

# On Generalized Non-Equilibrium Thermodynamics of Rarefied Gas

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**Abstract.** The problem of the conservation of Onsager symmetry after the exclusion of fast variables is studied. The violation of Onsager symmetry observed earlier when fast variables were formally excluded is shown to be related to incorrectly writing phenomenological equations. If the entropy production and phenomenological equations from which fast variables are excluded are written correctly, the Onsager relations remain valid. Additional thermodynamic fluxes then appear that ensure the conservation of Onsager symmetry. The relation between the general formalism and the Chapman-Enskog method of the kinetic theory of gases is discussed

## INTRODUCTION

The method of non-equilibrium thermodynamics was used in [1-3] to study a system with substantially different thermodynamic variable relaxation times. A quite rigorous analysis showed that the exclusion of fast variables from phenomenological equations led to the violation of the Onsager symmetry relations between kinetic coefficients. The “defect” value determining the difference of kinetic cross coefficients was proportional to the ratio between the fast and slow relaxation times. This defect appeared as a consequence of a time “jump” at the initial relaxation stage, which could not be described using slow variables only.

This cause of Onsager symmetry violation is similar to that considered in [4], where symmetry relation violation in moving to the higher Chapman-Enskog theory approximation was discussed. Recall that, in classic monograph [5], where the scope of the applicability of non-equilibrium thermodynamics was analyzed, the higher Chapman-Enskog theory approximations were considered to be outside this scope. It was at the same time shown in [6-10] that Onsager symmetry was retained also in the higher Chapman-Enskog theory approximations in the quasi-stationary case, when phenomenological equations did not contain time variables.

In this work, we use a procedure that enables Onsager symmetry violated by the exclusion of fast variables to be restored. Moreover, our concern will be dynamic equations of non-equilibrium thermodynamics containing the derivatives of thermodynamic parameters with respect to time. We consider first the general formalism and then the Chapman-Enskog scheme, which is a procedure for excluding fast variables.

## GENERAL FORMALISM

A non-equilibrium state of a thermodynamic system will be considered by a set of variables  $a_1, a_2, a_3 \dots a_n$ . The change in the entropy caused [5] by an arbitrary deviation of the system from equilibrium is given by the quadratic form

$$\Delta S = -\frac{1}{2} \mathbf{a} \cdot \mathbf{g} \cdot \mathbf{a}, \quad (1)$$

where  $\mathbf{g}$  is symmetrical positive definite matrix, and  $\mathbf{a}$  is the vector

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}. \quad (2)$$

After a weak deviation from equilibrium, the relaxation of thermodynamic parameters is described [5] by the linear equation

$$\dot{\mathbf{a}}(t) = -\mathbf{M} \cdot \mathbf{a}(t). \quad (3)$$

Let us introduce vector  $\mathbf{b}$  conjugated to  $\mathbf{a}$  by the equation

$$\mathbf{b} = -\frac{\partial \Delta S}{\partial \mathbf{a}} = \mathbf{g} \cdot \mathbf{a}. \quad (4)$$

Equation (3) then takes the form

$$\dot{\mathbf{a}}(t) = \mathbf{L} \cdot \mathbf{b}(t), \quad (5)$$

where

$$\mathbf{L} = -\mathbf{M} \cdot \mathbf{g}^{-1}. \quad (6)$$

Onsager showed that the  $\mathbf{L}$  matrix should be symmetrical

$$\mathbf{L}^T = \mathbf{L}, \quad (7)$$

where symbol  $T$  denotes transposition.

Consider a situation when the vector  $\mathbf{a}$  includes slow variables  $x_i$  and fast variables  $y_i$ ,  $\mathbf{a} = \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}$ . The structure of relaxation equations is then

$$\frac{d}{dt} \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{y}(t) \end{pmatrix} = -\mathbf{M}(\varepsilon) \cdot \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{y}(t) \end{pmatrix} = -\left\{ \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{F} \end{pmatrix} + \varepsilon \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \right\} \cdot \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{y}(t) \end{pmatrix}, \quad (8)$$

where  $\varepsilon$  is small parameter and the  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ ,  $\mathbf{D}$ , and  $\mathbf{F}$  matrixes are of the same order of magnitude.

