

Solving of the Boltzmann Equation in Some Problems with Moving Interface Surface

A. P. Kryukov, A. K. Yastrebov

*Department of Low Temperatures, Moscow Power Engineering Institute,
Krasnokazarmennaya 14, 111250, Moscow, Russia*

Abstract. Processes of heat and mass transfer in vapor films were studied with help of numerical solution of the Boltzmann kinetic equation. Formulation of problem was more complicated in comparison with previous study, film thickness was assumed to be variable. Corresponding problems were solved with help of dynamic adaptation method developed for Stefan problem. Solution was obtained for small velocities of the liquid surface motion. At determination of pressure inside the vapor film heat flux plays main role in comparison with the influence of mass flux. Results of calculation show that for finding of the difference between the vapor pressure near the liquid surface and the saturation pressure at the surface temperature known stationary kinetic correlation can be used.

Keywords: Boltzmann equation, transfer processes in vapor films.

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INTRODUCTION

In this paper we studied processes of heat and mass transfer in vapor films in application to different problems, such as vapor explosion, superfluid helium boiling etc. Earlier we obtained solution of the simplified problem; it was assumed that the film thickness was constant [1,2]. We used numerical solution of the Boltzmann kinetic equation for moderate Knudsen numbers (from 0.005 and higher). If Knudsen number is small then calculation time increases sharply, so in such cases we used joint numerical solution of the Boltzmann equation and the Navier – Stokes equations. Results presented in [1,2] show that increase of the heater temperature causes vapor condensation in studied range of film thicknesses. Duration of the considered non-stationary process does not exceed some microseconds. Vapor state can be assumed to be stationary in different applications; stationary kinetic correlations can be used to describe it. In this paper we made formulation of this problem more complicated and assumed that the surface of liquid moved with the constant velocity.

FORMULATION OF PROBLEM

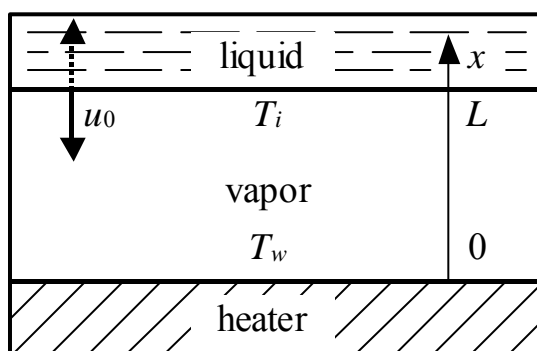


FIGURE 1. Scheme of studied problem.

Considered system is shown in Fig. 1, where T_w is temperature of the hot body (or heater) surface, T_i is temperature of the cold liquid surface, x is the coordinate and L is the film thickness. Process of the vapor film formation is complicated, and we didn't take it into account. We assumed that the vapor film existed at the initial moment of time; the film thickness was given. This film was limited by two flat parallel surfaces. First of them was not penetrable for mass flux, it was the surface of the heater, and second one was penetrable, it was the surface of the liquid. Temperatures of both surfaces were identical and equal to T_i before beginning of the time-dependent process. At the initial moment of time ($t = 0$) temperature of the heater surface increases and reaches the value of T_w , and the surface of the liquid begins to move with the constant velocity u_0 . Solution of this problem was obtained for both growth and collapse of the vapor cavity.

The Boltzmann kinetic equation in one-dimensional time-dependent formulation was used for description of transfer processes in the vapor film:

$$\frac{\partial f}{\partial t} + \xi_x \frac{\partial f}{\partial x} = J(f) \quad (1)$$

Here f is the velocity distribution function of vapor molecules, t is the time, ξ is the molecular velocity, x is the coordinate, and J is the collision integral. The splitting scheme [3] was used for solving the equation (1). It means that on each time step the equation of free molecular flow

$$\frac{\partial f}{\partial t} + \xi_x \frac{\partial f}{\partial x} = 0, \quad (2)$$

and the equation of spatially uniform relaxation

$$\frac{\partial f}{\partial t} = J(f) \quad (3)$$

were solved one after another. Solution of (2) was used as the initial condition for (3), and solution of (3) was used as the initial condition for (2) on next time step. The conservative discrete ordinates method [3] was used for calculation of the collision integral. Essential feature of this method is the exact fulfillment of conservation laws. The collision integral for the Maxwell (equilibrium) distribution can be guaranteed to be equal to zero exactly when this method is used. The velocity distribution function is obtained as result. Parameters of vapor (density, pressure, temperature, mass and energy fluxes) are calculated as moments of this function.

