Numerical Investigation on Models of the Boltzmann Equation for Gas Mixtures

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Abstract. New model of the Boltzmann equation for gas mixtures, which satisfies the H-theorem, has been proposed recently by Andries, *et al.* [*J. Stat. Phys.*, **106**, 993 (2002)]. In the present study, this model is compared numerically with other existing models (proposed by Hamel and by Garzó, *et al.*) as well as the Boltzmann equation for the Maxwell molecules. In order to make proper comparisons, some generalizations for the Hamel and Garzó models are introduced. The numerical comparisons are performed in a half-space problem of a vapor condensing onto a plane condensed phase in the presence of a noncondensable gas, after parameters concerning the molecular interaction are adjusted appropriately.

INTRODUCTION

The BGK model [1] of the nonlinear Boltzmann equation for single-component gases satisfies the fundamental properties of the Boltzmann equation including the H-theorem, has been widely used, and has been contributing to a great extent to the development of the rarefied gas dynamics. In the meantime, similar model equations for rarefied gas mixtures have been proposed [2, 3, 4]. For example, the model proposed by Hamel [3] and that by Garzó *et al.* [4] were constructed so that the collisional transfer of momentum and energy between the component gases of the mixture coincides with that of the Boltzmann equation for the Maxwell molecules. They have been widely used, whereas they have no proof for the H-theorem.

Only recently new model for the mixtures, which has a proof for the H-theorem, was proposed by Andries *et al.* [5]. In the present study, we will report on the numerical comparison of their model with other existing models (proposed by Hamel and by Garzó *et al.*) as well as the Boltzmann equation for the Maxwell molecules.

BOLTZMANN EQUATION

Let us first summarize the main notations. In the sequel, the Greek letters α, β, and γ will be used symbolically to indicate the gas species. Let t be the time, X_i (or $\boldsymbol{X}$) the space coordinate, ξ_i (or $\boldsymbol{\xi}$) the molecular velocity, $f^\alpha(t,\boldsymbol{X},\boldsymbol{\xi})$ the velocity distribution function of molecules of gas α, and m^α the molecular mass of gas α. The molecular number density, flow velocity, pressure, and temperature of gas α are, respectively, denoted by n^α, u_i^α (or $\boldsymbol{u}^\alpha$), p^α, and T^α and their counterparts of the total mixture by n, u_i (or $\boldsymbol{u}$), p, and T [6].

If there is no external force, the Boltzmann equation for gas mixtures may be written as

$$\frac{\partial f^\alpha}{\partial t} + \xi_i \frac{\partial f^\alpha}{\partial X_i} = \sum_\beta J^{\beta\alpha}(f^\beta, f^\alpha), \tag{1}$$

where, and in the sequel, the summation with respect to β (or γ) is taken over the whole gas species. The collision term $J^{\beta\alpha}$ is defined by

$$J^{\beta\alpha}(f,g) = \int \left[f(\boldsymbol{\xi}_*^{\beta\alpha}) g(\boldsymbol{\xi}^{\beta\alpha}) - f(\boldsymbol{\xi}_*) g(\boldsymbol{\xi}) \right] B^{\beta\alpha}\left(\frac{|\boldsymbol{e}\cdot\boldsymbol{V}|}{|\boldsymbol{V}|}, |\boldsymbol{V}| \right) \mathrm{d}\Omega(\boldsymbol{e}) \mathrm{d}^3\xi_*, \tag{2}$$

with

$$\boldsymbol{\xi}^{\beta\alpha} = \boldsymbol{\xi} + \frac{\mu^{\beta\alpha}}{m^\alpha}(\boldsymbol{e}\cdot\boldsymbol{V})\boldsymbol{e}, \quad \boldsymbol{\xi}_*^{\beta\alpha} = \boldsymbol{\xi}_* - \frac{\mu^{\beta\alpha}}{m^\beta}(\boldsymbol{e}\cdot\boldsymbol{V})\boldsymbol{e}, \quad \boldsymbol{V} = \boldsymbol{\xi}_* - \boldsymbol{\xi}, \quad \mu^{\beta\alpha} = \frac{2m^\beta m^\alpha}{m^\beta + m^\alpha}. \tag{3}$$

Here $\boldsymbol{e}$ is a unit vector, $\mathrm{d}\Omega(\boldsymbol{e})$ the solid angle element in the direction of $\boldsymbol{e}$, and $\mathrm{d}^3\boldsymbol{\xi}_* = \mathrm{d}\xi_{*1}\mathrm{d}\xi_{*2}\mathrm{d}\xi_{*3}$. The integration in Eq. (2), and in the sequel unless otherwise stated, is carried out over the whole space of its integration variables. The $B^{\beta\alpha}(=B^{\alpha\beta})$ is a nonnegative function whose functional form is determined by the intermolecular force between species β and α (see Ref. [7]).

