# Asymptotical Analysis Of Translational Nonequilibrium In The Hypersonic Flow Near The Flat Plate With The Sharp Leading Edge

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**Abstract.** The applicability region determination as applied to different models based on continuum Navier-Stokes equations for hypersonic flows in a transition regime is investigated. On the basis of a kinetic equation for gases with internal degrees of freedom an asymptotical analysis of hypersonic rarefied gas flows is conducted. Maxwell transport equations are used for derivation of closed system of macroscopic equations describing gas flows in viscous shock layer and hypersonic boundary layer. Specifically the singularity and nonmonotonic distributions of the some macro characteristics of hypersonic flow near to the sharp leading edge of flat plate analytically is investigated. The results obtained are compared to known continuum medium as Direct Simulation Monte Carlo data.

## THE THIN-LAYER VERSION OF KINETIC MOMENT EQUATION

It is well known that the Navier-Stokes equation (NS), which provides a continuum description of viscous fluid flow, begins to breakdown under rarefied conditions. Failure of the NS equation in hypersonic transitional flow occurs both in the shock front and in the region of shock layer immediately adjacent to the body surface (i.e. Knudsen layer). In this regions the continuum equations are not valid and the particle simulation technique such as the direct Simulation Monte-Carlo method (DSMC) usually are used. However the main reason for not applying the DSMC method to all flows is its prohibitive numerical cost at high-density conditions. For the study of normal hypersonic shock wave structures were the Burnett equations employed, too. However this equations set is more difficult to solve numerically than the NS and will in any case fail when the degree of rarefaction is sufficiently high. There are also questions about appropriate boundary conditions for the Burnett equations.

In the articles of Russian authors [1-4] and in the professor's H.K.Cheng et al. (USA) articles [5] the new methods to the solving of viscous shock layer problem in continuum-transition regime was developed. These methods based on asymptotic relations are deduced from the thin-layer version of moment equations of the gas-kinetic theory. In contradistinction to the Burnett equation the asymptotic simplified moment equations have the same leading order and the same number of boundary conditions as the NS equations. In contradistinction to the NS equations the new moment approaches have the nonlinear relation-ships for shear stress and heat transfer, what concise with the NS relation-ships by the small degree of rarefaction only. Moreover in contradistinction to the Burnett and super-Burnett approaches this new relation-ships have all corrections of higher order usually calculated on the Chapman-Enskog method.

It can therefore be surmised that the asymptotic simplified moment equations will accurately reproduce shock wave and shock layer structures.

However for the study of these problems it is necessary to modify these governing moment equations.

That is the main aim of this paper to extend the method based on asymptotic simplified moment equations to solve slender-body problems in continuum-transition regime.

## The Newtonian Limit And General Parameters Of Similarity

In this connection let's consider the rarefied flow past the leading edge of a sharp flat plate, whose surface is parallel to an oncoming uniform flow. The theoretical investigation of this problem can be divided into two groups depending on whether a kinetic theory, or a continuum flow approach is used. Our investigation holds an intermediate position between these groups. It is the theory based on asymptotic Newtonian limit in infinite system of equations for the kinetic moments. It is assumed that the model fluid is a perfect polyatomic gas with many numbers of internal freedoms in general case.

A flow model is shown in figure 1.

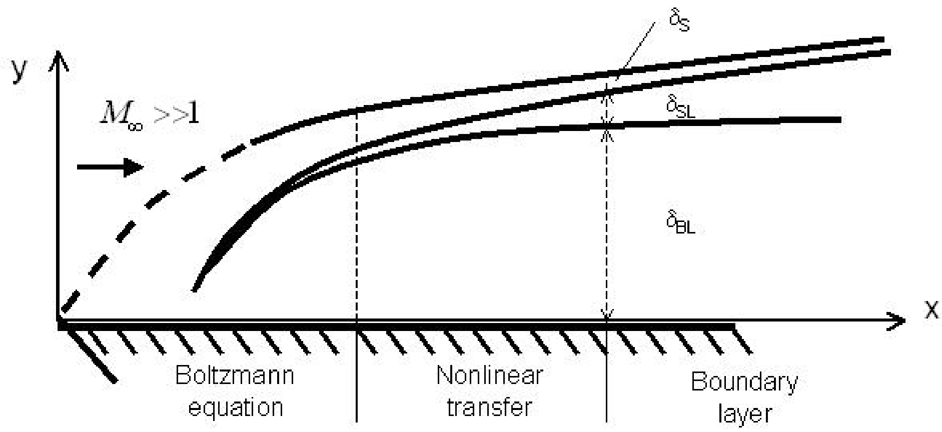


Figure 1. Hypersonic rarefied flow past a sharp leading-edge problem

Generally, the first region near leading plate edge in Fig.1, i.e. "Boltzmann equation", is not thin. However, for asymptotical analysis we'll assume that all three regions are thin, as in [6].

