On the Dependence of Rarefied Gas Flow Stability in Channels on Interaction Parameters

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Abstract. Obtained earlier by the methods of nonlinear dynamics analytical limit solutions (attractors) of describing the flow iteration equation for plane, cylindrical and wedge-shaped channels are tested numerically. Monte-Carlo simulations are applied taking into account collisions between gas atoms, i.e. for finite Knudsen numbers. The results confirm that negligible change of the parameters of scattering function can lead to substantial difference in the characteristics of gas flow in a channel. It is shown, that the regions of the parameters of gas-surface interaction corresponding to different types of attractors can be determined analytically and applied to numerical calculations.

INTRODUCTION

The effect of instability of rarefied gas flows in channels we studied earlier analytically using the methods of nonlinear dynamics [1]. Successive reflections of free-molecular gas atoms from channel walls have been considered, and for the corresponding iterative scheme attractors and bifurcations have been found. In practical applications it means that the gas flow is instable. The instability is comprehended here as a sharp deviation of flow characteristics by small variation of parameters. We consider as the gas-surface interaction parameters, as the parameters determining flow conditions.

The main purpose of the present investigation is to prove the instability numerically. Also we reveal that the flow keeps instable in spite of significant modification of conditions. To obtain the analytical results we imposed limitations on the basic statement of problem. In particular, free-molecular flow has been considered and ray model of gas atoms scattering from the walls proposed. Our new numerical investigations allow us to lift the restrictions. In particular, the instability keeps intact by changing the scattering function of gas atoms from the surface. Also we study the instability by varying geometrical shape of the channel and by modifying the Knudsen number, taking into account the joint interaction of gas particles.

ITERATIVE EQUATIONS AND ATTRACTORS

The basic assumptions are analogous to [1]. Rarefied gas flow in a channel at Knudsen numbers much more than one ($\mathrm{Kn} \gg 1$) is considered and geometrical shape of the channel is supposed not very different from flat or cylindrical. Describing the reflection of gas atoms from walls we apply trigonometric approximations of momentum exchange coefficients, which are used in local methods of aerodynamic calculations of convex bodies moving at high velocities in transitional regime. The possibility to apply these expressions to internal gas flows is proved in [2] on the base of approximation theory. In particular trigonometric approximations can be used to flows in very narrow channels, which are now vastly researched because of great practical advantages of microelectromechanical systems and different filter devices.

Surface geometry determines the unique connection between the angles of reflection θ'_m and of incidence in the next collision θ_{m+1}

$$\theta_{m+1} = \psi(l_m, \theta'_m), \tag{1}$$

where l_m is the coordinate of a point of previous collision of atom with a wall (see Fig.1).

Each iteration contains two main stages: the reflection from the walls according to selected scattering model and the motion of a gas atom between two points on the walls of the channel. Last stage includes possible interaction with another gas atoms.

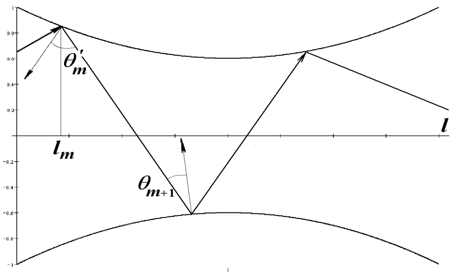

FIGURE 1. Iterative scheme in a nozzle.

Surface geometry defines second stage and sets the function ψ deterministically (nonrandom). Analytical results in [1] were obtained supposing ray model of gas atom from the surface, which also determines the connection between the angles θ_m and θ_m' nonrandom: $\theta_m' = \varphi(\theta)$. The function φ can be expressed in the terms of the momentum exchange coefficients p and τ

$$\varphi(\theta) = \arctan \frac{2\sin\theta\cos\theta - \tau(\theta)}{p(\theta) - 2\cos^2\theta}. \qquad (2)$$

Selecting the coefficients p and τ, we receive the parameters of the function φ. Hence, modifying the parameters of the function φ, it is possible to receive any values of p and τ.

