Numerical Simulation of Rarefied-Gas Flows about a Rotating Cylinder

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Abstract. Subsonic and supersonic flows of nitrogen, carbon dioxide, and argon near a spinning cylinder have been investigated numerically with the Direct Simulation Monte-Carlo technique under transitional rarefied conditions (Knudsen numbers from 0.003 to 10). The rarefaction factor (Knudsen number), spin rate, specific heat ratio, and upstream Mach number affect strongly both the flow structure and aerodynamic characteristics. It has been found that the lift force on a spinning cylinder ("the Magnus effect") at subsonic upstream conditions has different signs in the free-molecule and continuum regimes. The lift-force sign change occurs in the transition regime at Knudsen number of 0.2.

Keywords: Magnus Effect, Spinning Cylinder, Transition Rarefied-Gas Flows, Aerodynamic Coefficients.
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NOMENCLATURE

C_x = drag coefficient
C_y = lift coefficient
D = diameter of a cylinder, m
I_0, I_1 = modified Bessel functions
Kn_D = Knudsen number
k = Boltzmann's constant
M_∞ = Mach number
m = mass of molecule, kg
S = molecular speed ratio, $(0.5\gamma)^{1/2}M_\infty$
T = temperature, K
t_w = temperature factor, T_w/T_0
$\boldsymbol{u}_\infty$ = freestream flow velocity vector, m/s
W = roll parameter, $\Omega D/2u_\infty$
x = Cartesian coordinate in the direction of the freestream flow velocity vector, m
y = Cartesian coordinate in the direction of the vector $[\boldsymbol{\Omega}\times\boldsymbol{u}_\infty]$, m
γ = ratio of specific heats
σ_t = coefficient of accommodation of the tangential momentum, 1
$\boldsymbol{\Omega}$ = angular rotation vector of the spinning circular cylinder along its symmetry axis, rad/s

subscripts

FM = free-molecular-flow parameter
i = incident molecules
r = reflected molecules
w = wall condition
0 = stagnation flow parameter
∞ = freestream flow parameter.

INTRODUCTION

Incompressible flows around spinning bodies of revolution were studied in detail years ago (see reviews of Prandtl and Tietjens [1] and Lugt [2]). It was found that in the case of potential flow, the lift generated on the body has an opposite direction to the vector $[\boldsymbol{\Omega}\times\boldsymbol{u}_\infty]$ (the Magnus effect). In this flow regime, three flow patterns past a spinning circular cylinder can be identified [1, 2] by the value of the governing similarity parameter, which is the roll parameter, $W = \Omega D/2u_\infty$. The patterns depend on the location of the points of separation and attachment.

In unsteady flow of a viscous incompressible fluid, the flow pattern becomes a function of both the roll parameter and the Reynolds number, Re [2]. At $Re < 1.3\times10^5$ and $W < 0.5$, the Magnus force becomes negative [3].

In the case of free-molecule flow, a different result was observed in Refs. 4-6. The lift of the spinning body under the free-molecule flow conditions should be opposite to the vector of lift under the continuum potential flow conditions. The lift and drag coefficients $C_{y,FM}$ and $C_{x,FM}$, respectively, can be calculated using the formulae [6]:

$$C_{y,FM} = C_y(0) + C_y(W),\ \ C_y(W) = (\pi/2)\,\sigma_t\,W \tag{1}$$

$$C_{x,FM} = C_x(0) + C_x(W),\ \ C_x(W) = 0 \tag{2}$$

where the parameter σ_t is the coefficient of accommodation of the tangential momentum, and it is assumed to be equal unity. The lift and drag coefficients $C_y(0)$ and $C_x(0)$ in the nonspinning-cylinder case of free-molecule flow have been calculated by using the following expressions [7]:

$$C_y(0) = 0,\ \ C_x(0) = C_{x,\mathrm{i}}(0) + C_{x,\mathrm{r}}(0) \tag{3}$$

$$C_{x,\mathrm{i}}(0) = \pi^{1/2}\,e^{-(S\times S/2)}\,\{\mathrm{I}_0(S^2/2) + (0.5 + S^2)[\,\mathrm{I}_0(S^2/2) + \mathrm{I}_1(S^2/2)]\}/S \tag{4}$$

