

On the existence of the negative mass flows in evaporation and condensation problems

– Effects of the finite thermal conductivities of the condensed phases –

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Abstract. A peculiar phenomenon in evaporation and condensation problems would be the existence of the so-called *negative mass flow* in the sense that the direction of the mass flows may become opposite to what one would usually expect to be. This phenomenon, which may manifest itself in particular situations in which the latent heat parameter is quite small under the existence of some degree of rarefaction of gas phase, was first noticed by Sone and Onishi in their asymptotic analysis for steady gas flows of general type based on the linearized kinetic equation. The detailed study on this phenomenon, which has long been left untouched, has recently been carried out by Onishi. However, the fact that the thermal conductivity of the condensed phase is finite is not taken into account, in spite of the well-known fact that the finite thermal conductivity of the condensed phase may change drastically the role of the latent heat parameter and, hence, the mass and energy flows, leading to the existence of their maximum values. It is the present study to clarify its effects on the existence and the magnitude of this *negative mass flow* by taking up a simple two-surface problem of evaporation and condensation.

INTRODUCTION

One of the features in evaporation and condensation problems would be the existence of the so-called *negative mass flow* in the sense that the direction of the mass flows due to evaporation and condensation processes may become opposite to what one would usually expect to be. This negative mass flow, which was first noticed by Sone and Onishi [1] in their asymptotic analysis for steady gas flows of general type based on the linearized version of the kinetic equation [2], is a flow phenomenon manifesting itself in particular situations in which the latent heat parameter, the ratio of the latent heat to the enthalpy of the vapor, is quite small under the existence of some degree of rarefaction of the gas phase. However, this phenomenon has attracted no attention for a long time and, therefore, a question of whether or not the negative mass flow phenomenon is a real thing or a fake has long remained unsolved. Recently, Onishi [3], by taking up a so-called two-surface problem of evaporation and condensation, has made a simulation analysis on this negative mass flow based on the kinetic equation [2] and has clarified that the appearance of the phenomenon is not necessarily restricted to the steady states of the flow fields nor the weakness of evaporation and condensation processes. It is likely to appear at early stages of the flow fields, when the latent heat parameter is rather small in magnitude, then either disappearing gradually as time goes on or persisting further on until the steady states are established, depending on the magnitude of the parameter. However, in his analysis the thermal conductivity of the condensed phase being finite compared to that of its vapor has not been taken into account and, therefore, it is the present work that tries to show how it affects the negative mass flow phenomenon. The reason is that, as one of the significant effects of the finite thermal conductivity of the condensed phase, the roles of the latent heat parameter involved in the problem may drastically be changed, when compared to cases so far done in which the condensed phase has always been assumed to have infinite thermal conductivity. This fact has been pointed out and clarified by Onishi and Yamada [4] in their recent work on a half-space problem of evaporation and condensation and further detailed information given by Onishi & Ooshida [5] later on a two-surface problem of evaporation and condensation. Actually, a certain value of the latent heat parameter gives the maximum strength of the phase change process under the finite thermal conductivity of the condensed phase. The fact that the maximum values of the mass and energy flows exist for a certain value of the latent heat parameter is in distinctive contrast to our knowledge so far we have obtained on this

kind of problems. Actually, the role of the latent heat parameter is known to be quite simple; it increases simply the mass and energy flows as it becomes large when the thermal conductivity of the condensed phase is sufficiently large compared to that of its vapor and, consequently, it may well be assumed to be infinite. With this background in mind, the manifestation of the negative mass flow phenomenon and the effects on it of the finite thermal conductivity of the condensed phase will be studied by taking up a two-surface problem here also.

GOVERNING KINETIC EQUATIONS

Consider a two-surface problem in which a vapor is contained between the two plane condensed phases placed in parallel. The condensed phases occupy regions of $-D_1 \leq x \leq 0$ and $L \leq x \leq L + D_2$, respectively, their interface surfaces being at $x = 0$ and $x = L$. Initially, the vapor phase and its condensed phases are in complete equilibrium at a temperature T_0 . Let the pressure and density (or number density) of the vapor at this state be P_0 and ρ_0 (or N_0), respectively. Suppose that at a certain time, say $t = 0$, the temperatures of the edge surfaces of the condensed phases at $x = -D_1$ and $x = L + D_2$ are suddenly changed to T_{c1} and T_{c2} , respectively. The heat flows then occur through the condensed phases and after a certain elapse of time the temperatures of the interface surfaces start to be changing with time. This leads to the onset of phase change processes at the interfaces, giving then rise to a transient motion of the vapor in between. This is what the present study is concerned with (see Fig. 1).