Without loss in generality, it can be assumed that the  $\mathbf{g}$  matrix has the structure

$$\mathbf{g} = \begin{pmatrix} \mathbf{g}_{xx} & \\ & \mathbf{g}_{yy} \end{pmatrix}, \quad (9)$$

where  $\mathbf{g}_{xx}$  and  $\mathbf{g}_{yy}$  are symmetrical matrices.

Formally solving (8) with respect to  $\mathbf{y}(t)$  and excluding these variables from the equations for  $\mathbf{x}(t)$  shows that, with an accuracy to within  $\varepsilon^3$ , the equation for the slow variables takes the form

$$\dot{\mathbf{x}}(t) = -\mathbf{M}_{\text{red}} \mathbf{x}(t), \quad (10)$$

where

$$\mathbf{M}_{\text{red}} = \varepsilon \mathbf{A} - \varepsilon^2 \mathbf{B}(\mathbf{F} + \varepsilon \mathbf{D})^{-1} \mathbf{C} - \varepsilon^3 \mathbf{B} \mathbf{F}^{-2} \mathbf{C} \mathbf{A} + O(\varepsilon^4). \quad (11)$$

It is easy to see that the symmetry relations are hold in the first and in the second approximations. It is shown in [1-3] that symmetry relations are not satisfied in the higher approximations. This fact was treated in [1-3] as the approximate nature of the Onsager relation.

Now we turn to a procedure for restoring symmetry. Symmetry violation occurs because the following important circumstance is ignored while excluding fast variables: the thermodynamically non-equilibrium state of a system is then determined not only by the  $\mathbf{x}(t)$  variables but also by their derivatives with respect to time.

Really, the integration of the equations (8) allows  $\mathbf{y}(t)$  to be represented in the form of the infinite series

$$\mathbf{y}(t) = (\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \mathbf{x} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \dot{\mathbf{x}} + (\mathbf{F} + \varepsilon \mathbf{D})^{-3} \varepsilon \mathbf{C} \ddot{\mathbf{x}} - \dots \quad (12)$$

The relaxation equations then take the form

$$\begin{aligned} \dot{\mathbf{x}}(t) &= -\varepsilon \mathbf{A} \mathbf{x}(t) - \varepsilon \mathbf{B} \mathbf{y}(t), \\ \mathbf{y}(t) &= (\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \mathbf{x} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \dot{\mathbf{x}} + (\mathbf{F} + \varepsilon \mathbf{D})^{-3} \varepsilon \mathbf{C} \ddot{\mathbf{x}} - \dots \end{aligned} \quad (13)$$

Note that, if the first term of the expansion of  $\mathbf{y}(t)$  is retained, Onsager symmetry is conserved, whereas the second term already causes its violation. Importantly, the first term only contains  $\mathbf{x}(t)$ , whereas  $\dot{\mathbf{x}}(t)$  appears in the second. It follows that the symmetry is violated when the non-equilibrium thermodynamic state is characterized not only by the  $\mathbf{x}(t)$  variable but also by  $\dot{\mathbf{x}}(t)$ .

