DSMC computations of a rarefied gas interacting with light

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Abstract.
By examples of the light-induced drift effect and optical trap of the gas, the possibility of numerical studying of the rarefied gas flows with allowance for the interaction with light radiation by the DSMC method is examined. It is shown that the DSMC method can be appropriately modified for both cases of interaction with radiation of near-resonant and far-from-resonance frequency, and the result of the simulation is rather good.

Keywords: Direct simulations, interaction with light, optical trap, light induced drift

INTRODUCTION

Interaction of a gas with light radiation is a well-studied process. The theory of resonant and nonresonant interaction of the gas and light at the microscopic level has been well developed. To describe rarefied gas flows interacting with light in the general case, one has to use the kinetic approach (solve the Boltzmann equation). An effective method of solving problems of rarefied gas dynamics is the DSMC method. The possibility of the description of rarefied gas flows with allowance for various physical effects is a challenging direction of expanding the applicability of the DSMC method.

In the present paper, we study the possibilities of describing gas flows with strong influence of light radiation. Results of DSMC investigations of gas flows with allowance for their interaction with light of resonant and nonresonant frequency are presented. In the first part of the paper, the light-induced drift in the gas is considered as a possible application of the DSMC method. A comparison with some analytical solutions is performed. In the second part of the paper, the DSMC method is used to study the optical trap of the gas by radiation of nonresonant frequency. The effect of collisions on the optical trapping is investigated.

LIGHT INDUCED DRIFT IN GAS

The light-induced drift (LID) in gas is a fine kinetic effect, which arises in a gas when collisional interactions of one species with other species in the gas mixture are state-dependent. In this part of the paper, we describe extension of the DSMC method to the cases of resonant gas-light interaction, and some numerical results on the modeling of the LID effect are presented.

To explain the basic idea of LID, we consider a binary mixture of two level particles embedded in a buffer gas. Suppose that a narrow-band laser is detuned from resonance frequency such that it excites particles with a particular velocity component along the laser beam. A field of light with a frequency close to the atomic transition frequency induces a transition of electrons of the gas molecule with absorption of a photon into a state with a higher energy. This deforms the velocity distribution of the particles in the ground (state 1) and excited state (state 2). The peak and valley created in the profiles are known as the Bennet structures. These yield opposing fluxes of ground- and excited-state particles. Only if the collisional interaction of the ground- and excited-state particles with respect to the buffer gas is different, a macroscopic drift of the absorbing gas arises. Denoting the total flux of the absorbing gas by $J = j_1 + j_2$, we can express the drift velocity as $u_d = J/n$, where $n = n_1 + n_2$ is the total number density of absorbing particles. The LID effect occurs when light induces velocity-selective excitation of atoms in a gas mixture and when collisional interactions with other species in the gas mixture are state-dependent. The LID effect was discovered by Gelmukhanov and Shalagin [1]. Now the LID effect is widely used in different applications, for example, for isotope separation.

The frequency of absorption of a photon (transition from the ground to the exited state) per time unit can be expressed through the intensity of light I, light frequency ω, and cross section of photon absorption σ as $I\sigma/\hbar\omega$. The cross section of absorption depends on the detuning frequency and on the Doppler shift of the radiation frequency owing to molecular velocity:

$$\sigma = \frac{\sigma_r \Gamma^2}{(\Omega - (\mathbf{v},\mathbf{k}))^2 - \Gamma^2}. \tag{1}$$

Here $\sigma_r = \lambda^2 A/\pi\Gamma$ is the cross section in resonance, λ and $\mathbf{k}$ are the wave length and wave vector of laser radiation, A is the spontaneous emission probability, $\Omega = \omega - \omega_{12}$ is the frequency detuning from resonance, Γ is the linewidth of the resonance-absorption band. A change in the electron shell can lead to a significant difference between the total collision cross section in the ground state σ_1 and excited state σ_2. The associated difference in collisional frequencies ν_1 and ν_2 for molecules in states 1 and 2 with the buffer gas leads to a drift of gases in the opposite directions. If the mixture is in a closed cell, then the force corresponding to the LID effect leads to separation of buffer and light-absorbing components. The gas flow with allowance for the LID effect can be described at the analytical level only with involving of approximations. The general description of such a flow required the solution of Boltzmann equation.