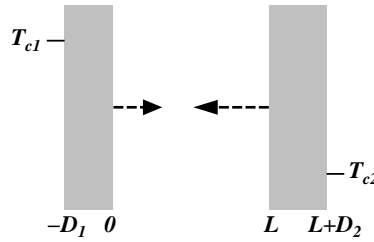


FIGURE 1. A schematic view of the possible occurrence of the *negative mass flow* in a certain situation in which the latent heat parameter is very small.

Owing to the thermal conductivity of the condensed phase being finite compared to that of the vapor phase, the temperature field is formed within the condensed phase, so that the simple equation of heat conduction with constant substance properties is used for that description, which may be written as

$$\left. \begin{aligned} \frac{\partial \tilde{T}_1}{\partial t} - \kappa_c \frac{\partial^2 \tilde{T}_1}{\partial x^2} &= 0 & (-D_1 < x < 0) \\ \frac{\partial \tilde{T}_2}{\partial t} - \kappa_c \frac{\partial^2 \tilde{T}_2}{\partial x^2} &= 0 & (L < x < L + D_2) \end{aligned} \right\} \quad (1)$$

where t is the time; x is the coordinate; κ_c is the thermal diffusivity of both of the condensed phases (assumed constant) and \tilde{T}_i represents the temperature field of the i -th condensed phase. For the description of the motions of the vapor, on the other hand, the Boltzmann equation of BGK type [2] is used, which may be written as

$$\frac{\partial f}{\partial t} + \xi_x \frac{\partial f}{\partial x} = N \nu_c (F_e - f) \quad (2)$$

$$F_e = \frac{N}{(2\pi RT)^{3/2}} \exp \left\{ -\frac{(\xi_x - u)^2 + \xi_y^2 + \xi_z^2}{2RT} \right\} \quad (3)$$

$$\left[\begin{array}{c} N \\ Nu \\ \frac{3}{2} N k T \end{array} \right] = \iiint \left[\begin{array}{c} 1 \\ \xi_x \\ \frac{1}{2} m [(\xi_x - u)^2 + \xi_y^2 + \xi_z^2] \end{array} \right] f d\xi_x d\xi_y d\xi_z \quad (4)$$

$$P = N k T = \rho R T \quad (5)$$

where (ξ_x, ξ_y, ξ_z) is the molecular velocity vector; f is the molecular velocity distribution function, F_e being the local Maxwellian distribution characterized by the local fluid dynamic quantities; N , u , T , P and ρ are, respectively,

the number density, the velocity, the temperature, the pressure and the density of the gas; m is the molecular mass; k is the Boltzmann constant and $R = k/m$ the gas constant per unit mass of gas. ν_c is a constant associated with the collision frequency ($N\nu_c$ is the local collision frequency) and, hence, can be calculated either from the viscosity μ or from the thermal conductivity λ of the gas at a certain reference state, say, at the initial equilibrium state, by the following relation

$$N_0 \nu_c = \frac{P_0}{\mu_0} = \frac{5}{2} R \frac{P_0}{\lambda_0} \quad (6)$$

the suffix 0 being understood to indicate the quantities at the initial state. It may be noted here that, in the BGK model equation, the relation $\lambda = (5/2)R\mu$ holds and, hence, the Prandtl number Pr is always unity for this model equation.

The initial conditions for the present problem may be specified as follows:

$$\tilde{T}_1 = \tilde{T}_2 = T_0 \quad (-D_1 \leq x \leq 0, \quad L \leq x \leq L + D_2) \quad (7)$$

for the temperatures of the condensed phases and

$$f = \frac{N_0}{(2\pi RT_0)^{3/2}} \exp \left\{ -\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT_0} \right\} \quad (0 < x < L) \quad (8)$$

for the distribution function of the molecules of the gas phase, the components of the molecular velocity vector (ξ_x, ξ_y, ξ_z) having all possible values. The boundary conditions at the edges of the two condensed phases and at the interface surfaces between the gas phase and the condensed phases become as follows:

$$\left. \begin{array}{ll} \tilde{T}_1 = T_{c1} & \text{at } x = -D_1 \\ \tilde{T}_2 = T_{c2} & \text{at } x = L + D_2 \end{array} \right\} \quad \text{and} \quad \left. \begin{array}{ll} \tilde{T}_1 = T_{W1} & \text{at } x = 0 \\ \tilde{T}_2 = T_{W2} & \text{at } x = L \end{array} \right\} \quad (9)$$