We will show that the introduction of a new thermodynamic variable enables the validity of the symmetry relations to be retained in the higher approximations also. We apply the results obtained in [5], according to which the use of various linear combinations of the thermodynamic fluxes and forces as new variables does not violate Onsager symmetry. With an accuracy to within  $\varepsilon^3$ , we have

$$\begin{aligned} \Delta S &= -\frac{1}{2} \mathbf{x} \mathbf{g}_{xx} \mathbf{x} - \frac{1}{2} [(\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \mathbf{x} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \dot{\mathbf{x}}] \times \\ &\quad \mathbf{g}_{yy} [(\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \mathbf{x} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \dot{\mathbf{x}}]. \end{aligned} \quad (14)$$

The entropy production takes the form

$$\begin{aligned} \Delta \dot{S} &= -\frac{1}{2} \dot{\mathbf{x}} \mathbf{g}_{xx} \mathbf{x} - \frac{1}{2} [(\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \dot{\mathbf{x}} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \ddot{\mathbf{x}}] \times \\ &\quad \mathbf{g}_{yy} [(\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \mathbf{x} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \dot{\mathbf{x}}]. \end{aligned} \quad (15)$$

The new thermodynamic variable is

$$(\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \mathbf{x} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \dot{\mathbf{x}}, \quad (16)$$

On the basis of entropy production (14), we can write the phenomenological equations of non-equilibrium thermodynamics in the form

$$\begin{aligned} \dot{\mathbf{x}} &= -\varepsilon \mathbf{A} \mathbf{x} - \varepsilon \mathbf{B} [(\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \mathbf{x} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \dot{\mathbf{x}}], \\ [(\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \dot{\mathbf{x}} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \ddot{\mathbf{x}}] &= -\varepsilon \mathbf{C} \mathbf{x} - (\mathbf{F} + \varepsilon \mathbf{D}) [(\mathbf{F} + \varepsilon \mathbf{D})^{-1} \varepsilon \mathbf{C} \mathbf{x} - (\mathbf{F} + \varepsilon \mathbf{D})^{-2} \varepsilon \mathbf{C} \dot{\mathbf{x}}]. \end{aligned} \quad (17)$$

It is easy to see that the symmetry relations are satisfied.

To summarize, symmetry relations are not violated if not only  $\mathbf{x}(t)$  variables but also their combinations with  $\dot{\mathbf{x}}(t)$  are used as new thermodynamic variables. Clearly, symmetry relations also hold in the higher approximations if combinations of  $\mathbf{x}(t)$  with their derivatives of the corresponding orders are used as thermodynamic variables.

## THE BOLTZMANN EQUATION AND THE CHAPMAN-ENSKOG METHOD

Let us apply the formalism developed above to the particular case of solving the Boltzmann kinetic equation by the Chapman-Enskog method [11]. The non-equilibrium state of a gas can be described using the velocity distribution of gas molecules

$$f(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{v})[1 + \varphi(\mathbf{r}, \mathbf{v}, t)], \quad (18)$$

where  $f_0(\mathbf{v})$  is the absolute Maxwell distribution,  $\mathbf{v}$  is the velocities of molecules,  $\mathbf{r}$  is the spatial coordinate, and  $\varphi(\mathbf{r}, \mathbf{v}, t)$  is the correction to the Maxwell distribution. The correction  $\varphi(\mathbf{r}, \mathbf{v}, t)$  satisfies the linearized Boltzmann equation [11] (we consider the case of the absence of external forces)

$$\frac{\partial \varphi}{\partial t} + \mathbf{v} \frac{\partial \varphi}{\partial \mathbf{r}} = \int f_0 K(\mathbf{v}, \mathbf{v}') \varphi(\mathbf{v}') d\mathbf{v}', \quad (20)$$

where the collision operator  $K(\mathbf{v}, \mathbf{v}')$  is symmetrical with respect to variables  $\mathbf{v}$ ,  $\mathbf{v}'$ .