The gas flow can be described using the equations [2]

$$\frac{\partial f_2}{\partial t} + (\mathbf{v},\nabla) f_2 = \frac{I\sigma}{\hbar\omega}(f_1 - f_2) - \gamma f_2 + S_2(\mathbf{v}), \quad \frac{\partial f_1}{\partial t} + (\mathbf{v},\nabla) f_1 = -\frac{I\sigma}{\hbar\omega}(f_1 - f_2) + \gamma f_2 + S_1(\mathbf{v}),$$
$$\frac{\partial f_0}{\partial t} + (\mathbf{v},\nabla) f_0 = S_0(\mathbf{v}), \tag{2}$$

where γ is the frequency of spontaneous emission and $S_{0,1,2}$ are the collisional integrals for the buffer gas and the gas in states 1 and 2, respectively.

The first two analytical expressions for drift velocity were obtained by Gelmukhanov and Shalagin in [3]. They are based on the assumption that $\Gamma \gg kv_T$ (approximation 1) and $\Gamma \ll kv_T$ (2). These approximations correspond to non-uniform and uniform broadening of the spectral line, respectively. The third expression for LID velocity under the assumption that $n_{1,2} \ll n_0$ and $m \ll m_b$ was proposed by Dykhne, Starostin in [2]. These approximate solutions will be used in the paper for verification of the proposed modification of the DSMC method, which can be treated as the numerical solution of system 2 and does not involve approximations.

DSMC simulation of LID. The DSMC method for simulation of rarefied gas flows is described in detail in [4]. The simulation procedure by the DSMC method usually consist of two subsequent stages: free motion of molecules at distances $v\Delta t$ (where Δt is a small time step and v is the velocity) and realization of collisions corresponding to the time step. The majorant frequency scheme [5] was used for performing collisions. In the present work, we added a stage of interaction with light. During this stage, each molecule transits independently from the ground to the excited state (from state 1 to 2) (the process of *photon absorption*) with given frequency $I\sigma/(\hbar\omega)$ and from state 2 to state 1 (the processes of *stimulated emission* and *spontaneous emission of radiation*). The probabilities of photon absorption and emission during the time step depend on light properties and molecule velocity through Eq. (1).

The problem of a spatially uniform gas flow with the LID effect was considered. The arising LID of the gas in this case corresponds well to the drift of the gas in an open cell. Initially, all molecules of the absorbing gas were in the ground state; the fraction of molecules in state 2 increased during the simulation and finally became almost constant. When the steady flow was reached, the sampling of macroparameters was performed.

The following parameters were used for simulation. The mass of the molecule of the absorbing gas was $m = 3.35 \times 10^{-26}$ kg, and the mass of the buffer gas was 100 times heavier. The ratio of the collision frequency of molecules in state 2 and state 1 with the buffer gas was $\nu_2/\nu_1 = 2$. The number density of the buffer gas was ten times higher than that of the absorbing gas: $n_b/n = 10$. The ratio of the absorption frequency in resonance to the collision frequency was $(I\sigma_r/\hbar\omega)/\nu_1 = 6.813 \times 10^{-2}$. These parameters satisfy approximation 3. In the present work, the process of spontaneous emission of radiation is not considered, $\gamma = 0$. We performed a series of computations with a varied resonance band Γ to compare the numerical result for LID velocity with approximations 1 and 2. The parameter $\Gamma/(kv_T)$ (which is the ratio of velocity selectivity to temperature velocity of the absorbing gas $v_T = \sqrt{2k_BT/m} = 353.1$ m/s) was varied through a range from 0.02 to 4; therefore, the conditions of approximations 1 and 2 were satisfied for large and small values of Γ, respectively.

