Quantifying the Effects of Rarefaction in High Velocity, Slip-Flow Regime

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Abstract. The present work follows two former papers by the first author where the effects of rarefaction in transitional regime were considered. In the first paper these effects were evaluated on the aerodynamic coefficients of a re-entering sphere-cone capsule. In the second paper parameters, quantifying non-equilibrium (anisotropy and thermodynamic non equilibrium), were proposed. Both studies were carried out changing at the same time velocity and altitude. In the present paper a sensitivity analysis, for identifying the most critical parameter for non-equilibrium, and an evaluation of the influence of non-equilibrium, slip velocity and temperature jump (slip effects) on the drag and heat transfer coefficients are carried out. Parameters quantifying the slip effects are also proposed. Computer simulations are made on a sphere in slip-flow regime by a: i) DSMC code (DS2V) for the sensitivity analysis and for quantifying the influence of non-equilibrium, ii) Navier-Stokes code (FLUENT) for the evaluation of the slip effects. The sensitivity analysis showed that the influence of the free stream velocity on the non-equilibrium parameters is much stronger than the one of the free stream thermodynamic parameters. Therefore the quantification was carried out changing the free stream velocity. In order to point out only the influence of non-equilibrium and slip, excluding the fluid-dynamic effects, for each variation of the free stream velocity the thermo-dynamic parameters were adapted in such a way to keep the same value of the free stream Mach, Reynolds, Knudsen, Prandtl and Lewis numbers. The results showed that the influence of non-equilibrium and slip on the aerodynamic coefficients is of the order of one percentage unit. A more consistent difference, between the aerodynamic coefficients from DS2V and from FLUENT, is probably due to the failure of the phenomenological equations of Newton, Fourier and Fick.

INTRODUCTION

Even though the balance and the conservation equations of fluid-dynamics are valid in all rarefaction regimes, the solution of the Navier-Stokes equations becomes lacking with increasing rarefaction. In fact, numerical integration of these equations relies also on the computation of shear stress, heat flux and concentration of chemical species. In low-density regimes phenomenological equations of Newton, Fourier and Fick are no longer valid. Furthermore, as the density decreases, the intermolecular collisions in the gas get too few for maintaining the isotropy of the pressure tensor, the conventional no-slip boundary condition can be no longer applied and finally effects such as thermal and pressure diffusion, usually not included in the Navier-Stokes solvers, become more prominent.

The solution of the transitional regime could rely on the extension of the Boltzmann equation to higher density flows or of the Burnett equations to lower density flows. Unfortunately, the numerical solution of the above-mentioned equations shows overwhelming difficulties. A kinetic approach such as the Direct Simulation Monte Carlo (DSMC) method [1] provides a sound solution of this regime.

The present paper is the logical continuation of two former papers by the first author [2, 3] where some of the above mentioned rarefaction phenomena were considered. In both papers computer simulations were made on a typical sphere-cone capsule, re-entering to Earth from an interplanetary mission in the altitude interval between 70 and 120 km; the free stream Knudsen number ranged from 2.4×10^{-4} to 1.1. More specifically, Zuppardi and Paterna verified [2] the influence of rarefaction on the aerodynamic coefficients. Zuppardi introduced [3] parameters for the quantification of non-equilibrium (anisotropy and thermodynamic non-equilibrium) and verified their correctness. Both studies were carried out only in terms of altitude or changing, at the same time, velocity, density, temperature and gas composition. As expected, non-equilibrium increases with altitude; this is due both to an increase of the shock wave intensity (produced by an increase of velocity) and to an increase of rarefaction. In fact, restoring equilibrium occurs only through intermolecular collisions; thus as rarefaction increases, the collision rate decreases leading to a higher level of non-equilibrium.

The aim of the present work is continuing this investigation. More specifically, the effects of non-equilibrium and the effects of the slip velocity and temperature jump (slip effects) will be focused. The study will consist of: i) a sensitivity analysis for the identification of the most critical fluid-dynamic quantity for non-equilibrium. This stage of the work is fulfilled by the DSMC code DS2V [4], ii) an evaluation of the influence of the non-equilibrium and slip effects on the aerodynamic forces and heat transfer. This stage of the work is fulfilled by DS2V for non-equilibrium and by the Navier-Stokes (NS) code FLUENT [5] for slip. Two parameters for the quantification of the slip effects, similar to those for non-equilibrium and anisotropy [3], are also proposed and verified.

