“Anomalous” Shock Structure In Weakly Ionized Gas Under Strong Thermal Nonequilibrium

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Abstract. Ambipolar ion diffusion across a plane shock structure in weakly ionized argon is considered in the case when the electron temperature is much higher than the gas temperature. An analytical description of the shock structure in terms of the ion fraction and ambipolar electric field caused by the gas density gradient is given in the entire flow field including the precursor zone. Ionization factor due to electrostatic field before the shock front is estimated and found to be not negligible.

Keywords: Weakly Ionized Gas, Shock Wave, Ambipolar Ion Diffusion, Ambipolar Electric Field, Ionization Factor.
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INTRODUCTION

In [1] the structure of a plane ion-acoustic shock wave in the case when the electron temperature is much higher than gas temperature ($T_e >> T_I \approx T_g$) has been considered in terms if the ion density and speed. Quite long precursor motion of the charged component was predicted and the length of the precursor zone was estimated. Though in [1] the electrostatic field, which appears within shock structure due to charge separation [2], was not analyzed. “Anomalous” structures of the ion fraction and ambipolar electric field with maximum inside the shock wave were obtained in [3, 4] through CFD modeling of the supersonic stagnation line flow of weakly ionized argon in the case $T_e/T_g>>1$. The mechanism of pumping ions out the shock layer up to the free stream was explained by the action of the ion diffusion driving force cased by the pressure gradient and gradient of the ratio T_e/T_g, and high value of the ambipolar diffusion coefficient [3, 4].

In the present paper the structure of the shock propagating in weakly ionized argon in terms of the ion fraction and ambipolar electric field is considered on the basis of an analytical solution of the ion diffusion equation using an ambipolar approach.

THE FORMULATION OF THE ION DIFFUSION PROBLEM

We consider the shock structure in weakly ionized quasi-neutral 3-species (*A* – atoms, *I* – ions and *e* – electrons) two-temperature plasma under assumptions as follows:

1. Plasma is weakly ionized: $c_I T_e / T << 1$;
2. Ion and gas temperatures are equal, there is no energy exchange between electrons and heavy particles, electron temperature is constant and much higher than gas temperature: $\theta = T_e / T >> 1$;
3. External electric field and electric current equal zero (ambipolar diffusion);
4. Motion of the charged particles does not affect the shock wave propagating in gas with given parameters;
5. Flow is chemically frozen.

The problem consists of in determination of the fraction of the ions and ambipolar electric field in the entire flow field including shock structure and upstream zone in front of the shock.

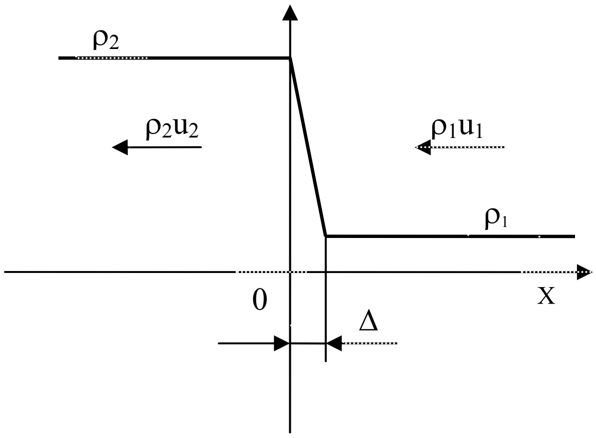

FIGURE 1. Schematic of the density profile in the plane shock wave and coordinate system.

In one-dimensional plane flow the coordinate axis X is connected with the shock structure as shown in Fig. 1. In the ambipolar approach for diffusion in 3-species plasmas under above assumptions the only independent unknown is the ion mass fraction c_I and single governing equation for the ion diffusion has to be solved. The key point of the problem is an expression for the ion mass diffusion flux J_I. Here we use the expression for the ambipolar ion mass diffusion flux obtained in [3, 4]. In the case under consideration the governing equation for the ion mass fraction becomes the form as follows ($-\propto<X<\propto$)

$$-\rho u \frac{dc_I}{dX} + \frac{dJ_I}{dX} = o, \ \ \rho u = \rho_1 u_1 \ , \tag{1}$$

$$J_I = -\rho D_a \left(\frac{dc_I}{dX} + c_I \frac{d \ln p}{dX} + c_I \frac{d \ln \theta}{dX} \right).$$

The ambipolar diffusion coefficient is expressed as

$$D_a = \theta D_{IA}(1), \quad D_{IA}(1) = \frac{3}{8n} \frac{(\pi k T / m_A)^{1/2}}{Q_{IA}^{11}}.$$

Boundary conditions for the ion mass fraction read as

$$c_I(-\propto) = c_{I1}, \ \ c_I(\propto) = c_{I1}, \tag{2}$$

where c_{I1} is the ion mass fraction in the supersonic free stream.