The results of DSMC simulations are presented in figure 1. The LID velocity of the absorbing gas u_d (in m/s) versus the parameter $\Gamma/(kv_T)$ is plotted as the solid curve and points. As can be seen from the plot, the LID velocity increases with increasing Γ, there is a maximum value approximately at $\Gamma/(kv_T) \approx 0.2$, and u_d decreases with further growth of Γ.

A comparison of the LID velocity obtained by the DSMC method with *approximation 1* and *2* (plotted by the bold solid curve and bold dashed curve in figure 1, respectively) shows that the result of simulation is close to the

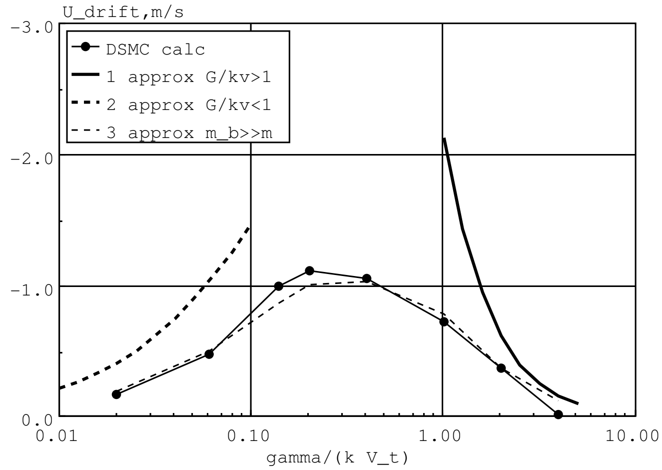

FIGURE 1. LID velocity. Comparison between DSMC and approximate expression of Gelmukhanov, Shalagin (1) and (2), and Dykhne, Starostin (3)

analytical result for an appropriately chosen value of the parameter $\Gamma/(kv_T)$. The statistical error of the presented results, however, does not allow us to estimate this asymptotical behavior of the numerical solution. Note that the temperature velocity v_T of molecules of the absorbing gas is of the order of hundred times greater than the LID velocity u_d; therefore a further decrease in the statistical error of LID velocity estimation would be very computationally expensive in this case.

The comparison of the DSMC result with analytical solution based on the *approximation 3* shows excellent conformity at the maximum value of LID velocity as well as far from maximum.

A comparison shows good agreement of simulation results with analytical calculations for appropriate parameters (both analytical calculations require satisfaction of approximation conditions: a large fraction of the buffer gas in comparison with the light-interacting gas or a small linewidth of the resonance band). Hence, the model used reproduces the LID effect and allows one to estimate the LID velocity in a wide range of parameters, but does not use approximations. This makes the DSMC method a good tool for studying gas flows with the LID effect in the general case.

OPTICAL TRAP

If the frequency of light radiation is substantially different from the resonant frequency, the probability of photon absorption by the molecule and the molecule transition to an excited state can be neglected. In a nonuniform electric field, polarized atoms and molecules are affected by a force directed along the electric field gradient, which is the so-called gradient force. It arises owing to variation of the Stark shift of molecule energy in a nonuniform field. If the polarized molecule is placed into the field of a traveling wave, formed by two intersecting laser beams, the periodic gradient force alters the molecule velocity, thus, trying to keep the molecule in the region with the highest field intensity (optical trapping on an interference grating, [6]). For high intensities of laser radiation (of the order of $10^{17}\mathrm{W/m^2}$), the energy of nonresonant interaction with laser radiation becomes of the order of the thermal energy of molecules at room temperature ([7],[8]). The use of laser radiation of higher intensity can lead to an adverse effect: breakdown in the gas and, hence, ionization. In the present work, the radiation intensity is assumed to be insufficient for the breakdown, and the effects associated with ionization are not considered. Shneider et al. [9] considered momentum and energy transfer from light radiation to a gas; the effect of collisions on the flow was substantial only outside the region of trapping by the optical trap, since the mean free path λ was significantly greater than the interference grating period $2\pi/q$.