Computer simulations are made on a sphere in air at an altitude of 100 km and at the free stream velocity of 6000 m/s; these results are used as reference data both for the sensitivity analysis and for the evaluation of the non-equilibrium and slip effects. The sensitivity analysis consists in evaluating the effects of percentage variations, in the range from –40% to 40%, of each single thermo-fluid-dynamic parameter. In order to avoid spoiling the analysis by the inclusion of fluid-dynamic effects, the free stream thermodynamic parameters and even the chemical composition are changed in such a way to keep the same value of the free stream Mach, Reynolds, Knudsen, Prandtl and Lewis numbers.

The influence of both non-equilibrium and anisotropy on the drag and heat flux (at the sphere stagnation point), of slip velocity on the drag and of temperature jump on the heat flux coefficients is evaluated in terms of the parameters quantifying non-equilibrium, anisotropy, slip velocity and temperature jump.

ANISOTROPY AND THERMODYNAMIC NON-EQUILIBRIUM

The physical process, generating non-equilibrium inside a shock wave, is due to the free stream kinetic energy being primarily converted, by means of molecular collisions, to the thermal energy in the direction perpendicular (x) to the shock wave. The thermal energy is then transferred, by subsequent collisions, to the parallel directions (y, z) and finally to the internal degrees of freedom of the molecules (rotation and vibration) [6]. The components of the translational temperature (T_t: T_{tx}, T_{ty}, T_{tz}), of the pressure tensor and of the components of diffusion velocity of each chemical species are different (anisotropy). Rotational (T_r) and vibrational (T_v) temperatures are also different from each other and from the translational one (thermodynamic non-equilibrium).

In the non-equilibrium processes there are two opposite effects, linked to the free stream parameters (velocity, density, temperature and gas composition): 1) the increase/decrease of velocity increases/decreases the intensity of the shock wave therefore increases/decreases non-equilibrium, 2) the increase/decrease of density increases/decreases the number of collisions therefore the capability of evolving toward equilibrium. Temperature (according to the Variable Hard Sphere model [1]) and gas composition influence the collision cross section, therefore the number of intermolecular collision and finally the capability of restoring equilibrium.

The parameters proposed in [3] for quantifying: i) thermodynamic non-equilibrium, were the differences of the maximum values of the translational and the vibrational temperatures ($\vartheta_{t\text{-}v}=T_{tmax}-T_{vmax}$) and of the translational and the rotational temperatures ($\vartheta_{t\text{-}r}=T_{tmax}-T_{rmax}$), ii) anisotropy, were the differences of the maximum values of the components of the translational temperature ($\vartheta_{x\text{-}y}=T_{txmax}- T_{tymax}$). These parameters have been taken into account in the present paper.

In principle, using the relative differences such as $\vartheta_{x\text{-}y}=(T_{txmax}-T_{tymax})/T_{tmax}$, $\vartheta_{t\text{-}v}=(T_{tmax}-T_{vmax})/T_{vmax}$ and so on, as parameters for quantifying anisotropy and non-equilibrium, is more proper. However, for the tests conditions met both in [3] and in the present paper, that are typical of a re-entry trajectory, the orders of magnitude of the maximum value of temperatures are: T_{txmax} and $T_{tmax}\equiv O(10^4)$, T_{tymax}, T_{vmax} and $T_{rmax}\equiv O(10^3)$. Since the order of magnitude of the values involved in the tests are always the same, it is has been considered more effective quantifying non-equilibrium and anisotropy with parameters based on the absolute differences.

SLIP VELOCITY AND TEMPERATURE JUMP

The slip effects are due to a non-zero bulk velocity (u_s) on the surface of the body and a temperature (T_s), different from that of the body surface. These effects are produced in the Knudsen layer where the molecules approaching the wall collide with those reflected from the wall. A correct evaluation of these parameters should rely on knowledge of the distribution functions of both incoming and reflected molecules [7].

