DSMC Method as Applied to Investigation of Slip Flow of Plasma-Rise Gas in Hollow Cathode

Boris I. Yudin*, Ivan B. Yudin†

*Physics Department of Novosibirsk State Technical University, K. Marx prosp 20, 630092 Russia

†Instutute of Theoretical and Applied Mechanics of Siberian Department of Russian Academy of Science 4/1, Institutskaya str, 630090, Novosibirsk, Russia

Abstract. Rarefied gas flow in transition regime in cylindrical channels of different configurations at Knudsen numbers 0.005÷0.05 is analyzed by the method of direct statistical modeling according to Bird scheme. It calculated on tube diameter as Kn=λ/L, where λ is free path length. In Knudsen layer of a channel with regular cross-section effects of gas slippage on a wall was studied if reflection of molecules is diffusive. A cylindrical channel with extension was investigated to model orifice gas flow in a hollow cathode. Slip factor – Knudsen number relations well agreed with experimental data has been obtained. Analysis of radial and longitudinal functions of hydrodynamic parameters of gas such as velocity, pressure, temperature and density has given laws of shaping initial hydrodynamic path, self-similar flow and output path for flowing to the low-pressure region in relatively short tubes *L/d*~10. It is proved that parabolic profile of velocity for Knudsen numbers 0.01÷0.05 is formed at the distance of 2 or 3 calibers. Modeling of gas flow in a hollow cathode has shown that a supersonic jet having complicated multi-mooring structure with several Mach disks and blast waves is formed in axile part of the cathode hollow. Effect of intensive heating of the cathode wall on change of gas flow in the hollow of the cathode has been investigated.

INTRODUCTION

Short cylindrical channels are applied in many technological devices. Diversity of hydrodynamic and thermal conditions of gas flow in channels of an irregular shape makes it necessary to study different effects in such channels not only by means of experimental but computational methods. For this purpose various modifications of the method of direct statistical modeling of molecular gas dynamics have comprehensive possibilities. The present paper demonstrates feasibilities of the DSMC method to investigate technological devices.

When gas is flown by moving material walls flow characteristic profoundly effected by gas slippage at the solid bound. This effect is analyzed in many papers. Nevertheless development of sufficiently simple and reliable engineering procedure of design of slipping gas flow is an actual problem. It is known that the considered effect appears at low pressure of gas. Here transport properties of gas are determined by Knudsen number. It is established experimentally without sufficient theoretical explanation that the flow characteristic of disk ventilators (DV) decreases at 30 torr pressure. The present paper allows finding gas velocity step at solid surface under definite conditions which is the main cause of velocity decrease in DV at low pressure. The method of direct statistical modeling can be used for a computational experiment to obtain necessary functions of Knudsen number, analyzing effect of medium rarity on gas flow regime. Knudsen number in DV is calculated on length between disks – minimal distance between two surfaces. Because of irregular shape of the channel of DV and limited resources of a computer it is necessary to use more simple models for numeric analysis. Modeling of rarefied monatomic gas flow was made for a straight cylindrical tube. It is supposed that the established laws of rarefied gas flow in a tube are valid for DV.

STATEMENT OF PURPOSE

The numeric model is an axisymmetrical design area with internal hollow of the cylinder which is a solid cylindrical surface. Temperature of the wall is supported 300 K. The temperature of the input gas flow is T_{in}=300 K, the input pressure is p = 146 Pa (1.1 torr). The radius of the cylinder is R_0 = 2.5 mm or the typical size is d=5mm, the length is L = 140 mm. Therefore, rarefied gas flow in a cylinder tube was modeled at low pressure with respect to the initial hydrodynamic path.

Flow was calculated by direct Monte-Carlo method according to Bird scheme. The solid balls model has been selected as a model of particles impaction. It is simple, takes into account laws of impulse and energy conservation and describes ideal rarefied monatomic gas. Increase of gas density leads to erroneous apprehension of molecules as solid balls. However, the method of particles provides relatively low accuracy because all numeric subroutines in programs have low accuracy. Such approach saves significantly calculation time. In spite of low calculation accuracy the particles method reproduces adequately many effects in initial mathematical models inaccessible in other numeric models.