After integrating Eq. (1) from X to $\propto$ with taking into account the boundary condition (2) at $X=\propto$, we arrive to the first order linear ordinary differential equation with the boundary condition at X = $-\propto$, which in the dimensionless form appears as

$$\rho^0 \frac{D_a}{\Delta u_1} \frac{dC_I}{dx} + \left(\frac{D_a}{\Delta u_1} \frac{d\rho^0}{dx} + 1 \right) C_I(x) - 1 = 0\,, \tag{3}$$

$$C_I(-\propto) = 1\,,$$

$$\Delta = \frac{n_1}{n_2}\psi(M_1)l\,,\; l = \frac{1}{\sqrt{2}n_1 Q_{AA}^{22}},\quad \psi(M_1) = 1 - \exp(1 - M_1),$$

$$I = \frac{J_I}{\rho_1 u_1 c_{I1}},\quad C_I = \frac{c_I}{c_{I1}},\quad x = \frac{X}{\Delta},\quad \rho^0 = \frac{\rho}{\rho_1}.$$

Here $M_1 > 1$ is Mach number in the gas free stream, $n_{1,2}$ is the number density of atoms before and after shock, the shock wave thickness Δ and the mean free path l are defined according to [5].

THE SOLUTION OF THE ION AMBIPOLAR DIFFUSION EQUATION

In the case under consideration the density gradient appears as the diffusion driving force, which acts on ions. Behind the shock structure the density is constant so, the problem (3) has the obvious solution

$$-\infty < x \le 0 : C_I(x) \equiv 1\,. \tag{4}$$

Let us approximate the distributions of the density and gas temperature within the shock structure (Fig. 1) by linear functions ($0 \le x \le 1$):

$$\rho^0(x) = \frac{1}{k}[1 - (1-k)x],\;\; t(x) = \frac{1}{\tau}[1 - (1-\tau)x],$$

$$k = \frac{\rho_1}{\rho_2} = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{\gamma + 1}\frac{1}{M_1^2},\quad \tau = \frac{T_1}{T_2} = \left(1 + \frac{\gamma - 1}{2}M_1^2\left(1 - k^2\right)\right)^{-1}.$$

We assume as well that the ratio of the atom-atom viscosity cross section Q_{AA}^{22} to the ion-atom charge-transfer cross section Q_{IA}^{11} is constant (does not depend on the gas temperature). Under above assumptions the exact solution of the problem (3) in the interval $0 \le x \le 1$ with the boundary condition $C_I(0) = 1$ is as follows

$$C_I(x) = 1 + \frac{1-k}{(1-\tau)[1-(1-k)x]}G(x), \tag{5}$$

$$G(x) = (1-\tau)x - 1 +$$
$$+ \exp\left\{A[1-(1-\tau)x]^{3/2}\right\}\left\{\exp(-A) + A^{-2/3}\left[\Gamma\left(\frac{5}{3}, A[1-(1-\tau)x]^{3/2}\right) - \Gamma\left(\frac{5}{3}, A\right)\right]\right\}$$

$$A = \frac{2}{3}\frac{B}{\tau^{1/2}(1-\tau)},\; B = \frac{8}{3\sqrt{2\pi}}\sqrt{\gamma}k\psi(M_1)M_1\frac{Q_{IA}^{11}}{Q_{AA}^{22}}\frac{1}{\theta_1},\;\; \theta_1 = \frac{T_e}{T_1}.$$

The maximum of the ion mass fraction C_I^* occurs at the shock front $x = 1$:

$$C_I^* = C_I(1) = 1 + \frac{1-k}{(1-\tau)k} G(1), \tag{6}$$

$$G(1) = -\tau + \exp\left(A\tau^{3/2}\right)\left\{\exp(-A) + A^{-2/3}\left[\Gamma\left(\frac{5}{3}, A\tau^{3/2}\right) - \Gamma\left(\frac{5}{3}, A\right)\right]\right\}.$$

