Macroscopic Models for Non-equilibrium Flows of Monatomic Gas and Normal Solutions

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Abstract. Description of macroscopic models (objective, species of modeling flows and principles of models creation) is presented. The role of exact, Hilbert normal series and numerical (DSMC) solutions of kinetic equations for analysis of transport processes and for models construction is analyzed. The proposed model, based on conservation equation and on nonlinear (in first order gradients of velocity and temperature) constitutive relations are developed. Application to determination of normal stress in high velocity flow past thin plane plate at zero angle of attack is demonstrated.†

The suitable mathematical model for non-equilibrium flows of monatomic one-component gas is Boltzmann equation [1 - 3]. The most applicable method of solution for this equation is DSMC method [4]. The goals of design more simple approximate models are more fundamental understanding properties of Boltzmann equation itself and properties of flows. Particularly derivation of approximate similarity parameters for processing DSMC data in complex flows.

If Knudsen number of the order of unity and if there is no possibility to separate thin boundary Knudsen layers, macroscopic models lead to finite value of inaccuracy, which tend to zero only with infinite complication of the models. In this case it is known two types of macroscopic models. In the one type of model design, in moment method, it is used the various sequences of integral consequences of kinetic Boltzmann equation under various approximations of velocity distribution function with set of macroscopic parameters. In another type, on the first step of macroscopic model design, there is change of Boltzmann collision integral by relaxation term with finite number of macroscopic parameters. At the second step, these model kinetic equations are transformed to integral form, to system of nonlinear integral equations for macroscopic parameters.

If values of local Knudsen number are small ones in a gas flow (excluding thin Knudsen layers), then the solution of kinetic Boltzmann equation degenerates to solution in Hilbert normal series [5, 1]. In this case velocity distribution function depends on time and spatial coordinates only through dependence upon density, velocity and temperature (and its gradients of any order). Unfortunately, normal solutions in closed form are found only for some singular flows (example will be done lower) and only one simple boundary problem (it will be discussed later). The lack of normal solution in closed form (as a “sum” of normal series) leads to using for description of flows with small values of Knudsen number mainly first term of normal series. Macroscopic model – Navier-Stokes equations - corresponds to this first order approximation. Research of flow in thin Knudsen boundary layers leads to formulation the slip boundary conditions for Navier-Stokes equations, which make wider their applicability (for larger value of Knudsen number) in comparison with usual using zero velocity boundary condition.

Special attention of investigator to the second approximation of the normal series (to Burnett equations [6, 2]) is explained by two reasons. On the one hand, it is explained by the hope to make wider applicability of macroscopic description for larger value of Knudsen number and to give more exact description of flows in the cases where Navier-Stokes equations give harmless approximation. On the other hand, it is explained by the hope to discover new unknown effects with the help of these equations. What is about refinement of description, most typical example is the flow in weak shock wave. In fact, it was shown that Burnett equations give more exact description of the profile of shock wave: true effect of asymmetry [7] and true distance between density and temperature profiles [8]. The examples of new effects revealed with the help of Burnett equations are the following ones:

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1) existence the component of heat flux, perpendicular to temperature gradient [9]; 2) development the theory of slow non-isothermal flows [10-12] with new type effects of free convection, negative drag force acting on strongly heated sphere, repulsive force between heated or cooled particles.

However, mentioned merits of Burnett equations are compensated by its demerits: instability of solution for steady equilibrium state [13], absence the elaborated theory of boundary conditions for these equations and so on. In this connection the attempts to construct new macroscopic models are continued. On this path the modification of Burnett equations, modification of 13-moment equations and another approaches are proposed.

Let us start our brief review of the principles of models construction from semi-empirical modifications the second order term of normal series – Burnett equations [14-15]. These modifications intend for mathematical modeling the strong shock wave flow only. These modifications use part of super Burnett terms in constitutive relations (for stress and heat flux) for stabilizing Burnett equations and improvement the other its properties in the first modified version. The iteration method is used in the second version. In this version Navier –Stokes and part of Burnett terms are included in equations at the first step. At the second step the same equations are solved, but with source term – other terms of Burnett and super-Burnett stress and heat flux, calculated by the solution of first step. These modifications lead to equations, for which the solution exists at any Mach number. The profiles of density and temperature, calculated by these modifications, are closer one to DSMC solution, than Navier-Stokes profiles. But the problem of limit point for hypersonic shock wave is not removed.

Remember, that problem of limit point firstly was discovered by V.V. Sychev for Navier-Stokes equations. It consists in undisturbed flow in upwind part of shock wave and is explained by the trend of temperature, and as a consequence the viscosity coefficient, to zero in upwind part of flow as Mach number tends to infinity. The profiles of macroscopic parameters, calculated by all known macroscopic models, based on differential form (modification of normal series terms or finite number of differential moment equations) have this singularity. Urge of researchers towards testing new macroscopic models by the problem of shock wave, though this problem can not be described by Hilbert normal solution is explained by absence of boundary conditions at finite distance in it.

