The Normal Momentum Accommodation Effect on Thin Plate Oscillations in a Rarefied Gas

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Abstract. The problem of free damping plate oscillations in a rarefied gas has been studied by variation method. An influence of gas-surface interaction on oscillation process has been considered. Analytical expression for damping coefficient of the plate vibration as a function of normal momentum accommodation coefficient has been obtained.

Keywords: Gas – Surface Interaction, Normal Momentum Accommodation, Plate natural vibrations, Variation method.
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INTRODUCTION

Gas-surface interaction study in aspects of energy and momentum exchange between gas molecules and surface atoms as well as chemical adsorption is a topical basic problem that has many important applications. As an example one can refer to the development of new gas driven micro devices and MEMS technologies.

An attempt to apply a new approach to the problem of normal momentum exchange using vibration technique is realized in [1, 2]. The main idea of this approach consists in processing data on silicon single crystal plate oscillations in a rarefied gas. The oscillations initially created by variable electric field are damped due to gas-surface interaction and inelastic processes in the plate. The main measuring parameter that characterizes behavior of the system is damping coefficient. The preliminary measurements based on the use of this experimental approach have demonstrated significant change in a damping process as a result of gas-surface interaction. In particular, the frequency shift and decay curve alteration have been observed for a wide range of Knudsen numbers. This fact can be interpreted as a confirmation of high sensitivity of the developed method to gas-surface interaction parameters. However for an effective use of this approach the further development of the theory is required. In particular, it is necessary to reconsider some theoretical assumptions used at drawing up of the basic equation for the plate vibrations and to specify contribution of geometrical factors and balance of the forces applied to the plate from the gas.

The goal of this research is specification of the basic equation describing process of an elastic plate oscillation in a gas and obtaining expression for main parameter measured in experiment as a function of normal momentum accommodation coefficient.

BASIC EQUATION

Due to relative simplicity combined with high efficiency variation methods are widely used for studying an elastic body deformation. In order to write down the equation describing process of the plate movement in a gas it is convenient to construct functional of the moving plate and determine its extreme by the use of variation Ritz method.

Functional variation equation written for whole volume of the elastic body has the following form

$$\delta\int_{t_0}^{t}\int_{v}\left(\rho\frac{\upsilon^2}{2}+U(\vec{X})\right)dv\,dt+\delta B=0\,, \tag{1}$$

$$\delta B = \int_{\Omega} (\vec{p}^{\,n})^i \delta w_i d\sigma + \int_{v} \rho F_i \delta w_i dv - \int_{v} \rho s \delta T dv\,, \tag{2}$$

where $U(\vec{X})$ -potential energy of volume element of the body, ρ -material density, t_0, t - initial and present time respectively, $\vec{\upsilon}$ -velocity of the volume element, $\delta w_i(\vec{X})$-displacement deviations, dv-volume element, δT - temperature deviations of elasticity body, $\vec{F}$ -mass force applied to the body, s -entropy of volume element, $\vec{X} = (x_1, x_2, x_3)$, Ω - surface area,

$$(\vec{p}^{\,n})^i = p_{ij}\vec{n}_j\,,$$

here $\vec{n}_j$ - normal vector of j axis, p_{ij} - stress tensor.

Taking into account that body temperature is constant last item in expression for δB can be discarded. Some principal components of p_{ij} and $U(\vec{\mathrm{X}})$ must be determined.

FORCE APPLIED TO THE PLATE

In this study the free damping oscillations of the square thin plate with rigidly fixed edges are considered. The plate is placed in the rarefied gas. Free molecular gas conditions are provided by high ratio of mean free path of gas molecules to linear plate dimensions.

Accepting small curvature of the plate for the first degree of approximation one can neglect tangential stress to the plate surface.

According to the assumption regarding smallness of the plate curvature the normal vector to any element of the surface can be considered as parallel to z-axis (Figure 1). Inside the moving plate there is so-called “neutral layer” [3], where any tensions and compressions are absent. The neutral layer point’s movement is described by transverse displacement of the in-plate co-ordinates x and y as $\vec{\omega}$.