The dimensionless ion mass diffusion flux and ambipolar electric field are expressed through $C_I(x)$ as

$$I(x) = C_I(x) - 1, \tag{7}$$

$$E_a = -\frac{kT_e}{e\Delta}\left(\frac{d\ln C_I\rho^\circ}{dx}\right) = \frac{8}{3\sqrt{\pi}}\sqrt{\gamma}\left\{\frac{[1-(1-\tau)x]}{\tau}\right\}^{1/2}\left(1 - \frac{1}{C_I(x)}\right)\frac{p_1 Q_{IA}^{11}}{e} M_1. \tag{8}$$

By the way, it is interesting that in the case of ambipolar diffusion in 3-species weakly ionized quasi-neutral plasmas there is correlation between the ion electric current j_I, the electron electric current j_e and the ambipolar electric field similar to Ohm's law

$$j_I = -j_e = \sigma_a E_a, \sigma_a = \frac{e^2 n_I}{kT_e} D_a \ , \tag{9}$$

where σ_a is ambipolar coefficient of the electrical conductivity.

The governing equation for the ambipolar diffusion of ions from the shock to upstream reads as

$$1 \le x < \infty: \ \frac{dC_I(x)}{dx} + BC_I(x) = B, \ C_I(1) = C_I^*. \tag{10}$$

The exact solution of Eq. (10) gives the precursor ion mass fraction and diffusion flux distributions in the free stream as follows

$$C_I(x) = 1 + \left(C_I^* - 1\right)\exp[-B(x-1)], \tag{11}$$

$$I(x) = \left(C_I^* - 1\right)\exp[-B(x-1)]. \tag{12}$$

The precursor ambipolar electric field equals

$$E_a(x) = \frac{8}{3\sqrt{\pi}}\sqrt{\gamma}\frac{\left(C_I^* - 1\right)\exp[-B(x-1)]}{1 + \left(C_I^* - 1\right)\exp[-B(x-1)]}\frac{Q_{IA}^{11} p_1}{e} M_1. \tag{13}$$

This solution gives us an estimation of the length of the ambipolar ion diffusion and electrostatic precursor zone L_{ap} as

$$L_{ap} \approx \frac{\Delta}{B} = \frac{3\sqrt{\pi}}{8\sqrt{\gamma}} \frac{kT_e}{p_1 Q_{IA}^{11}} \frac{1}{M_1}. \tag{14}$$

ANALYSIS OF RESULTS

Calculations have been carried out for weakly ionized argon (γ = 5/3) flow in the Mach number range M_1 = 1.5-10 at T_1 = 300 K, θ_1 = 100. Values of the cross sections Q_{IA}^{11} and Q_{AA}^{22} were taken from [6]. Fig. 2 shows the ion mass fraction within the shock structure (a) and before the shock front (b). This ion fraction distribution is found to be in good agreement with our prior numerical results for the shock wave structure in supersonic weakly ionized argon flow at M_∞ = 3 and θ_1 = 100 in terms of the maximum ion mass fraction at the shock front [3]. The maximum of the ion fraction occurs at the shock front (x = 1) and rises a value 3.5 with respect to the ion fraction in the hypersonic free stream. Quite long precursor ion diffusion is clear visible in Fig. 2,b.

It is clear from Eq. (8) that the maximum of the ambipolar electric field occurs not at the shock front, but within shock structure. Separated locations of the maxima of the ion fraction and ambipolar electric field indicate the double charged layer within the shock structure, but this double layer is not resolved through ambipolar diffusion approach. Calculation of the maximum of the ambipolar electric field at M_1 = 6 and the pressure in the free stream $p_1 = 10^3$ Pa gives the value about $8 \cdot 10^4$ V/m. For comparison, the static electric field inside of the strong shock propagating through fully ionized hydrogen plasma in [2] was estimated as $2.5 \cdot 10^6$ V/m.