More general semi-empirical modification of Burnett equations was proposed in [16], where third order linear terms with gradients of density and temperature are added for stabilizing the numerical solution of Burnett equations (it named as “augmented Burnett equations”). This modification was applied not only to shock wave problem, but also to axially symmetric hypersonic flow past blunt body [17]. The comparison augmented Burnett equations solution on stagnation streamline with DSMC solution shows [18, 19] the deficiency profiles of density and temperature for small Knudsen number ($M_\infty = 10,\ Kn_\infty = 0.1$). There it is seen, that problem of limiting point for strong shock wave exists in this case also. In [19] besides augmented Burnett equations were analyzed “BGK-Burnett equations” also. Last term introduced by authors because they derived their modified version of Burnett equations with the help of BGK kinetic model equation. Their procedure is modified Chapman-Enskog method. This modification differs from augmented Burnett equations in a small part.

One more version of modified Burnett equations is proposed in [20]. It consists in linear dependence of stress on velocity gradient and heat transfer on temperature gradient with coefficients, second-order dependent on gradients invariants. The goal of this paper is consistency with generalized Clausius-Duhem inequality. In this paper general properties of modified equations are researched and result is compared with exact singular solution of Boltzmann equation only.

These approaches use some part of normal solution of Boltzmann equation for modeling: conservation equations supplemented by various approximations of constitutive relations for stress and heat flux. These constitutive relations express stress and heat flux through gradients of velocity, temperature and pressure (or density).

Another approach, based on truncated set of moment equations, goes back to Grad’s systems (especially to 13-M approximation). In [21] Burnett constitutive relations transformed to differential equations for stress and heat flux (constitutive equations) by substitution these quantities instead gradients of velocity and temperature in linear terms. This substitution is not unique. Chosen version gives an opportunity to author to investigate the problem of nonlinear acoustics.

More elaborated approach is presented in set of papers [22-24]. In these papers closure of 13-M equations for stress and heat flux is carried out not by Grad’s procedure, but with the using the higher moment equations. Consequence of approximation is achieved by introduction the fictitious small parameter – time relaxation of higher moments of distribution function. This parameter at the end of derivation is equal to unity.

Transition place in these approaches is filled by the approach, which uses all of these properties [25-27]. The thermodynamic arguments, 13-M equations, semi-empirical transformation of equations and choice of parameters are exist in this approach. Unfortunately the profile of shock wave, calculated by these equations has little in common with DSMC profile: the problem of limit point particularly expressive one.