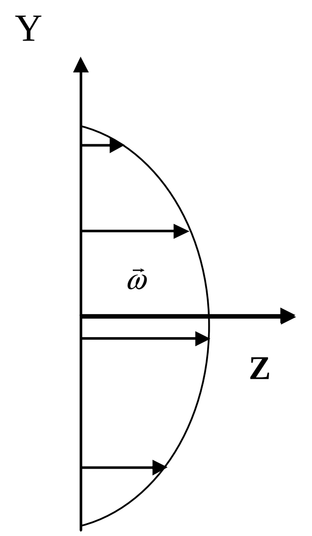

FIGURE 1. Profile the neutral layer point’s displacements of plate.

Therefore, the vector of displacements of points of the neutral layer can be presented as

$$\vec{\omega} = (0,0,\omega_0)\,, \tag{3}$$

where $\omega_0 = f(x,y,t)$. For small displacement the points of the neutral layer move in perpendicular directions relatively to the plane of initial plate position.

Let us introduce the coordinate system $x^{'}, y^{'}, z^{'}$ connected with the moving plate surface.

If to take into account that unit normal to any surface element is parallel to z-axes the components of molecule velocities can be written as

$$\begin{cases} \vartheta_x = \vartheta_{x^{'}} \\ \vartheta_y = \vartheta_{y^{'}} \\ \vartheta_z = \vartheta_{z^{'}} \pm \dfrac{\partial \omega_0}{\partial t} \end{cases} \tag{4}$$

The different sign at $\dfrac{\partial \omega_0}{\partial t}$ connected with the opposite plate sides [4].

The normal momentum flux to the elementary surface area can be expressed as

$$p_i = \frac{\rho_\infty R T_\infty}{\pi^{1/2}} (B \exp(-B^2) + \pi^{1/2} (\frac{1}{2} + B^2)(1 + erf(B)), \tag{5}$$

where $\beta = \dfrac{1}{\sqrt{2RT_\infty}}$; $B = \pm \dfrac{1}{\sqrt{2RT_\infty}} \dfrac{\partial \omega_0}{\partial t}$, ρ_∞ and T_∞ - density and temperature of undisturbed gas correspondingly, R - gas constant.

According to accepted Knudsen definition of accommodation coefficient the equation for normal momentum flux created by reflected molecules can be presented as

$$p_r = (1 - \alpha_n) p_i + \alpha_n p_w . \tag{6}$$

The expression for total normal momentum flux applied to the plate has a view as

$$p = p_i + p_r = (2 - \alpha_n) p_i + \alpha_n p_w \tag{7}$$

where α_n - so-called normal momentum accommodation coefficient, p_i - normal momentum flux created by incident molecules, p_w - normal momentum flux caused by the influence of reflected molecule having Maxwell distribution function with zero macroscopic velocity and with the temperature of the surface.

Assuming velocity loss of diffuse reflected molecules that leave the surface with equilibrium distribution function following to [5] one could write p_w as

$$p_w = \frac{m}{2} \sqrt{2\pi R T_\infty} N_i , \tag{8}$$

where m - molecule mass, N_i - number of molecules striking the unit surface per unit time:

$$N_i = n_\infty \sqrt{\frac{RT_\infty}{2\pi}} (\exp(-B^2) + \sqrt{\pi} B (1 + erf(B)), \tag{9}$$

where n_∞ - gas numerical density at infinity.

Oscillations are believed to be at quasi-steady condition. Hence, it can be considered that $B^2 << 1$.

Finally, the normal pressure to the surface unit becomes

$$P = P_\infty \pm \rho_\infty \sqrt{\frac{2RT_\infty}{\pi}} \frac{\partial \omega_0}{\partial t} (\alpha_n \frac{\pi}{4} + 2 - \alpha_n), \tag{10}$$

P_∞ - gas pressure at infinity, $\rho_\infty = n_\infty m$.

POTENTIAL ENERGY OF THE PLATE

Elastic body presents the plate of cubic syngony with (100) orientation.

