+\frac{2\beta}{3(\beta+\nu)}-\frac{1}{3}\right)\tau\right]+O(\tau^2)\ ,\qquad p_w=\sin^2\alpha-\frac{2r_w\cos\alpha}{3(1+\nu)R}\tau+O(\tau^2) \tag{6}$$

$$\text{Regime I:}\quad \tau=(\sigma^{1-\omega}\varepsilon\,\mathrm{Re}\,\beta^*)^{1/(1+\omega)},\quad \phi=\frac{1}{2-\omega}\qquad\qquad \text{Regime III:}\quad \tau=(\varepsilon\,\mathrm{Re}\,T_w^{1-\omega}\beta^*)^{1/2},\quad \phi=1 \tag{7}$$

$$\text{Regime II:}\quad \begin{aligned}&\tau=(\sigma^{1-\omega}\varepsilon\,\mathrm{Re}\,\beta^*)^{1/(1+\omega)}(1+\lambda)^{(1-\omega)/(1+\omega)},\quad \phi=\frac{(1+\lambda)^{2-\omega}-\lambda^{2-\omega}}{(2-\omega)(1+\lambda)^{1-\omega}}\\ &T_w=(\sigma^2\varepsilon\,\mathrm{Re}\,\beta^*)^{1/(1+\omega)}\lambda(1+\lambda)^{(1-\omega)/(1+\omega)}\end{aligned} \tag{8}$$

$$\lim_{\mathrm{Re}\,\varepsilon\to 0}c_H=\sin\alpha\qquad \lim_{\mathrm{Re}\,\varepsilon\to 0}c_f=2\sin\alpha\cos\alpha\qquad \lim_{\mathrm{Re}\,\varepsilon\to 0}p_w=\sin^2\alpha \tag{9}$$

Regime I $T_w << (Re\varepsilon\,\beta^*)^{1/(1+\omega)}$ corresponds to a strongly cold wall and solution in this case does not depend on T_w. Regime I can be considered as limit case of regime II at $\lambda \to 0$.

For c_H prediction both TVSL and correct TVSL can be used, because at low Re number tangential pressure gradient has not an influence on c_H, c_H depends only on parameter τ and surface geometry, it does not depend on parameter χ and approaches its correct free-molecule limit at a unit accommodation coefficient [13] as Re, or $\tau \to 0$. For c_f prediction (6) correct TVSL has been used for correct transition to free-molecule flow. In usual TVSL c_f depends on τ and χ because of tangential pressure gradient influence. In that case, for example in regime I ($\chi \sim \varepsilon$), c_f is:

$$c_f = 2\sin\alpha\cos\alpha\left[1-\left(\frac{(1+\nu)}{(2-\omega)(\beta+\nu)}+\frac{2\beta}{3(\beta+\nu)}-\frac{1}{3}\right)\tau+\frac{2\varepsilon\sigma\beta}{\sin^2\alpha(\beta+\nu)}\right]+O\left(\tau^2,\varepsilon^2,\varepsilon\tau\right) \tag{10}$$

$$\lim_{\mathrm{Re}\,\varepsilon\to 0}c_f=2\sin\alpha\cos\alpha(1+2\sigma\varepsilon\beta/(\sin^2\alpha(\beta+\nu)));\quad \lim_{\substack{\mathrm{Re}\,\varepsilon\to 0\\ \varepsilon\to 0}}c_f=2\sin\alpha\cos\alpha$$

So as $Re \to 0$, c_f approaches a free-molecule limit at $\chi \to 0$ or on condition that the correct TVSL model is used.

VANISHING THIN VISCOUS SHOCK LAYER ANALYTICAL SOLUTION

In limit case, when one neglects terms $O(\chi^2)$, $O(\chi)$ and $O(\tau)$ in NS equations, taking into account only terms $O(1)$, the NS equations are reduced to the "local" Reynolds equations with vanishing inertia and pressure forces, which are called vanishing thin viscous shock layer (VTVSL) equations. Taking into account in TVSL equations (1) and boundary conditions (2) only terms $O(1)$ and neglecting terms $O(\tau)$, one obtains VTVSL equations with boundary conditions at a shock $\zeta = 1$ and at a wall $\zeta = 0$:

$$\frac{\partial}{\partial\zeta}\left(\mu\rho\frac{\partial u}{\partial\zeta}\right)=0,\quad \frac{\partial}{\partial\zeta}\left(\mu\rho\frac{\partial T}{\partial\zeta}\right)=0,\quad \frac{\partial p}{\partial\zeta}=0,\quad \rho=\frac{p}{\varepsilon T}\qquad \zeta=0:\ \ u=0,\quad T=T_w$$