The Boltzmann equation (1) (or its collision term $J^{\beta\alpha}$) has the following basic properties (see Ref. [5]); (i) the mass of each species and the momentum and energy of the total mixture are conserved during the molecular collisions; (ii) the equilibrium solution is the Maxwellian with common flow velocity and temperature; (iii) the H-theorem holds; (iv) the indifferentiability principle holds: when the molecules of all the component gases are mechanically identical (i.e., all the mass m^α and function $B^{\beta\alpha}$ are identical), one obtains the single-species Boltzmann equation for the total distribution $f = \sum_\beta f^\beta$.

Molecular models

We shall give below the specific forms of $B^{\beta\alpha}$ for the Maxwell and the variable hard-sphere [8] molecular models, which will be used later in numerical comparisons with the model equations.

The Maxwell molecular model assumes the potential $U^{\beta\alpha}$ of the intermolecular force between β- and α-species molecules to be given by $U^{\beta\alpha} = a^{\beta\alpha}r^{-4}$ with r being the distance between the molecules and $a^{\beta\alpha}$ a positive constant. The $B^{\beta\alpha}$ for this model is independent of $|\boldsymbol{V}|$ and is written as [7]

$$B^{\beta\alpha}(\cos\theta, |\boldsymbol{V}|) = \Big(\frac{a^{\beta\alpha}}{\mu^{\beta\alpha}}\Big)^{1/2}\frac{g(\theta)}{\sin\theta}\frac{\mathrm{d}g}{\mathrm{d}\theta}, \quad (0 \le \theta < \frac{\pi}{2}). \tag{4}$$

Here g is the monotonically increasing function of θ defined by the integral

$$\theta = \int_0^{y_c(g)} [1-(y/g)^4 - y^2]^{-1/2}\mathrm{d}y, \quad y_c = \Big[\frac{-g^4 + (g^8+4g^4)^{1/2}}{2}\Big]^{1/2}. \tag{5}$$

Note that $g(0) = 0$ and $g(\pi/2) = \infty$.

The variable hard-sphere (VHS) molecule is a special kind of the hard-sphere molecule whose diameter varies according to the relative collision energy between the colliding molecules. The $B^{\beta\alpha}$ is given by (see Refs. [7, 8])

$$B^{\beta\alpha}\Big(\frac{|\boldsymbol{e}\cdot\boldsymbol{V}|}{|\boldsymbol{V}|}, |\boldsymbol{V}|\Big) = \frac{1}{2}(d^{\beta\alpha})^2|\boldsymbol{e}\cdot\boldsymbol{V}|, \qquad d^{\beta\alpha} = d_*^{\beta\alpha}\Big[\frac{\mu^{\beta\alpha}|\boldsymbol{V}|^2}{4(2-\omega)kT_*}\Big]^{-\omega/2}, \quad d_*^{\beta\alpha} = \frac{d_*^\beta + d_*^\alpha}{2}, \tag{6}$$

where k is Boltzmann's constant, T_* the reference temperature, and d_*^α the reference molecular diameter of species α at temperature T_*. The ω is a constant: $\omega = 0$ corresponds to the classical hard-sphere molecule and $\omega = 1/2$ to the Maxwell molecule (VHS-Maxwell molecule). In the latter case, Eq. (6) reduces to

$$B^{\beta\alpha}(\cos\theta, |\boldsymbol{V}|) = \frac{1}{2}(d_*^{\beta\alpha})^2\Big(\frac{6kT_*}{\mu^{\beta\alpha}}\Big)^{1/2}\cos\theta, \quad (0 \le \theta \le \frac{\pi}{2}). \tag{7}$$

That is, $B^{\beta\alpha}$ for the VHS-Maxwell molecule is independent of $|\boldsymbol{V}|$ as in the case of the Maxwell molecule.

Collision frequency and transfer coefficient

For the later convenience, we shall show below the molecular collision frequency and the collisional transfer of momentum and energy between the component species in the case of the Maxwell and VHS-Maxwell molecules.