Thus, combining both stages (1) and (2) of iterations, we receive an iterative equation establishing the relationship between angles of incidence for two successive collisions of a gas atom with walls. This equation can be rewritten for variables $x_m = \mathrm{tg}\,\theta_m$ and $x_{m+1} = \mathrm{tg}\,\theta_{m+1}$ in the form

$$x_{m+1} = \tan\left(\psi\left(l_m, \arctan\frac{\sin 2\arctan x_m - \tau}{p - 1 - \cos 2\arctan x_m}\right)\right). \qquad (3)$$

To reduce the number of parameters we apply trigonometric approximations of momentum exchange coefficients p and τ

$$p(\theta) = p_1\cos\theta + p_2\cos^2\theta, \quad \tau(\theta) = \tau_0\sin\theta\cos\theta, \qquad (4)$$

considered also by Galkin, Erofeyev and Tolstykh (see references in [2]). Regime coefficients of local approximation p_1, p_2 and τ_0 we can get from experimental data [2]. Thus, after denoting $a = \frac{p_1}{2-\tau_0}$ and $b = \frac{2-p_2}{2-\tau_0}$ the iterative equation (3) transforms to

$$x_{m+1} = \tan\left(\psi\left(l_m, \arctan\frac{x_m}{a\sqrt{1+x_m^2} - b}\right)\right). \qquad (5)$$

The investigation of iterative schemes obtained by means of methods of nonlinear dynamics has shown that, depending on flow parameters, all types of attractors can appear: stationary attractive points, cycles, infinite bifurcations cascade and chaotic attractor [1]. To test numerically obtained analytical attractors, we consider the statement of problem, which is generalized in three ways.

First, we use in calculation more general scattering model, than ray scattering function. Thus the basic iterative equation (5) is valid only partially. Details of applying this scattering function are discussed in our paper “Stability of rarefied gas flow in a channel for ray-diffuse scattering function” in this book.

Second, the interaction of gas atoms is accounted. This factor involves increase of computation time. However, only large Knudsen numbers (more than about 3) are necessary to obtain the instability of the same type as calculated analytically.

Third, we consider different shapes of the channel. But the stability of the flow depends on the change of the incline of channel walls complicatedly, and it is very difficult to find the values of parameters corresponding to the bifurcation. We solve the problem of searching these parameter values applying the analytical results.

MONTE-CARLO SIMULATION RESULTS

Numerical calculations were performed for N=5000 gas particles in the channel with length l=100 relative to its width.

The comparison of numerical and analytical results shows that, for different regions of a, b, the flow becomes unstable by a negligible modification of the parameters at the same values of a and b. Next four figures (2,3,4 and 5) demonstrate, that the flow instability takes place in a very small interval of values of a independent of the type of scattering function. Each figure contains two graphs, which differ only in the values of a. The values of a and b are selected from analytical investigations on the base of iterative equation (5) as most indicative. However, similar results are obtained by various parameter values of gas-surface approximation.

We can see, that very small difference in a causes very large changes in the flow, as well for ray scattering function (fig. 2) as for more general ray-diffuse scattering function (fig. 3). Here l is the coordinate along the channel, θ is average scattering angle in a section across the channel, and N is average number of gas atoms along the channel. Note that flow instability can be observed only by specific values of a and b. For example, even large decrease of a less than 0.78 causes very insignificant modification of the results from left graphs in these figures.