For the tube channel calculation the axial-symmetrical network is used. Number of cells was about 150 000. Number of iterations for the first non-stationary regime was about 130 000 time steps and then statistical data was collected. The statistical sampling for the results has the value about 10 000. Modeling of the flow was carried out with about three million particles in the calc region simultaneously. For the collision the hard sphere model was used. The same parameters were also for the cathode channel because these values are close to computational limit of our computer.

The results of particle stationary motion modeling are processed after long enough calculation time. Calculation time is assumed as the time of flow setting and accumulation of statistical date on particles number per cell and their velocities through definite time intervals. During theses time intervals information obtained at previous intervals becomes out of date. Thus necessary statistical accuracy is provided to calculate such gas-dynamic parameters as velocity, pressure, density and temperature. Moreover deviation of gas flow from equilibrium is established from the results of processing of the temperature components $T_{\|}$ and $T_{\perp}$ by means of the function of molecules distribution on velocities. Macro-parameters of flow and the function of molecules distribution on velocities allow to reproduce the complete picture of flow and to investigate intrinsically non-equilibrium effects.

Flow in the whole design area has cylindrical space symmetry and 2D3V dimension in the velocity space. Calculations were iterative with the time step Δt. Time step was taken as the time of free passage of a molecule in a non-disturbed region. If calculations are dimensionless Δt is taken as one. The input design area of the problem was filled with particles from non-disturbed molecular flow according to known pressure (density) and equilibrium temperature. Particles out of design area are not considered and their indexes in arrays are set to new particles. Function of distribution of incoming particles of the average velocity v_0 and incoming gas temperature T has the following form [1]:

$$F = v_x f(v_x)\, dv_x, \quad \text{where} \quad f(v_x) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{(v_x - v_0)^2}{2RT}}$$

Particles was moving rectilinearly for the time step Δt. Contacting the surface particles was totally diffusively reflected according to the cosine law with respect to the laws of conservation of impulse and energy. Such interaction of ideal molecules with a wall is typical for rarefied gases.

Because of fluctuations the results of the numeric calculations should be correctly interpreted. Relatively small number of particles modeling motion of real molecules lead of substantial fluctuations of the calculated values especially velocity. In real gas velocities of separate molecules also fluctuate near the average but these fluctuations cannot be measured as any measurement device with certain dimensions measures average velocity. Therefore, the method of direct modeling gives more information on particles motion. However, to compare calculation results with experimental data it is necessary to average date as on time as on space. Time average is made by multiple calculation of particles velocities in each point of the design area in the stationary mode. Space average is made in a volume of several lengths of free passage. Thus fluctuations are reduced and smoother functions of concerning values are obtained for the further comparison with experimental data.

GAS SLIPPAGE OF ON A WALL

Slip flow was considered and described by many authors and will be of interest of scientists studying the rare gas flow problems. In [5] authors considered works devoted to slip flow on the plate and many works where gas slippage has influence to the flow close to wall. In main works were considered are devoted to the flows where the slippage coefficients of velocity and temperature are the boundary condition for the flow between the walls. To calculate ones usually the Boltzmann equation are used. In this study the effort of numerical value obtaining of slip coefficient for the rare gas flow in the tube was made.

According to Knudsen, slip factor was calculated from the velocity step and normal derivative of velocity on a wall:

$$\Delta V = \zeta \frac{dV}{dn} . \quad (1)$$

This formula allows to express ζ through the dynamic viscosity (η) and the friction factor on a wall $\xi : \zeta = \eta / \xi$. The DMSC computational experiment allows calculating velocity step and derivative directly from modeling results.

The dependence of the velocity step ΔV on tube length at various Knudsen's numbers is plotted for two values of Kn=0.01 and Kn=0.05 (fig. 1). Knudsen number here was calculated on tube diameter value. It expressed in units of the most probable velocity calculated from input temperature equal for all calculations $V_{mp} = \sqrt{2kT/m} = 352\ m/s$.