$$C_{x,r}(0) = \pi^{3/2}/(4\,u_\infty\,h_\mathrm{r}^{1/2}),\ \ S = (0.5\gamma)^{1/2}\,M_\infty,\ \ h_\mathrm{r} = m/(2kT_\mathrm{r}) \tag{5}$$

where M_∞ is the Mach number, S is the molecular speed ratio, γ is a ratio of specific heats, m is a mass of molecule, k is Boltzmann's constant, T is temperature, and I_0 and I_1 are modified Bessel functions. Subscripts i and r refer to incident and reflected molecules, respectively. The Cartesian coordinate x is in the direction of the freestream flow velocity vector $\boldsymbol{u}_\infty$, and the coordinate y is in the direction of vector $[\boldsymbol{\Omega}\times\boldsymbol{u}_\infty]$.

Karr and Yen [4] showed that the effect of spin on drag is of second order in W, and the component of lift $C_y(W)$ has been found to be proportional to W and is analogous to the Magnus effect with the opposite sign [6]. The expressions of the momentum characteristics can be found in Ref. 6.

In the present study, the aerodynamic coefficients of a spinning infinite cylinder have been evaluated numerically for a range of the similarity parameters: Knudsen number $Kn_{\infty,D}$, spin-rate W, and a ratio of specific heats γ. The analysis of the coefficients of the spinning cylinder is based on the numerical results that were obtained using direct simulation Monte-Carlo (DSMC) technique [8, 9]. The results are compared with free-molecule flow data [6].

DSMC METHOD

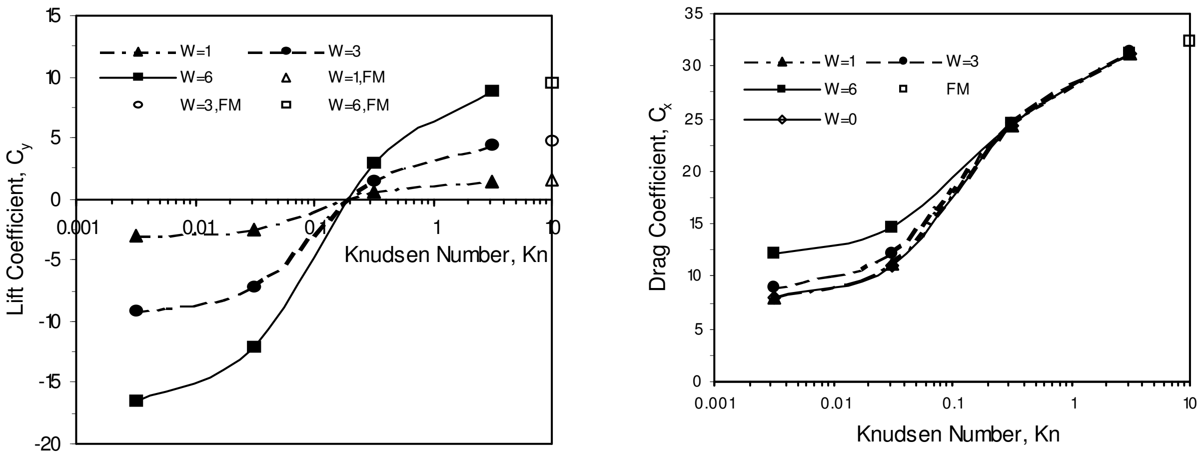
The DSMC method [8] has been used in this study as a numerical simulation technique for low-density gas flows. The flow parameters are calculated using a two-dimensional DSMC code [9]. Molecular collisions in nitrogen, argon, and carbon dioxide are modeled using the variable hard sphere molecular model [8]. The gas-surface interactions are assumed to be fully diffusive with full energy and moment accommodation (σ_t = 1). The code validation was tested [10] in comparing numerical results with experimental data [10] for simple-shape bodies.