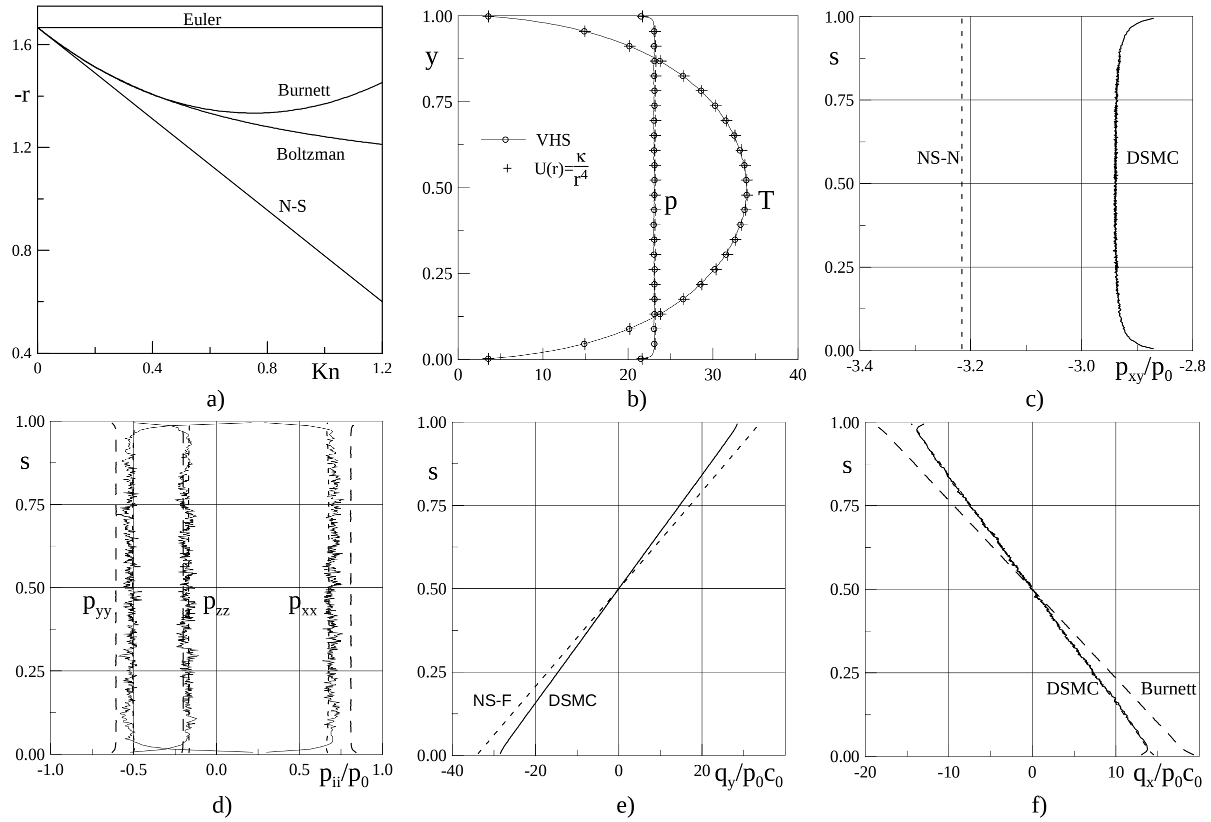

FIGURE 1. Analysis of pressure in singular flow and transport parameters in Couette flow: a) pressure in flow (1); b) comparison of VHS-DSMC solution and true potential DSMC solution for Couette flow ($Kn_0 = 0.00438,\ S_w = 20,\ T_{w2}/T_{w1} = 1$, parameters the same ones for figures b) – f)); c) comparison of DSMC result for shear stress and Navier-Stokes (Newton) constitutive relation, calculated by DSMC results for velocity and temperature; d) comparison of DSMC result for normal stresses and Burnett constitutive relation, calculated by DSMC results for velocity and temperature (long dashed line) and by DSMC result for shear stress (short dashed line); e) comparison of DSMC result for transverse heat flux and Navier-Stokes (Fourier) constitutive relation, calculated by DSMC result for temperature; f) comparison of DSMC result for longitudinal heat flux and Burnett constitutive relation, calculated by DSMC results for velocity and temperature (long dashed line) and for shear stress and transverse heat flux (short dashed line). s – modified transverse coordinate (in dimensional form): $d/ds = \mu\, d/dy$. Burnett constitutive relations for Couette flow see in (2).

DSMC AS A TOOL FOR TRANSPORT PROPERTIES RESEARCH

The possibility to test various approximation of normal solution or another macroscopic model by exact solution of Bolzmann equation is confined by singular flows. For example consider flow, in which only velocity proportional to coordinate, **x**, and all other moments depend only on time, t, [28-30]. Then for one-dimensional flow the normal solution may be written down in closed form (in non-dimensional variables for special initial condition):

$$u_x = x(1+\alpha)^{-1} \quad , \quad \alpha = const \quad , \quad p(t) = p_0[1+(t/\alpha)]^r \quad , \quad r = -(1/2Kn)\{1+4Kn-[1+4Kn((1/3)+Kn)]\} \qquad (1)$$

On Fig.1a) the solution (1) is shown with three terms of normal series. As it is seen Burnett approximation better, than Navier-Stokes one for small Knudsen number (for small non-equilibrium parameter), but deviate from normal solution in closed form at strong non-equilibrium conditions.

Comparisons of density, velocity and temperature fields, calculated by DSMC method and by some macroscopic model are used usually for more complex flows. For these comparisons it is necessary to have boundary conditions for applied models. At initial stage of research or owing to complexity of model equations these boundary conditions are frequently absent (this is one reason to test the models by the shock wave problem). But the research of transport properties directly gives more information about applicability of constructed model to any flow. The first step was done in [31], where heat flux, calculated by DSMC method, was compared with Fourier constitutive relation for it. The elaboration of this approach to analysis of the stress and the heat flux for Couette flow and shock wave flow was

