SIMPLE RAREFIED GAS FLOWS IN POROUS REGIONS

L.M. de Socio, L. Marino

Dip. Meccanica e Aeronautica
Via Eudossiana, 18 I-00184 Roma
Universitá "La Sapienza"

Abstract. The case of a fibrous material was investigated both experimentally and by numerical simulations to obtain new results in a physical situation which left many open questions in the literature. A geometric configuration was devised which was intended to provide a possible reference for future researches dealing with a very peculiar permeable medium of increasing interest in applications. The test probe was obtained by compacting thin discs of a high density plastic net in a tube with their cylindrical symmetry axis parallel to the axis of their container. The pressure distribution was measured along the wall of the probe at various nitrogen flow rates and in more or less rarefied gas conditions. A Knudsen effect, related to the material conductance, was obtained similar to the one in a bed of spheres and in a capillary tube. Furthermore, a physical model for the experimented fibrous bed flow was proposed and a simulation code, based on a MonteCarlo Direct Simulation procedure, was run with very good agreement with the experimental data.

INTRODUCTION

In their fundamental survey paper of the year 1986, Jackson and James [1] wrote that the literature concerning the experiments on fibrous materials was - at that time - much richer than the one referable to the theoretical studies. However the number of reliable experimental data had, in that circumstance, to be limited to 25 media since many of the available results could not be scientifically organized and compared. This was the consequence of the great spread of different approaches, different applications and even different definitions which pertain to the domain of a very complex phenomenology. Indeed the various physical situations where heterogeneous materials as the fibrous media are present have been considered and studied by scientists with extremely different interests. Just to cite a few of them, materials as textiles, glass and metal wool, stratified ceramic matrices and even Chinese and blond hair have been considered (see, for example, [2]).

In twenty years from the day that survey appeared, things have rapidly changed under the powerful impulse of the growing applications of the fibrous materials. A strong effort has been spent to give rigorous mathematical and physical substance to the rationalization of the basic definitions and mathematical tools concerning the heterogeneous media in a conspicuous series of papers and books (see, e.g.: [3, 4, 5, 6]). In spite of all this effort, it is curious enough that, even at the present time, many authors (including perhaps the ones of this work) seem to ignore results which appeared in journals not strictly pertaining to their fields of specialization.

A noticeable contribution to the studies of fibrous materials has come from physicists and mathematicians, on one hand, and, on the other hand, from the increasing capabilities of the numerical simulations, so that the gap between experimental and analytical-numerical results is being rapidly filled. One notes that the numerical simulations, performed on almost standard configurations of the fibrous media, very often do not refer to the experimental evidences anymore but rather compare their results and find their motivation on mere numerical reasons ([7, 8, 9]). The purpose of this paper is twofold. On one side we wish to present a simple model configuration, for an anisotropic fibrous material, which is not reducible to the widely considered cubic array geometry but can be of interest in many practical situations and can serve as a reference for further studies. For a gas flow through this simple microstructure we obtained experimental results and carried out numerical simulations with excellent comparisons in a range of Knudsen numbers which corresponds to the free molecular flow in the microscale. On the other side we recovered the "Knudsen effect", first obtained in capillary tubes and later in a bed of spheres ([10, 11]). The existence of these minima opens the way to new considerations about the flows in microstructures.

Turning now to the description of the physical model and of the relative probe, the fibrous configuration is obtained by piling two-dimensional rigid nets of square meshes of side l. The nets lie in planes parallel to the plane (y,z) of a Cartesian system of coordinates, are made of straight rods of small diameter d with respect to l, and are piled up along their normal axis x at random as far as the orientation of the fibers in the plane (y,z) is concerned. All the rods are kept isothermal at a temperature T.

The experimental model and its physico-mathematical counterpart, with their well defined structure, somehow overcome the first approach to thin filtering fibrous structures in [12].

Let a stream of gas of number density n, mass m, and viscosity μ move along x under the action of a pressure gradient, ∇p, and let the mean free molecular path be several times the thickness d of each net, so that the flow can be considered, in this respect, rarefied. Then the molecules entering into a net either hit the walls of a mesh or pass through to the next one without collisions. We subsequently adapt to this situation a concept which follows from the dusty gas approach to the porous media, where the solid and the fluid phases are dealt with as a mixture of heavy and large particles and light and small molecules. The fraction of the total number of molecules crossing each net and hitting the fibers (the heavy molecules) which represent its walls loses its momentum and energy. After impinging the molecules are diffusely re-emitted, in the half space facing each wall, according to a Maxwellian velocity distribution function at the temperature T. The mean velocity components of the molecules in the plane of the net, namely V and W, along y and z respectively, are zero. All the remaining molecules, *i.e.*: those which do not impinge against the walls, proceed across the apertures while keeping their values, at the entrance of the net, of the mean velocity U along x and of the thermal speed components u,v,w. The molecules do not collide among themselves due to their great value of lambda. At the exit of each net, the statistical new values of the state of the gas, U and p, are evaluated over the entire number of flowing molecules. As a consequence, the molecules entering the next fibrous sheet have mean velocity U and a Maxwellian u,v,w distribution corresponding to T. From the evaluation of the specific flow rate $Q = nmUA$ and of the pressure drop, the local permeability k in x direction can be calculated by $k = -U/(|\nabla p|/\mu)$.