In the present calculations, one region is used with a total of 2700 cells. The 29,800 molecules are unevenly distributed while providing an overall average of 11 molecules per cell. Following the recommendations of Refs. 8 and 9, reliable results are obtained for an average of at least 10 molecules per cell in the most critical region of the flow. In all cases the usual criterion [8] for the time step Δt_m has been realized: $2\times10^{-7} \le \Delta t_m \le 1\times10^{-6}$ s. Under these conditions, aerodynamic coefficients and gas-dynamic parameters have become insensitive to the time step. The location of the external boundary with the upstream flow conditions varies from 1.0D to 1.5D for different flow conditions. Calculations were carried out on a personal computer. The computing time of each variant was estimated to be approximately 5 - 20 h.

RESULTS

Subsonic Rarefied-Gas-Flow Regime

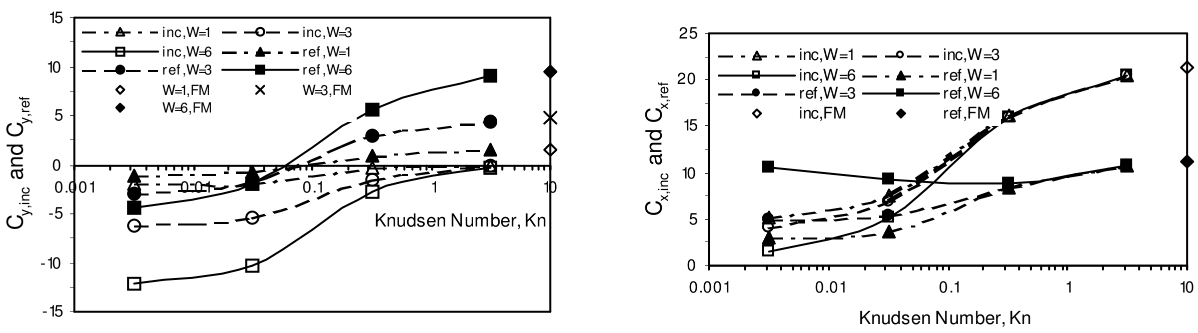
At subsonic flow conditions, the speed ratio, S, becomes small, and the aerodynamic coefficients become very sensitive to its magnitude [7, 11, 12]. In the present paper, the transition flow regime has been studied numerically at $M_\infty = 0.15$, $\gamma = 7/5$ (nitrogen), $\gamma = 5/3$ (argon gas), $\gamma = 9/7$ (carbon dioxide), $t_w = 1$, and $W = 1$, 3, and 6.



(a) lift coefficient C_y **(b) drag coefficient C_x**

FIGURE 1. The aerodynamic coefficients of a spinning cylinder vs. Knudsen number $Kn_{\infty,D}$ in the flow of nitrogen at $M_\infty = 0.15$ and different spin rates: $W = 1$, $W = 3$, and $W = 6$.

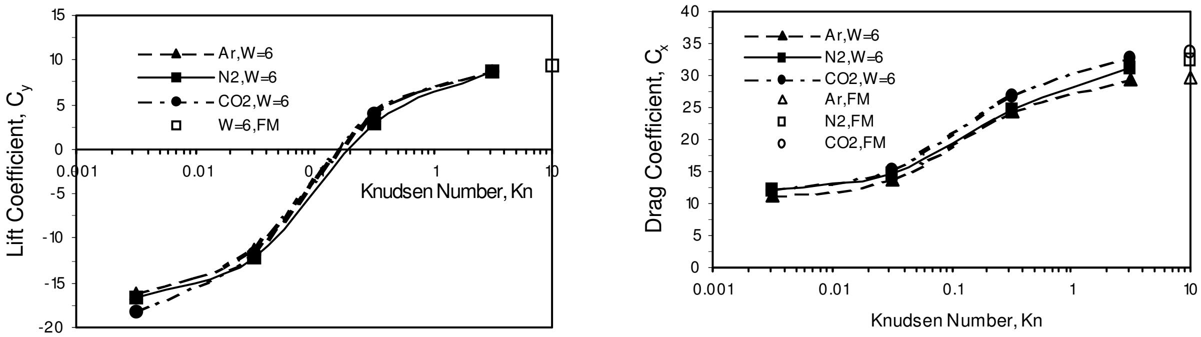
The lift and drag coefficients of a spinning cylinder in the supersonic flow of nitrogen are shown in Figs. 1a and 1b, respectively. In the transition flow regime ($Kn_{\infty,D} > 0.03$), both the incident and reflected molecules significantly influence the lift. The incident molecules dominate when $Kn_{\infty,D} < 0.1$, and the reflected molecules dominate when $Kn_{\infty,D} > 0.1$ (see Fig. 2a). Under these conditions, the lift coefficient changes sign for the cylinder spinning in counter-clockwise direction, and the drag coefficient becomes a function of the spin rate (see Figs. 1a, 1b). The values of $C_{y,r}$ and C_x at $Kn_{\infty,D} > 3$ are near the magnitudes of the lift $C_{y,FM}$ and drag $C_{x,FM}$ coefficients calculated from Eqs. (1-5) for the free-molecule flow.