Density of strain energy U in Hookian approximation is a quadratic function of components of the deformation tensor [6]:

$$U = \frac{1}{2}\sum_{\lambda=1}^{6}\sum_{\mu=1}^{6} C_{\lambda\mu} e_\lambda e_\mu \, , \tag{11}$$

where e_λ, e_μ - deformation tensors, λ, μ indexes are determined as

$$1 = xx\, ,\ 2 = yy\, ,\ 3 = zz\, ,\ 4 = yz\, ,\ 5 = zx\, ,\ 6 = xy\, ,$$

$C_{\lambda\mu}$ - components of the elasticity tensor (cubic syngony) can be written in a matrix form:

$$C_{\lambda\mu} = \begin{pmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{pmatrix}.$$

Hence,

$$U = \frac{1}{2}c_{11}(e_{xx}^2 + e_{yy}^2 + e_{zz}^2) + \frac{1}{2}c_{44}(e_{yz}^2 + e_{zx}^2 + e_{xy}^2) + c_{12}(e_{yy}e_{zz} + e_{zz}e_{xx} + e_{xx}e_{yy})\, . \tag{12}$$

Taking into account assumptions made above and using correspondent expressions from [3] one can obtain

$$e_{xx} = -z\frac{\partial^2\omega_0}{\partial x^2}\, ;\ e_{yy} = -z\frac{\partial^2\omega_0}{\partial y^2}\, ;\ e_{xy} = -2z\frac{\partial^2\omega_0}{\partial x\partial y}\, ;\ e_{zz} = e_{xz} = e_{yz} = 0\, . \tag{13}$$

Expression for U can be obtained by replacing (13) in (12)

$$U = \frac{1}{2}c_{11}z^2\left(\left(\frac{\partial^2\omega_0}{\partial x^2}\right)^2 + \left(\frac{\partial^2\omega_0}{\partial y^2}\right)^2\right) + 2c_{44}z^2\left(\frac{\partial^2\omega_0}{\partial x\partial y}\right)^2 + c_{12}z^2\frac{\partial^2\omega_0}{\partial x^2}\frac{\partial^2\omega_0}{\partial y^2}\, . \tag{14}$$

Finally for the whole plate it becomes:

$$U = \frac{h^3}{3}\iint\left(\frac{1}{2}c_{11}\left(\left(\frac{\partial^2\omega_0}{\partial x^2}\right)^2 + \left(\frac{\partial^2\omega_0}{\partial y^2}\right)^2\right) + 2c_{44}\left(\frac{\partial^2\omega_0}{\partial x\partial y}\right)^2 + c_{12}\frac{\partial^2\omega_0}{\partial x^2}\frac{\partial^2\omega_0}{\partial y^2}\right)dxdy\, . \tag{15}$$

RESULTS

Present consideration of the variation task is based on following assumptions.

a) The damping of the plate oscillations caused by internal stresses is negligible in comparison with the damping due to external forces due to gas-surface interactions.
b) The plate produces the small oscillations in strongly perpendicular directions to the plane of unmoved plate.
c) The oscillations are believed to have a single harmonic.

Equations (1) and (2) are transformed to the following form:

$$\delta\int_{t_0}^{t_1}(T + U)dt + 2\int_{t_0}^{t_1}\iint_{\Omega}\rho_\infty\sqrt{\frac{2RT}{\pi}}\frac{\partial\omega_0}{\partial t}\left(\alpha_n\frac{\pi}{4} + 2 - \alpha_n\right)\delta\omega_0\, dxdydt = 0\, , \tag{16}$$

where T -kinetic energy of the plate determined as

$$T = \frac{\rho h}{2}\iint_{\Omega}\left(\frac{\partial\omega_0}{\partial t}\right)^2 dxdy\, , \tag{17}$$

here h - plate thickness.

Determination of the extreme of the functional (16) is made by the Ritz method.

The minimization form is taken in the following view:

$$\omega_0(x,y,t)=\sum_{n,m} a_{nm}(t)\sin^2\frac{\pi xn}{a}\sin^2\frac{\pi ym}{a}, \tag{18}$$

where a - side length of the plate, $n, m \in (0,\infty)$.

This form must satisfy to boundary conditions corresponding to the case of the plate with rigidly fixed edges.

$$\omega_0 = 0 \text{ at } x=0, x=a \text{ and } y=0, y=a;$$

$$\frac{\partial\omega_0}{\partial x}=0 \text{ at } x=0, x=a; \quad \frac{\partial\omega_0}{\partial y}=0 \text{ at } y=0, y=a.$$

In addition to the conditions mentioned above, function $\omega_0(x,y,t)$ in continuous region $G\in[0,a]\times[0,a]$ at $n=1, m=1$ has a single extreme point in the center of the plate $(\frac{a}{2};\frac{a}{2})$, which corresponds to the oscillation condition with single harmonic and the basis functions $\sin^2\frac{\pi xn}{a}$ and $\sin^2\frac{\pi ym}{a}$ have the spanning property.