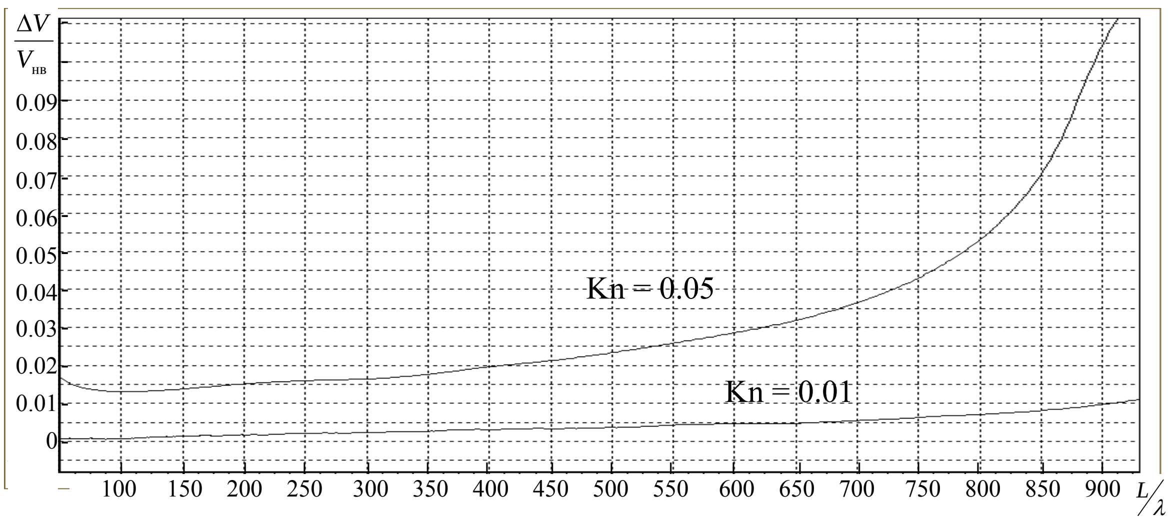
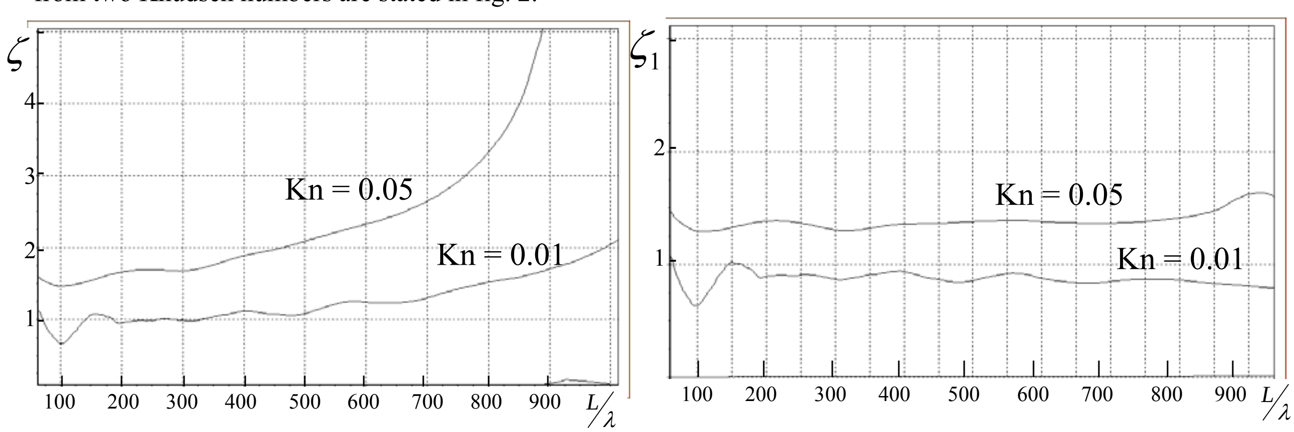


FIGURE 1 Dependence of the velocity step ΔV on tube length at various Knudsen's numbers. Plots are stated form two values of Kn=0.01 and Kn=0.05 in units of the most probable velocity calculated from the input temperature T_{in}=300 K equal for all calculations.

Velocity step intrinsically depends on Knudsen number and is 6÷15% from maximal velocity at flow axis and 12÷50% from mass-average velocity.