In the present paper, we consider a gas flow in the field of an interference grating for the case with $\lambda \leq 2\pi/q$. To solve the problem, we modified and applied the DSMC method, which can be interpreted as a numerical method of solving the Boltzmann equation [10]. In gas-flow modeling, the influence of the gradient force was taken into account at the stage of molecule transport. The goal of the present activities was the DSMC study of the influence of intermolecular collisions on momentum and energy transfer from radiation to the gas in the optical lattice.

Theory of the optical trap. An atom with polarizability α in an electric field of intensity E acquires a dipole moment $d = \alpha E$. The Stark shift of the energy of the polarized atom placed into a nonuniform electric field induces a force acting on the atom. If the polarization effect is linear, this force can be written as

$$\mathrm{F} = \frac{1}{2}\alpha\nabla\mathrm{E}^2. \tag{3}$$

The motion of an atom under the action of this force can be considered as the motion of a particle in potential field $U = -\alpha\mathrm{E}^2/2$.

Let us consider the particle motion in a field formed by two laser beams directed toward each other along the x axis. The intensity of the electric field induced by the first laser with an angular frequency ω_1 is $E_1(x,t) = E_1(t)\sin(k_1x - \omega_1 t)$, and that of the second laser is $E_2(x,t) = E_2(t)\sin(k_2x - \omega_2 t)$. We denote the difference in the wave vectors by $q = |k_1 + k_2|$ and the difference in laser frequencies by $\Delta\omega = \omega_1 - \omega_2$. Assuming that $\Delta\omega << \omega_1, \omega_2$, we can write the resultant field in the form[8]

$$E^2(x,t) = E_1(t)E_2(t)\cos(qx - \Delta\omega t). \tag{4}$$

The resultant *traveling wave* has a phase velocity $V_f = \Delta\omega/q$ and a period $2\pi/q$. The equation of motion of an atom in the traveling wave field (4) can be written as

$$m\ddot{x} = \frac{\alpha q E_1 E_2}{2}\sin(qx - \Delta\omega t). \tag{5}$$

In the lattice-fitted reference system $x' = x - V_f t$, the total energy of the atom remains unchanged:

$$\mathscr{E}' = \frac{m\dot{x}'^2}{2} + \frac{\alpha E_1 E_2}{2}\cos(qx') = \mathrm{const}.$$

If the particle has an initial velocity $\dot{x}_0'$ and position x_0', and the energy satisfies the inequality $\mathscr{E}' < \frac{\alpha E_1 E_2}{2}$, the domain of possible motion of the particle is bounded by the region $x' : \alpha E_1 E_2 \cos(qx')/2 \le \mathscr{E}'$, and the particle is trapped by the optical field. The equilibrium position is determined by the minimum potential energy $x'_{eq} = \pi/q$ with periodicity $2\pi/q$. The trapped particles fluctuates around the equilibrium position, moving with a mean velocity $\langle\dot{x}\rangle = V_f$ together with the lattice. The angular frequency of small fluctuations around the equilibrium position is (see [11] for details) $\omega_{osc} = q\sqrt{\alpha E_1 E_2/(2m)}$. The value of the maximum velocity $\max\{\dot{x}'\} = V_{trap}$ of the trapped particle at the minimum of potential energy x'_{eq} is determined as $V_{trap} = \sqrt{2\alpha E_1 E_2/m}$.