Let us denote the collision frequency of an α-species molecule with velocity $\boldsymbol{\xi}$ with respect to collisions with β-species molecules as $\nu^{\beta\alpha}(\boldsymbol{\xi})$. As was seen in the previous section, $B^{\beta\alpha}$ for the Maxwell and VHS-Maxwell molecules are independent of $|\boldsymbol{V}|$. In such a case, $\nu^{\beta\alpha}$ does not depend on $\boldsymbol{\xi}$ and is expressed as

$$\nu^{\beta\alpha} = K^{\beta\alpha}n^\beta, \quad K^{\beta\alpha} = 4\pi\int_0^{\pi/2} B^{\beta\alpha}\sin\theta\mathrm{d}\theta. \tag{8}$$

Since $K^{\beta\alpha}$ for the Maxwell molecule diverges, we introduce the angular cutoff. That is, we assume $B^{\beta\alpha}=0$ for $\theta_c^{\beta\alpha}\le\theta\le\pi/2$, which corresponds to ignoring the grazing collisions whose deflection angle (the angle between $\boldsymbol{\xi}_*-\boldsymbol{\xi}$ and $\boldsymbol{\xi}_*^{\beta\alpha}-\boldsymbol{\xi}^{\beta\alpha}$) is less than $\pi-2\theta_c^{\beta\alpha}$. Then we have

$$K^{\beta\alpha}=\begin{cases}2\pi(a^{\beta\alpha}/\mu^{\beta\alpha})^{1/2}(g_c^{\beta\alpha})^2, & \text{with } g_c^{\beta\alpha}=g(\theta_c^{\beta\alpha}) \text{ for the Maxwell molecule,}\\ \pi(6kT_*/\mu^{\beta\alpha})^{1/2}(d_*^{\beta\alpha})^2, & \text{for the VHS-Maxwell molecule.}\end{cases} \tag{9}$$

We next show the collisional transfer of momentum and that of energy between β and α species per unit time. When $B^{\beta\alpha}$ is independent of $|\boldsymbol{V}|$, they can be expressed as follows [9]:

$$\begin{aligned}&\int m^\alpha\boldsymbol{\xi}J^{\beta\alpha}(f^\beta,f^\alpha)\mathrm{d}^3\xi=\chi^{\beta\alpha}\mu^{\beta\alpha}n^\beta n^\alpha(\boldsymbol{u}^\beta-\boldsymbol{u}^\alpha),\\ &\int\frac{1}{2}m^\alpha|\boldsymbol{\xi}|^2J^{\beta\alpha}(f^\beta,f^\alpha)\mathrm{d}^3\xi=\chi^{\beta\alpha}\mu^{\beta\alpha}\frac{n^\beta n^\alpha}{m^\beta+m^\alpha}\left[3k(T^\beta-T^\alpha)+(m^\beta\boldsymbol{u}^\beta+m^\alpha\boldsymbol{u}^\alpha)\cdot(\boldsymbol{u}^\beta-\boldsymbol{u}^\alpha)\right],\end{aligned} \tag{10}$$

with

$$\chi^{\beta\alpha}=4\pi\int_0^{\pi/2}B^{\beta\alpha}\sin\theta\cos^2\theta\mathrm{d}\theta. \tag{11}$$

Let us call $\chi^{\beta\alpha}$ the *transfer coefficient* in the sequel. For the Maxwell and VHS-Maxwell molecules, we have

$$\chi^{\beta\alpha}=\begin{cases}2\pi(a^{\beta\alpha}/\mu^{\beta\alpha})^{1/2}A(g_c^{\beta\alpha}), & \text{with } A(z)=2\displaystyle\int_0^z g\cos^2\theta\mathrm{d}g \text{ for the Maxwell molecule,}\\ (\pi/2)(6kT_*/\mu^{\beta\alpha})^{1/2}(d_*^{\beta\alpha})^2, & \text{for the VHS-Maxwell molecule.}\end{cases} \tag{12}$$

Note that $\chi^{\beta\alpha}$ has its meaning only when $\beta\neq\alpha$ [see Eq. (10)] and $A(\infty)$ is finite ($=0.597$). It is also notable that Eqs. (9) and (12) lead to $\chi^{\beta\alpha}/K^{\beta\alpha}=A(g_c^{\beta\alpha})/(g_c^{\beta\alpha})^2$ for the Maxwell molecule and $1/2$ for the VHS-Maxwell molecule. In other words, $\chi^{\beta\alpha}$ is not independent of $K^{\beta\alpha}$ in the latter case, whereas it can take any value in the range $0<\chi^{\beta\alpha}\le K^{\beta\alpha}$ by adjusting the cutoff parameter $g_c^{\beta\alpha}$ (or $\theta_c^{\beta\alpha}$) in the former case (see Fig. 1).