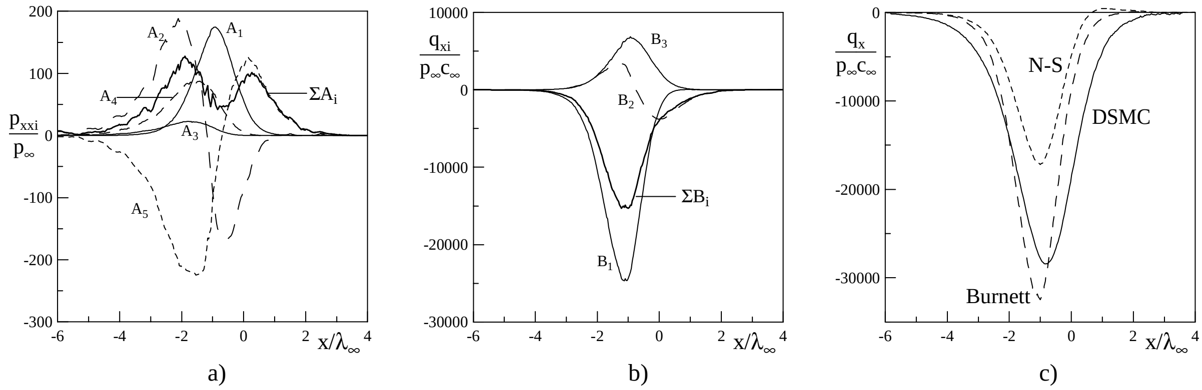

FIGURE 2. Burnett stress (a) and heat flux (b) components (notation see in equations (3)) in shock wave with $M_\infty = 50$ and hard-sphere molecular model; heat flux for this shock wave:1 – DSMC solution, 2 – Navier Stokes (Fourier) constitutive relation, 3 – Burnett constitutive relation. All constitutive relations are calculated by the DSMC results for velocity, temperature and pressure.

presented in [32-35]. The results for Couette flow for moderate Knudsen number were presented in [32]. The results for small Knudsen number and VHS-model of molecular interaction are shown on Fig. 1b)-1f). On Fig. 1b) the acceptability of this model for Couette flow with stated flow parameters is shown. It is done by comparison of VHS results and results, calculated by real Maxwell's potential. On Fig.1c)-1d) the analysis of stress components and on Fig 1e)-1f) the analysis of heat flux components are carried out. For shear stress and transverse heat flux there are no Burnett correction terms to Navier-Stokes (Newton-Fourier) constitutive relations. Therefore errors of these quantities are small for small Knudsen number. For normal component of stress tensor and longitudinal component of heat flux Navier-Stokes (Newton-Fourier) constitutive relations give zero values. The Burnett values of these quantities for Maxwell molecular model are presented below:

$$p_{xx}^B = 1.6\,(p_{xy}^N)^2 / p + O(Kn^4) \quad , \quad p_{yy}^B = -1.2\,(p_{xy}^N)^2 / p + O(Kn^4) \quad , \quad q_x^B = 3.5\,q_y^F \cdot p_{xy}^N / p + O(Kn^4) \qquad (2)$$

It is interesting to note that though errors of Burnett approximations for considered Knudsen number is about 20 %, the error of nonlinear relations between normal and shear stresses and transverse and longitudinal heat fluxes, calculated by DSMC results, (if change Newton value of shear stress and Fourier value of transverse heat flux by the DSMC values) is much less. The analogous research of Couette flow transport properties by DSMC method was carried out at the same time in [36].

The application of DSMC method to analysis of transport properties for shock wave flow [33] lead to conclusion that Hilbert normal solution can describe it only in subsonic part of hypersonic shock wave. The analysis of Burnett constitutive relations [35] in strong shock wave for Maxwell molecular model shows that there is no possibility to neglect any term in Burnett's stress or heat flux. Here (see Fig.2) the same conclusion may be done for the case of hard-sphere molecular model (only third term in stress is less then others terms):

$$p_{xx}^B = p_{xxi}^B \equiv \Sigma A_i, \quad A_1 \propto \left(\frac{du}{dx}\right)^2, \;\; A_2 \propto \frac{d^2T}{dx^2}, \;\; A_3 \propto \left(\frac{dT}{dx}\right)^2, \;\; A_4 \propto \frac{dp}{dx}, \quad A_5 \propto \frac{d^2p}{dx^2}$$

$$q_x^B = q_{xi}^B \equiv \Sigma B_i\,, \quad B_1 \propto (dT/dx)\,(du/dx), \quad B_2 \propto (d^2u/dx^2), \quad B_3 \propto (dp/dx)(du/dx) \qquad (3)$$

But it was shown in [32] that all these terms may be transformed to dependence of stress and heat flux on the first order gradients of velocity and temperature. The analogous analysis and conclusions were carried out for the flows of spherical and cylindrical expansions into the vacuum [34, 35].

Now the investigation of hypersonic two-dimensional flow past thin flat plate at zero angle of attack is presented. On Fig. 3 Burnett component of normal stress, sum of Burnett terms and Navier-Stokes (Newton) term of normal stress, and DSMC result for the same quantities are shown at distance between plate (y=0) and shock wave ($y/L \approx 0.12$). The distance from leading edge is determined by the value $\mathrm{Re}_x = 8000$. Kinetic computation was carried out for hard-sphere molecular model. The continuum constitutive relations with corresponding viscosity-temperature dependence were calculated by the fields of velocity, temperature and pressure, determined in DSMC calculation. Difference between DSMC results and continuum results at distance about y/L=0.06 may be explained by errors of calculation the first and second derivatives of flow fields (ill-posed problem!). It is seen, that some terms in Burnett approximation (particularly terms proportional to composition of first order derivatives of temperature and pressure) is far less than the other ones. But the sum of these terms gives nonzero contribution to overall sum.