Usually, the evaluation of the slip parameters relies on a simpler method, independent of knowledge of the distribution functions. This method is based on the concept of the mean free path [7]; u_s reads:

$$u_S = A\lambda_W \left(\frac{\partial u}{\partial z}\right)_W \quad (1)$$

where: A=(2-σ)/σ, σ is the accommodation coefficient of the tangential momentum, λ_w is the mean free path at the wall conditions (w) and z is the coordinate of the local normal to the body surface. It is possible to define, in an analogous way, the slip temperature:

$$T_S = T_W + B\frac{2\gamma}{\gamma+1}\frac{1}{Pr_W}\lambda_W\left(\frac{\partial T}{\partial z}\right)_W \quad (2)$$

where: B=(2-α)/α, α is the energy momentum accommodation coefficients, γ is the ratio of constant pressure and constant volume specific heats and Pr is the Prandtl number. A review of more specific values of A and B are reported by Street [8]. Usually the slip temperature is provided in terms of temperature jump: $\Delta T=T_s-T_w$.

Equations 1 and 2 provide the macroscopic boundary conditions usually used in the integration of the NS equations and have the advantage of providing the continuum boundary conditions when λ_w goes to zero. As the free molecule path technique is rather rough, in the present computations, the slip velocity and the temperature jump, input to FLUENT as boundary conditions, are provided by DS2V.

ANALYSIS OF THE RESULTS

Computer simulations have been carried out considering a sphere (diameter 1 m) at an altitude (h) of 100 km [9] and a free stream velocity (V_∞) of 6000 m/s. The free stream Mach (Ma_∞), Reynolds (Re_∞) Knudsen (Kn_∞), Prandtl (Pr_∞) and Lewis (Le_∞) numbers are about 21, 253, 0.12, 0.73 and 0.96, respectively. The Reynolds number downstream a normal shock wave (Re_2) is about 14. Both the Knudsen and the Reynolds numbers are based on the sphere diameter. According to the well-known and accepted classification reported in literature, Kn_∞ and Re_2 agree in stating the flow field to be in slip regime, defined by: $Kn_\infty \equiv O(10^{-1})$ and $Re_2 \equiv O(10)$.

To save space, the profiles along the stagnation line of the translational temperature and its components and of the vibrational and rotational temperatures are not shown here; they are very similar to those reported in [3].

The current values of the non-equilibrium and anisotropy parameters are ϑ_{t-v}=15070 K, ϑ_{t-r}=11918 K and ϑ_{x-y}=31782 K, respectively. These values show that, in the present case, anisotropy is comparable with the thermodynamic non-equilibrium: $\vartheta_{x-y}/\vartheta_{t-v}\cong 2.1$, $\vartheta_{x-y}/\vartheta_{t-r}\cong 2.7$. These values of the non equilibrium and anisotropy parameters as well as of the drag coefficient C_D=1.6013 and of the heat flux coefficient (at the stagnation point) $C_h(0)$=0.6962 are assumed to be reference values in the following sensitivity analysis.

Sensitivity Analysis

A sensitivity analysis has been carried out considering the percentage deviation of non-equilibrium parameters as a function of the percentage variations of each thermo-fluid-dynamic parameter in the range from –40% to 40% (Figs. 1a, b). Symbol P is, in turn, for the free stream velocity (V_∞), temperature (T_∞), density (ρ_∞) and gas composition (α_∞). More specifically, α_∞ represents the mole fraction of nitrogen; the percentage mole fraction of oxygen adapts accordingly.

As shown in Figs.1a and 1b, the free stream velocity is the most critical parameter; the percentage variation of ϑ_{t-v} and ϑ_{x-y} due to velocity, ranges practically in the interval between –60% and 80%. On the other hand the variation of the thermo-dynamic parameters produces only negligible variations. In fact, the maximum deviation, produced by the variation of density, is between –5% and 5%. As already said, restoring isotropy and equilibrium occurs through intermolecular collisions. The variation of the free stream, average number of collisions, produced by the variation of density, range between –60% and 100%. This variation of collisions is not sufficient for producing appreciable effects on the non-equilibrium and anisotropy parameters.