Let us consider a spatially uniform gas flow with allowance for interaction with the optical lattice. To determine flow parameters, following [7], we write the Boltzmann equation in the form

$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + \frac{F}{m}\frac{\partial f}{\partial v_x} = \left(\frac{\partial f}{\partial t}\right)_{coll}, \tag{6}$$

where $\left(\frac{\partial f}{\partial t}\right)_{coll}$ is the collision integral. The initial conditions are set in the form of a Maxwellian distribution for a quiescent gas with a temperature T_0: $f(v,0) = \exp\{-(\mathbf{v}/v_T)^2\}/(v_T\sqrt{\pi})^3$. Here $v_T = \sqrt{2k_BT_0/m}$ is the characteristic velocity of thermal motion of molecules. Let us convert Eq. (6) to dimensionless coordinates $(x', \mathbf{v}', t')$: $x = \lambda x', \mathbf{v} = v_T\mathbf{v}', t = t'/\nu$, to obtain

$$\frac{\partial f'}{\partial t'} + v_x'\frac{\partial f'}{\partial x'} + \left[\frac{\alpha E_1 E_2}{4k_BT_0}\right]\{\lambda q\}\sin(x'\lambda q - \frac{\Delta\omega}{\nu}t')\frac{\partial f'}{\partial v_x'} = \left(\frac{m}{2k_BT\lambda}\right)^2\left(\frac{\partial f}{\partial t}\right)_{coll}, \tag{7}$$

where λ is the mean free path and $\nu = \frac{v_T}{\lambda}$. It is seen that the flow depends on two dimensionless parameters: 1) λq, i.e., the parameter is proportional to the ratio of the mean free path to the lattice period (for free-molecular flow and low gas density, $\lambda q \to \infty$) and 2) $\frac{\alpha E_1 E_2}{4k_BT_0}$, i.e., the parameter is proportional to the ratio of the potential energy to the energy of thermal motion (a higher value of the parameter corresponds to a field with a deeper potential well, which is

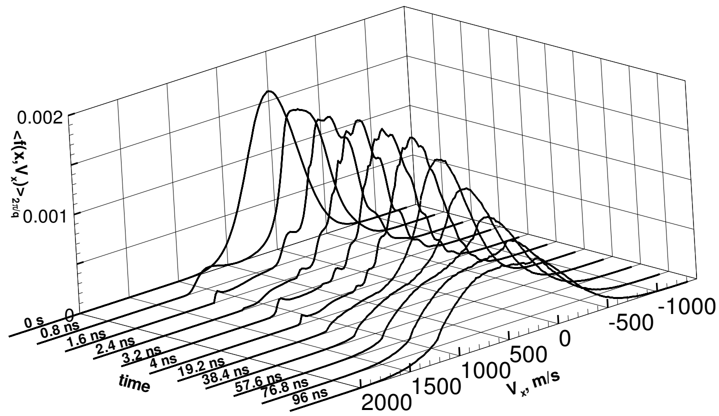

FIGURE 2. Profile of the distribution function. Case 1 ($\lambda q/2\pi = 10$).

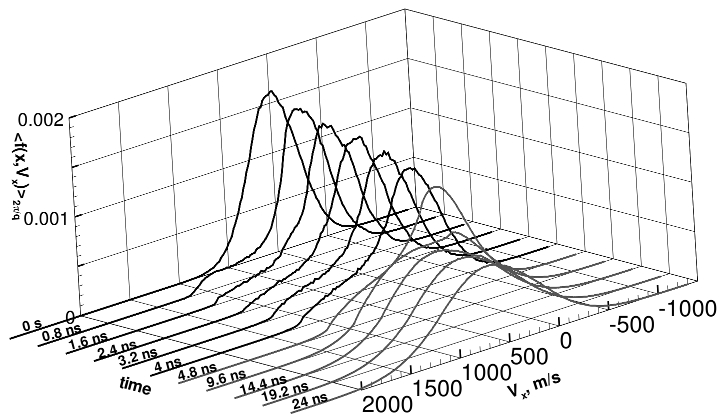

FIGURE 3. Case 2 ($\lambda q/2\pi = 1$).

determined by intensity of laser radiation $I = \varepsilon_0 c E^2/2$). Gas flows with rather wide ranges of the parameters λq and $\frac{\alpha E_1 E_2}{4k_B T_0}$ are considered in the literature. Thus, Shneider et al. [12] considered trapping of gaseous methane at a pressure of 10torr and initial temperature of 300K by laser radiation with an intensity of $6\times10^{16}\mathrm{W}/m^2$ and a lattice period of 400nm. Assuming that $\alpha = 2.9\times10^{-40}$ $\mathrm{C^2 m/N}$ for methane and a diameter $d = 2.73\times -10$ m, the estimates yields $\lambda q = 47.1$ and $\frac{\alpha E_1 E_2}{4k_B T_0} = 0.079$. The third parameter for a uniformly moving lattice with a velocity V_f and initially quiescent gas is determined as 3) V_f/v_T, i.e., it defines the velocity of the optical lattice with respect to the thermal velocity of molecular motion.

