ANALYSIS OF FREE MOLECULAR FLOW IN A 3-D TURBOMOLECULAR PUMP WITH NON-PARALLEL BLADES

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Abstract. Turbomolecular pumps are widely used in scientific and industrial applications. The three-dimensional simulation of free molecular flow within a row of blades of a turbomolecular pump is solved using test-particle Monte Carlo method. The solution performed in an inertial reference frame where molecular paths are straight line. The following of molecule paths in both rotor and stator is done in a similar system of coordinates. Good agreement between numerical results and quasi similar previous simulations confirm the validity of the presented algorithm. In this simulation, effects of the flow leakage due to existence of clearance between blade tip wall and casing is considered and the blade thickness is ignored. In addition the flow analysis shows effects of determination of sizes, significant effects of clearance and more importantly the effects of blade's angle on turbo molecular pump's fundamental characteristics. The simulation of turbo molecular pump with the non-parallel blades is done and compared with similar parallel blades.

Keywords Free molecular flow – Test particle Monte Carlo – Turbomolecular pump.

INTRODUCTION

The turbomolecular pumps (TMPs) are widely used in scientific and industrial applications for their provision of higher and cleaner vacuum compared to oil diffusion pumps. Kruger [1] first investigated the pumping performance of TMPs in free molecular flow regime experimentally and theoretically. His study was based on parallel flat-plate blades with infinite height, and calculations were made on single-row and multi-row blades by both numerical and Monte Carlo methods. He studied the effects of the blade geometrical characteristics on the performance of the single rotor and provided TMPs performance curve. Sawada et al. [2] studied flat blades with finite height for a single rotor using an integration method. The method based on some geometrical calculations for the transmission of molecules from elements of the blade and integration of these elements on the blade boundary. However a closed-form solution for integral equations could not be found due to complication of multiple reflections between the blades, therefore the solution was obtained using a numerical approach. Sawada [3] extended his work to analyze the rotor–stator combinations by considering the effects of changing the mean mass velocity by collision with adjacent rows, interception of molecules by the blade thickness, and molecules passing through the clearance between rows. Then he evaluated pumping performance of multi-row TMPs by multiplication of the transmission probabilities of individual blade rows [4]. Iida and Kimura [5] analyzed the performance of a pump with blade row of wedge type profile, and compared the results with those of a flat-plate blade pump. According to their results, the wedge type blades present much larger pumping speed comparing to flat-plate ones. On the other hand the maximum pressure ratio of wedge type blades is only a few percent smaller than that of parallel wall type.

Chu and Hua [6] evaluated the modified velocity of molecules passing through an infinite blade height by statistical mechanics method. Their results are expressed in analytical forms, which are easier to apply for designing the TMPs. Katsimichas et al. [7] simulated free molecular flow within a single rotor machine with a three-dimensional flat-plate blade using the Monte Carlo method. Their calculations were done in the rotational reference frame where the molecular paths were not straight lines. They also neglected the effects of clearance between the tip of the blade and the pump casing. The maximum compression ratio was found to be higher than that calculated via two-dimensional simulation especially at high rotational speed and when pumping heavy gases. Skovorodko [8] considered the effect of clearance between the blade tip and the pump casing. Neglecting the blade thickness, he simulated free molecular flow in a couple of rotor–stator stages using the inertial frame of reference and Monte

Carlo Method. Chang and Jou [9] calculated the pumping characteristics of a three-dimensional single rotor from molecular to transition regime using the DSMC method. Also Hosseinalipour et al. [10] and Amoli et al. [11] simulated free molecular flow within three-dimensional single rotor and multi-row TMPs in an inertial reference of frame regarding the real topology of the blades. Also Hosseinalipour et al. [12] investigated the effects of non-parallel blades on performance of TMPs.