Non-Equilibrium and Slip Effects

Preliminary runs of DS2V and FLUENT preceded the evaluation of the non-equilibrium and slip effects. These runs have been made with V_∞=6000 m/s and at h=65 km, where the test sphere is characterized by a near continuum flow field ($Kn_\infty \cong 4.3\times10^{-4}$ and $Re_2 \cong 3.6\times10^{3}$). This is the lowest possible altitude to run properly DS2V by the available computer (2 Gb RAM, 2.8 Ghz). As expected, the non-equilibrium and the anisotropy parameters are lower than those at 100 km. In fact at 65 km the values of $\vartheta_{t\text{-}r}$ and $\vartheta_{x\text{-}y}$ are 32 and 13037 K, at 100 km are 12164 and 32927 K, respectively.

The evaluation of the non-equilibrium and slip effects has been carried out changing the free stream velocity from 4800 to 8100 m/s (or from –20% to 35% of the reference free stream velocity). Thermodynamic data and chemical composition were set in such a way to limit the percentage variations of Ma_∞, Re_∞, Kn_∞, Pr_∞ and Le_∞ in the range ±0.6% of the values met at 100 km. Furthermore in order to avoid the influence of chemical reactions, air has been considered not-reactive and not-vibrating (ideal gas). Chemical reactions and vibration reduce non- equilibrium.

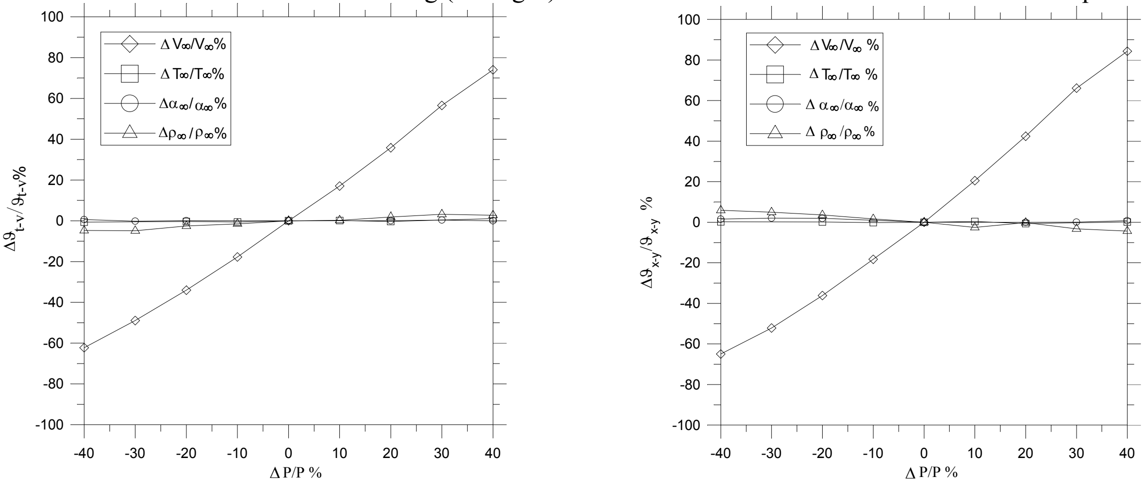

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Figure 1. Percentage variation of the non-equilibrium parameter $\vartheta_{t\text{-}v}$ (a) and of the anisotropy parameter $\vartheta_{x\text{-}y}$ (b) as a function of the percentage variation of a generic thermo fluid-dynamic parameter P: V_∞, T_∞, α_∞, ρ_∞

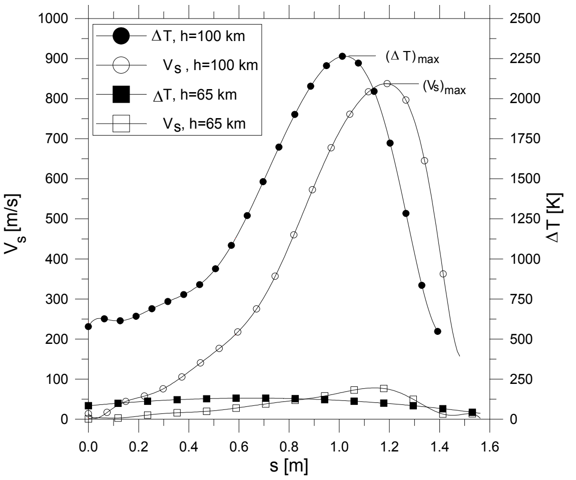

Figure 2. Profiles of slip velocity and temperature jump along the sphere surface as a function of the curvilinear abscissa: V_∞=6000 m/s