The experimental probe was made by piling circular nets with l = 2 mm and d = 0.1 mm in a tube of diameter 40 mm and length 170 mm. The probe was then installed in a rig instrumented with flowmeters and flow controllers, thermocouples, and absolute and differential manometers. The flow, coming from a nitrogen bottle was driven through vacuum pumps. Along the probe wall a series of pressure measurements could be taken.

On the basis of the model discussed above, a simulation was carried out which takes advantage of a MonteCarlo numerical code. The details of the code will not be reported here since it was already described in full in [11]. The overall dimensions of the calculation domain correspond to those of the experimental cylindrical probe and its content of nets was simulated by sorting at random the angular orientation around x of each net. The calculation domain, symmetrical with respect to x, was then divided into cells, N_x and N_r along the longitudinal axis and the radius, respectively, and a proper choice of the representative particles n_p was made. In all calculations, as in the experiments, the gas was nitrogen and the temperature constant and equal to the experimental T. $n_p = 10^6$, $N_x = 100$ and $N_r = 60$ demonstrated to be the best choice in terms of accuracy and computational time.

Erofeev *et al*. [13] while dealing with a supersonic flow through a perforated membrane of vanishing thickness assumed that, the probability of hitting/hitting not the membrane by a molecule should be taken equal to the solidity φ of the membrane, equal to $1-\varepsilon$, where ε is the porosity (area of the void over the total area). In our case we made use of the same approach of [11]. In particular, for each cell of the calculation domain, one knows the mean free path of the incoming flow. The molecules which will not collide among themselves and against the walls, while crossing the porous medium, will be a fraction β of the total number of molecules, and $1-\beta$ will be the fraction hitting the walls. The probability β comes out from the expression $\beta = \exp[-\sin^{-1}(Kn^{-1})]$ which holds for the equivalent tube of length λ in which flows a free molecular flow. The diameter of this tube is calculated from the cross sectional area of the cell and the porosity of the medium. Note that, for the boundary cells of the finite domain, the molecules impinging against the cylindrical wall are diffusely re-emitted according to the very usual rule.

EXPERIMENTS AND SIMULATIONS

We begin with a few experimental data and their comparisons with the results of the corresponding numerical simulations (DSMC). Figure 1 shows the pressure distribution along the longitudinal coordinate x made dimensionless with respect to the length of the probe. The calculated values of p are represented both at the cylindrical wall, where the measurements were carried out, and along the axis of symmetry. The less favourable case Fig. 1.a corresponds to the most rarefied conditions of the flow at the inlet of the probe, since the accuracy of the simulations, with respect to the experiments, increases as one moves toward the continuum regime Fig. 1.b. In any case the comparisons are very good. This means that a satisfactorily representation of a sufficiently structured fibrous medium can provide a useful investigation tool for heterogeneous, three-dimensional configurations.

Figure 2 reports the full set of experimental data in terms of pressure distributions *vs* x at various flow rate values. The mean free path at the entrance of the probe ranged between 0.3 and 6 mm with the Knudsen number, referred to the diameter of the wire or, in other words, to the thickness of a net, ranging between 3 and 60. The permeability values along p, calculated from the experimental set of data, is shown in Fig. 3 together with an interpolating curve. As one moves towards the continuum regime, k decreases and reaches values of the same order of magnitude of the numerical data for the cubic structures considered in [7]. Furthermore the experiments enrich the global presentation of the rather old survey in [14] in a region of rather scarce data from the literature.

A final comment concerns one of the motivations of this work. In a paper which appeared long time ago [14] the authors wrote about the possibility that the conductance of porous media might present the same behavior of capillary tubes at low pressures, an effect first shown by Knudsen. However in that article the few results dealing with the fibrous media were not conclusive. After that very preliminary attempt no more investigation on this point was carried out, either experimental or numerical. In [11] the authors showed that the Knudsen effect is present in porous media made of randomly closed packed spheres or, to improve what in [3] is called an "ill defined idea", made of a maximally random jammed configuration of hard spheres. Dealing now with the fibrous medium we report in Fig. 4 the results and their interpolating curves concerning the ratio K_d over K_r, where K_d is the conductance of a porous medium according to a widely accepted definition, $K_d = Up/(|\nabla p|)$, and K_r is given by $\sqrt{RT}d$. In the figure are presented both cases of the bed of spheres and of the experimented fibrous material. It is worth noting the fact that both curves present minimum values with a difference in the range of the experimental errors and occurring at the same value of the pressure. However, above and below the point of minimum the curves show well defined differences.