$$\zeta=1:\quad \frac{\mu\rho}{\mathrm{Re}\,\Delta\sin\alpha}\frac{\partial u}{\partial\zeta}=1,\quad \frac{\mu\rho}{\sigma\,\mathrm{Re}\,\Delta\sin\alpha(1-T_w)}\frac{\partial T}{\partial\zeta}=1,\quad p=\sin^2\alpha,\quad \int_0^1 u\,\mathrm{d}\zeta=\frac{r_w}{(\nu+1)\Delta\cos\alpha} \tag{11}$$

The solution of problem (11) for $\mu = T^{\omega}$ was obtained: for $\omega = 1$ – the exact solution; for regimes I, III – approximate solutions by expansion procedure. Temperature T and shock layer thickness $y_s(x)$ are

$$\omega=1:\quad T=T_w+\sigma^*\sqrt{\beta^*\varepsilon\,\mathrm{Re}}\,\zeta\ ,\quad y_s(x)=\sqrt{\frac{\beta^*\varepsilon T_w^{1+\omega}}{\sin^2\alpha\,\mathrm{Re}}}+\frac{\beta^*\sigma^*\varepsilon}{2\sin\alpha} \tag{12}$$

$$\text{Regime III}:\quad T=\left(T_w^{\omega}+\omega\sigma^*\sqrt{\frac{\beta^*\varepsilon\,\mathrm{Re}}{T_w^{1-\omega}}}\,\zeta\right)^{1/\omega},\quad y_s(x)=\sqrt{\frac{\beta^*\varepsilon T_w^{1+\omega}}{\sin^2\alpha\,\mathrm{Re}}}+\frac{\beta^*\sigma^*\varepsilon}{2\sin\alpha} \tag{13}$$

$$\text{Regime I}:\quad T=\left(\frac{\beta^*(1+\omega)\sigma^{*2}\,\varepsilon\,\mathrm{Re}}{2}\right)^{1/(1+\omega)}\zeta^{1/\omega}\ ,\quad y_s(x)=\frac{\beta^*\sigma^*\varepsilon}{2\sin\alpha} \tag{14}$$

$$u=\frac{1}{\sigma^*}\left(T-T_w\right),\quad \sigma^*=\sigma(1-T_w) \tag{15}$$

Expressions (13) and (14) in case of $\omega = 1$ coincide with expressions (12) (for regime I (12) at $T_w = 0$).

Wall pressure, heat-transfer, skin friction coefficients solutions in the framework of VTVSL exactly coincide with free-molecule flow solution at unit accommodation coefficient: $p_w=\sin^2\alpha$, $c_H=\sin\alpha$, $c_f=2\sin\alpha\cos\alpha$.