$$f = \frac{N_{W1}}{(2\pi RT_{W1})^{3/2}} \exp \left\{ -\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT_{W1}} \right\} \quad \text{for } \xi_x > 0 \quad \text{at } x = 0 \quad (10)$$

$$f = \frac{N_{W2}}{(2\pi RT_{W2})^{3/2}} \exp \left\{ -\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT_{W2}} \right\} \quad \text{for } \xi_x < 0 \quad \text{at } x = L \quad (11)$$

for molecules leaving each of the interface surfaces, respectively, where T_{Wi} ($i = 1, 2$), which is an unknown parameter, represents the temperature of the interface surface itself and will eventually be determined as part of the solution. N_{Wi} is the saturated vapor number density at the temperature T_{Wi} of the surface of the condensed phase to be determined by the Clapeyron-Clausius relation as

$$N_{Wi} = \frac{P_{Wi}}{k T_{Wi}}, \quad \frac{P_{Wi}}{P_0} = \exp \left\{ -\Gamma \left(\frac{T_0}{T_{Wi}} - 1 \right) \right\} \quad \text{with } \Gamma \equiv \frac{h_L}{R T_0} \quad (12)$$

where Γ is a nondimensional parameter, h_L being the latent heat of vaporization per unit mass.

In addition to the above specification of the conditions at the boundary surfaces, the condition of the continuity of energy flow across each of the interfaces has to be imposed, which may be expressed as

$$\mp \lambda_c \frac{\partial \tilde{T}_i}{\partial x} = \pm E \pm \rho u (h_L - c_p T) \quad (13)$$

the upper sign applying to the interface at $x = 0$ and the lower to that at $x = L$, respectively, where λ_c is the thermal conductivity of the condensed phase itself (assumed constant). c_p is the specific heat at constant pressure. E is the component of the energy flux of the gas in the x -direction defined by

$$E = \frac{1}{2} m \iiint \xi_x (\xi_x^2 + \xi_y^2 + \xi_z^2) f d\xi_x d\xi_y d\xi_z \quad (14)$$

which is here to be evaluated at the interfaces.

CHARACTERISTIC PARAMETERS

For the analysis of the present problem, we have introduced L , the distance between the condensed phases, as the length scale and τ_0 taken as

$$\tau_0 \equiv \frac{L}{(2RT_0)^{1/2}} = \left(\frac{\gamma}{2}\right)^{1/2} \frac{L}{c_0} = \frac{1}{N_0 \nu_c} = \frac{\mu_0}{P_0} \quad \text{with} \quad c_0 \equiv (\gamma RT_0)^{1/2} \quad (15)$$

as the time scale, where c_0 , taken as one of the velocity scales here, is the sound speed of the gas at the initial state, γ being the specific heat ratio ($\gamma = 5/3$ here). With the fluid dynamic quantities at the initial state together with these length, velocity and time scales, the system of the governing heat conduction and kinetic equations and the initial and boundary conditions is appropriately nondimensionalized, giving the following non-dimensional parameters characterizing the present flow field

$$\frac{T_{c1}}{T_0}, \quad \frac{T_{c2}}{T_0}, \quad \frac{D_1}{L}, \quad \frac{D_2}{L}, \quad \frac{\lambda_c}{\lambda_0}, \quad \frac{\kappa_c}{\kappa_0}, \quad \Gamma \equiv \frac{h_L}{RT_0}, \quad Kn \equiv \frac{l_0}{L} \quad (16)$$

where λ_0 and $\kappa_0 \equiv \lambda_0/(\rho_0 c_p)$ are the thermal conductivity and the thermal diffusivity of the gas at the initial state, respectively. Kn is the Knudsen number, l_0 being the mean free path of the gas molecules at the initial state defined by $l_0 = (\mu_0/P_0)(8RT_0/\pi)^{1/2}$. It may be noted that the Reynolds number Re defined by $Re \equiv \rho_0 c_0 L/\mu_0$, μ_0 being the viscosity of the gas at the initial state, is related here to the Knudsen number as $Re = (8\gamma/\pi)^{1/2}(1/Kn)$.