Computation results . The unsteady gas flow was modeled in a one-dimensional formulation with periodic boundary conditions at the input and output boundary. Intermolecular collisions were assumed to occur in the approximation where molecules are assumed to be hard spheres (hard sphere model) with the molecule diameter $d = 4.092\times10^{-10}$ m, molecule mass $m = 6.64\times10^{-26}$ kg, and polarizability $\alpha = 5\times10^{-41}$ $\mathrm{C^2 m/N}$.

The parameters of the potential field corresponded to an optical lattice formed by two laser beams, each having an intensity $I = 2.198\times10^{17}$ $\mathrm{W/m^2}$. The depth of the potential well $\alpha E_1 E_2 = 8.28\times10^{-21}$ J corresponds to the trapping velocity $V_{trap} = 500$ m/s. The period of the interference grating was $2\pi/q = 400$ nm, and the phase velocity of the grating was $V_f = 700$ m/s.

At the beginning, molecular velocities were sampled according to the Maxwellian distribution with a temperature $T_0 = 300$ K ($V_f/v_T = 1.416$); the dimensionless parameter was $\frac{\alpha E_1 E_2}{4k_B T_0} = 0.5$. Collisions were implemented by the majorant frequency scheme[10]. The computational domain included one period of the grating $2\pi/q$ and was divided into 200 identical collision cells. The time step in the computations was chosen from the condition $v_t \Delta t/\Delta x \lessapprox 1$ and

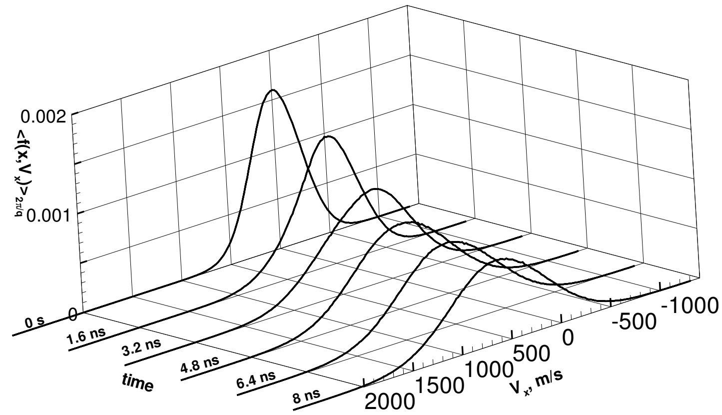

FIGURE 4. Case 3 ($\lambda q/2\pi = 0.1$).

amounted to $\Delta t = 1.6 \times 10^{-12}$ s [13].

The computations were performed for three cases: 1) $\lambda q/2\pi = 10$, gas density $n = 3.3605 \times 10^{23}$ m^{-3} – close to the free-molecular regime; 2) $\lambda q/2\pi = 1$, gas density $n = 3.3605 \times 10^{24}$ m^{-3} – transitional regime; 3) $\lambda q/2\pi = 0.1$, gas density $n = 3.3605 \times 10^{25}$ m^{-3} – regime with strong collisions (initial pressures $p_0 = 1392, 13920$, and 139200 Pa, respectively).