NUMERICAL SOLUTION AND DISCUSSION OF RESULTS

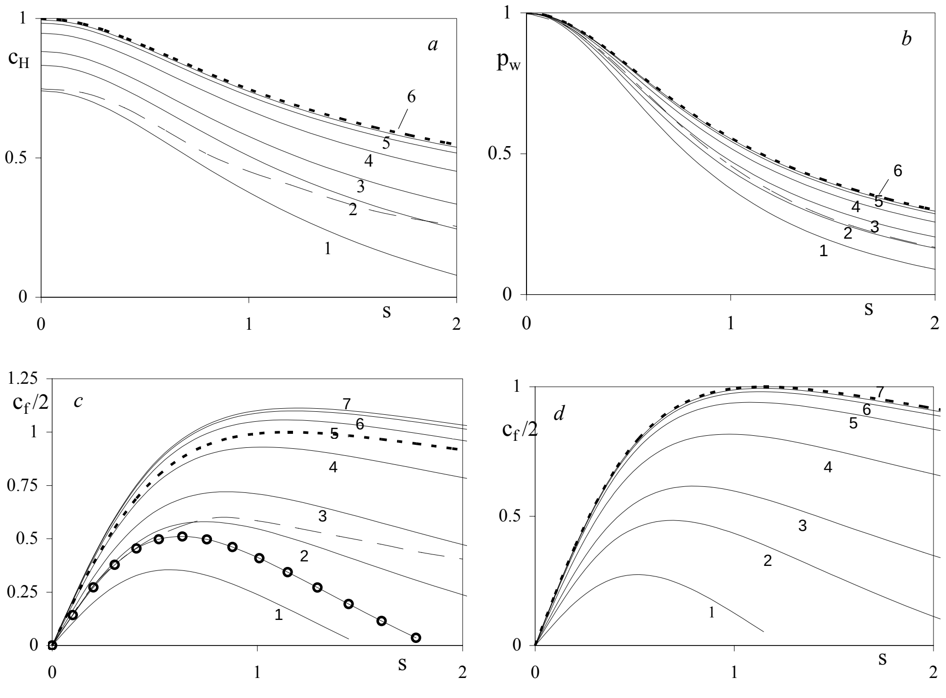

FIGURE 1. Distributions of c_H , p_w , c_f along paraboloid in dependence on distance along the surface $s=\xi$. Solid lines 1-7 – asymptotic solutions at Re = 2.4, 1, 0.5, 0.1, 0.01, 0.001, 0.0001; chain lines –solution [8] at Re = 2.4; line with circles - c_f* at Re = 2.4; dotted lines – free-molecule flow; $\gamma = 1.33$, $\omega = 1$, $\sigma = 0.71$. c and d – calculation c_f by formulas (10) and (6).

Theoretical results were verified by numerical VSL and TVSL solutions, obtained by using the low-iterative high-resolution fully coupled implicit space-marching procedure [14]. To take into account upstream influence, the accelerated method of global iterations on an elliptical component of pressure gradient was elaborated. New splitting of a tangential pressure gradient into hyperbolic and elliptic components was employed. The following continuum flow models were solved by this method: 1) VSL; 2) TVSL; 3) VSL with unit Lame coefficients - VSL-L1; 4) TVSL without assumption that a shock shape is equidistant to a body shape - TLSL-ES.

Distributions of heat transfer coefficient (*a*), wall pressure (*b*) and skin-friction coefficient with (*c*) and without (*d*) taking into account tangential pressure gradient along paraboloid for various *Re* numbers are shown in Fig. 1. Asymptotic solution is compared with free-molecule flow solution and numerical solution [8]. Solution [8] is obtained with neglecting term $\sim u^2$ in the first equation (1), so for correct comparison, c_f* was obtained on the same assumption. Asymptotic solution is in agreement with numerical solution in a stagnation region with high degree of rarefaction. As s increase, parameter τ also increases through influence of geometrical parameter β^* and an error arises in asymptotic solution since assumption of small τ is broken. Parameter τ defines a range of applicability of asymptotic solution. For lower *Re* number asymptotic solution is valid on longer distance from a stagnation point. As Re$\rightarrow$0, c_H, p_w, c_f (*d*) approach to free-molecule limit.

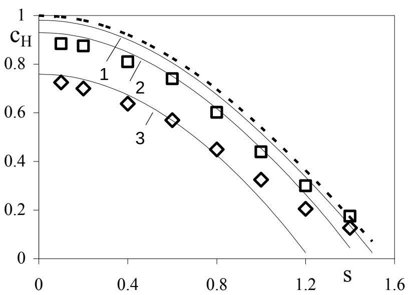

FIGURE 2. Comparison of asymptotic c_H solutions (solid lines *1,2,3*) on cylinder for Shuttle reentry trajectory at 110, 100, 90 km with DSMC results [4]: squares -100 km, rhombuses -90 km; dotted line - free-molecule flow. $V_\infty = 7.5$ *km/s*, $R_0 = 0.0254$ *m.*