Distribution functions for molecules moving from surfaces are boundary conditions for the Boltzmann equation. We assumed that these molecules had the Maxwell distribution with the temperature of surfaces. On the surface of the heater ($x = 0$, $\xi_x > 0$) parameters of the Maxwell distribution function were $n = n_w$ и $T = T_w$:

$$f = \frac{n_w}{(2\pi RT_w)^{3/2}} \exp\left(-\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT_w}\right), \quad (4)$$

on the surface of the liquid ($x = L$, $\xi_x < 0$) these parameters were $n = n_i$, $T = T_i$:

$$f = \frac{n_i}{(2\pi RT_i)^{3/2}} \exp\left(-\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT_i}\right). \quad (5)$$

In both formulas f is the distribution function, n is the numerical density (number of molecules in volume unity), T is the surface temperature (parameters n and T are written with corresponding subscripts), R is the individual gas constant, and ξ_x , ξ_y and ξ_z are the velocity components. The value of n_w was found using condition of non-penetrability of the heater surface, mass flux at $x = 0$ should be equal to zero. It was assumed that the condensation coefficient on the liquid surface was equal to unity, so the value of n_i was equal to the numerical density of saturated vapor at the temperature T_i . Thus reflection of vapor molecules from the heater surface was diffusive, reflection of molecules from the liquid surface was absent, and all molecules moving to this surface condensed on it.

METHOD OF DYNAMIC ADAPTATION

Presence of a moving surface leads to additional complexity of a problem formulation and a mathematical description. In our paper these problems were solved with help of the dynamic adaptation method [4] developed for the Stefan problem. Base of this method is the procedure of transition to a non-stationary coordinate system. In general case equations for motion of nodes are added to equations for transfer processes. If velocity of surface is given, then coordinates of nodes can be calculated previously without solving of equations.

We assumed that motion of the liquid surface occurred only on the free molecular flow stage because molecules do not move between coordinate nodes on the spatially uniform relaxation stage. Transformation of the free molecular flow equation (2) was done as follows.

The velocity of the liquid surface is constant and equal to u_0 . The coordinate grid x_i changed with time:

$$x_i = x_{0i} + u_0 t x_{0i} / L_0, \quad (6)$$

where x_{0i} and L_0 are the nodes coordinates and the film thickness at $t = 0$ respectively. The grid was lengthened or pressed depending on direction of the liquid surface motion. It is expediently to solve the equation (2) in such coordinate system (for such grid y_i) in which the film thickness is constant and equal to L_0 . Then nodes y_i were nodes x_{0i} and they were defined as

$$y_i = x_i / (1 + u_0 t / L_0) \quad (7)$$

The equation of free molecular flow should be written in the new coordinate system. Let's take x_i , t as initial independent variables and y_i , τ as independent variables in new coordinate system ($t = \tau$). When transition from initial coordinate system to a different one occurred then derivatives should be replaced as follows:

$$\begin{cases} \frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} + \frac{Q}{\Psi} \frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} = \frac{1}{\Psi} \frac{\partial}{\partial y} \end{cases}, \quad (8)$$

where

$$\begin{cases} Q = -\partial x / \partial t \\ \Psi = \partial x / \partial y \end{cases}. \quad (9)$$

For the transformation of grid following the correlation (7) we had

$$\begin{cases} Q = -u_0 y / L_0 \\ \Psi = 1 + u_0 t / L_0 \end{cases} \quad (10)$$

Substituting (10) into (8) and replacing the derivatives in the equation (2) following the correlation (8) we obtained the equation of free molecular flow for grid y_i :

$$\frac{\partial f}{\partial \tau} + \frac{\xi_x - u_0 y / L_0}{1 + u_0 \tau / L_0} \frac{\partial f}{\partial y} = 0. \quad (11)$$

The coefficient before space derivative changed. In the initial equation (2) this derivative described only changing of the distribution function due to motion of molecules. In (11) changing of the distribution function due to mutual motion of nodes x_i and y_i was added. So we wrote the equation of free molecular flow for such coordinate system in which the liquid surface did not move. The nodes y_i and x_i are connected by the correlation (7) identically. Vapor parameters in the nodes y_i can be found as solution of (11), and then we can find coordinates of the nodes x_i , in which parameters are the same.

RESULTS AND DISCUSSION

Results are presented in the non-dimensional view. Non-dimensional values were defined as follows:

$$T = T_D / T_i \quad (12)$$

$$\rho = \rho_D / \rho_s (T_i) \quad (13)$$

$$x = x_D / l_i > \quad (14)$$

$$j = j_D / \rho_s (T_i) \sqrt{RT_i} \quad (15)$$

$$t = t_D \sqrt{RT_i} / l_i > \quad (16)$$

$$u_0 = u_{0D} / \sqrt{RT_i} \quad (17)$$

Here the subscript “D” means “dimensional”, $\rho_s(T_i)$ is the saturated vapor pressure at the liquid surface temperature T_i , $\langle l_i \rangle$ is the mean free path of saturated vapor molecules at T_i , R is the individual gas constant.