(a) lift coefficients $C_{y,incident}$ and $C_{y,reflected}$ **(b) drag coefficients $C_{y,incident}$ and $C_{y,reflected}$**

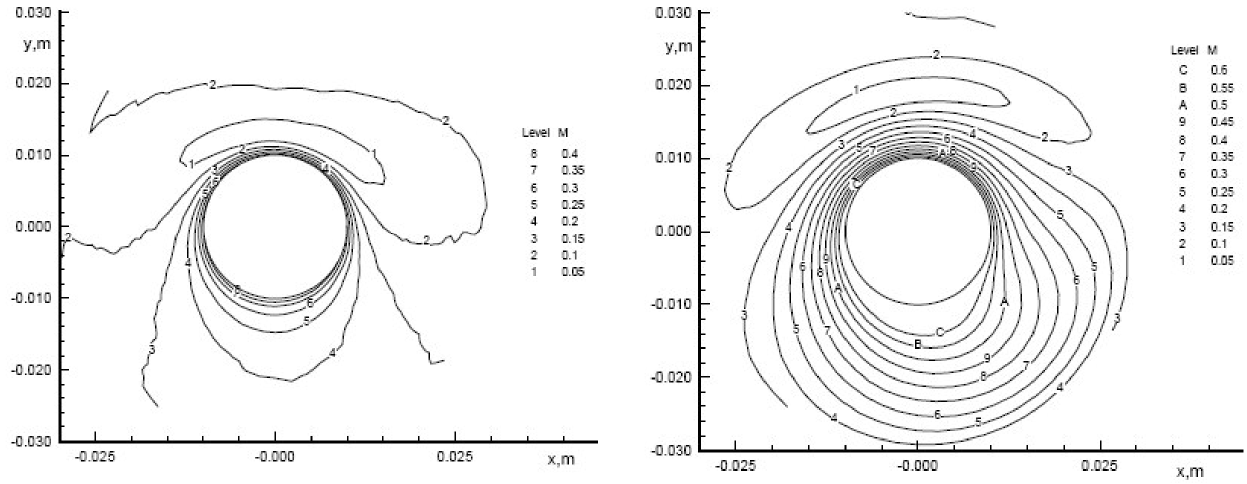
FIGURE 2. Components of lift and drag coefficients of a spinning cylinder vs. Knudsen number $Kn_{\infty,D}$ in the flow of nitrogen at $M_\infty = 0.15$ and different spin rates: $W = 1$, $W = 3$, and $W = 6$.

The influence of a ratio of the specific heats γ on aerodynamic coefficients of a spinning cylinder is insignificant. The drag coefficient is sensitive to the parameter γ in near-free-molecule flow regimes [about 15%] (see Fig. 3b). Both parameters W and γ influence more significantly the lift in continuum-flow regimes (see Fig. 3a).



(a) lift coefficient C_y **(b) drag coefficient C_x**

FIGURE 3. The aerodynamic coefficients of a spinning cylinder vs. Knudsen number $Kn_{\infty,D}$ in the flows of argon, nitrogen, and carbon dioxide at $M_\infty = 0.15$ and $W = 6$.