CONCLUSIONS

In this paper a series of experimental results have been presented concerning a simple fibrous materials which can be of some relevance for applications and for mathematical studies microstructured materials. A model was also put forward for the direct flow simulation of the problem which was implemented in a MonteCarlo code with excellent results.

Further simulations are necessary to evaluate the influence of the porosity on the permeability of the medium.

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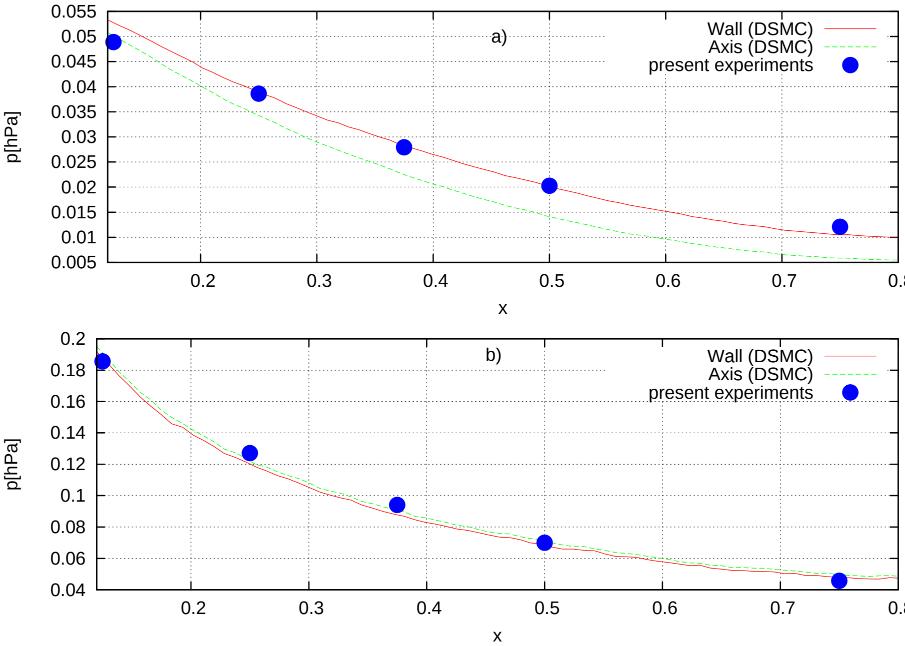

FIGURE 1. Pressure distributions *vs x*. Experimental and simulated results. a): $Q = 1\,sccm$; b):$Q = 5\,sccm$