The velocity distribution function of the molecules in the laser-beam direction for case 1 at different times is shown in Fig.2. The distribution function is averaged over the period of the optical lattice: $\langle f(x,V_x)\rangle_{\frac{2\pi}{q}} = \int_0^{2\pi/q} f(x,V_x)dx/\left(\frac{qn}{2\pi}\right)$. It is seen from the graph that a significant portion of trapped molecules acquire a velocity $V_x \approx 1000$ m/s at the time $t = 0.8$ ns. Such a rapid change in the distribution function is caused by the vibrational motion of particles in the optical field (with a period of small oscillations $T_{osc} = 1.602$ ns). During the time of the order of T_{osc}, the temperature increases approximately by 100 K. During further evolution of optical trapping, at times when the influence of intermolecular collisions may be neglected ($t = 1.6, 2.4, 3.2$, and 4 ns), a plateau is formed in the plot of the distribution function in the region of trapped molecules 200m/s $< V_x <$ 1200m/s. At times t greater than the mean time between the collisions t_c, the distribution function becomes smoother ($t = 19.2$ ns). At $t = 38.4, 57.6$, and 76.8 ns, a significant fraction of molecules start moving faster than the trapping velocity $V_f + V_{trap} = 1200m/s$, which indicates the *pumping of energy* transferred from the moving optical lattice to the translational degree of freedom of molecules V_x.

At high times, the maximum of the distribution function $\langle f(V_x)\rangle$ is shifted to $V_x = 700$ m/s, and the distribution function acquires a shape close to symmetrical. It is seen from the plot in Fig.2 (figure at the top) for $t = 96$ ns that the gas moves with a mean velocity close to the lattice velocity and is heated. Gas heating at high times of interaction with the optical lattice can be easily explained by considering trapping in a coordinate system fitted to the optical lattice. The initial energy of translational motion of the gas as a whole $\rho V_f^2/2$ in this coordinate system is completely transferred to the thermal energy of the gas owing to collisions, which induces gas heating by $\Delta T = mV_f^2/3k_B = 785.5$ K. Note that ΔT depends only on the phase velocity of the optical lattice (difference in laser frequencies $\Delta\omega$). This indicates a possibility of *strong local heating of the gas*, the magnitude of heating under the assumptions used being independent of the depth of the potential well (intensity of laser radiation). The increase in temperature occurs owing to heat transfer between the groups of trapped and non-trapped atoms.

For $\lambda q/2\pi = 1$ (case 2), the characteristic time between the collisions is of the order of the oscillation period T_{osc}. It is seen from the plot $\langle f(V_x)\rangle$ that the distribution function is smooth even at small times $t = 0.8$ ns (Fig.3). Because of the evolution of gas trapping in the presence of collisions, energy pumping proceeds faster. As compared with case 1, the plateau in the distribution function in the trapping region is less expressed, and thermalization of the function occurs faster: already at $t = 9.6$ ns, the maximum $\langle f(V_x)\rangle$ at $V_x = 0$ m/s is only weakly expressed. At $t = 24$ ns, the distribution function is symmetric and wide, which corresponds to gas heating. The gas temperature at the time $t = 24$ ns reaches 1114 K, and the gas heating is substantially more intense than that in case 1 at the corresponding time.

With a further increase in the level of collisions $\lambda q/2\pi = 0.1$ (case 3), the plateau in the distribution function profile disappears. As is seen from Fig.4, gas heating and acceleration occur simultaneously with the evolution of trapping. The profile of the distribution function is close to symmetrical both in the beginning of optical trapping ($t = 1.6$ ns in the plot) and at $t = 8$ ns.

From these results, we may conclude that the high level of collisions in the case of optical trapping of a dense gas ($\lambda q \leq 1$) ensures intense energy and momentum exchange between the moving optical lattice and the gas, which results in acceleration of the gas *as a whole* to a velocity equal to the velocity of the optical lattice and in significant heating of the gas. Apparently, the mechanism of this strong influence of radiation implies the exchange between trapped and non-trapped particles owing to collisions. Because of collisions, the trapped particle leaves the trapping region in the space of velocities, and particles that were not initially trapped may enter the trapping region. This exchange between trapped and non-trapped particles in the case of a high level of collisions (case 3) may be responsible for the absence of an expressed molecular beam of trapped molecules, but intensifies the momentum and energy transfer from the optical lattice to the gas. Hence, the high level of collisions allows accelerating the gas as a whole and heating the gas to high temperatures.