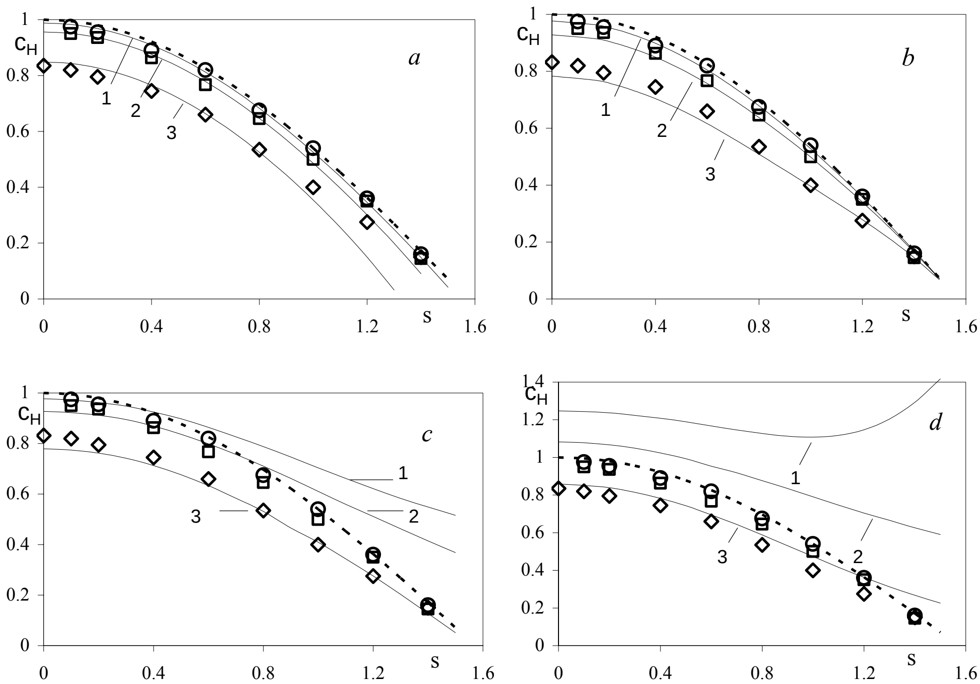

FIGURE 3. Comparison of various continuum c_H predictions along sphere for Shuttle reentry trajectory at altitudes 110, 100, 90 km (solid lines *1,2,3*) with DSMC results [3] - circles, squares, rhombuses. Dotted line - free-molecule flow. *a* - asymptotic solution; *b* – TVSL, *c* – TVSL-ES and VSL-L1, *d* – VSL numerical solutions. $V_\infty = 7.5$ *km/s*, $R_0 = 0.0254$ *m.*

For plane flow comparison of asymptotic TVSL c_H solution on cylinder for Shuttle reentry trajectory with DSMC results [4] is shown in Fig. 2. An agreement between analytical and DSMC solutions is observed at 100 km and at 90 km up to s ~ 1. At high altitudes – 110 km and higher asymptotic solution approaches free-molecule flow solution.

In TVSL c_H and c_f depend on shock layer thickness y_s only through tangential pressure gradient, and at low *Re* number tangential pressure gradient does not influence on heat-transfer coefficient c_H. So when y_s increase, it does not influence on c_H (and on c_f in case that correct TVSL model is used). In other flow models (VSL, NS) y_s increase leads to c_H , c_f increase because y_s influences on c_H, c_f also through Lame coefficients, which are equal unit in TVSL. It was shown [11] that VSL-L1 with unit Lame coefficients give good c_H prediction near a stagnation point. Here VSL-L1 and TVSL-ES models are verified at a distance from a stagnation point.

Comparison of various c_H predictions (TVSL asymptotic solution; TVSL, TVSL-ES, VSL-L1, VSL numerical solutions) along sphere for Shuttle reentry trajectory at altitudes 110, 100, 90 km with DSMC results [1] and free-molecule flow solution is shown in Fig. 3. A good agreement between asymptotic and numerical TVSL solutions and DSMC results is observed. At altitude 90 km with increasing of distance from a stagnation point (for $s > 1$), an error in asymptotic solution arises since assumption of small τ in asymptotic solution is broken because of influence of β^*. Fig. 3 illustrates also that TVSL asymptotic and numerical c_H solutions approach free-molecule flow solution.

VSL, VSL-L1 and TVSL-ES are more full models than TVSL and more exact for high *Re* number. But for low Re number the more simple TVSL model is more exact. VSL, VSL-L1, TVSL-ES solutions are in agreement with DSMC results at 90 km, but at 100 km and higher they are incorrect. It is caused by growth of shock layer thickness as *Re* decrease and *s* increase. At that VSL-L1 and TVSL-ES give correct c_H prediction near stagnation point.

CONCLUSION

Three regimes of 2D hypersonic rarefied flow over blunt body are founded in dependence on Re, ε, ω, T_w and geometric parameter β^*. TVSL asymptotic solution is obtained at low *Re* number and similarity parameter τ is founded for each regime. Simple analytical expressions for pressure, heat-transfer and skin-friction coefficients are given in dependence on parameter τ and body geometry. As $Re \rightarrow 0$ these coefficients approach the free-molecule flow solution. Numerical solutions of VSL, TVSL, VSL-L1, TVSL-ES are obtained. Comparison between asymptotic and numerical continuum solutions and DSMC results showed, that TVSL – analytical and numerical solutions – can be used for heat transfer, pressure and skin friction prediction in hypersonic transitional flow.

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