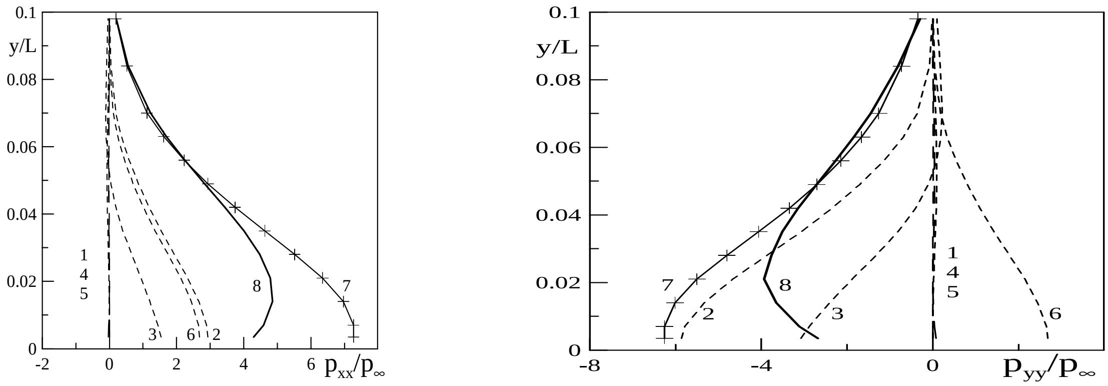

FIGURE 3. Burnett components of normal stress in flow past flat plate at zero angle of attack. Notations: $p_{xx} \propto$ 1, 6 $-\nabla u \nabla u$; 2 $-\nabla\nabla p$; 3 $-\nabla\nabla T$; 4 $-\nabla p \nabla T$; 5 $-\nabla T \nabla T$; 7 – sum of all Burnett terms and Navier-Stokes term; 8 – DSMC result. Parameters of shock wave: $M_\infty = 23$, $t_w = T_w / T_0 = 0.2$, $\mathrm{Re}_L = 10000$, $\mathrm{Re}_x = 8000$.

MACROSCOPIC MODEL WITH FIRST ORDER GRADIENTS

The beginnings of this model are in two sets of papers [37, 38] and [39-40], in which plane Couette flow with strong gradients was researched. In the first set it was shown that shear stress and transverse heat flux, determined by normal solution of Boltzmann equation with Maxwell molecular model have the simple structure. The stress depends only on velocity gradients and the heat flux is proportional to temperature gradient and depends in nonlinear form on velocity gradient. In other words each term of asymptotic series $p_{xy}^{(k)} / p_{xy}^N$ and $q_y^{(k)} / q_y^F$ depend only on ratio $a = p_{xy}^N / p$. Here index k is the order of the term in normal series, and p_{xy}^N, q_y^F are stress and heat flux in Navier-Stokes approximation, defined by Newton-Fourier constitutive relations. In the second set this result was considerably advanced. There closed form of normal solution for BGK model kinetic equation was found for constitutive relations and for distribution function respectively. Authors of these papers have continued these researches and carried out super-Burnett coefficients for constitutive relations on the base of Boltzmann equation with Maxwell model of molecular potential [41]. Besides, the constitutive relations in the whole [42, 36] and for various terms of Chapman-Enskog approximations [43, 44] were investigated by the DSMC method for Maxwell and hard-sphere molecular models. Our results [32, 34] confirm the data of [36] for Maxwell molecular model and give additional information on structure and values of constitutive relations for all transport quantities. Below we use directly the results of [34], therefore necessary data are presented here.