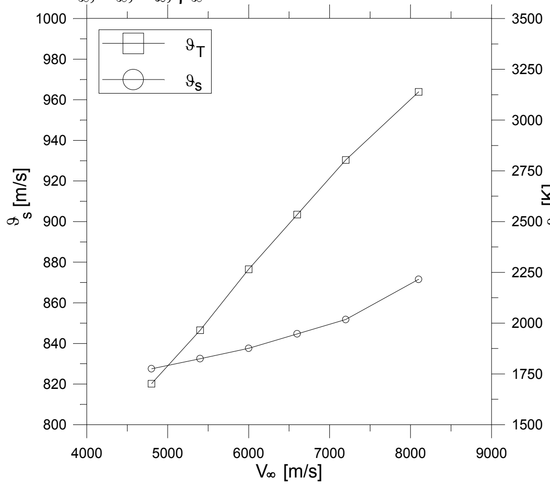

Figure 3. Profiles of the slip parameters as a function of the free stream velocity: h=100 km

In fact $\vartheta_{t\text{-}r}$ and $\vartheta_{x\text{-}y}$, computed at 100 km without chemical reactions and vibration, are lower than those computed with chemical reactions and vibration of about 3.5% and 2%, respectively. This is due to the endothermic effect of the vibrational excitation and of the molecule dissociation.

Figure 2 shows the profiles of the slip velocity and the temperature jump along the sphere surface. In analogy to the definition of the parameters $\vartheta_{x\text{-}y}$, $\vartheta_{t\text{-}r}$, etc., the maximum values of the slip velocity and of the temperature jump can be considered as parameters for quantifying the slip effects: $\vartheta_s=(V_s)_{max}$, $\vartheta_T=(\Delta T)_{max}$. Figure 3 verifies the validity of these parameters. In fact, ϑ_s and ϑ_T increase with V_∞: ϑ_s from 828 to 872 m/s and ϑ_T from 1701 to 3139 K. This had to be expected because these parameters are linked, through $\partial u/\partial z$ and $\partial T/\partial z$ (Eqs. 1 and 2), to the wall skin friction (τ) and to the heat flux ($\dot{q}$), respectively; in the present cases the maximum values of τ and $\dot{q}$ (at the stagnation point) increase from 2.95 to 10.71 N/m^2 and from 1.9×10^4 to 1.2×10^5 W/m^2. Furthermore, both quantities increase of about one order of magnitude from 65 km to 100 km.

In order to evaluate the incidence of the slip effects on drag and heat flux, FLUENT run with and without slip velocity and temperature jump. At 65 km, C_D and $C_h(0)$ from DS2V are 0.9194 and 0.0698. The same coefficients from FLUENT are 0.9283 and 0.0631 for the no-slip condition and 0.9285 and 0.0589 for the slip condition. It has to be pointed out that at 65 km the drag and the heat coefficients from FLUENT (with slip and with no-slip conditions) and from DS2V are rather close.

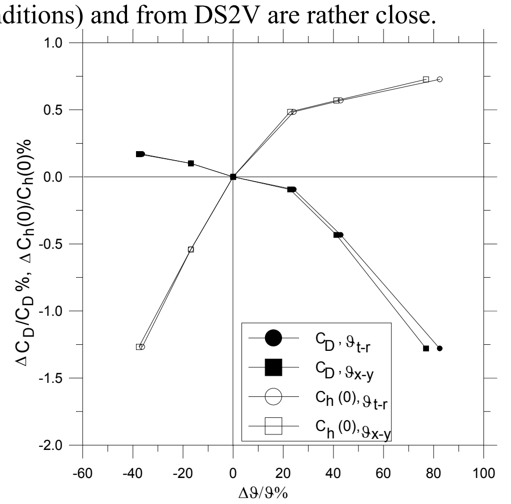

Figure 4. Influence of the non-equilibrium and anisotropy parameters on the drag and heat flux coefficients: $\vartheta = \vartheta_{x-y}$, ϑ_{t-r}