In an inertial frame the moving path of a molecule is a straight line and following this path is done both in rotor and stator in a similar system of coordinates. In this paper TPMC method is employed for computation of a one rotor blade row of an axial turbo-molecular pump with wedge profiles. A three-dimensional simulation of a combination of rotor and stator with flat plate blades is studied. The effects of blade thickness, blade-casing clearance, and clearance between rotor and stator are considered. Since this simulation is performed in an inertial frame of reference, an algorithm is developed for molecular path tracking, when molecules go back and forth between stator and rotor.

Theoretical Model

schematic view of a rotor–stator row is shown in Figure (1) . Cascade theory is used, and therefore the flow through one of passages is modeled and its schematic is illustrated in Figure 2. This figure a single blade row consists of a geometrically three-dimensional row of convergent blade moving between two infinite regions of a three-dimensional gas flow.

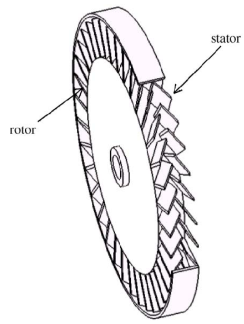

FIGURE 1. A rotor–stator row.

The cross section of a this blade row is shown in Figure (3). In these Figs, h is blade height (distance between root and tip of the blade), b is blade chord and d is clearance between blade tip wall and casing (or rotor body in stator), α is blade angle (angle between the blade and line normal to the rotor axis), ε is wedge angle and $L = b\sin\alpha$ is blade length.

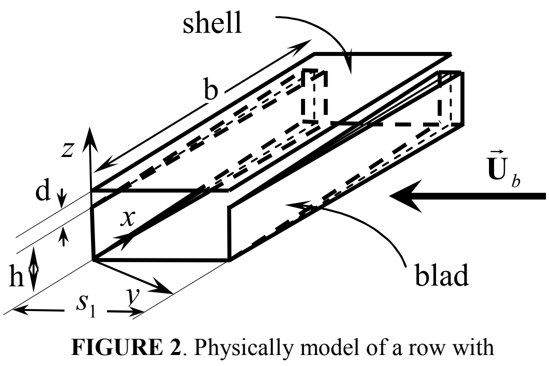

FIGURE 2. Physically model of a row with non-parallel blade (3D)

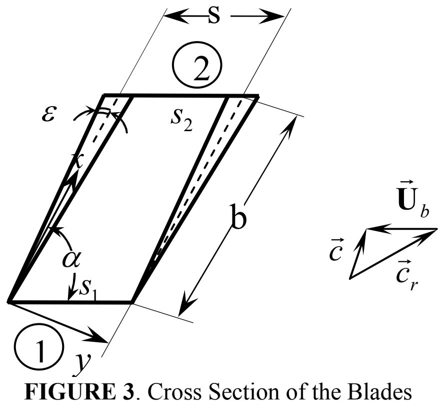

FIGURE 3. Cross Section of the Blades

Let $\sum_{12}$ be the fraction of those entire molecules incident on the blade row from side 1 (up stream) that pass ultimately to side 2 (down stream) most likely after several collisions with the blades. Similarly $\sum_{21}$ is the fraction of those entire molecules incident on the blade row from side and flow toward side 2. It can be shown that pressure ratio of combination of rotor–stator would be as follows:

$$\frac{p_2}{p_1} = \left(\frac{s_1}{s_2}\right)\frac{\Sigma_{12} - W}{\Sigma_{21}} \tag{1}$$

Where p_1 and p_2 are pressures in side 1 and 2 respectively, and W is non-dimensional pumping speed as the net number of molecules, which flow from side 1 towards side 2 per unit area and per unit molecular flux number incident upon the side 1.