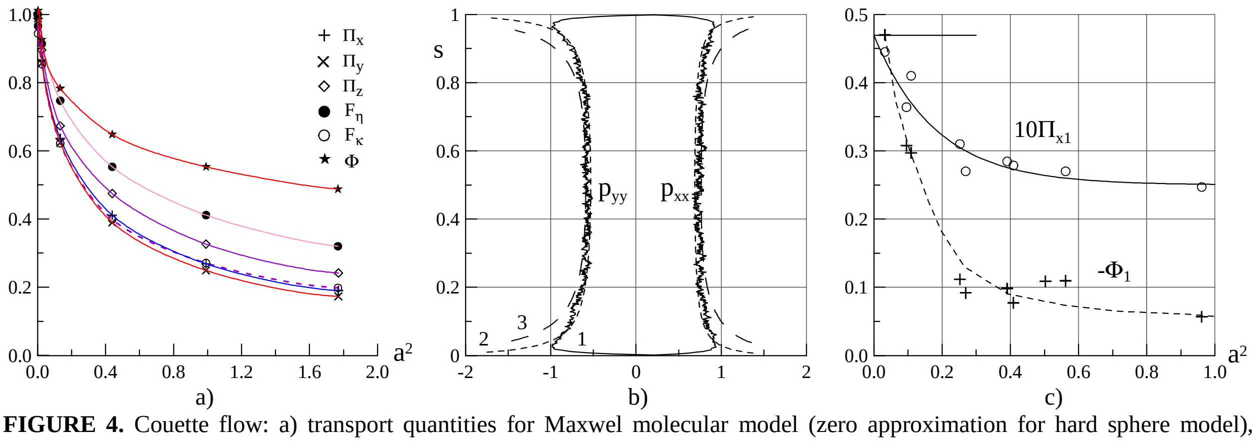

FIGURE 4. Couette flow: a) transport quantities for Maxwel molecular model (zero approximation for hard sphere model), calculated by DSMC; b) profiles of normal stresses for hard sphere molecular model (notation the same as in Fig. 1d), flow parameters: $S_w = 20, T_{w2} / T_{w1} = 1, Kn = 0.032$); c) first order approximation for transport quantities for the case of hard-sphere molecular model; notation for a) and c) see in Eq. (4).

$$F_\eta^{MM}(a^2)=\frac{p_{xy}}{p_{xy}^N}=F_{\eta 0},\quad F_k^{MM}(a^2)=\frac{q_y}{q_y^F}=F_{k0},\quad \Phi^{MM}(a^2)=\frac{q_x}{q_x^B}=\Phi_0,\quad \Pi_x(a^2)=\frac{p_{xx}}{p_{xx}^B}=\Pi_{x0},\quad a=\frac{p_{xy}^N}{p}$$

$$F_\eta^{HS}(a,b)=F_{\eta 0}(a^2)+b^2F_{\eta 1}(a^2)\ ,\quad F_k^{HS}(a,b)=F_{k0}(a^2)+b^2F_{k1}(a^2)\ ,\ \Phi^{HS}(a,b)=\Phi_0(a^2)+b^2\Phi_1(a^2)\ , \qquad (4)$$

$$\frac{p_{xx}}{p}=\Pi_{x0}(a^2)A_x a^2+b^2\Pi_{x1}(a^2)\ ,\ p_{xy}^N=-\mu\frac{\partial u_x}{\partial y},\quad q_y^F=-k\frac{\partial T}{\partial y}\quad b=\frac{q_y^F}{p\sqrt{2RT}}\ ,\ A_x^{HS}=1.51,\ A_y^{HS}=-1.163$$

q_x^B, p_{xx}^B - are defined by (2), the relations for p_{yy} are similar ones to relations for p_{xx}, except $\Pi_{y1}=-2\Pi_{x1}$.

Upper we see the degeneration of normal constitutive relations to nonlinear relations on velocity gradients for Couette flow only †. The analogous results were shown for strong shock wave flow [33], for spherical [34] and cylindrical [35] one-dimensional expansion flow into vacuum. Now we consider hypersonic flows past cold bodies (in these cases Knudsen boundary layers are thin ones). The essential hypothesis of proposed macroscopic model for these flows is: constitutive relations are functions, depending on velocity and temperature gradients of first order only. Additional demand is: constitutive relations for 3D flows must coincide with established constitutive relations for considered one-dimensional flows. Let us consider the structure of modeled stress tensor in flow past plate.

At the absence of others limitations it is need to consider arbitrary tensor function, depending on symmetric (u^s) and antisymmetric (u^{as}) parts of gradient velocity tensor and on temperature gradient:

$$u_{ij}^s=\frac{1}{2}(u_{i,j}+u_{j,i}),\quad u_{ij}^{as}=\frac{1}{2}(u_{i,j}-u_{j,i}),\quad t_{ij}=T_{,i}T_{,j},\quad u_{i,j}=\frac{\partial u_i}{\partial x_j},\quad T_{,i}=\frac{\partial T}{\partial x_i} \qquad (5)$$

As a generalization of Hamilton-Caley theorem stress tensor may be only polynomial function on its arguments, on tensors u^s, u^{as}, t. The order of polynomial on arguments is the second one on u^s, u^{as} and the first one on t; overall order on arguments is five. Stress tensor is deviator; this property gives additional limitation on the tensor structure. Let us give complete set of monomials (brackets denote symmetrical and traceless tensor) first, second third, forth and fifth orders:

$$M_1^1=\langle u^s\rangle, M_2^1=\langle t\rangle; M_1^2=\langle u^s u^s\rangle,\quad M_2^2=\langle u^s u^{as}\rangle,\quad M_3^2=\langle u^{as}u^{as}\rangle,\quad M_4^2=\langle u^s t\rangle,\quad M_5^2=\langle u^{as}t\rangle$$