MODEL EQUATIONS

We shall show below the models $J_M^{\beta\alpha}$ of the collision term $J^{\beta\alpha}$ proposed by Hamel [3], Garzó, *et al.* [4], and Andries, *et al.* [5].

$$J_M^{\beta\alpha}=K_M^{\beta\alpha}n^\beta\times\begin{cases}\left(n^\alpha M^\alpha(\boldsymbol{\xi};\boldsymbol{u}^{\beta\alpha},T^{\beta\alpha})-f^\alpha\right), & \text{for the Hamel model,}\\ \left(n^\alpha M^\alpha(\boldsymbol{\xi};\boldsymbol{u},T)\Phi^{\beta\alpha}-f^\alpha\right), & \text{for the Garzó model,}\\ \left(n^\alpha M^\alpha(\boldsymbol{\xi};\boldsymbol{u}^{(\alpha)},T^{(\alpha)})-f^\alpha\right), & \text{for the Andries model,}\end{cases} \tag{13}$$

where $K_M^{\beta\alpha}(=K_M^{\alpha\beta})$ is a positive constant, $M^\alpha(\boldsymbol{\xi};\boldsymbol{a},b)=(m^\alpha/2\pi kb)^{3/2}\exp(-m^\alpha|\boldsymbol{\xi}-\boldsymbol{a}|^2/2kb)$, and

$$\Phi^{\beta\alpha}=1+\frac{m^\alpha(\boldsymbol{u}^{\beta\alpha}-\boldsymbol{u})\cdot(\boldsymbol{\xi}-\boldsymbol{u})}{kT}+\left(\frac{T^{\beta\alpha}-T}{T}+\frac{m^\alpha|\boldsymbol{u}^{\beta\alpha}-\boldsymbol{u}|^2}{3kT}\right)\left(\frac{m^\alpha|\boldsymbol{\xi}-\boldsymbol{u}|^2}{2kT}-\frac{3}{2}\right). \tag{14}$$

Here we define $\boldsymbol{u}^{\beta\alpha}$ and $T^{\beta\alpha}$ appearing in the Hamel and Garzó models not by their original expressions in Refs. [3, 4] but by the following ones which contain another positive constant $\chi_M^{\beta\alpha}$ $(=\chi_M^{\alpha\beta})$:

$$\boldsymbol{u}^{\beta\alpha}=\boldsymbol{u}^\alpha+\frac{2\chi_M^{\beta\alpha}}{K_M^{\beta\alpha}}\frac{m^\beta}{m^\beta+m^\alpha}(\boldsymbol{u}^\beta-\boldsymbol{u}^\alpha),\quad T^{\beta\alpha}=T^\alpha+\frac{2\chi_M^{\beta\alpha}}{K_M^{\beta\alpha}}\frac{\mu^{\beta\alpha}}{m^\beta+m^\alpha}\left[T^\beta-T^\alpha+\frac{m^\beta}{3k}\left(1-\frac{\chi_M^{\beta\alpha}}{K_M^{\beta\alpha}}\right)|\boldsymbol{u}^\beta-\boldsymbol{u}^\alpha|^2\right]. \tag{15}$$

If one sets $\chi_M^{\beta\alpha}=K_M^{\beta\alpha}/2$, the original definitions of $\boldsymbol{u}^{\beta\alpha}$ and $T^{\beta\alpha}$ in Refs. [3, 4] are restored. Note that $0<\chi_M^{\beta\alpha}\le K_M^{\beta\alpha}$ must be satisfied to prevent $T^{\beta\alpha}$ from being negative. The $\boldsymbol{u}^{(\alpha)}$ and $T^{(\alpha)}$ in the Andries model are given by

$$\begin{aligned}&\boldsymbol{u}^{(\alpha)}=\boldsymbol{u}^\alpha+\frac{2}{C^\alpha}\sum_\gamma\frac{\chi_M^{\gamma\alpha}m^\gamma n^\gamma}{m^\gamma+m^\alpha}(\boldsymbol{u}^\gamma-\boldsymbol{u}^\alpha),\\ &T^{(\alpha)}=T^\alpha-\frac{m^\alpha}{3k}|\boldsymbol{u}^{(\alpha)}-\boldsymbol{u}^\alpha|^2+\frac{2}{C^\alpha}\sum_\gamma\frac{\chi_M^{\gamma\alpha}\mu^{\gamma\alpha}n^\gamma}{m^\gamma+m^\alpha}\left(T^\gamma-T^\alpha+\frac{m^\gamma}{3k}|\boldsymbol{u}^\gamma-\boldsymbol{u}^\alpha|^2\right),\end{aligned} \tag{16}$$

where $C^\alpha = \sum_\gamma K_M^{\gamma\alpha} n^\gamma$.