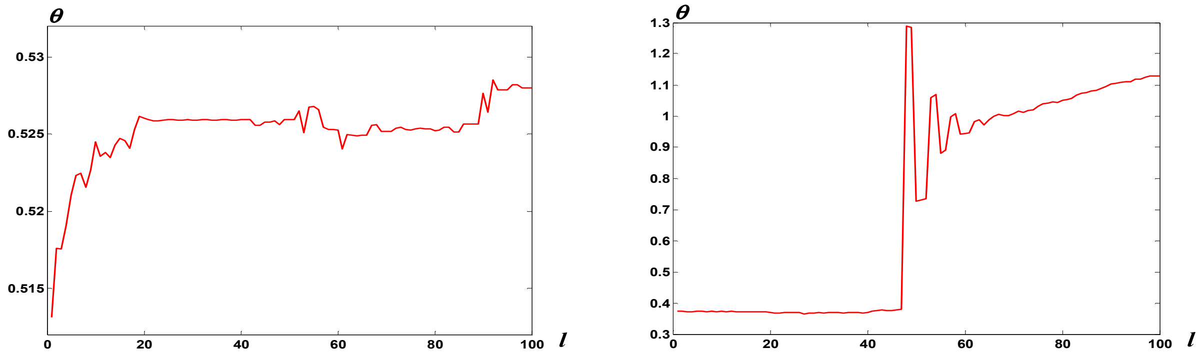

FIGURE 2. The change of average scattering angle θ'_m along the channel by a negligible modification of the parameter a from $a = 0.78$ (left graph) to $a = 0.79$ (right graph) by constant $b = 0.9$ for ray model of scattering.

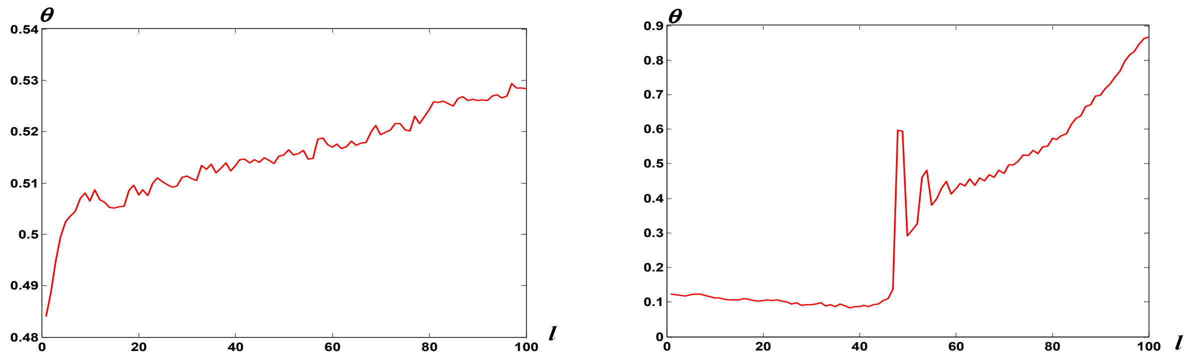

FIGURE 3. The change of average scattering angle θ'_m along the channel by a negligible modification of the parameter a from $a = 0.78$ (left graph) to $a = 0.79$ (right graph) by constant $b = 0.9$.

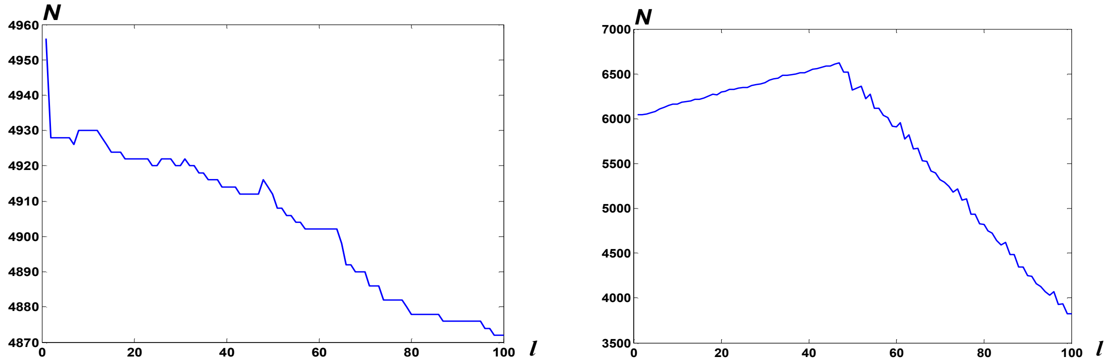

FIGURE 4. The change of the number N of gas atoms along the channel by a negligible modification of the parameter a from $a = 0.78$ (left graph) to $a = 0.79$ (right graph) by constant $b = 0.9$ for ray model of scattering.