RESULTS AND DISCUSSION

The simulation has been carried out for so many cases. However, only one of the flow fields will be shown here through the pressure and temperature distributions in Fig. 2, in order to get a rough idea about how a transient flow field of a vapor develops toward its steady state with the eventual negative mass flow. In this case, the velocity of the vapor gradually decreases, becoming very small in magnitude and, hence, the diffusion process becomes prevailing in the flow field, as a consequence of which the temperature, as one can see in the figure, assumes a very simple and almost straight line type of distribution at the steady state (of course, not exactly a straight line because of the temperature dependency of the thermal conductivity of the gas). Now let us look at the distributions of the mass and energy fluxes,

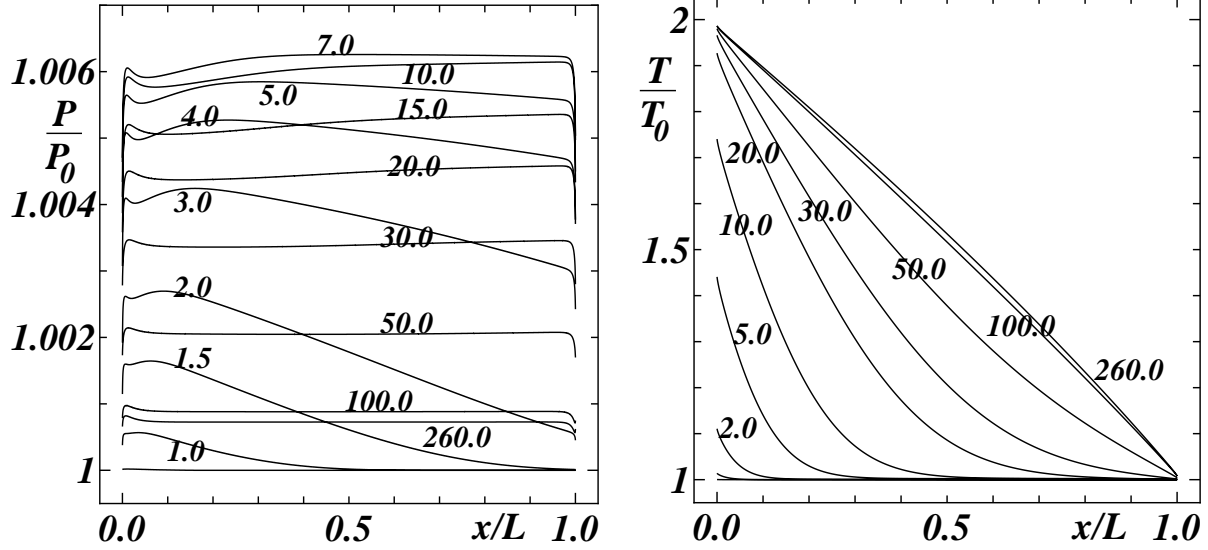


FIGURE 2. Transient to steady distributions of the pressure P/P_0 , and the temperature T/T_0 of a vapor between the two condensed phases placed in parallel at $x/L = 0$ and $x/L = 1$, respectively. $T_{c1}/T_0 = 2.0$, $T_{c2}/T_0 = 1.0$, $D_1/L = D_2/L = 0.1$, $\lambda_c/\lambda_0 = 30.0$, $\kappa_c/\kappa_0 = 0.32$, $\Gamma = 0.001$, $Kn = 0.005$ ($Re = 412.03$). The numbers in the figures indicate the time t/τ_0 .

transient to steady, which are shown in Fig. 3. One can see there the negative mass fluxes near the hotter condensed

phase and positive in the rest of the flow region at early stages of the flow. The mass flux, then, gradually becomes smaller and smaller and eventually becomes negative in the whole flow region, whereas the energy flux, increasing at early stages and then gradually becoming smaller, keeps always its positive value. The negative mass flux persisting at the steady state is in general very small and, in this case, its value is $\rho u/\rho_0 c_0 = -0.00116$ (see Table 1). It may also be interesting to see how the distribution functions at the interfaces look like when the negative mass flow phenomenon is manifesting itself. To see this, one of the so-called reduced distribution functions defined by

$$g^A(t, x, \xi_x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, x, \xi_x, \xi_y, \xi_z) d\xi_y d\xi_z \quad (17)$$