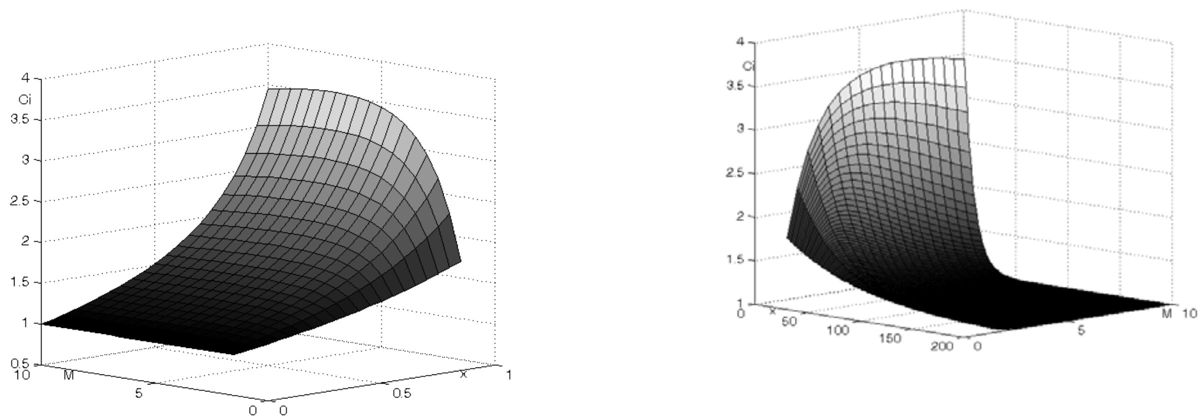

FIGURE 2. Ion mass fraction within the shock structure (a) and before the shock front (b).

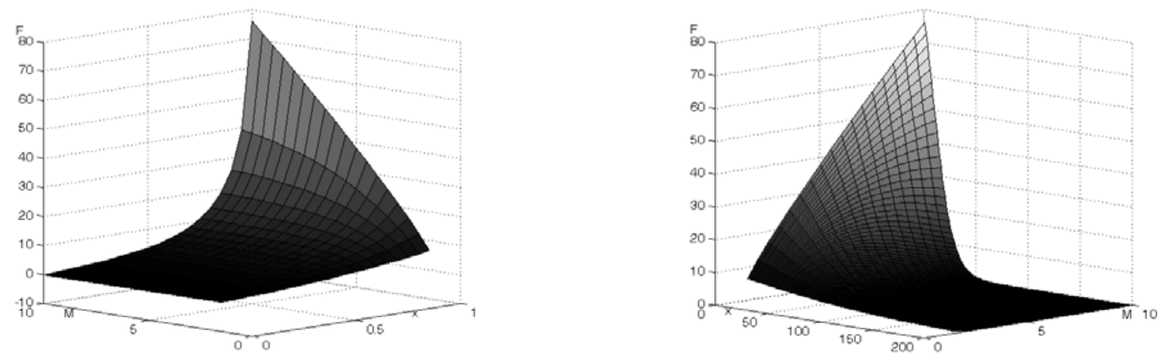

FIGURE 3. Ionization factor within the shock structure (a) and before the shock front (b).

In Fig. 3 the ionization factor $F = E_a/p$ within the shock structure (a) and before the shock front (b) is shown. This factor is proportional to Mach number M_1, reaches the maximum at the shock front and it is not very sensitive to the parameter θ_1. The ionization factor appears to be not negligible in very thin layer ~ 1/4Δ behind the shock front. At the same time the ambipolar electric field penetrates from the shock front quite far up stream depending on the parameter θ_1. The precursor length in terms of the electric field is the same as for precursor ion diffusion. At the shock front the ionization coefficient α/p can reach a value about 1 $m^{-1}Pa^{-1}$. At θ_1 = 100 the ion precursor length is longer than the shock thickness with a factor of about 50. For example, the precursor length at M_1 = 3, $p_1 = 10^2$ Pa, T_1 = 300 K, T_e = 30000 K according to Eq. (14) can be estimated as $L_{ap} \sim 6{\cdot}10^{-4}$ m. If M_1 increases, the length of the precursor zone decreases – this trend is visible in Figs. 2 and 3.

CONCLUSIONS

The inherent relationships for the ion ambipolar diffusion and electric field within the shock structure and in the precursor zone in front of the shock propagating in weakly ionized gas are established and the two basic dimensionless parameters A and B which control the ambipolar diffusion and the length of the precursor zone are defined. Taking into account the ionization factor and length of the precursor zone, it looks possible that at some sufficiently high electron temperature and low pressure in the supersonic free stream the ambipolar electric field caused by the charge separation can somewhat enhance the ionization before the shock. Because the ionization factor and the precursor length have different trends with respect to the Mach number, we expect that there is the optimal range of the moderate Mach number in terms of possible precursor ionization.

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