The influence of the variation of non-equilibrium on the aerodynamic coefficients is shown in Fig.4. In order to leave out the slip effects, the aerodynamic coefficients for this part of the analysis are those from DS2V. As said before, the percentage variation of C_D and $C_h(0)$ are correlated to the percentage variations of the non-equilibrium and anisotropy parameters (ϑ is in turn for ϑ_{t-r} and ϑ_{x-y}). The analysis verified that the influence of the thermodynamic non-equilibrium and anisotropy is negligible. In fact, in the interval from -37% to 79% of the non equilibrium parameters, the percentage variation of the drag and of the heat flux coefficients range from 0.2% to -1.3% and from –1.3% to 0.7%, respectively. Furthermore, the influence of non equilibrium on the drag and heat flux coefficients is practically the same for both parameters.

The influence of the slip effects is evaluated by FLUENT. The slip velocity and the temperature jump, computed by DS2V, were input as boundary conditions. A quantification of the slip effects can be carried out from Figs.5a, b where the percentage variations of C_D and $C_h(0)$ between the slip and no-slip values are reported as functions of ϑ_s and ϑ_T, respectively. Also the influence of the slip is negligible. The average percentage variation of C_D and $C_h(0)$, computed with and without slip effects, is -0.42% for C_D and –1.53% for $C_h(0)$) in the whole interval of ϑ_s and ϑ_T.

Figures 6a, b show the drag and the heat flux coefficients at the sphere stagnation point as functions of the percentage variation of the free stream velocity. As expected, both C_D and $C_h(0)$ with slip effects are lower than those computed in no-slip conditions; the maximum percentage reduction for the drag and for the heat flux coefficients are 0.83% and 1.84%, respectively.

The mismatch of the FLUENT results, obtained using the slip effects, with those from DS2V, ranges from 2.8% to 3.3% for C_D and from 11.3% to 5.7 % for $C_h(0)$. This points out that taking into account the slip effects is not sufficient for considering completely the influence of rarefaction.

CONCLUSIONS AND FURTHER DEVELOPMENTS

The incidence of rarefaction on the aerodynamic coefficients (drag and heat flux) has been quantified. More specifically, in the present paper the effects of non-equilibrium (anisotropy and thermodynamic non-equilibrium) and of slip (slip velocity and temperature jump) have been focused. Tests have been carried out on a sphere (1 m diameter) at an altitude of 100 km, velocity of 6000 m/s and in slip flow regime: $Kn_\infty=0.12$, $Re_2=14$.

A sensitivity analysis has been carried out in order to identify the most critical thermo-fluid-dynamic parameters for non-equilibrium. The analysis has been made by a DSMC code, considering a percentage variation from –40% to 40% of the free stream parameters. The analysis showed that the influence of velocity is almost one order of magnitude higher than those of the thermodynamic parameters.

The non-equilibrium and slip effects have been evaluated by a DSMC and a NS code, respectively. The slip data computed by a DSMC code have been input, as boundary conditions, to the NS code. According to the results from the sensitivity analysis, tests have been made changing the velocity in the range from –20% to 35%. For each variation of velocity, the thermodynamic parameters and air composition have been changed in such a way to keep practically the same value of the free stream Mach, Reynolds, Knudsen, Prandtl and Lewis numbers.

The analysis verified that the incidence of both non-equilibrium and of the slip effects is of the order of one percentage unit, therefore practically negligible. The comparison of the results obtained with and without slip effects verified that these effects tend to reduce the drag and the heat transfer coefficients. Furthermore, the results showed that considering the slip effects in a NS code is not sufficient to correctly compute skin friction and heat flux. The mismatch with the DSMC results could be attributed to the failure of the phenomenological equations