The slip factor ζ significantly depend from gas flow velocity and increases on tube length together with velocity. As pressure on tube length decreases product of slip factor by pressure should remain constant. These dependences from two Knudsen numbers are stated in fig. 2.



a) *b*)

FIGURE 2. *a*) Slip factor ζ on tube length at Kn = 0.01 and 0.05. *b*) Product of slip factor by pressure $\zeta_1 = \zeta \cdot p$.

Slip factor obtained experimentally by Brown & Co [4] is proportional to free passage length and is equal to 1.38λ. The calculated slip factor ζ is in agreement with experimental data at moderate velocities. Where the flow has Poiseuille velocity distribution (at beginning of tube) the value ζ has approximate values to Brown's obtained one. The value $\zeta \cdot p$ is theoretically constant on tube length. It also confirmable result by Fig. 2 b.

Radial velocity profiles plotted along the flow are stated in fig. 3. The parabolic velocity profile of rarefied gas is formed at the distance of one caliber from tube input.

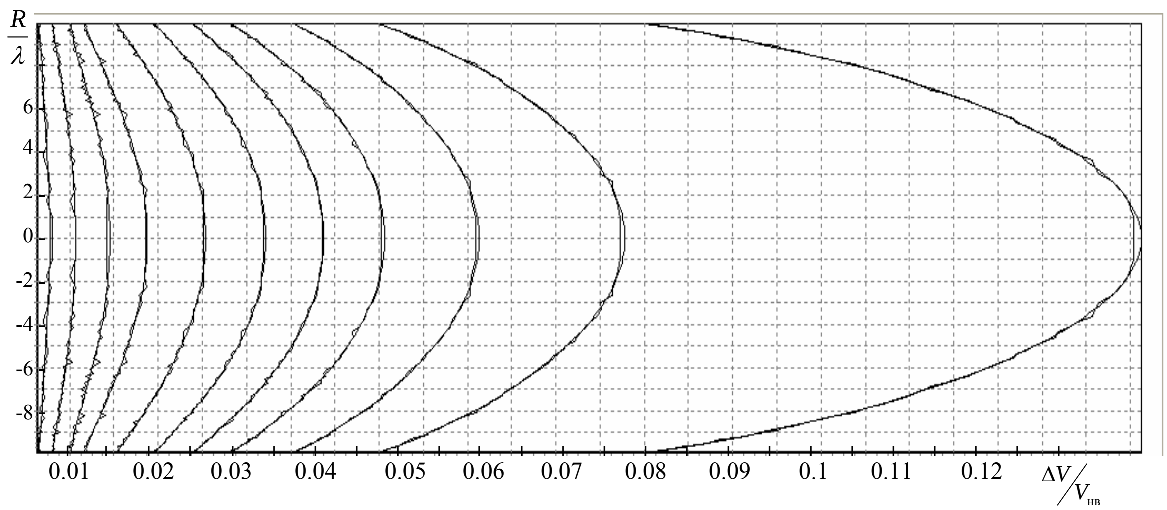


FIGURE 3. Various velocity profiles at the distance of one caliber from tube input.

GAS FLOW IN THE HOLLOW CATHODE

DSMC was applied for research of plasma generation gas in the hollow cathode. The purpose of this numeric investigation was to study the structure of gas flow in the cathode hollow before plasma generation, when cathode walls are heated by glow discharge to temperature enough for intensive thermal emission. As experiments show [3] for hollow cathodes of vacuum plasmathrones there are significant difference of self-nascent gas pressures in cathode cavity and outside one when plasma-rise gas is inflowing in cathode cavity. One can suppose that in this regime *neutral gas* is dominantly flowing in the cathode hollow. Hence it is possible not to consider electrodynamical forces and to model plasma generation gas flow by the method of molecular dynamics. The results of numeric modeling have allowed to interpret the experimental data and to explain conditions of stable discharge ignition including flow rate, pressure and geometrical ratios.

As it was shown in the previous section, gas slippage on the wall noticeably changes rarefied gas flow in tubes as opposed to dense gas with satisfied adhesion condition. Especially it concerns gas flow in channels with drastically changing cross-section. Sudden change of the general dimension (channel diameter) leads to change of free path length of gas molecules and Knudsen number, resulting to drastic flow regime change.