Examples of obtained results are shown in Fig. 2–4 for both growth and collapse of the vapor film. The dependencies of density and temperature on coordinate at $t = 500$ are presented; dependence of mass flux on the liquid surface is shown also. We selected the value of $t = 500$ because if the problem is solved in simplified formulation (without taking into account motion of liquid surface) this time is the duration of the time-dependent stage.

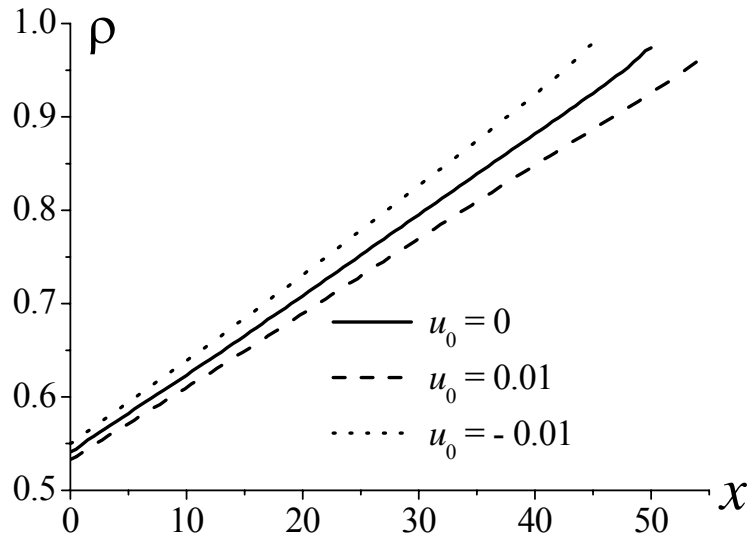


FIGURE 2. Dependency of density on coordinate ($t = 500$).

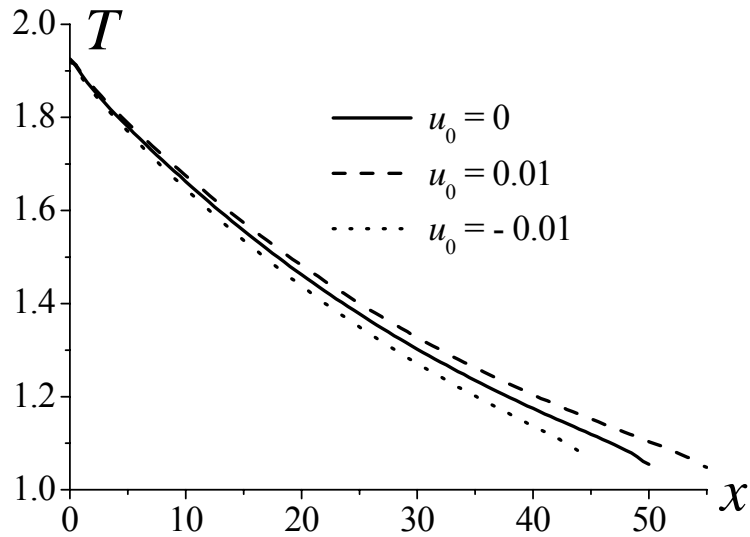


FIGURE 3. Dependency of temperature on coordinate ($t = 500$).

If the velocity of the liquid surface motion is small, then this motion affects dependencies of density and temperature weakly. Presented in Fig. 2 and 3 dependencies for $u_0 \neq 0$ are very near to ones obtained for the corresponding film thickness when motion of the surface was not taken into account (for example, at $u_0 = 0.01$ for $L = 55$). Difference is about 1–2%. So transfer processes can be considered as stationary when some application is analyzed.

Let's consider influence of the initial disturbances on phase transitions on the liquid surface. Motion of the surface leads to liquid evaporation during growth of the vapor cavity; and it leads to vapor condensation during collapse of the film. Vapor pressure in the film increases due to increase of the heater temperature and becomes higher than saturation pressure at the liquid temperature. This is condition for condensation. So if the vapor film collapses than condensation occurs always. Mass flux on this surface is always greater than zero, it is confirmed by our calculations (see Fig. 4). If the vapor film grows then both evaporation and condensation are possible. Our results show that liquid evaporates during initial stage of process. When the disturbance caused by increase of the heater temperature reaches the liquid surface then vapor begins to condense. After some time inverted transition from condensation to evaporation occurs.