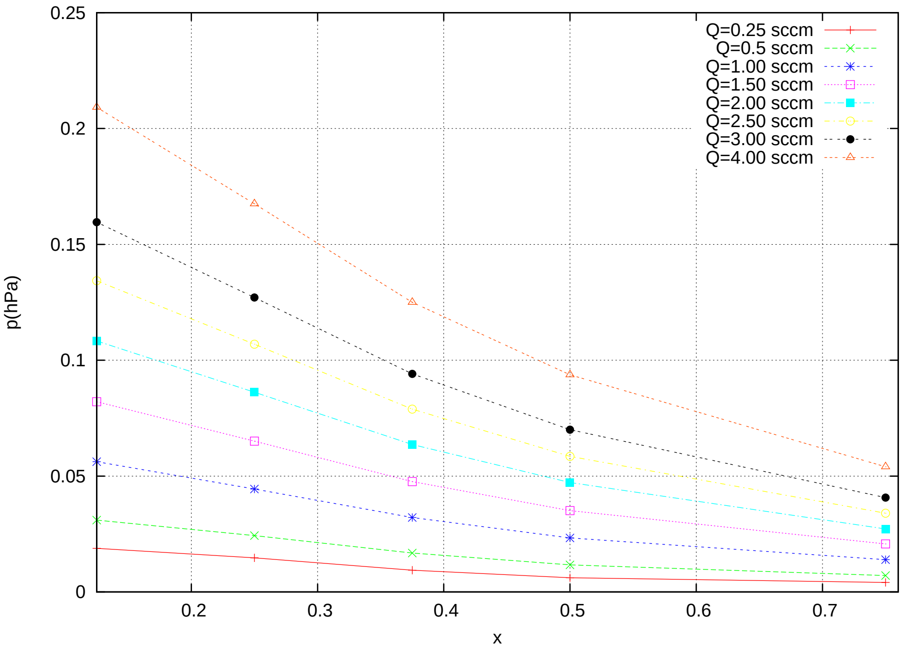

FIGURE 2. Experimental pressure distributions *vs x*

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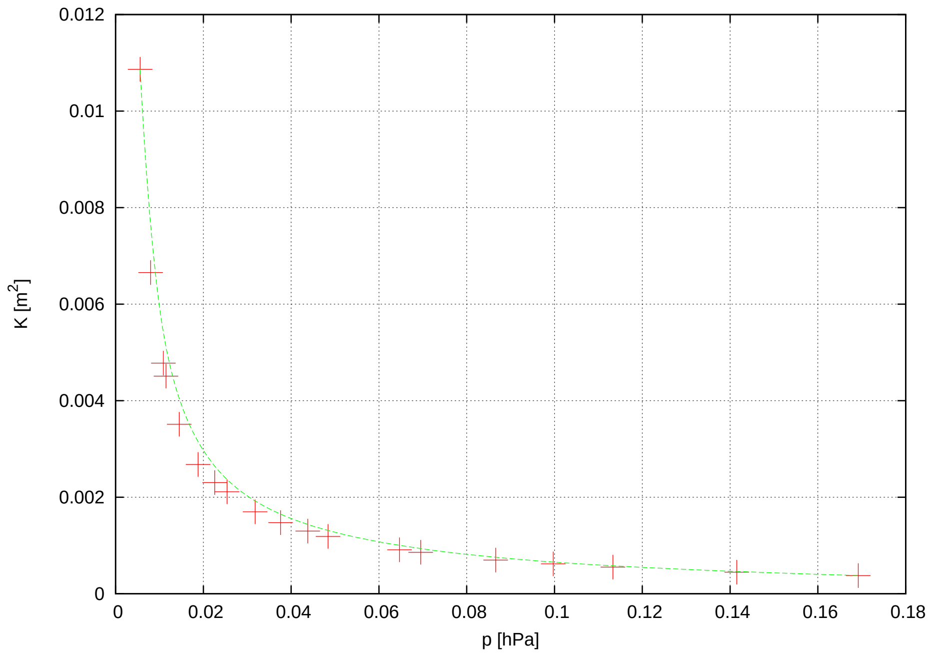

FIGURE 3. Experimental permeability distributions *vs* p

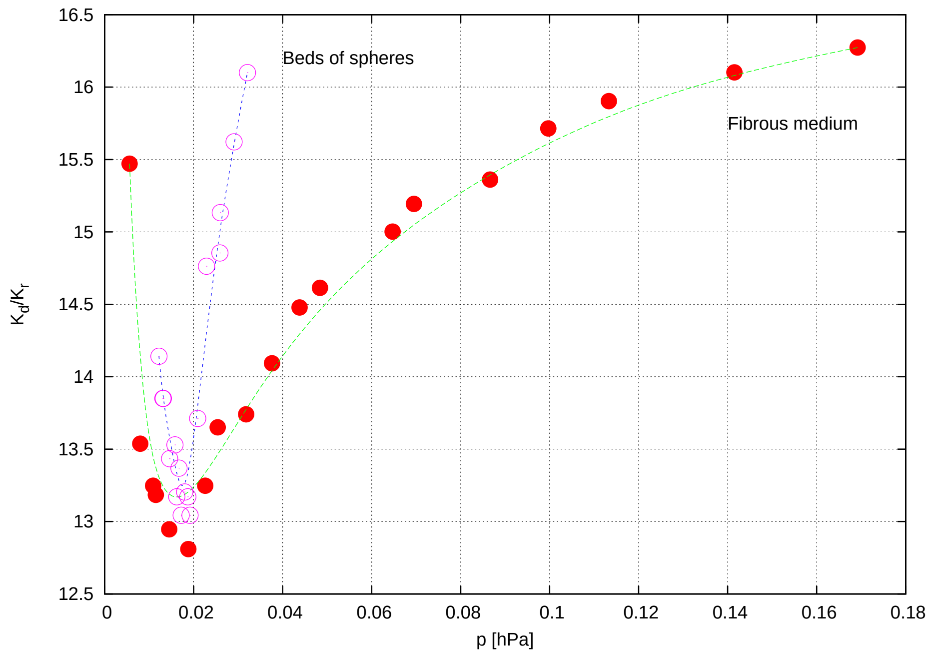

FIGURE 4. Experimental conductance K_d/K_r distributions *vs* p