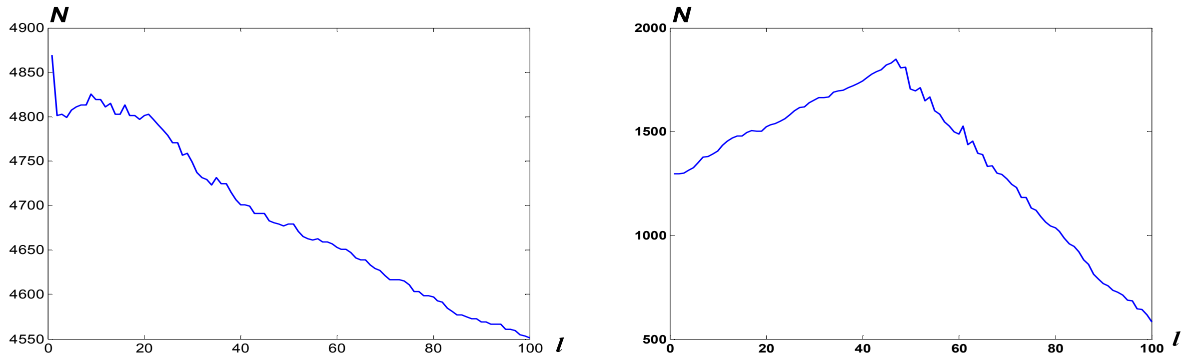

FIGURE 5. The change of the number N of gas atoms along the channel by a negligible modification of the parameter a from $a = 0.78$ (left graph) to $a = 0.79$ (right graph) by constant $b = 0.9$.

The irregularities on all figures appear in consequence to the selection of the parameter values a and b near the points of Feigenbaum cascade of bifurcations, i.e. near the singularities.

Next three figures demonstrate the influence of changing Knudsen number Kn on the same average characteristics θ'_m and N of gas flow. The DSMC computation for finite Kn takes much more time, therefore presented results are carried out for 50 gas atoms instead of 5000 for free-molecular flow. Different values of the parameters a and b are more indicative in this case.

Comparing the figs. 6 and 7 we can see that the difference between graphs corresponding to modified value of a becomes weaker for Kn=2 (and as demonstrate further calculations negligible for Kn=1). Hence, the effect of instability is substantial only for large Kn.

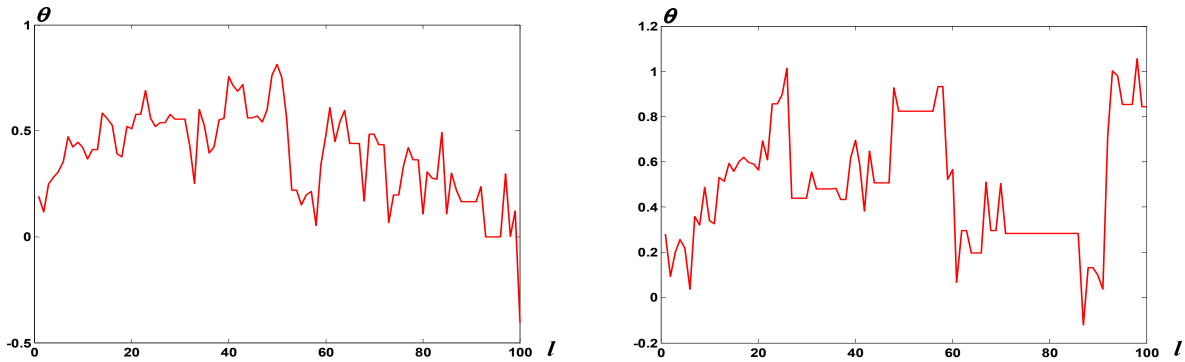

FIGURE 6. The change of average scattering angle θ'_m along the channel by a negligible modification of the parameter a from $a = 1.00$ (left graph) to $a = 1.05$ (right graph) by constant $b = 1.5$, Kn=2.