Let's consider more in detail intermediate from three basic regions of flow field. We named this region as "Equation Nonlinear Transfer".

Limiting forms of the solutions for such a flow are studied as:

(1) The free-stream Mach number goes to infinity;

(2) The stagnation-stream Reynolds number, based upon the distance from the leading edge, RL, goes to infinity; and

(3) The "Newtonian parameter" ε = (γ –1)/(γ –1), where γ is the ratio of specific heats, goes to zero; such that the translational Nonequilibrium parameter N (we named it Nicol'sky parameter) remains the finite magnitude.

In continuous theory parameter N defines the magnitude of ratio of the shear stress value $P_{xy}$ to the thermodynamic pressure value P:

$$\underline{N} = \frac{1}{\varepsilon\sqrt{\mathrm{Re}_{BL}}} \approx \frac{P_{xy}}{P}$$

where the Reynolds number is based on the characteristic scales of boundary layer.

Moreover, this parameter includes many another parameters, namely: the strong interaction parameter $\overline{\chi}$ and the rarefaction parameter $\overline{V}$ .

$$\underline{N} = \frac{\sqrt{\overline{\chi}}}{(M_\infty\sqrt{\varepsilon})^{5/4}} = \frac{\sqrt{\overline{V}}}{(M_\infty^{1/5}\sqrt{\varepsilon})^{5/4}}$$

In kinetic theory parameter N defines the magnitude of ratio of the convective term to the collision term of Boltzmann equation. One may also define N as the ratio of the Knudsen number $K_{BL}$ to the fractional power of "Newtonian parameter" ε. One should note that N may be a number of the first order, while Knudsen number's value is small.

$$\underline{N} = \frac{1}{\varepsilon\sqrt{\mathrm{Re}_{BL}}} \approx \frac{\mathrm{K}_{\mathrm{BL}}}{\varepsilon^{3/2}} \approx \frac{\frac{d\,f}{d\,t}}{J_{st}}$$

Here $f$ is a molecular distribution function, $J_{st}$ is a collision integral.

## The Equations Of Nonlinear Transfer

Let's consider the metrics of a slender body shock layer.

$$y = y^0 / \delta_B, \quad x = x^0 / L, \quad u = u^0 / V_\infty, \quad v = v^0 / v_B,$$
$$\rho = \rho^0 / \rho_B, \quad p = p^0 / p_B, \quad \left|\vec{C}_k\right| = \left|\vec{C}_k^0\right| / v_{TB}, \quad v_{TB} = \sqrt{\varepsilon} V_\infty$$
$$v_B = V_\infty \overline{\delta_B}, \quad \overline{\delta_B} = (\delta_B / L) = \underline{N}\,\varepsilon, \quad \rho_B = \rho_s (\overline{\delta_B})^2, \quad p_B = \rho_\infty V_\infty^2 (\overline{\delta_B})^2.$$

Here dimension values are marked with a subscript a zero, boundary values are marked with an subscript B.

After nondimensionalization of the Boltzmann equation, Df = J(f,f1) it will contain the parameter N.

Here

$$Df = -\underline{N} C_y \frac{\partial u}{\partial y} \cdot \frac{\partial f}{\partial C_x} - \underline{N}\sqrt{\varepsilon}\left(\frac{du}{dt} \cdot \frac{\partial f}{\partial C_x} - C_y \frac{\partial f}{\partial y}\right) + o(\underline{N}\varepsilon)$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_k \frac{\partial f}{\partial r_k}, \quad f = f(\vec{C}_1 E_{j,i}, \vec{r}, t), \quad J = \frac{M^t - f}{\tau_{el}} + \frac{M^{eq} - f}{\tau_{in}}$$

Where $E_{ji}$ – the internal energy of j-th state of i-th degree of freedom, $\tau_{el}$ and $\tau_{in}$ – the elastic and inelastic collision time respectively, $M^t$ and $M^{eq}$ – are maxwellians with nonequally ($T_t \neq T_i$) and equally ($T_t = T_i = T^{eq}$) temperature of translational ($T_t$) and internal ($T_i$) degrees of freedom respectively.