from which the number density N and the velocity u of the vapor can be calculated, has been introduced and its non-dimensional version $\hat{g}^A = [(2RT_0)^{1/2}/N_0] g^A$ is plotted against $\hat{\xi}_x = \xi_x/(2RT_0)^{1/2}$ in Fig. 4, where the enlarged graphs are also included. From Fig. 4, it may be clear that the number density $N^{(-)}/N_0$ of the molecules going into the interface, say at $x = 0$, is slightly larger than the number density $N^{(+)}/N_0$ of the molecules leaving the interface by looking at the areas bounded by \hat{g}^A for $\hat{\xi}_x < 0$ and for $\hat{\xi}_x > 0$, which are given by the integrals $\int_{\hat{\xi}_x < 0} \hat{g}^A d\hat{\xi}_x$ and $\int_{\hat{\xi}_x > 0} \hat{g}^A d\hat{\xi}_x$, respectively. Of course, this does not directly mean that the number flux of the incoming molecules is larger than that of the outgoing molecules because it is the number density that we have just dealt with not the number flux. However, one may be able to imagine that the incoming molecules are likely to be more than those outgoing from it, contrary to what one would generally expect. The detailed discussion of this phenomenon itself and the explanation of its occurrence may be found in a recent work of Onishi [3].

In contrast to the cases treated by Onishi [3], the flow fields of the vapor formed in the present cases are weaker in general except the temperature fields at the steady state. The final temperature fields are almost the same in both cases because they are governed almost totally by the diffusion. The differences or the jumps in the pressure and temperature at the interfaces, which are the driving forces for the phase change processes to occur, are quite small in the present cases because the energy to produce eventually the motions of the vapor is supplied by the heat conduction through the condensed phases. However, the magnitudes of the negative mass flows, if they occur, are likely to be larger than those for the infinite thermal conductivity cases. One can recognize this from the comparison between Table 1 here listed with the corresponding table given in Ref. [3], from which the order of magnitudes of the negative mass flows may be grasped in both cases. Of course, the appropriate comparison of the results between the two cases is difficult but, as far as the comparison from the viewpoint of the latent heat parameter is concerned, the above can be stated.

TABLE 1. Some of the sets of the parameters associated with the existence and the non-existence of the negative mass flow phenomenon. $T_{c1}/T_0 = 2.0$, $T_{c2}/T_0 = 1.0$ and $Kn = 0.005$. The negative value of $\rho u/\rho_0 c_0|_{x=0}$ indicates the existence of the negative mass flow phenomenon at the steady state.

$\Gamma \equiv h_L/(RT_0)$	λ_c/λ_0	κ_c/κ_0	$\rho u/\rho_0 c_0 _{x=0}$	$E/P_0(2RT_0)^{1/2} _{x=0}$
0.001	30.0	0.32	$-0.1158379 \times 10^{-2}$	0.4048206×10^{-2}
0.01	30.0	0.32	$-0.2450742 \times 10^{-3}$	0.7218703×10^{-2}
0.05	30.0	0.32	0.3698902×10^{-2}	0.2181820×10^{-1}
0.001	50.0	0.5	$-0.1162925 \times 10^{-2}$	0.4063541×10^{-2}
0.01	50.0	0.5	$-0.2521902 \times 10^{-3}$	0.7255878×10^{-2}
0.02	50.0	0.5	0.7526458×10^{-3}	0.1084914×10^{-1}
0.001	80.0	0.5	$-0.1169074 \times 10^{-2}$	0.4071373×10^{-2}
0.05	80.0	0.5	0.3740663×10^{-2}	0.2202654×10^{-1}

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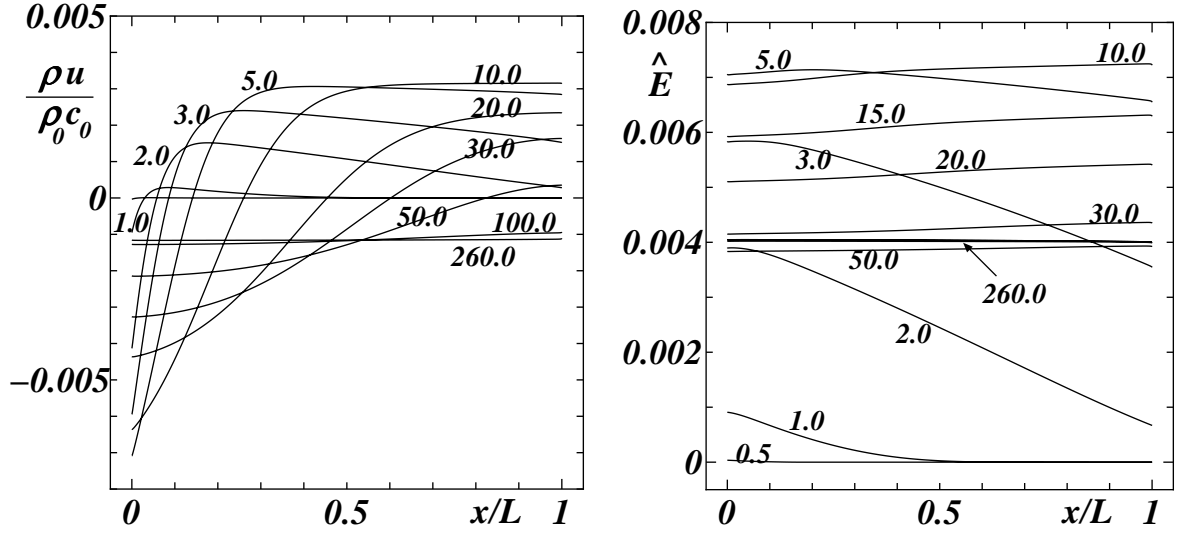