The entropy of the non-equilibrium system is determined by the equation [5, 11]

$$\Delta S = -\frac{1}{2} \iint f_0 \varphi^2(\mathbf{r}, \mathbf{v}, t) d\mathbf{r} d\mathbf{v}.$$

Following [6, 7], we consider  $\varphi(\mathbf{r}, \mathbf{v}, t)$  as a generalized thermodynamic variable. The value conjugate to it is then

$$-\frac{\delta \Delta S}{\delta \varphi} = f_0 \varphi(\mathbf{r}, \mathbf{v}, t). \quad (21)$$

Clearly, the entropy production has the form

$$\Delta \dot{S} = -\iint f_0 \varphi \frac{\partial \varphi}{\partial t} d\mathbf{r} d\mathbf{v}. \quad (22)$$

The phenomenological equation obtained on the basis of entropy production (22) has a fairly simple form

$$\frac{\partial \varphi(\mathbf{r}, \mathbf{v}, t)}{\partial t} = \Lambda(f_0 \varphi(\mathbf{r}, \mathbf{v}, t)), \quad (23)$$

where the kinetic coefficient  $\Lambda$  is the operator

$$\Lambda = -\iint \left[ \frac{1}{\sqrt{f_0(\mathbf{v})f_0(\mathbf{v}')}} \frac{\partial}{\partial \mathbf{r}} \delta(\mathbf{r} - \mathbf{r}') \delta(\mathbf{v} - \mathbf{v}') \right] d\mathbf{r}' d\mathbf{v}' + \iint K(\mathbf{v}, \mathbf{v}') \delta(\mathbf{r} - \mathbf{r}') d\mathbf{r}' d\mathbf{v}'. \quad (24)$$

Here  $\delta(\mathbf{r})$  is the Dirac delta function, and the Onsager symmetry relations clearly holds for the operator  $\Lambda$ ; that is  $\Lambda(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}') = \Lambda(\mathbf{r}', \mathbf{v}'; \mathbf{r}, \mathbf{v})$ . It is easy to see that phenomenological equation (23) obtained by non-equilibrium thermodynamic methods coincides with the Boltzmann kinetic equation [Eq. (20)].

Let us perform the Chapman-Enskog expansion of the entropy production

$$\Delta \dot{S} = -\iint f_0 \varphi \frac{\partial \varphi}{\partial t} d\mathbf{r} d\mathbf{v}. \quad (25)$$

According to the Chapman-Enskog scheme [11], not only the distribution function but also its derivative with respect to time are expanded in powers of a small parameter,

$$\begin{aligned} \varphi(\mathbf{r}, \mathbf{v}, t) &= \varphi_0(\mathbf{r}, \mathbf{v}, t) + \varepsilon \varphi_1(\mathbf{r}, \mathbf{v}, t) + \varepsilon^2 \varphi_2(\mathbf{r}, \mathbf{v}, t) + \dots \\ \frac{\partial}{\partial t} &= \frac{\partial_0}{\partial t} + \varepsilon \frac{\partial_1}{\partial t} + \varepsilon^2 \frac{\partial_2}{\partial t} + \dots \end{aligned} \quad (26)$$

For  $\varphi_0(\mathbf{r}, \mathbf{v}, t)$  we have the representation [11]

$$\varphi_0(\mathbf{r}, \mathbf{v}, t) = \sum_{l=1}^5 a_l^{(0)}(\mathbf{r}, t) \Phi_l(\mathbf{v}), \quad (27)$$

where  $\Phi_l(\mathbf{v})$  are the eigenfunctions of collision operator (they correspond to zero eigenvalues at  $l = 1, \dots, 5$ ). We likewise have

$$\varphi_n(\mathbf{r}, \mathbf{v}, t) = \sum_{l=1}^{\infty} a_l^{(n)}(\mathbf{r}, t) \Phi_l(\mathbf{v}), \quad (28)$$

for the other  $\varphi_n$  functions. In the Chapman-Enskog scheme, it is assumed that

$$a_l^{(n)} = 0 \quad \text{for } l = 1 \dots 5 \quad \text{and } n > 0. \quad (29)$$