Assuming a similar molecular distribution function in both sides and also an isothermal motion for molecules through the blade row, the ultimate pressure ratio is obtained when $W \to 0$. In addition, the maximum pumping speed is achieved when a large amount of gas is introduced to the inlet of the pump and the gas pressures are still equal at the upstream and downstream, i.e. $p_2 / p_1 = 1$

$$\left(\frac{p_2}{p_1}\right)_{\max} = \frac{s_1}{s_2}\frac{\Sigma_{12}}{\Sigma_{21}} \tag{2}$$

$$W_{\max} = \Sigma_{12} - \frac{s_2}{s_1}\Sigma_{21} \tag{3}$$

Both analytical and Monte Carlo methods for a single rotor [1,2,7] or combinations of rotor and stator [3,4,8] are used to calculate these coefficients.

Test Particle Monte Carlo Scheme

The Monte Carlo method is peculiarly suitable for problems of rarefied gas dynamics, in as much as the numerous samples in this method are direct analogs of the individual molecules of the gas flow. For the analog to be valid, the samples must be statistically representative of the distribution functions of the gas molecules. In an axial turbomolecular pump, the samples are individual molecules incident upon the blade row. Each molecule is followed as it enters the passage, undergoes successive reflections, and finally emerges into region 1 or region 2. The fraction of all such molecules transmitted through the blade row gives the probability of transmission Σ_{12} or Σ_{21}.

In Figure (3) the solution domains are shown for a rotor and stator row. Each domain is a channel between two adjacent blades that is symmetrically repeated around the periphery of the rotor body. As it is seen in Figure3, there are three types of boundary conditions: (1) solid boundaries include rotor and stator blade walls, rotor body, and casing; (2) cyclic boundary conditions that are the symmetric planes of each blade and are shown by dashed lines; and (3) inlet and outlet boundaries which are the planes where molecule enters or emerges from and are surrounded by solid and cyclic boundaries.

To perform an exact computation for collision point of a molecule with each boundary surface, the analytical equations of these surfaces are needed. The molecular flux function at the blade upstream for the fraction of molecules with velocity c and angles θ, ϕ and $\beta = 1/(2RT)$ (inverse of most probable of molecule velocity) is expressed as [5]

$$\frac{d\dot{N}}{n} = \left(\frac{\beta^3}{\pi^{3/2}}\right) c^3 \exp\left(-\beta^2 c^2\right) \sin\theta \cos\theta \, d\theta \, d\phi \, dc \tag{4}$$

Velocity vector of a molecule at blade inlet is computed from the following expression:

$$\vec{C} = c\left(\cos\theta \ \hat{i} + \sin\theta \ \cos\phi \ \hat{j} + \sin\theta \ \sin\phi \ \hat{k}\right) \tag{5}$$

If R_f is selected from a random number table, the corresponding angles of emission may be computed from random numbers theories. According to Eq. (5) polar (θ) and tangential angle (ϕ) can be chosen as follows:

$$\begin{aligned} \theta &= \cos^{-1}(\sqrt{R_f}) \\ \phi &= 2\pi R_f \end{aligned} \tag{6}$$

In general, velocity of c is determined by Acceptance-Rejection method using random numbers [13] from distribution function of Eq. (5). Considering the coordinate system is joined to rotor and because of the molecules have velocity of $-\vec{\mathbf{U}}_b$ relative to blade in inlet, velocity vector of $-\vec{\mathbf{U}}_b$ is added to velocity of Test Particle Monte Carlo, therefore

$$c_{rx} = u - \mathbf{U}_b \cos(\alpha)$$
$$c_{ry} = v - \mathbf{U}_b \sin(\alpha) \quad (7)$$
$$c_{rz} = w$$

Having velocity vector neglecting the intermolecular forces in a Cartesian coordinate system, the motion of a molecule through the blade row would occur on a straight line with the following equations:

$$\begin{cases} x = x_i + c_{rx} t \\ y = y_i + c_{ry} t \\ z = z_i + c_{rz} t \end{cases} \quad (8)$$

Where (c_{rx}, c_{ry}, c_{rz}) are cosines and (x_i, y_i, z_i) present the initial coordinates at the entrance of the blade or when a molecule is reemitted from solid surfaces.