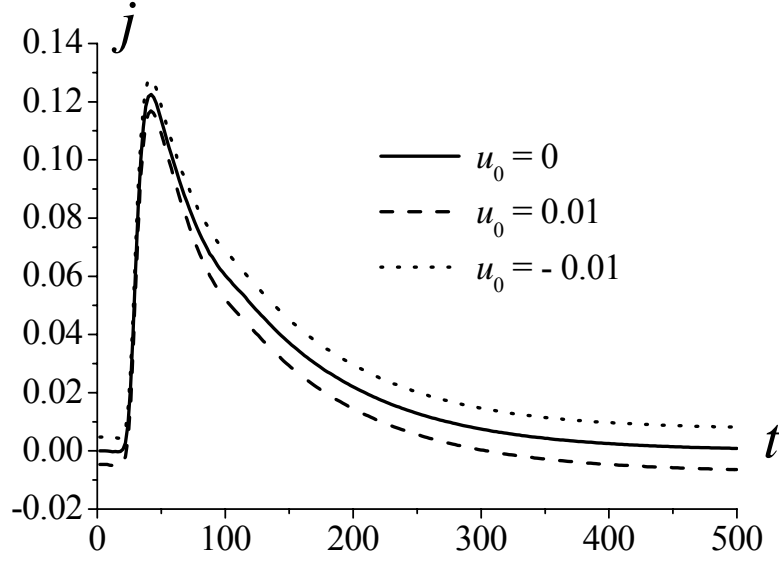


FIGURE 4. Dependency of mass flux on liquid surface on time.

In both cases (growth and collapse) there is a quasi-stationary value of mass flux at $t = 500$. Our calculations show that this value depends only on the velocity of the liquid surface movement.

For stationary processes of heat and mass transfer on interphase surfaces there is the following correlation obtained with help of Boltzmann equation solving by moment method [5]:

$$\frac{p_D - p_{sD}(T_{iD})}{p_{sD}(T_{iD})} = 2,127 \frac{j_D}{\rho_{sD}(T_{iD})\sqrt{2RT_{iD}}} + 0,443 \frac{q_D}{p_{sD}(T_{iD})\sqrt{2RT_{iD}}} \quad (18)$$

Here j and q are mass and heat fluxes on the surface respectively, p is vapor pressure near the surface, $p_s(T_i)$ is the saturation pressure at the surface temperature. All values in the correlation (18) are written with subscript “D”, so they are dimensional. We estimated influence of the liquid surface motion on the difference between vapor pressure and saturation pressure $p_s(T_i) - p$ defined by (18).

If the surface does not move, than stationary mass flux is equal to zero. We obtained this result in our paper [1]. For moving surface $j \neq 0$ and depends on value of u_0 . Let's consider following initial conditions: liquid is the superfluid helium at the temperature 2 K, the temperature of the heater surface is 4 K, the film thickness is 1.5 micron, the velocity of the liquid surface motion is -0.64 m/s (the vapor cavity collapses). Heat transfer in the superfluid helium is very effective, so it can be assumed that heat supply to the liquid does not cause changing of its temperature. In this case the items in (18) are equal to 0.012 and 0.034 respectively, so influences of the mass and heat fluxes are comparable. The heat flux in vapor phase and condensation heat are comparable also: $q = 2 \cdot 10^4$ W/m², $j \cdot r = 9 \cdot 10^3$ W/m². But in real cases the velocity of the liquid surface motion is much less (order of 1 cm/s), mass flux should be much less also, and its influence should be negligible. We did not carry out calculations for such small velocities, because disturbances produced by motion of the liquid surface are near to the error of the numerical solution. We estimated that for $u_0 = 1$ cm/s item with mass flux in (18) is about $2 \cdot 10^{-4}$. It means that influence of the mass flux is much less than influence of the heat flux.

CONCLUSION

Study of time-dependent processes of heat and mass transfer was carried out for vapor films produced by interaction of hot bodies with liquids. The formulation of the problem was more complicated in comparison with the previous study, the film thickness was assumed to be variable. Results of calculation show that influence of the mass flux is negligible, the heat flux plays main role if the difference between the vapor pressure near surface and the saturation pressure at the surface temperature is calculated using the stationary kinetic correlation. This conclusion is true for small velocities of the liquid surface motion.

We assumed that liquid temperature was constant. It could be right for the superfluid helium due to very high efficiency of heat transfer in this liquid. In general case additional factor should be taken into account; this factor is increase of the liquid temperature. Both motion of the liquid surface and heating of the liquid lead to evaporation during growth of the vapor film. In this case it can be necessary to solve the studied problem taking into account both factors. If collapse of the vapor film is considered, then the liquid should evaporate due to heat supply and the vapor should condense due to motion of the liquid surface. Total value of the mass flux can be lower than one obtained with taking into account only one factor. So in this case solving of the problem with taking into account one of the factors can be enough.

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