The above three models all satisfy the properties (i) and (ii) of the Boltzmann equation. As for the H-theorem [the property (iii)], only the Andries model has a rigorous proof. The Garzó model always satisfies the indifferentiability principle [the property (iv)], i.e., it reduces to the BGK model for the total distribution when the molecules of all the component species are mechanically identical. The Andries model has the same property only when $\chi_M^{\beta\alpha} = K_M^{\beta\alpha}$, but the Hamel model has no such property. It should be noted here that the Hamel model was originally developed for a binary gas mixture with disparate molecular masses.

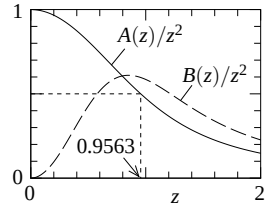


FIGURE 1. $A(z)/z^2$ and $B(z)/z^2$

The collision frequencies $\nu^{\beta\alpha}$ for the model equations are given by Eq. (8) with $K^{\beta\alpha} = K_M^{\beta\alpha}$. The collisional transfer of momentum and that of energy between β and α species per unit time are given by Eq. (10) (with $J^{\beta\alpha}$ in the left-hand side being replaced by $J_M^{\beta\alpha}$) with the transfer coefficient $\chi^{\beta\alpha} = \chi_M^{\beta\alpha}$. The ratio $\chi^{\beta\alpha}/K^{\beta\alpha}$ for the models, therefore, can take any value in the range $0 < \chi^{\beta\alpha}/K^{\beta\alpha} \leq 1$ as in the case of the Boltzmann equation for the Maxwell molecules. In contrast, the original Hamel and Garzó models [setting $\chi_M^{\beta\alpha} = K_M^{\beta\alpha}/2$ in Eq. (15)] give $\chi^{\beta\alpha}/K^{\beta\alpha} = 1/2$, which is the same fixed value as that obtained for the VHS-Maxwell molecules [see the sentences after Eq. (12)], and do not have freedom to chose $\chi^{\beta\alpha}$ independently. In this sense, the Hamel and Garzó models with Eq. (15) shown above can be said to be generalized versions of their originals.

NUMERICAL COMPARISON

In order to perform proper comparisons, we should adjust appropriately parameters concerning the molecular interaction contained in the respective molecular models and model equations. Here we adjust them in such a way that the collision frequency and the transfer coefficient become common in all comparisons. That is, assuming that the molecular mass m^α, the cutoff parameter $g_c^{\beta\alpha}$, and the constant $a^{\beta\alpha}$ in the intermolecular force law of the Maxwell molecule are given beforehand, we determine $K_M^{\beta\alpha}$ and $\chi_M^{\beta\alpha}$ of the model equations by Eqs. (9) and (12). We also determine $T_*^{1/4} d_*^\alpha$ of the VHS-Maxwell molecule by using Eq. (9). In this case, however, it is seen from what was mentioned after Eq. (12) that the VHS-Maxwell molecule does not give the correct value of $\chi^{\beta\alpha}$, unless $A(g_c^{\beta\alpha})/(g_c^{\beta\alpha})^2 = 1/2$ or $g_c^{\beta\alpha} = 0.9563$ (see Fig. 1).

Problem

The numerical comparison is performed in a half-space steady problem shown in Fig. 2. Let us consider a binary mixture of a vapor (species A) of a condensed phase and a noncondensable gas (species B) in a half space ($X_1 > 0$). The condensed phase is located at $X_1 = 0$ and kept at temperature T_w. The p_w is the saturation pressure of the vapor at temperature T_w. From the infinity ($X_1 \to \infty$), the equilibrium flow of the pure vapor with pressure p_∞, temperature T_∞, and velocity $\boldsymbol{u}_\infty$ $[= (u_{1\infty}, u_{2\infty}, 0); u_{1\infty} < 0, u_{2\infty} \geq 0]$ is flowing to the condensed phase, and a steady condensation is taking place at the surface of the condensed phase. The boundary condition at $X_1 = 0$ is the complete condensation for the vapor and the diffuse reflection for the noncondensable gas. The parameter Γ indicates the total amount of the noncondensable gas in the gas phase, whose definition is the same as that in Refs. [10, 11]. The comprehensive analysis of this problem for mixtures with mechanically identical molecules was performed on the basis of the Garzó model in the same references.