Since the blades are non parallel, the distance between blades at exit plane,s_2 is expressed by

$$s_2 = s_1 - \left(\sqrt{A_1^2 + b^2 - 2A_1 b \cos\varepsilon} + \sqrt{A_2^2 + b^2 + 2A_2 b \cos\varepsilon}\right) \quad (9)$$

$$\begin{cases} A_1 = b \sin(\alpha) / \sin(\alpha - \varepsilon) \\ A_2 = b \sin(\alpha) / \sin(\alpha + \varepsilon) \end{cases}$$

By introducing the equations of molecular trajectories into the analytical equations of the blades, the intersection points of the molecular path with blade walls are calculated. Although the mechanism of molecule-surface interaction is not completely understood but the experimental studies [14] indicate that molecules leave the surface in an approximately diffuse manner. The re-emission of molecules from solid surfaces is assumed fully diffusive, i.e. the direction of the emission is independent of the incidence direction. Therefore it is assumed that re-emitted molecules have a Maxwellian motion as if they came from behind the surface from an imaginary gas at rest with respect to the surface. Thus, the direction of reemission is independent of the direction of incidence and all incoming tangential momentum is lost to the surface. This leads to aforementioned Cosine distribution function. For reemitted molecules from moving parts of the blade row, reflected tangential velocity is superimposed by velocity of blade. When a molecule leaves the cyclic boundary, it returns from another one with the same velocity.

Numerical Results

a)Simulation results for parallel blades pump

Figure (4) depicts the transmission coefficient for a moving blade row as a function of blade non-dimensional speed $\overline{\mathbf{U}}_b = \mathbf{U}_b / \sqrt{2RT}$. The value is assumed to be $\varepsilon = 0$. The presented positive and negative values for $\overline{\mathbf{U}}_b$, are corresponding to $\sum_{12}$ and $\sum_{21}$ respectively. The geometrical parameters are $\alpha = 40^\circ$, s /b=1 and h/b>>1 .The comparison between these results and those computed by Kruger [1] shows a good agreement.

Figure (5) shows the numerical data for variation of transmission coefficient as a function of blade non-dimensional speed. The Geometrical parameters are same as Figure (4). According to this Figure, when non-dimensional speed is greater than zero, the spline have maximum point that means, for transmission coefficient, exist the optimum value of angle. The angle is obtained with dimensions of pump.

Variation of maximum pumping compression ratio (MPCR) for blades characteristics of $\alpha = 20^\circ$ ،*h/b*= 2 *and s/b*= 1 without clearance (d/b=0) and with it (d/b=0.1, d/b=0.2) depicted in Figure (6). As it can be seen in this Figure it is attenuated in the clearance region between blade-tip wall and pump casing due to leakage effect.

Figure (7) shows the effects of leakage and blade non-dimensional speed on MPCR with geometry of s/b=1 and h/b=2 and it contains two comparisons. According to the Figure, when the clearance increases, then MPCR decreases. Also effects of decline of with grow up clearance will be reduced (same as Figure (7)). When clearance trends to zero the spline Vs. angle don't have any maximum point. Also when clearance exists, the splines have maximum point that means, the optimum value of angle exists for every non-dimensional speed. The angle is obtained with dimensions of pump.

Figure (8) shows the variation of $W_{12\max}$ with respect to $\overline{\mathbf{U}}_b$. As it can be seen the maximum pumping speed doesn't change with increasing clearance (it reduce a little), But the angle has more sensible effect on maximum pumping speed. Effects of these angles are different. The optimum points also are seen in these figures. Also height of blade has don't effect on maximum pumping speed.

b)Simulation results for non-parallel blades pump

In this part the simulation results for performance prediction of a pump with blade row of wedge type profile where flow passage is convergent or divergent ($s_1 / s_2 > 1$, $s_1 / s_2 < 1$ in Figure (3) respectively) are presented and compared with that of parallel blade type.