The coefficients of the expansion of  $\varphi_0(\mathbf{r}, \mathbf{v}, t)$  are related to gas density and temperature deviations from equilibrium  $\Delta n$  and  $\Delta T$  and gas velocity  $\mathbf{u}$  as

$$a_1^{(0)} = \frac{1}{n_0} \Delta n(\mathbf{r}, t), \quad a_\alpha^{(0)} = \left( \frac{m}{kT_0} \right)^{1/2} u_\alpha, \quad a_5^{(0)} = \sqrt{\frac{3}{2}} \frac{1}{T_0} \Delta T(\mathbf{r}, t), \quad (30)$$

where  $n_0$  and  $T_0$  are the equilibrium gas density and temperature. Condition (30) therefor means that the higher corrections  $\varphi_n$  do not contribute to the density, velocity, and temperature.

Substituting expansion (26) into the equation for entropy production yields

$$\Delta \dot{S} = - \iint (\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \dots) \left( \frac{\partial_0}{\partial t} + \varepsilon \frac{\partial_1}{\partial t} + \varepsilon^2 \frac{\partial_2}{\partial t} + \dots \right) \times (\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \dots) f_0 d\mathbf{v} d\mathbf{r}, \quad (31)$$

Combining terms with equal power of  $\varepsilon$ , we obtain

$$\Delta \dot{S} = - \iint f_0 \left\{ \varphi_0 \frac{\partial_0 \varphi_0}{\partial t} + \varepsilon \left( \varphi_1 \frac{\partial_0 \varphi_0}{\partial t} + \varphi_0 \frac{\partial_1 \varphi_0}{\partial t} \varphi_0 + \frac{\partial_0 \varphi_1}{\partial t} \right) + \varepsilon^2 \left[ \varphi_2 \frac{\partial_0 \varphi_0}{\partial t} + \varphi_1 \left( \frac{\partial_0 \varphi_1}{\partial t} + \frac{\partial_1 \varphi_0}{\partial t} \right) + \varphi_0 \left( \frac{\partial_0 \varphi_2}{\partial t} + \frac{\partial_1 \varphi_1}{\partial t} \frac{\partial_2 \varphi_0}{\partial t} \right) \right] + \dots \right\} d\mathbf{r} d\mathbf{v}. \quad (32)$$

The phenomenological non-equilibrium thermodynamic equations can be obtained from entropy productions (31) and (28) by two methods. With (31) we can use the expressions  $(\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \dots) f_0$  as thermodynamic forces and the combination  $\left( \frac{\partial_0}{\partial t} + \varepsilon \frac{\partial_1}{\partial t} + \varepsilon^2 \frac{\partial_2}{\partial t} + \dots \right) (\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \dots)$  as thermodynamic fluxes. The phenomenological equations then take the form

$$\left( \frac{\partial_0}{\partial t} + \varepsilon \frac{\partial_1}{\partial t} + \varepsilon^2 \frac{\partial_2}{\partial t} + \dots \right) (\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \dots) = \Lambda (\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \dots), \quad (33)$$

where the operator  $\Lambda$  coincides with that in (24). However, taking into account the small parameter  $\varepsilon$  introduced, the terms of  $\Lambda$  have different orders of magnitude (in the dimensionless form), as follows from the representation

$$\Lambda = - \iint \left[ \frac{1}{\sqrt{f_0(\mathbf{v})} f_0(\mathbf{v}')} \frac{\partial}{\partial \mathbf{r}} \delta(\mathbf{r} - \mathbf{r}') \delta(\mathbf{v} - \mathbf{v}') \right] d\mathbf{r}' d\mathbf{v}' + \iint K(\mathbf{v}, \mathbf{v}') \delta(\mathbf{r} - \mathbf{r}') d\mathbf{r}' d\mathbf{v}' \leftrightarrow \hat{L} + \frac{1}{\varepsilon} \hat{K}. \quad (34)$$

Naturally, the kinetic coefficients in (33) remains symmetrical.