Flow of argon in the hollow cathode until anode-cathode path disruption and plasma generation is important for glow discharge. Conditions of the glow discharge include pressure, density and temperature distribution and the character of the gas velocity field in the cavity of the cathode and in the gas stream from the cathode to a discharge chamber. In the cathode cavity certain gradients of gas dynamic parameters are set to generate plasma and move it to the low pressure chamber. The anode closes the discharge circuit and limits the stream, resulting to gradients of density, pressure and temperature to cause disruption and glow discharge. The Knudsen number calculated on cathode diameter varies from 0.001 to 0.1 along the gas path.

The computer simulation was made by DSMC method with "No Time Counter" scheme [2]. The dimension of the model was 2D3V because of the cylindrical form of the cathode. The design area is determined by the geometrical parameters of the cathode. Gas is fed to the cathode cavity by a narrow tube. The cathode length L is from 3 to 4 of its diameters. The flat anode is perpendicular to the gas stream. The distance between the anode and the stream is equal to L. Velocities of simulated incoming particles were projected on the input cross-section of the cathode tube with respect to equilibrium density of the stream. Boundary conditions for diffuse reflection were set on material walls. The condition of particles absorption was set on the free bound of the stream. Occurrence of residual gas in the discharge chamber was simulated by launching particles through the free bound of the stream having certain temperature and density. The stationary mode was achieved by stabilization of flow rate. The number of simulated particles was more than $3 \cdot 10^6$ to have the average number of particles per cell more than 10.

The simulation gave us space distributions of pressure, density, temperature and velocity vector field patterns for two temperature modes of the cathode: isothermal wall with 300 K temperature; gas output part of the cathode having ¼ of its length warmed up to 2700 K. The second mode simply imitates wall heating by glow discharge. Pressure distribution along the stream axis is stated in Fig. 4. Fig. 5 shows isobars in the cathode cavity if output part of the cathode wall is heated. Both modes cause the underexpanded supersonic stream.

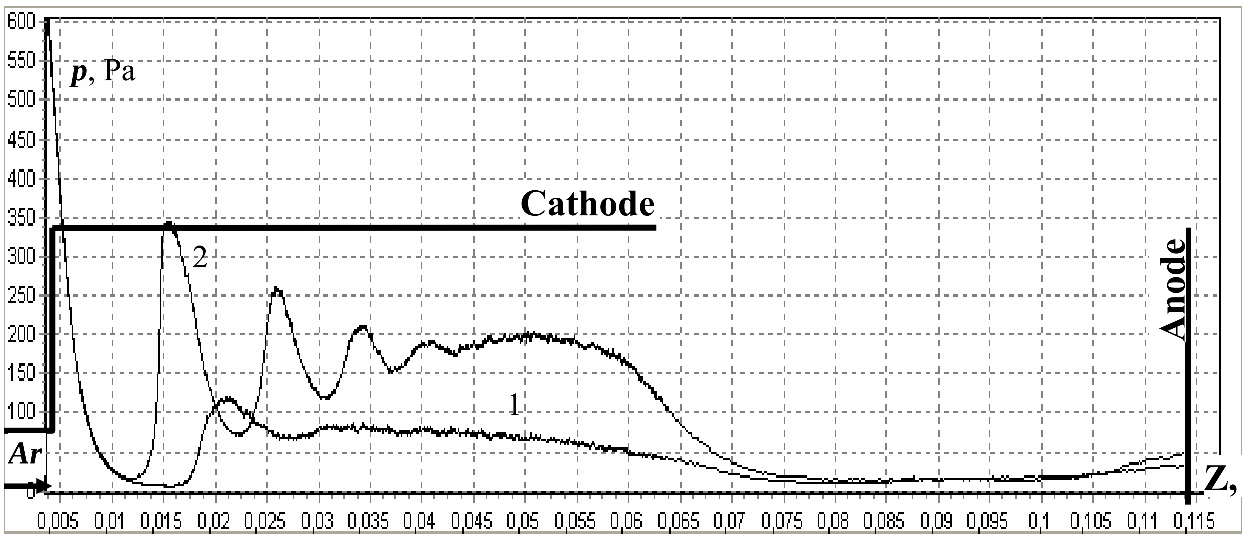


FIGURE 4. Dependence of pressure (p, Pa) on the long axis coordinate (z, m) for two temperature modes of gas flow: 1 – isothermal wall; 2 – warmed-up cathode. Cathode is cylindrical cathode, Anode is flat anode.