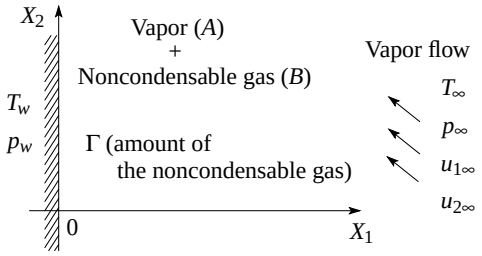


FIGURE 2. Problem

The problem described above may seem to be rather special, but it is a good sample problem to illustrate some nature of model equations for mixtures. Since the species B is a noncondensable gas, it can not penetrate into the condensed phase and its flow velocity along the X_1-axis is identically zero ($u_1^B \equiv 0$). Therefore one can adjust freely the strength of the diffusion (i.e., $|u_1^A - u_1^B|$) by changing the normal flow velocity at infinity $u_{1\infty}$. In addition, one can inspect the effect of the shear by putting a tangential flow at infinity, i.e., nonzero $u_{2\infty}$.

Results and discussions

The problem is characterized by eleven dimensionless parameters: T_∞/T_w, p_∞/p_w, $M_{n\infty}(=-u_{1\infty}/a_\infty)$, $M_{t\infty}(=u_{2\infty}/a_\infty)$, m^B/m^A, a^{BB}/a^{AA}, a^{BA}/a^{AA}, g_c^{AA}, g_c^{BB}, g_c^{BA}, and Γ, where $a_\infty[=(5kT_\infty/3m^A)^{1/2}]$ is the sound speed in the pure vapor at temperature T_∞ and $M_{n\infty}$ and $M_{t\infty}$ are the Mach number of the normal and tangential flow at infinity. Here we restrict ourselves to the case of $T_\infty/T_w=1$, $a^{BB}/a^{AA}=a^{BA}/a^{AA}=1$, and $g_c^{AA}=g_c^{BB}=g_c^{BA}$ ($\equiv g_c$), and seek the solutions changing the values of $M_{n\infty}(<1)$, $M_{t\infty}$, m^B/m^A, g_c, and Γ. Note that, in this case, the value of p_∞/p_w will be determined simultaneously with the solution (see Refs. [10, 11]). The Boltzmann equation is solved by the DSMC method and the model equations by a finite-difference method.

Figure 3 shows the results for a relatively weak condensation ($M_{n\infty}=0.1$) and Fig. 4 those for a strong condensation ($M_{n\infty}=0.5$) in the case of mechanically identical molecules ($m^B/m^A=1$). Here $\ell_w[=kT_w(8kT_w/\pi m^A)^{1/2}/p_wK^{AA}]$ is the mean free path of vapor molecules in the equilibrium state at rest with temperature T_w and pressure p_w. The T^B and u_2^B obtained by the DSMC simulation fluctuate at the far field from the condensed phase, since p^B is vanishing there and so the number of simulation particles for species B is very small.

The results of the VHS-Maxwell molecule are independent of the cutoff parameter g_c and agree well with those of the Maxwell molecule with $g_c=0.9563$. The pressures obtained by the model equations show good agreement with those by the Maxwell molecule, irrespective of g_c. The slope of profiles for $g_c=2$ by the Maxwell molecule becomes gradual compared with those for $g_c=0.9563$, whereas profiles of T^A, u_2^A, and u_2^B by the models are almost independent of g_c. Such behavior of the macroscopic quantities may be explained by the dependence on g_c of the transport coefficients, i.e., the mutual diffusion coefficient D_{AB}, viscosity μ, and thermal conductivity λ. Their first approximations in the Chapman–Enskog procedure are given by $D_{AB}=(kT/nm^AK^{AA})\hat{D}_{AB}$, $\mu=(kT/K^{AA})\hat{\mu}$, and $\lambda=(5k^2T/2m^AK^{AA})\hat{\lambda}$. Under the parameter setting in Figs. 3 and 4, one has $\hat{D}_{AB}=\hat{\mu}=(2/3)\hat{\lambda}=2$ for the VHS-Maxwell molecule, $\hat{D}_{AB}=g_c^2/A(g_c)$ and $\hat{\mu}=(2/3)\hat{\lambda}=g_c^2/B(g_c)$ for the Maxwell molecule, and $\hat{D}_{AB}=g_c^2/A(g_c)$ and $\hat{\mu}=\hat{\lambda}=1$ for the model equations, with $B(z)=6\int_0^z g\sin^2\theta\cos^2\theta\,\mathrm{d}g$. Consequently it is seen with the aid of Fig. 1 that (i) the Maxwell molecule and model equations have common D_{AB}, which increases as increasing g_c and coincides with that of the VHS-Maxwell molecule when $g_c=0.9563$; (ii) μ and λ of the Maxwell molecule have their minimum around $g_c\sim 0.8$, while those of the models are independent of g_c.