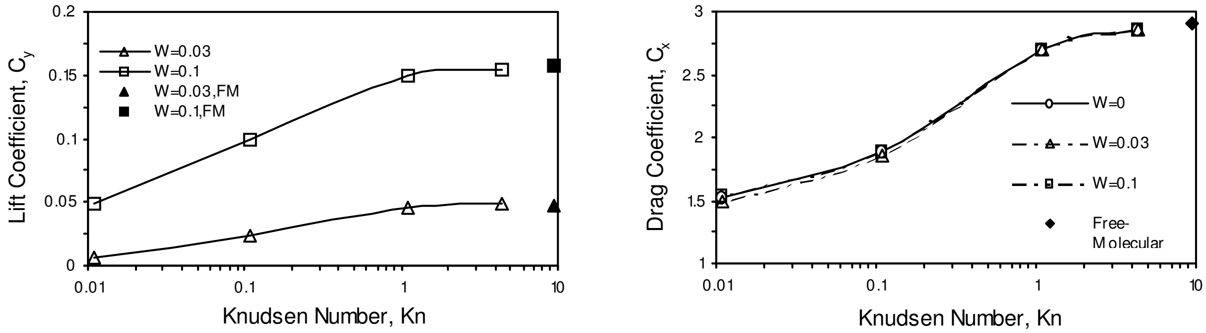
(a) $Kn_{\infty,D}$ = 3.18 **(b) $Kn_{\infty,D}$ = 0.032**

FIGURE 4. Contours of constant Mach numbers near a spinning cylinder in the flow of nitrogen at various Knudsen numbers, $M_\infty = 0.15$, and $W = 6$.

The flow-field patterns near a spinning cylinder at near-free-molecule ($Kn_{\infty,D} = 3.18$) and near-continuum ($Kn_{\infty,D} = 0.032$) flow regimes were studied by Riabov [11, 12] for flows of argon. The contours of constant Mach numbers near a spinning cylinder in the flow of nitrogen at various Knudsen numbers, $M_\infty = 0.15$, and $W = 6$ are shown in Fig. 4a ($Kn_{\infty,D} = 3.18$) and Fig. 4b ($Kn_{\infty,D} = 0.032$). The character of the flow is absolutely different in these cases. The zone of circulating flow is much wider in continuum-flow regime, and its width is comparable with the radius of a cylinder (see Fig. 4b). In the near-free-molecule flow regime, the asymmetry of the flow in upper and bottom regions is significant. The major disturbances of the flow parameters are concentrated in the vicinity of the spinning surface (see Fig. 4a). In the opposite case of near-continuum flow regime, the spinning effect changes significantly the flow pattern in the area far from the surface (see Fig. 4b). These differences in flow patterns dominate the character of molecule-surface interactions.

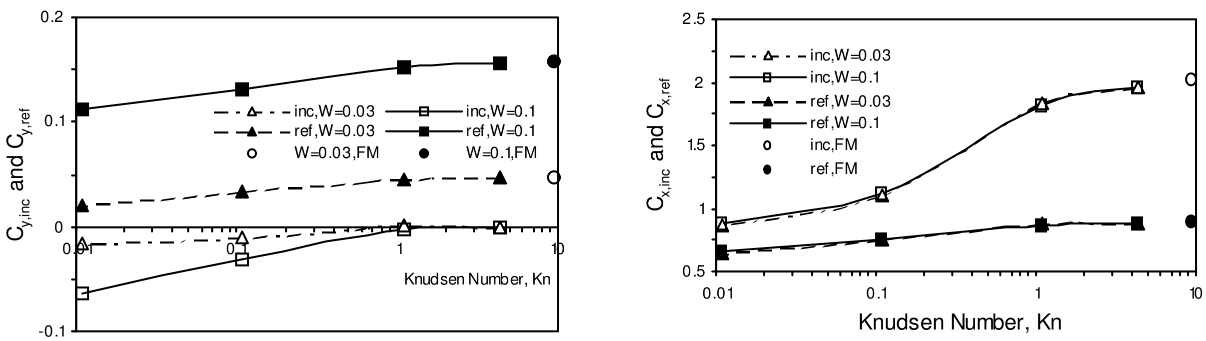
Supersonic Rarefied-Gas-Flow Regime

At supersonic flow conditions, the speed ratio, S, becomes large, and the aerodynamic coefficients become less sensitive to its magnitude [7, 11, 12]. In the present study, the transition flow regime has been investigated numerically at $M_\infty = 10$, $\gamma = 7/5$ (nitrogen), $\gamma = 5/3$ (argon gas), $\gamma = 9/7$ (carbon dioxide), $t_w = 1$, and $W = 0.03$ and 0.1.