Distribution of pressure, density and current lines shows that gas flow in the cathode hollow has the form of a strongly underexpanded stream with diameter a bit more than cathode tube dimension and length up to 2/3 cathode length.

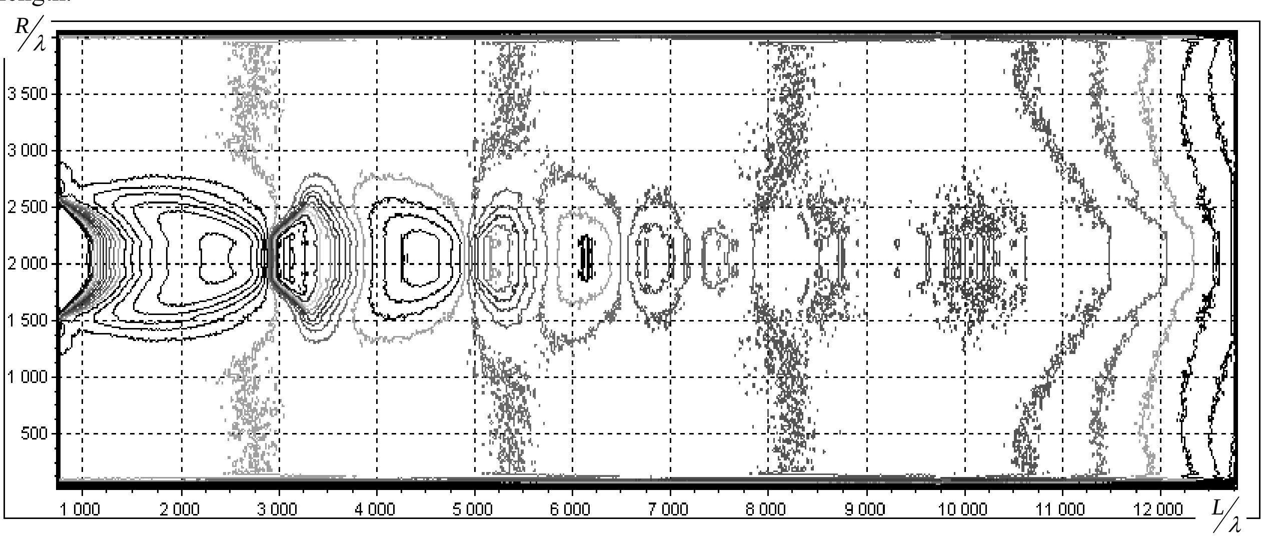


FIGURE 5. Axially symmetric supersonic stream in the cathode cavity on the coordinates in free path lengths.

In the output of the cathode where is intensive heat exchange with heated wall the stream is expanded and fills the whole volume of the cathode (Fig. 5). Change of Mach number on the stream axis and contour curves are stated in Fig.6, 7. Mach number changes in discrete steps and achieves 10 at the first expansion. The following steps become less. After stream disintegration central part of flow is supersonic, in other parts M<1.