The p_∞/p_w, which is determined simultaneously with the solution, is approximately given by p^A/p_w at the rightmost point in each figure and shows model dependence. Thus it provides one of the measure of the reliability of model equations. The dependence is slightly observed in Figs. 3 and 4, but is more clearly seen in the case of a disparate-mass mixture (Fig. 5) and in the case of a very strong condensation (Fig. 6). We also made comparisons with other values

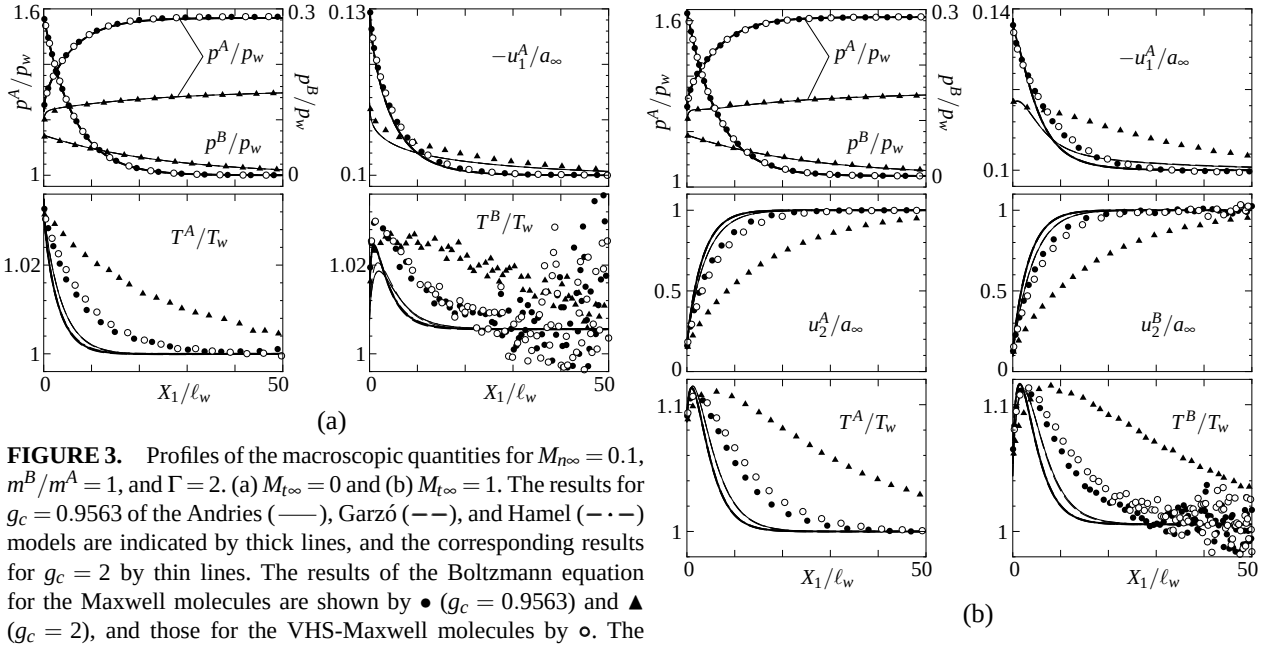
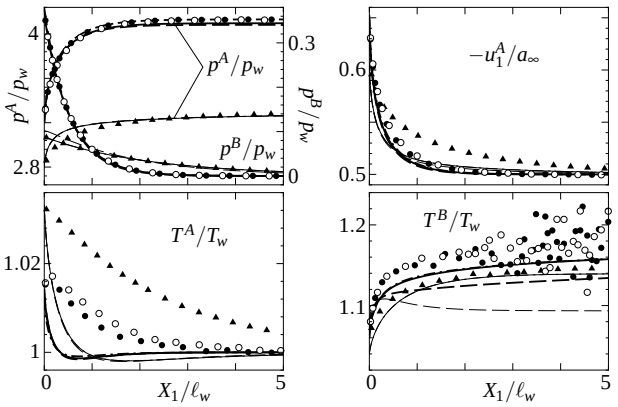


FIGURE 3. Profiles of the macroscopic quantities for $M_{n\infty}=0.1$, $m^B/m^A=1$, and $\Gamma=2$. (a) $M_{t\infty}=0$ and (b) $M_{t\infty}=1$. The results for $g_c=0.9563$ of the Andries (——), Garzó (− −), and Hamel (− · −) models are indicated by thick lines, and the corresponding results for $g_c=2$ by thin lines. The results of the Boltzmann equation for the Maxwell molecules are shown by • ($g_c=0.9563$) and ▲ ($g_c=2$), and those for the VHS-Maxwell molecules by ∘. The cutoff angle θ_c is 63.15° for $g_c=0.9563$ and 86.27° for $g_c=2$.