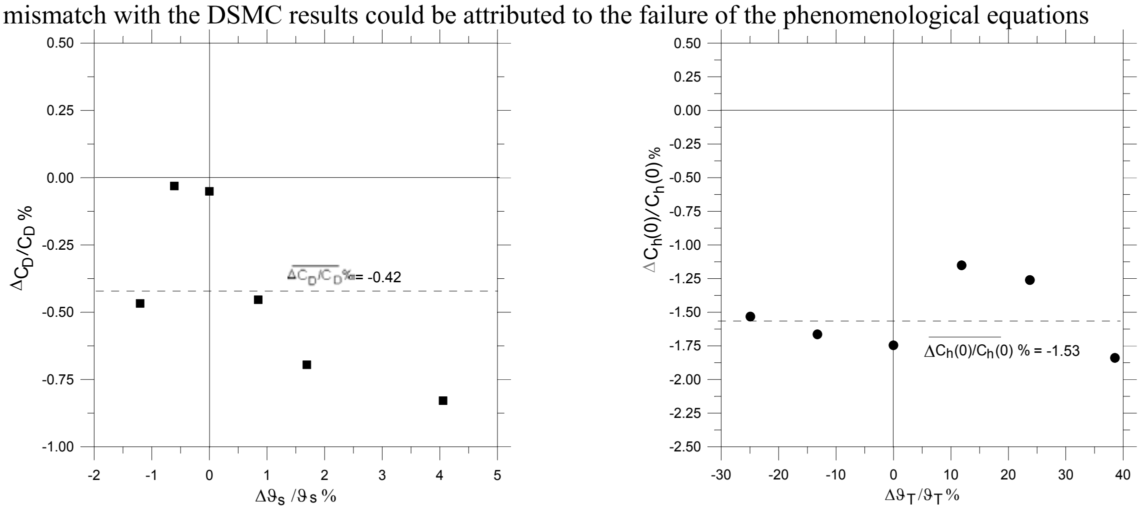

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Figure 5. Influence of the slip velocity (a) and the temperature jump (b) parameters on the drag and heat flux coefficients

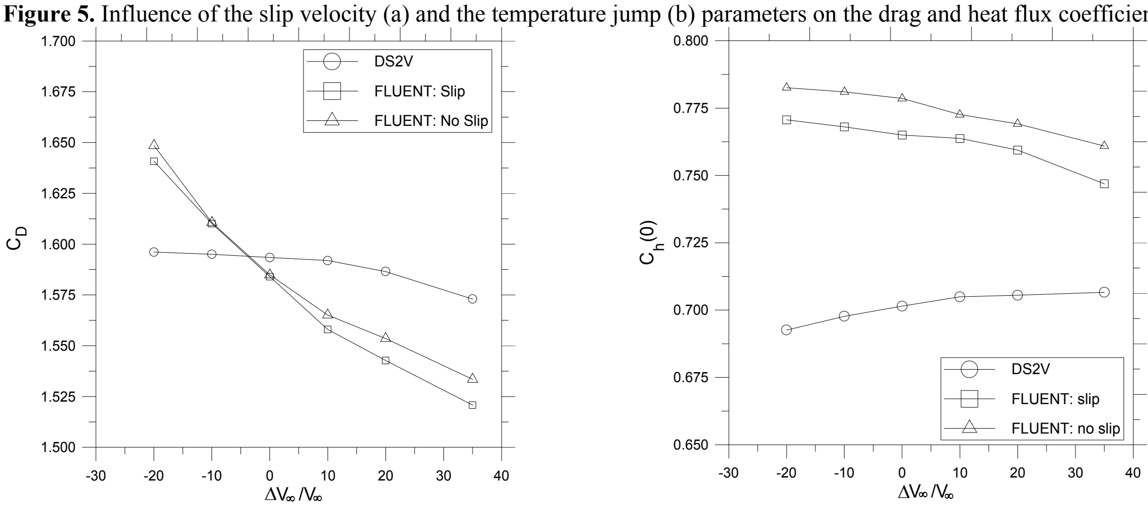

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Figure 6. Profiles of the drag coefficient (a) and of the heat flux coefficient (b) as a function of the percentage variation of the free stream velocity: comparison with the DS2V results

of Newton, Fourier and Fick. Further investigations are scheduled for the evaluation of the non-equilibrium and slip effects at different altitudes and for quantifying the influence of the failure of the phenomenological equations.

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