$$M_1^3=\langle u^s u^s u^{as}\rangle,\quad M_2^3=\langle u^s u^{as} u^s\rangle,\quad M_3^3=\langle u^s u^{as} u^{as}\rangle,\quad M_4^3=\langle u^{as} u^s u^{as}\rangle,\quad M_5^3=\langle u^s u^s t\rangle,\quad M_6^3=\langle u^s t u^s\rangle,$$

$$M_7^3=\langle u^s u^{as} t\rangle,\quad M_8^3=\langle u^s t u^{as}\rangle,\quad M_9^3=\langle t u^s u^{as}\rangle,\quad M_{10}^3=\langle u^{as} u^{as} t\rangle,\quad M_{11}^3=\langle u^{as} t u^{as}\rangle$$

$$M_1^4=\langle u^s u^s u^{as} u^{as}\rangle,\quad M_2^4=\langle u^s u^{as} u^s u^{as}\rangle,\quad M_3^4=\langle u^{as} u^s u^s u^{as}\rangle,\quad M_4^4=\langle u^s u^{as} u^{as} u^s\rangle,\quad M_5^4=\langle u^s u^s u^{as} t\rangle,$$

$$M_6^4=\langle u^s u^s t u^{as}\rangle,\quad M_7^4=\langle u^s t u^s u^{as}\rangle,\quad M_8^4=\langle t u^s u^s u^{as}\rangle,\quad M_9^4=\langle u^s u^{as} u^s t\rangle,\quad M_{10}^4=\langle u^s u^{as} t u^s\rangle;\quad M_{11}^4=\langle u^s u^{as} u^{as} t\rangle,$$

$$M_{12}^4=\langle u^s u^{as} t u^{as}\rangle,\quad M_{13}^4=\langle u^s t u^{as} u^{as}\rangle,\quad M_{14}^4=\langle t u^s u^{as} u^{as}\rangle,\quad M_{15}^4=\langle u^{as} u^s u^{as} t\rangle,\quad M_{16}^4=\langle u^{as} t u^s u^{as}\rangle.$$

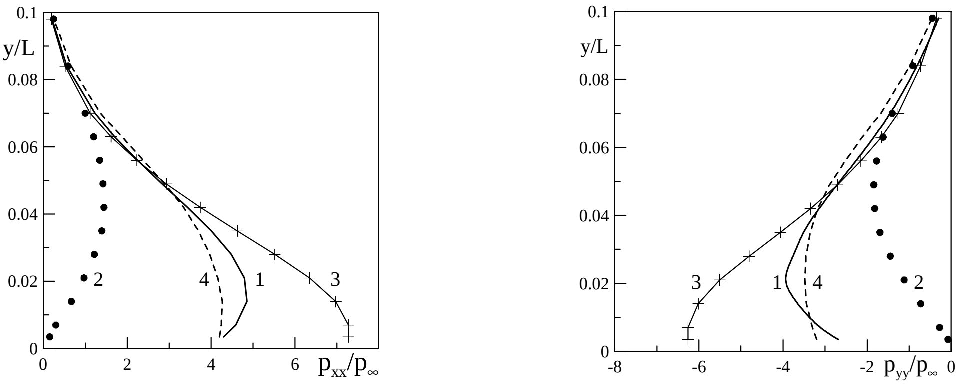

FIGURE 5. Normal stress components in the same flow as in Fig. 3. Notation: 1 – DSMC results, 2 – Navier-Stokes (Newton) approximation, 3 – sum of Newton and Burnett terms, 4 – present model, see equations (8).Parameters of flow see in Fig. 3.

†The results, similar to described upper ones, were carried out by modification of 13M approach in [45, 46], but constitutive relations in these papers are defined for limited values of p_{xy}^N / p only.

$$M_1^5 = <u^s u^s u^{as} u^{as} t>, \quad M_2^5 = <u^s u^s u^{as} t u^{as}>, \quad M_3^5 = <u^s u^s t u^{as} u^{as}>, \quad M_4^5 = <u^s t u^s u^{as} u^{as}>, \quad M_5^5 = <t u^s u^s u^{as} u^{as}>;$$

$$M_6^5 = <u^s u^{as} u^s u^{as} t>, \quad M_7^5 = <u^s u^{as} u^s t u^{as}>, \quad M_8^5 = <u^s u^{as} t u^s u^{as}>, \quad M_9^5 = <u^s t u^{as} u^s u^{as}>, \quad M_{10}^5 = <t u^s u^{as} u^s u^{as}>,$$

$$M_{11}^5 = <u^{as} u^s u^s u^{as} t>, \quad M_{12}^5 = <u^{as} u^s u^s t u^{as}>, \quad M_{13}^5 = <u^{as} u^s t u^s u^{as}>; \quad M_{14}^5 = <u^s u^{as} u^{as} u^s t>, \quad M_{15}^5 = <u^s u^{as} u^{as} t u^s>,$$

$$M_{16}^5 = <u^s u^{as} t u^{as} u^s>. \qquad p_{ij} = \sum_{i,j} f_j^i \left(M_j^i \right)$$

p_{ij} - total stress tensor, where coefficients f_i^j are scalar transport functions (generalized transport coefficients). Coefficients f_i^j depend on invariants of tensors u^s, u^{as}, t only. The complete set of invariants may be constructed from traces of tensors: self-invariants (I_1-I_5) and joint invariants (I_6-I_9).