(a)

FIGURE 4. Profiles of the macroscopic quantities for $M_{n\infty} = 0.5$, $m^B/m^A = 1$, $\Gamma = 0.2$, and $g_c = 0.9563$ and 2. (a) $M_{t\infty} = 0$ and (b) $M_{t\infty} = 1$. See the caption of Fig. 3.

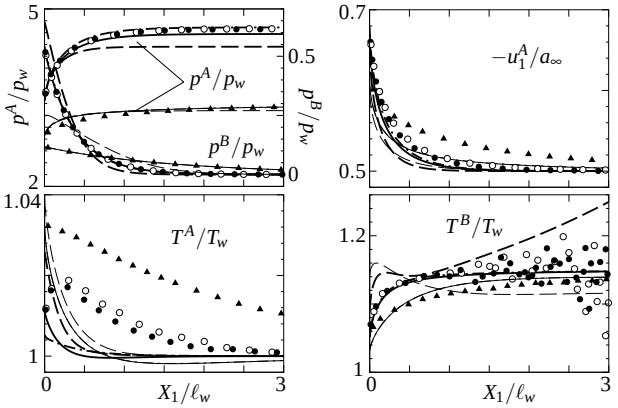


FIGURE 5. Profiles of the macroscopic quantities for $M_{n\infty} = 0.5$, $M_{t\infty} = 0$, $m^B/m^A = 5$, $\Gamma = 0.2$, and $g_c = 0.9563$ and 2. See the caption of Fig. 3.

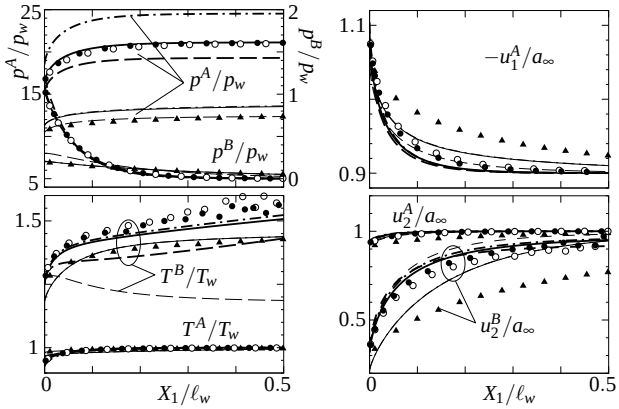
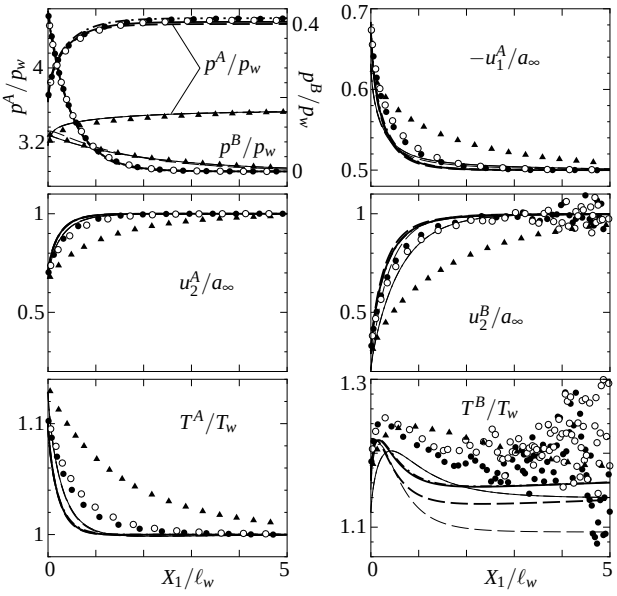


FIGURE 6. Profiles of the macroscopic quantities for $M_{n\infty} = 0.9$, $M_{t\infty} = 1$, $m^B/m^A = 1$, $\Gamma = 0.06$, and $g_c = 0.9563$ and 2. See the caption of Fig. 3.



(b)

of m^B/m^A and Γ. As a result, we have a feeling that the Andries model gives not always best but stably close approximations to results of the Boltzmann equation, whereas the other models may fail in some cases.

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