The time of gas acceleration to the velocity of the optical lattice V_f and gas heating in all cases is of the order of the mean free time. Note that the magnitude of heating depends only on the lattice velocity $\Delta T \approx mV_f^2/3k_B$, which indicates the possibility of strong local heating of a dense gas by laser radiation of lower intensity than that considered in the present work. It may be assumed, however, that the rate of gas heating by low-intensity radiation will depend not only on the collision frequency but also on intensity: heating will occur more slowly in the case of a smaller depth of the potential well. The issue of the dependence of the gas heating rate on the mean collision frequency and intensity of laser radiation has to be studied in more detail.

CONCLUSIONS

The DSMC method was modified to description of gas flows with allowance for the interaction with light. Effect of resonant effect of radiation illustrated by the example of light-induced drift in gas. It was shown, that DSMC results for LID velocity are close to the analytical estimations for appropriate parameters, but does not use approximations. This makes the DSMC method a good tool for studying gas flows with the LID effect in the general case.

For optical trap of gas three cases are considered: flow with low, moderate and high level of collisions. It was shown that It was shown, that high level of collisions (for $\lambda q \leq 1$) lead to more efficient transfer of moment and energy from radiation field to the gas.

ACKNOWLEGEMENT. This work has been supported by INTAS (grant Nr. 04-83-4010). This support is greatly acknowledged.

REFERENCES

1. F.Kh.Gelmukhanov, A.M. Shalagin, JETP Lett. 29, 243 (1979)
2. A.M.Dykhne, M.N Starostin Zh. Eksp. Teor. Fiz., 79, 1211 (1980)
3. F.Kh. Gelmukhanov, A.M. Shalagin Zh. Eksp. Teor. Fiz., 78, 1672 (1980).
4. Bird, G. A. Molecular gas dynamics and the direct simulation of gas flows. Corrected reprint of the 1994 original. With 1 IBM-PC floppy disk (3.5 inch; DD). Oxford Engineering Science Series, 42. Oxford Science Publications. The Clarendon Press, Oxford University Press, New York, 1995.
5. Ivanov MS, Rogasinsky SV. 1988. Analysis of the numerical techniques of the direct simulation Monte Carlo method in the rarefied gas dynamics. *Soviet J. Numer. Anal. Math. Modelling*, 3(6):453-465
6. HL Bethlem, G. Berden, FMH Crompvoets, RT Jongma, AJA van Roij, and G. Meijer, Nature (London) 406, 491 (2000)
7. P. F. Barker, M. N. Shneider, "Optical microlinear accelerator for molecules and atoms" Phys. Rev. A 64, 033408 (2001)
8. P. F. Barker, M. N. Shneider, "Slowing molecules by optical microlinear deceleration" PHYSICAL REVIEW A 66, 065402 (2002)
9. M. N. Shneider, S. F. Gimelshein, P. F. Barker, "Micropropulsion devices based on molecular acceleration by pulsed optical lattices", Journal of Applied Physics 99, 063102, 2006.
10. Ivanov, M.S., Markelov, G.N., Gimelshein, S.F. "Statistical simulation of reactive rarefied flows: numerical approach and applications," AIAA Paper 98-2669, June 1998.
11. Guangjiong Donga, Weiping Lub, P.F. Barker, M.N. Shneider, "Cold molecules in pulsed optical lattices" Progress in Quantum Electronics 29 (2005) pp.1-58
12. MN Shneider, C Ngalande, SF Gimelshein - 44 th AIAA Aerospace Sciences Meeting and Exhibit, 2006
13. A. A. Shevyrin, Ye. A. Bondar, and M. S. Ivanov, "Analysis of Repeated Collisions in the DSMC Method",RGD-24, AIP Conference Proceedings – May 16, 2005 – Volume 762, pp. 565-570