Figure (9) shows the MPCR as a function of $\overline{\mathbf{U}}_b$ and for different values of ε as a parameter. The results show higher values of MPCR for non-parallel blades comparing with those of parallel ones. For validation the results are compared with them that obtained by Hoseinalipour [12]. When the blade's wall is high, results shows a good agreement with Hoseinalipour's results. As see as possible, with increasing of the wedge angle (with attention to dimension of channel) MPCR increase. Also it can be seen agreement with parallel blade that increasing of non-dimension of blade speed due to growth on MPCR.

Figure (10) presents the effect of ε on $W_{12\max}$ as a function of $\overline{\mathbf{U}}_b$. According to these results, the convergent blades ($\varepsilon = 0$) show a larger value of $W_{12\max}$ comparing to parallel blade respectively. This conclusion is in correlation with that presented by Hoseinalipour.

Figure (11) presents the effect of clearance on MPCR. Geometries are $\alpha = 20^\circ$ ، s/b=0.5 and h/b>>1. According to Figure, effect of clearance is same as the parallel blade.

Figure (12) present the effects of h/b on MPCR for a pump with non-parallel blades. According to Fig., effects of height, angle of wedge and angle of blade have an optimized value. The results show that only positive angle of wedge in high blade's wall due to increase efficiency of pump and in ordinary dimensional makes it low.

Conclusions

Pumping performance of a rotor–stator row in free molecular regime is investigated using test particle Monte Carlo method. Pursuing the molecules is done in a stationary coordinate system through rotor. Good agreement between the presented results and the quasi similar previous simulations confirms validity of the presented algorithm. This algorithm is readily extended to multi-row turbomolecular pump. Some reductions in molecular concentration is observed in the clearance region between rotor and casing. The results show that pumping efficiency increases in tall blades with positive wedge angles.

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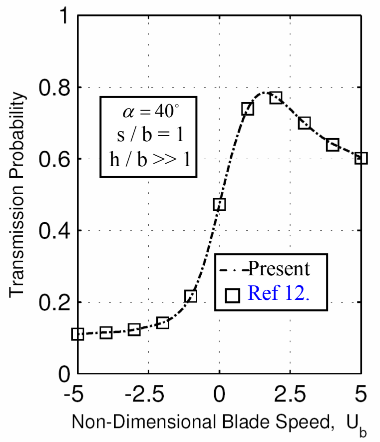