FIGURE 3. Transient to steady distributions of the mass and energy flows of a vapor between the two condensed phases placed in parallel at $x/L = 0$ and $x/L = 1$, respectively $T_{c1}/T_0 = 2.0$, $T_{c2}/T_0 = 1.0$, $D_1/L = D_2/L = 0.1$, $\lambda_c/\lambda_0 = 30.0$, $\kappa_c/\kappa_0 = 0.32$, $\Gamma = 0.001$, $Kn = 0.005$ ($Re = 412.03$). \hat{E} defined by $\hat{E} = E/P_0(2RT_0)^{1/2}$ represents the nondimensional energy flux of a vapor. The numbers in the figures indicate the time t/τ_0 .

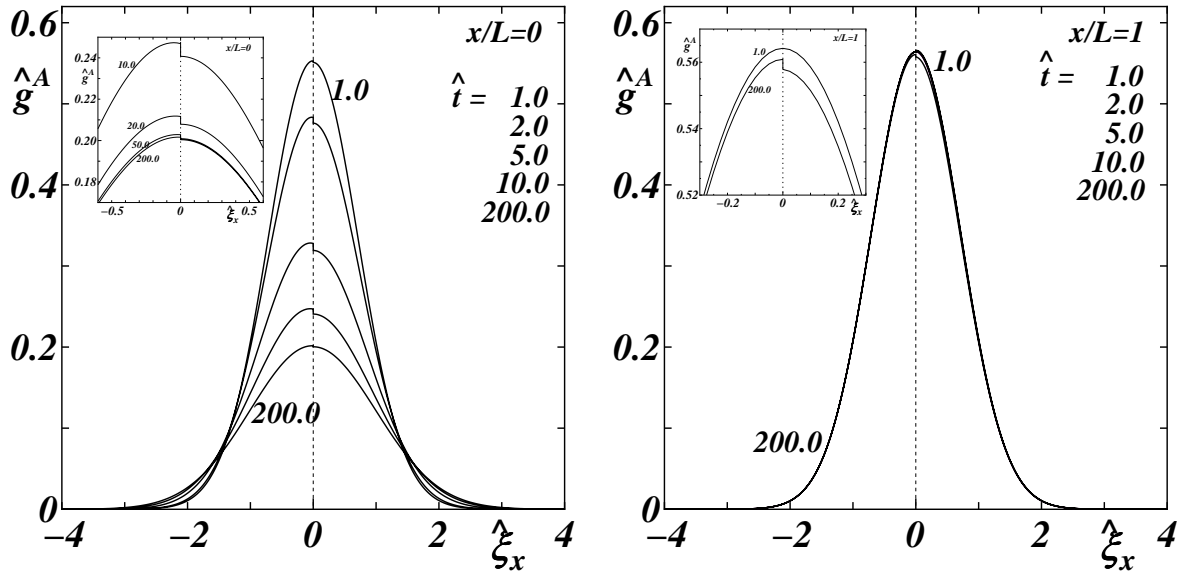


FIGURE 4. The reduced distribution function $\hat{g}^A = [(2RT_0)^{1/2}/N_0]g^A$ against the molecular velocity $\hat{\xi}_x = \xi_x/(2RT_0)^{1/2}$. $T_{c1}/T_0 = 2.0$, $T_{c2}/T_0 = 1.0$, $D_1/L = D_2/L = 0.1$, $\lambda_c/\lambda_0 = 30.0$, $\kappa_c/\kappa_0 = 0.32$, $\Gamma = 0.001$, $Kn = 0.005$ ($Re = 412.03$). The numbers in the figures indicate the time $\hat{t} \equiv t/\tau_0$. The enlarged graphs show clearly the discontinuity of the reduced distribution function g^A .