(a) lift coefficient C_y **(b) drag coefficient C_x**

FIGURE 5. Lift and drag coefficients of a spinning cylinder vs. Knudsen number $Kn_{\infty,D}$ in the flow of nitrogen at $M_\infty = 10$ and different spin rates: $W = 0.03$ and $W = 0.1$.

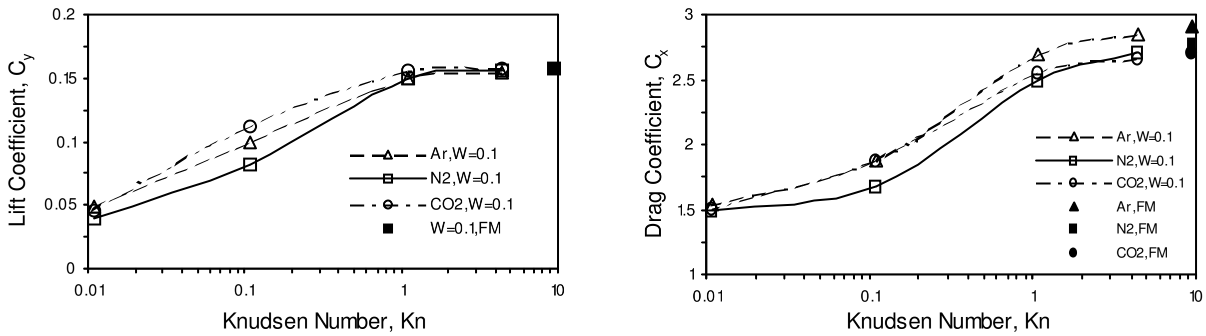


(a) lift coefficients $C_{y,incident}$ and $C_{y,reflected}$ **(b) drag coefficients $C_{x,incident}$ and $C_{x,reflected}$**

FIGURE 6. Components of lift and drag coefficients of a spinning cylinder vs. Knudsen number $Kn_{\infty,D}$ in the flow of nitrogen at $M_\infty = 10$ and different spin rates: $W = 0.03$ and $W = 0.1$.

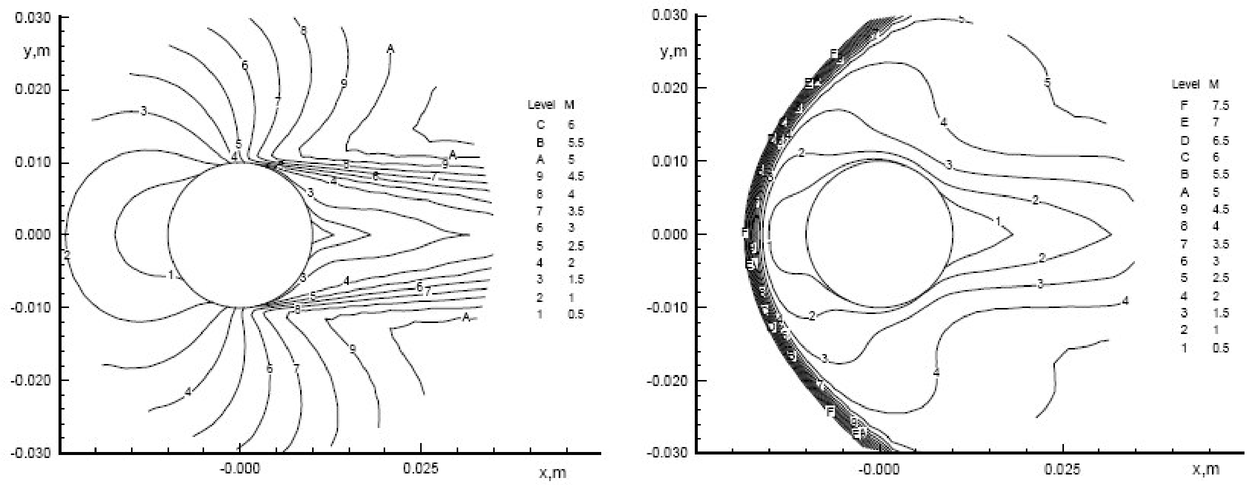
The lift and drag coefficients of a spinning cylinder in the supersonic flow of nitrogen are shown in Figs. 5a and 5b, respectively. For the lift coefficient, the influence of reflected molecules is dominant in the transition-flow regime ($Kn_{\infty,D} > 0.01$). The incident-molecule input becomes significant at Knudsen number $Kn_{\infty,D} < 0.1$ (see Fig. 6a). Under the considered flow conditions, the lift coefficient has a positive sign (which is opposite to the sign under the continuum flow regime) for the cylinder spinning in a counter-clockwise direction. The drag coefficient is insensitive to the spin rate (see Fig. 5b). Furthermore, the incident part of the drag coefficient $C_{x,i}$ predominates the magnitude of the total drag coefficient C_x (see Fig. 6b). The values of C_y and C_x at $Kn_{\infty,D} > 4$ are near the magnitudes of the lift $C_{y,FM}$ and drag $C_{x,FM}$ coefficients calculated from Eqs. (1-5) for the free-molecule flow.