$$I_1 = Tr(u^s), \quad I_2 = Tr(u^s u^s), \quad I_3 = Tr(u^s u^s u^s), \quad I_4 = Tr(u^{as} u^{as}), \quad I_5 = Tr(t)$$

$$I_6 = Tr(u^s u^{as} u^{as}), \quad I_7 = Tr(u^s u^s u^{as} u^{as}), \quad I_8 = Tr(u^s t), \quad I_9 = Tr(u^s u^s t)$$

Now we have no elaborated theory for selection monomials, therefore let us demonstrate possibility to construct macroscopic model by the example of stress in hypersonic flow past thin flat plate; parameters of flow see in Fig. 3 caption (example for heat flux was presented in [34, 35] †). As a criterion we use normal component of stress tensor (see Fig. 3). This flow and Couette flow are similar ones; this is the reason for determination of transport functions by the considering three independent relations for stress component established for Couette flow (4), (2). Therefore three monomials are used in the simplest model (nondimensional form is adopted):

$$\frac{p_{ij}}{p} = f_1 (-2\frac{\mu}{p}) < u^s > + f_2 \frac{\mu^2}{p^2} < u^s u^s > + f_3 \frac{\mu^2}{p^2} < u^s u^{as} > \tag{6}$$

Functions f_i may depend on five invariants for the case of 2D plane flow. The additional supposition is: in 2D flow transport functions depend only on the invariants, which are nonzero ones in one-dimensional Couette flow. But in Couette flow only two independent invariants exist. In dimensionless form these two invariants for 2D flow are:

$$I_2 = (\mu / p)^2 Tr(u^s u^s) = (\mu / p)^2 [u_{1,1}^2 + u_{2,2}^2 + (1/2)(u_{1,2} + u_{2,1})^2],$$

$$I_5 = (k^2 m / p^2 2 k_B T) Tr(t) = (k^2 m / p^2 2 k_B T)(T_{,1}^2 + T_{,2}^2)$$

For Couette flow these invariants are reduced to the form

$$I_2^C = (\mu / p)^2 (1/2) u_{1,2}^2 = (1/2)(p_{12}^N / p)^2 = (1/2) a^2 \quad , \quad I_5^C = (k^2 m / p^2 2 k_B T) T_{,2}^2 = (q_2^F)^2 (m / 2 k_B T p^2) = b^2$$

Equating dependences (6) in the case of Couette flow to relations (4), established for this flow, we determine functions f_i:

$$f_1 = F_{\eta 0}(2I_2) + I_5 F_{\eta 1}(2I_2) \quad , \quad f_2 = 6[A_{x,HS}\Pi_{x0} + A_{y,HS}\Pi_{y0} + (I_5 / 2I_2)(\Pi_{x1} + \Pi_{y1})],$$

$$f_3 = 2[-A_{x,HS}\Pi_{x0} + A_{y,HS}\Pi_{y0} + (I_5 / 2I_2)(-\Pi_{x1} + \Pi_{y1})] \tag{7}$$

The relations for stress components may be expressed by (6) and (7) in the obvious form:

$$p_{xy} = f_1 \mu u_{1,2}, \quad p_{xx} = (\mu^2 / p)\{ f_2 [(2/3) u_{1,1}^2 + (1/12)(u_{1,2} + u_{2,1})^2 - (1/3) u_{2,2}^2] + f_3 [-(1/4)(u_{1,2}^2 - u_{2,1}^2)]\}, \tag{8}$$

$$p_{yy} = (\mu^2 / p)\{ f_2 [(2/3) u_{2,2}^2 + (1/12)(u_{1,2} + u_{2,1})^2 - (1/3) u_{1,1}^2] + f_3 [(1/4)(u_{1,2}^2 - u_{2,1}^2)]\}$$

On Fig. 5 comparison of these results for normal components of stress tensor with DSMC results, Navier-Stokes and Burnett approximations is conducted. It is seen, that results of present model for plane flow are correlated with DSMC data better, than usual Navier-Stokes and Burnett approximations.

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† The errors in [35] will be checked in detailed paper.

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