Using correspondent replacements and some lemmas of variation calculus, one can obtain the definitive equation

$$\rho h a^2\frac{d^2a(t)}{dt^2}+2\rho_\infty\sqrt{\frac{2RT}{\pi}}(\alpha_n\frac{\pi}{4}+2-\alpha_n)a^2\frac{da(t)}{dt}+\frac{32h^3\pi^4}{27a^2}(3c_{11}+2c_{22}+c_{12})a(t)=0. \tag{19}$$

Finally, the damping coefficient of plate oscillations can be presented as a function of normal momentum accommodation coefficient by the following way

$$\beta=\frac{\rho_\infty}{\rho h}\sqrt{\frac{2RT}{\pi}}(\alpha_n\frac{\pi}{4}+2-\alpha_n). \tag{20}$$

This expression can be compared with the result obtained earlier [1]:

$$\beta=\frac{\rho_\infty}{\rho h}\sqrt{\frac{2RT}{\pi}}(2-\alpha_n). \tag{21}$$

The normal momentum accommodation coefficient dependence of β in relative units obtained from equations (20) and (21) is shown in Figure 2.

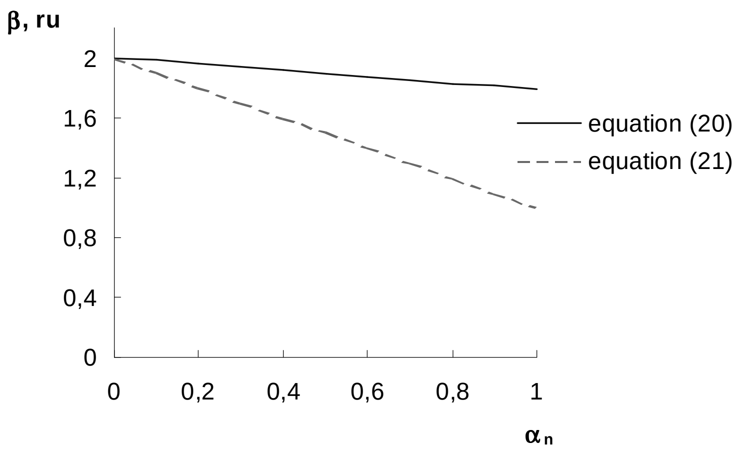

FIGURE 2. Normal momentum accommodation coefficient dependence of β related to $\rho_\infty\sqrt{\frac{2RT}{\pi}}\Big/\rho h$ for equations (20) and (21).

The contribution of normal momentum accommodation in damping coefficient according to equation (20) is about 20% at $\alpha_n \approx 1$. The difference between β obtained from equations (20) and (21) can reach up 78%.

Thus, equation (20) is more preferable to processing experimental data especially for the gases with normal momentum accommodation coefficient close to one.

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REFERENCES

1. P.J. Polikarpov et.al. *J. Appl. Mech. & Tech. Phys.*, **44**, 298-303 (2003).
2. A.P. Polikarpov, P.J. Polikarpov, S.F. Borisov "A Silicon Crystal Microbalance for Normal Momentum Transfer Study in a Gas-Surface System " in *24-th Int. Symp. on Rarefied Gas Dynamics*, edited by M.Capitelli, AIP Conference Proceedings 762, American Institute of Physics, Melville, NY, 2005, pp. 977-980.
3. L.D. Landau, E.M. Lifshitz, *Theory of elasticity*, Oxford-New-York: Pergamon Press, 1970.
4. N.T. Pashchenko, *J. Appl. Math. & Mech.*, **23**, 760-765 (1959).
5. V.P. Shidloskiy, *Introduction to the dynamics of rarefied gases*, New York: American Elsevier Publishing Company Inc., 1967.
6. C. Kittel, *Introduction to solid state physics*, New York: Wiley, 1972.