FIG 4. Transmission Coefficient of Parallel blade

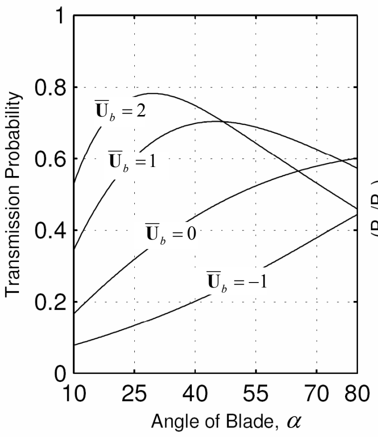

FIG 5. Transmission Coefficient of Parallel blade

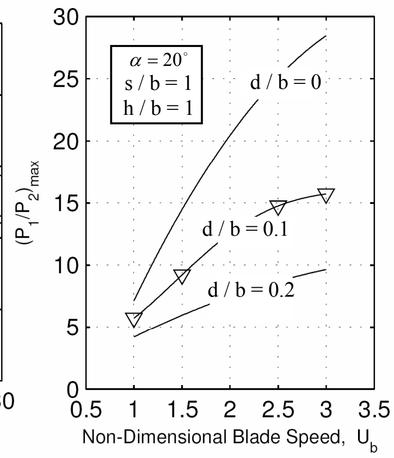

FIG 6. Effect of Clearance on MPCR

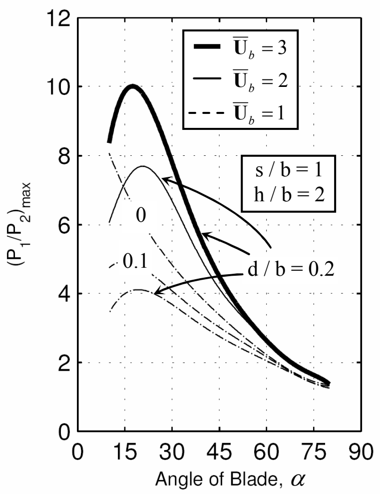

FIG 7. Effects of Leakage and Blade Speed on MPCR

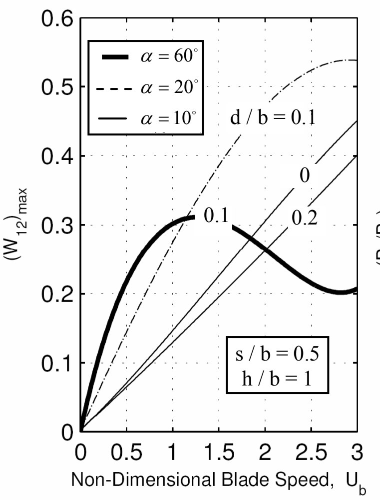

FIG 8. Effects of Leakage on Maximum Pumping Speed

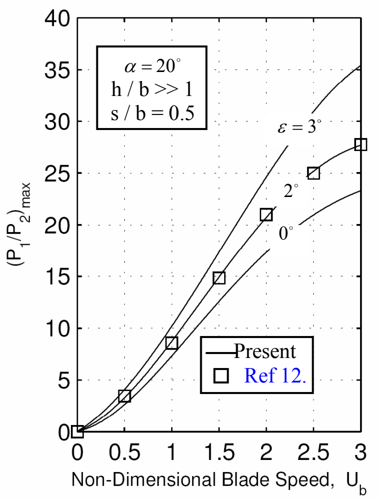

FIG 9. Effects of Wedge angle on MPCR

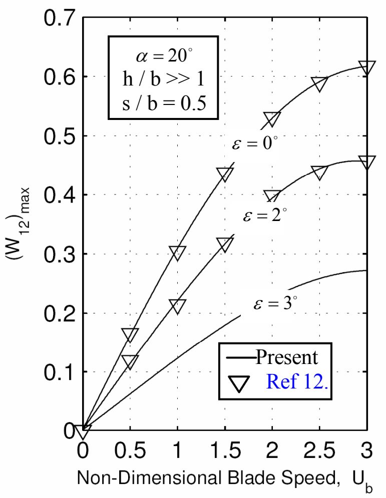

FIG 10. Effects of Wedge Angle on Maximum Pumping Speed

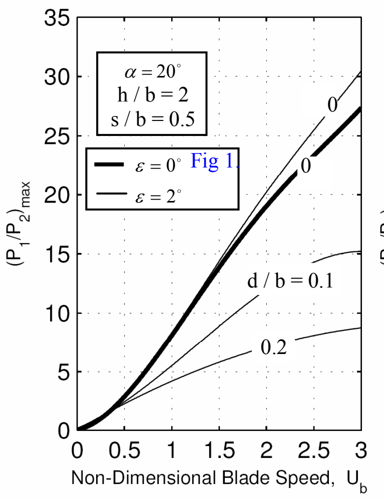

FIG 11. Effect of clearance on MPCR

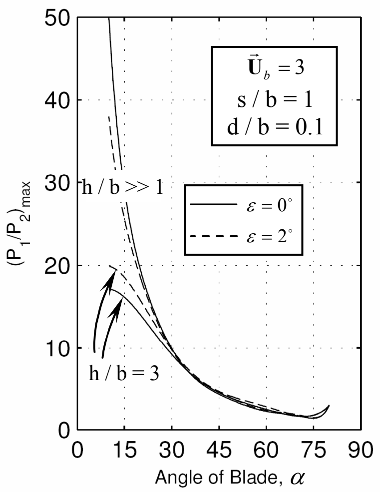

FIG 12. effects of h/b on MPCR