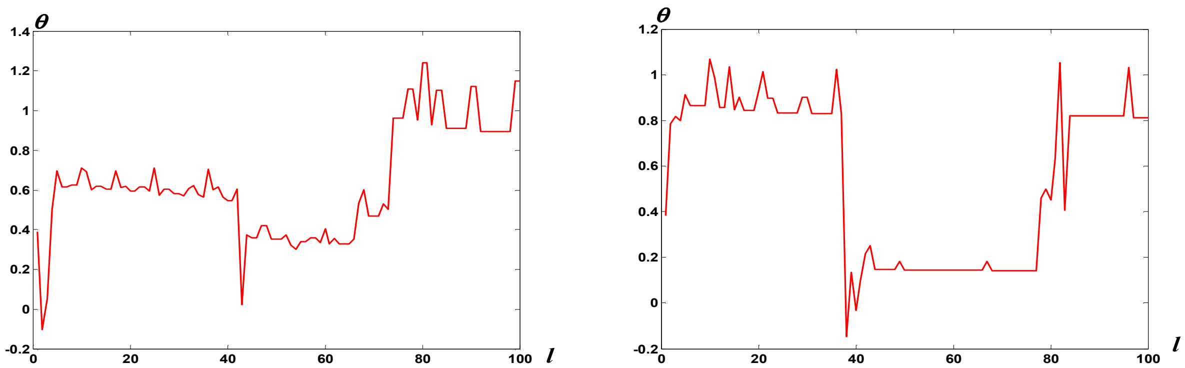

FIGURE 7. The change of average scattering angle θ'_m along the channel by a negligible modification of the parameter a from $a = 1.00$ (left graph) to $a = 1.05$ (right graph) by constant $b = 1.5$, Kn=∞.

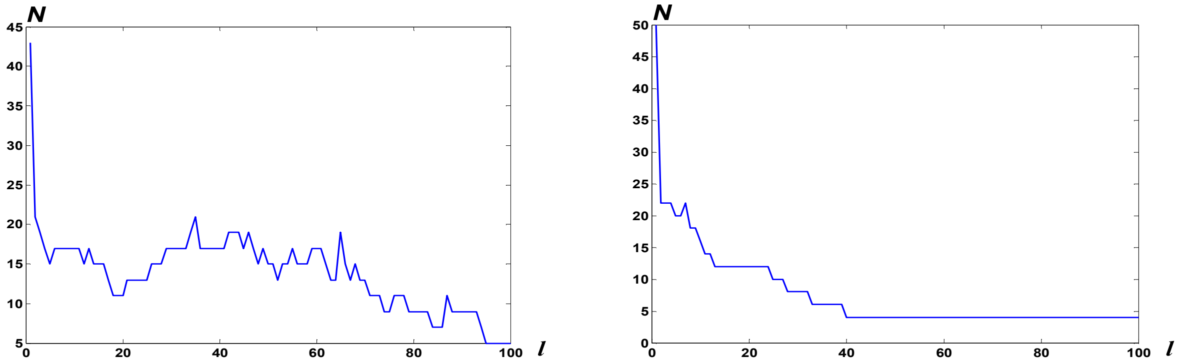

FIGURE 8. The change of the number N of gas atoms along the channel by a negligible modification of the parameter a from a =1.00 (left graph) to a = 1.05 (right graph) by constant b =1.5, Kn=2.

CONCLUSIONS

Thus, the attractors and the bifurcations in rarefied gas flows in channels are obtained analytically (using the methods of nonlinear dynamics) and numerically (applying DSMC computation). The effects of onstability (connected with bifurcations) are indicated numerically by the same values of gas-surface interaction parameters as in analytical solutions not only for modified scattering function, but also for different Knydsen numbers.

The results can be regarded as analytical exact in free-molecular flow and giving in applications approximate values of flow and surface parameters corresponding to flow instability appearing in consequence of instability of nonlinear dynamic system describing multiple gas atoms scattering from the walls of a channel.

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