If the value of parameter N is much less than unit, the Chapman-Enskog theory may be applied. As a result, from the Boltzmann equation the boundary layer equations will be followed. If the order of value of parameter N is equal to unit, the Chapman-Enskog theory is invalid. It has been shown, that the system of equations for kinetic moments may be asymptotically truncated. Specifically, the finite expression for components of shear stress and the vector components of heat flux have been obtained from third-order moment system. These expressions close the equations of conversation of mass, momentum, energy. They depend nonlinear on magnitude of a velocity gradient, normal to a plate, and may be transformed to the traditional form with dissipative coefficients, which depend on a square of magnitude of a normal component of the velocity gradient [1]. These closing expressions can be written as:

$$P_{yy} = P(1 - \zeta\Omega^2)^{-1}, \quad P_{xy} = -\mu_{eff} \frac{\partial u}{\partial y}, \quad q_y = -\lambda_{eff} \frac{\partial T}{\partial y}$$

$$\mu_{eff} = \mu_{N-S} / (1 - \zeta\Omega^2), \quad \lambda_{eff} = \lambda_{N-S} / (1 - \zeta\Omega^2), \quad \frac{T^t - T^{eq}}{T^t} = \frac{2}{3} \frac{C_V^{in}}{C_V} \frac{\frac{1+\alpha}{\alpha}\Omega^2}{1 + \frac{2}{3}\Omega^2}$$

here $C_V^{in}$ and $C_V$ - the internal degrees of freedom and total specific heat respectively, α is the ratio of inelastic to elastic collision time, k – constant of Boltzmann, $\mu_{N-S}$ and $\lambda_{N-S}$ dissipative coefficients in Navier-Stocks equations.

The similar relations are held for dissipative coefficients in phenomenological turbulent theory. H.K. Cheng has deduced the similar constitutive relations for monatomic gases also from Grad's 13-moment equations.

$$\mu_{N-S} = NP\tau/1+\alpha, \quad \lambda_{N-S} = \frac{C_P}{m} NP\tau/1+\alpha, \quad \zeta = \frac{2}{3}\left(\frac{C_V^{in}}{C_V}\frac{1+\alpha}{\alpha} - 1\right),$$

$$\Omega = \frac{N\tau}{1+\alpha}\frac{\partial u}{\partial y}, \quad \frac{3}{2}kT^t + \sum_i C_V^{(i)} T_i = C_V T^{eq}, \quad C_V = \frac{3}{2}k + C_V^{in}$$

Thus the system of kinetic moment equations asymptotically is reduced to a set of parabolic equations which can be solved using standard boundary techniques.

## Numerical Results

The modified Rankine-Hugoniot conditions (including the "shock slip") are used as outer boundary conditions, which is quite independent of the gas-kinetic model of the shock-transition zone, as noted early by Cheng [5]. The wall-slip effects are found to influence the shock-layer flow at the most by relative order $N\sqrt{\varepsilon T_w / T_0}$ ($T_w$ - is a wall temperature). They have been demonstrated to the negligibly small for the low $T_w / T_0$ relevant to most hypersonic applications. To distinguish from the equations of the viscous shock layer (VSL) based on Navier-Stokes model we refer our kinetic version (2) as an "equations of nonlinear transfer" (ENT). For slender body flows (unlike the nonslender case) the shock slip proves to be far less important than the translational nonequilibrium effect. This effect is to reduce significantly the streamline displacement, which affects the surface normal stress $P_n = P_{yy}$ and heat transfer rate, as well as the skin friction $P_{xy}$ , depending on the degree of nonequilibrium. The kinetic version of parabolized Navier-Stokes equations (PNS) is applied to the problem of rarefied hypersonic flow of rotationally exited N2 past the leading edge of a two-dimensional flat plate aligned with the free stream.

Figure 2 compares the $\overline{P}_n$ $(\overline{P}_n = P_{yy} / P_{fm})$ distribution along the sharp flat plate based on the kinetic version of PNS and DSMC calculations [7] for the freestream Mach number $M_\infty = 23$ are presented. Distance along the plate $\overline{x} = x / \lambda_{fb}$, length scale $\lambda_{fb}$ and free-molecular pressure $P_{fm}$ is defined in Ref.[7]. The kinetic version of PNS results agree better with DSMC calculations compared with hypersonic strong interaction theory (HSIT) results.

Figure 3 compares $P_{yy}$ – distribution along the plate obtained in ENT-theory (ENTT) with HSIT results, where $P_{yy}$=P. (Note, that the tangent wedge approximation on outer boundary layer edge was employed for numerical calculation of $P_{yy}$). The dimensionless distance $\xi_1$, parameter B and temperature factor $g_w$ showed in Fig. 3 are

$$\xi_1 = \frac{3}{2}\underline{N}^{-2}\int_0^x P_{yy}dx, \quad B = \frac{3M_\infty^4}{2\varepsilon^2 \mathrm{Re}_{BL}^2}, \quad g_w = \frac{T_w}{T_0}$$

($T_w$ and $T_0$ is wall and stagnation temperature respectively).