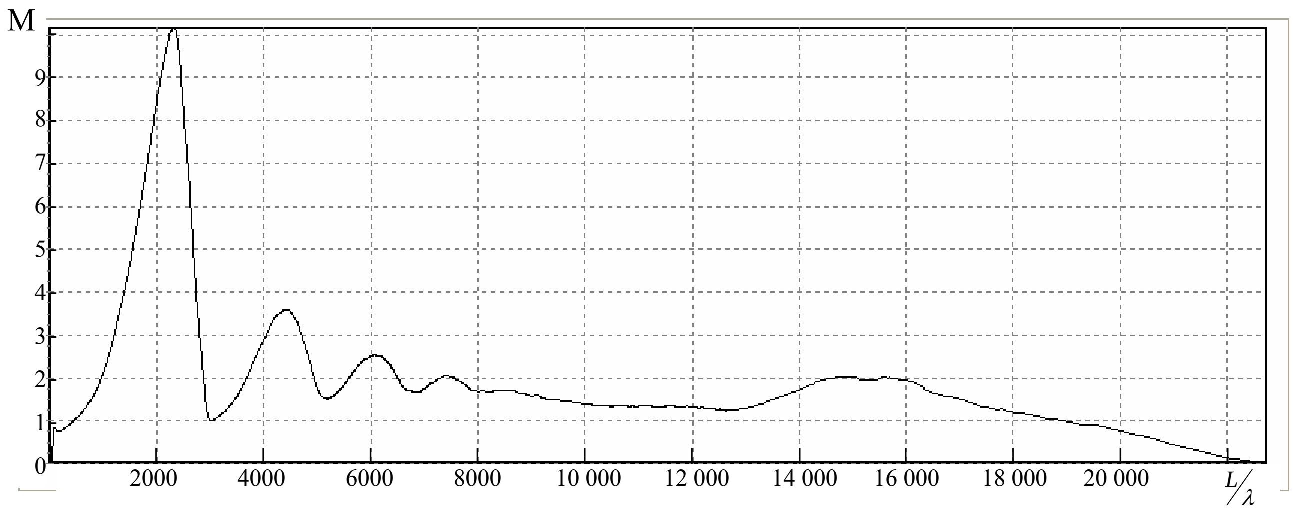


FIGURE 6. Longitudinal change of Mach number on the flow axis.

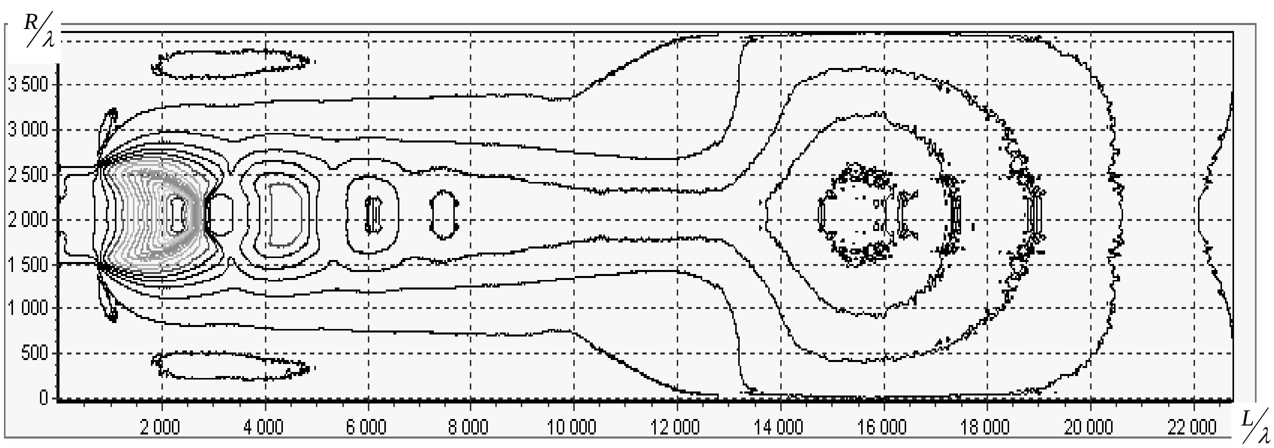


FIGURE 7. Contour curves of Mach number in the cathode and the gas stream between the cathode and anode.

CONCLUSION

As modeling has shown, in the cathode hollow strongly underexpanded supersonic stream is formed with the multi-mooring structure and high rarefaction before the front of the central step. Alternate disk blast waves of Mach disks with direct densification bound appear in the longitudinal direction on the stream axis. The zone of low intensity and low pressure backflow appears in the radial direction between the stream and cylindrical wall of the cathode. At the output of the cathode with wall temperature step change from 300 to 2700 K the stream is bluntly flowing on the radial direction and then flowing to the low pressure chamber at supersonic velocity.

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