The influence of a ratio of the specific heats γ on aerodynamic coefficients of a spinning cylinder is significant for supersonic transition rarefied-flow regimes (see Figs. 7a and 7b). The drag coefficient is sensitive to the parameter γ in near freemolecule flow regimes [about 10%] (see Fig. 7b). Both parameters W and γ influence more significantly the lift [about 30%] in transition flow regimes at $Kn_{\infty,D} \sim 0.1$ (see Fig. 7a).



(a) lift coefficient C_y **(b) drag coefficient C_x**

FIGURE 7. The aerodynamic coefficients of a spinning cylinder vs. Knudsen number $Kn_{\infty,D}$ in the flows of argon, nitrogen, and carbon dioxide at $M_\infty = 10$ and $W = 0.1$.



(a) $Kn_{\infty,D}$ = 1.1 **(b) $Kn_{\infty,D}$ = 0.011**

FIGURE 8. Contours of constant Mach numbers near a spinning cylinder in the flow of nitrogen at various Knudsen numbers, $M_\infty = 10$, and $W = 0.1$.

The patterns of supersonic flows of nitrogen near a spinning cylinder at near-free-molecule ($Kn_{\infty,D}$ = 1.1) and near-continuum ($Kn_{\infty,D}$ = 0.011) flow regimes are significantly different (see Figs. 8a and 8b, respectively). In both cases, for a small spin-rate, $W = 0.1$, the zones of circulating flow are located in the vicinity of the surface, and they do not affect significantly the flow zones located far from the surface.

In the case of near-continuum flow, the spinning effect changes significantly the flow pattern in the area near the surface (see also Ref. 11). The flow pattern becomes slightly asymmetrical in this case (see Fig. 8b). These differences in flow patterns dominate the character of molecule-surface interactions, and they characterize the differences in the performance parameters under significantly distinct flow conditions (see Figs. 5 and 6).

CONCLUSIONS

The aerodynamic coefficients of a spinning infinite cylinder have been evaluated numerically for a range of three similarity parameters: Knudsen number, roll parameter, and a ration of specific heats. It has been found that the lift force on a spinning cylinder at subsonic upstream conditions has different signs in the continuum and free-molecule flow regimes. The location of the sign change is in the transition rarefied-gas flow regime near $Kn_{\infty,D}$ ~ 0.1. The major factor of influence is the magnitude of momentum of the reflected and incident molecules, which depends on the value of the Knudsen number. The spinning parameter influences significantly the flow pattern around the cylinder as well as the force magnitude. At the supersonic upstream flow conditions, the lift coefficient has a positive sign in the transitional and free molecular regimes (which is opposite to the sign under the continuum flow regime) for the cylinder spinning in a counter-clockwise direction. The supersonic drag coefficient is insensitive to the spin rate, and the incident component dominates the magnitude of the total drag coefficient. Both lift and drag coefficients depend slightly on the ratios of the specific heats.

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