From the analysis of a figure 3 it is clear that the HSIT distribution of thermodynamic pressure P has a singularity near the sharp leading flat plate edge. On the contrary the ENTT distribution of the normal component of stress tensor $P_{yy}$ has no singularity. Moreover, the $P_{yy}$ distribution reaches first the weak maximum (Located on distance of several mean free path of the gas from the plate leading edge) and further decreases steadily along the plate. It may be shown that $\delta_{BL}$ varies as $x^{3/2}$ near leading plate edge. ($P_{yy}$ and $\frac{d\delta_{BL}}{dx}$ go to the zero as x →0). It is important to note, that ENTT does not take account the velocity and temperature jump effect unlike HSIT. Consequently the absence of singularity of $P_{yy}$ near leading plate edge causes only the nonlinear dependence of stress tensor and heat flux from normal to plate velocity gradient.

Thus ENTT may be successfully applied to solving the strong non-linear shear flows problem.

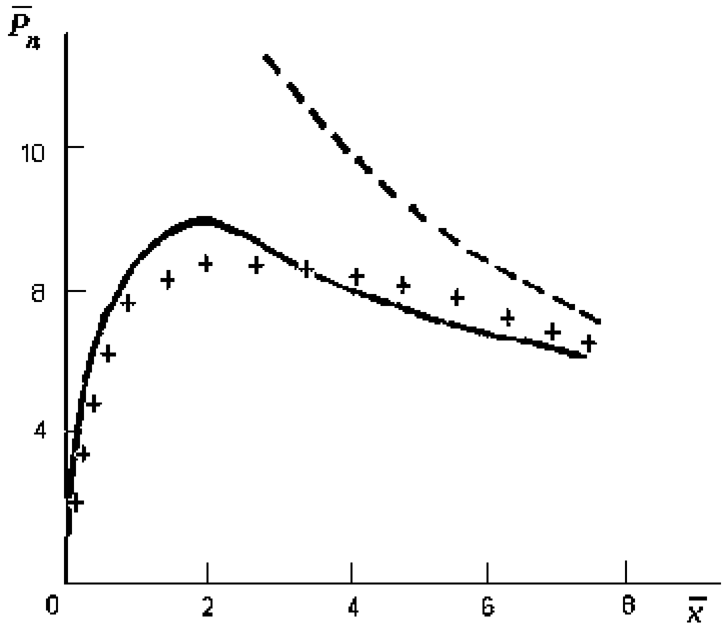


**FIGURE 2.** Comparison of the $P_n$ distribution along the sharp flat plate based on the kinetic version of PNS and DSMC calculations. Solid line – ENTT, dashed line – HSIT, dagger line – DSMC.

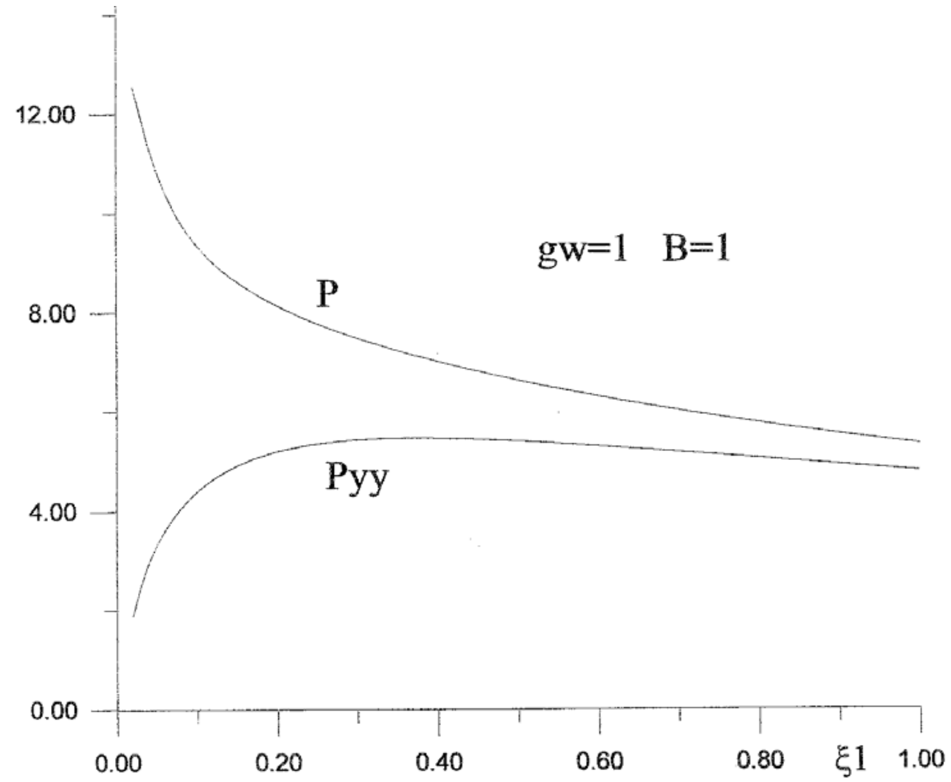


**FIGURE 3.** Comparison of the P and $P_{yy}$ distributions along the flat plate based on ENTT and HSIT calculations.

## ACKNOWLEDGMENTS

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