Equating terms at equal powers of  $\varepsilon$  in (33) yields the system of equation

$$\begin{aligned} \int d\mathbf{v}' K(\mathbf{v}, \mathbf{v}') f_0 \varphi_0(\mathbf{r}, \mathbf{v}', t) &= 0, \\ \left( \frac{\partial_0}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}} \right) \varphi_0 &= \int d\mathbf{v}' K(\mathbf{v}, \mathbf{v}') f_0 \varphi_1(\mathbf{r}, \mathbf{v}', t), \\ \left( \frac{\partial_0}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}} \right) \varphi_1 + \frac{\partial_1}{\partial t} \varphi_0 &= \int d\mathbf{v}' K(\mathbf{v}, \mathbf{v}') f_0 \varphi_2(\mathbf{r}, \mathbf{v}', t), \\ \left( \frac{\partial_0}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}} \right) \varphi_2 + \frac{\partial_1}{\partial t} \varphi_1 + \frac{\partial_2}{\partial t} \varphi_0 &= \int d\mathbf{v}' K(\mathbf{v}, \mathbf{v}') f_0 \varphi_3(\mathbf{r}, \mathbf{v}', t), \\ &\dots, \end{aligned} \quad (35)$$

which corresponds to the system of equation that appears in the Chapman-Enskog theory [11].

Let us consider the result of using a limited number of terms in the expansion given above. Specially, we will consider the second-order approximation. The entropy production takes the form

$$\Delta \dot{S} = - \iint (\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2) \left( \frac{\partial_0}{\partial t} + \varepsilon \frac{\partial_1}{\partial t} + \varepsilon^2 \frac{\partial_2}{\partial t} \right) \times (\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2) f_0 d\mathbf{v} d\mathbf{r} \quad (36)$$

and the phenomenological equation can be written as

$$\left( \frac{\partial_0}{\partial t} + \varepsilon \frac{\partial_1}{\partial t} + \varepsilon^2 \frac{\partial_2}{\partial t} \right) (\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2) = \Lambda (\varphi_0 + \varepsilon \varphi_1 + \varepsilon^2 \varphi_2). \quad (37)$$

Using the explicit form of  $\Lambda$  and equating terms of the same order of magnitude, we obtain the system of equations

$$\begin{aligned}
\int d\mathbf{v}' K(\mathbf{v}, \mathbf{v}') f_0 \varphi_0(\mathbf{r}, \mathbf{v}', t) &= 0, \\
\left(\frac{\partial_0}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}}\right) \varphi_0 &= \int d\mathbf{v}' K(\mathbf{v}, \mathbf{v}') f_0 \varphi_1(\mathbf{r}, \mathbf{v}', t), \\
\left(\frac{\partial_0}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}}\right) \varphi_1 + \frac{\partial_1}{\partial t} \varphi_0 &= \int d\mathbf{v}' K(\mathbf{v}, \mathbf{v}') f_0 \varphi_2(\mathbf{r}, \mathbf{v}', t), \\
\left(\frac{\partial_0}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}}\right) \varphi_2 + \frac{\partial_1}{\partial t} \varphi_1 + \frac{\partial_2}{\partial t} \varphi_0 &= 0.
\end{aligned} \tag{38}$$

Onsager symmetry holds because the  $\Lambda$  operator is symmetrical. However, a problem arises with the correctness of Eqs (38). Indeed, the last equation in (38) is clearly incomplete, and equations up to the first order of magnitude should only be considered correct. This is related to the form of the  $\Lambda$  operator, which makes the equations less exact by one order of magnitude.

## CONCLUSION

Our analysis shows that, in reality, the passage from the complete system of dynamical equations to equations for slow variables does not cause Onsager symmetry violation. It must be taken into account that equations for slow variables then only constitute part of phenomenological equations of non-equilibrium thermodynamics. The introduction of additional equations “restores” Onsager symmetry. The form of the additional equations is determined by the structure of entropy production and the method used for constructing them. The character of Onsager cross relations can then change substantially.

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