Time-dependent Processes in Diodes with Rarefied Plasma

Andrey Ya. Ender and Victor I. Kuznetsov

Ioffe Physico-Technical Institute of RAS, Polytekhnicheskaya St. 26, 194021 St. Petersburg, Russia

Abstract. Under study are the kinetic phenomena inherent for the time-dependent processes in 1D diodes, in which the charged particles are supplied by an emitter and move without any collision in the interelectrode spacing. Using an original numerical method (E,K-code), the ion and electron velocity distribution functions are built up and studied the structures forming during the nonlinear oscillations in the Knudsen Diode with Surface Ionization. It is shown that existence of the oscillations is related with development of the Pierce-type instability. Studied are the nonlinear structures, both the inherent time-independent as well as time-dependent solutions for the diodes with a monoenergetic electron flow moving through a background of uniformly distributed ions at different rates of the electron charge neutralization by the ions.

INTRODUCTION

Nevertheless the charged particles moving with no collisions in the diodes with rarefied plasma, there is an intensive energy exchange between them and the electric field during time-of-flight from one electrode to another. As a result, the particle velocity distribution functions (DF) turn out to be in strong non-equilibrium. It results in a complicated nonlinear dynamics of plasma. In this paper, the diodes of two types are under consideration, in which the charged particles are supplied by emitter's surface and move between the electrodes in a self-consistent field (see, Fig.1). The first type incorporates the thermionic energy converters for direct heat conversion into electric power and Q-machines using for modeling the dynamics of double layers in space plasma. Here, the ions and electrons are supplied from the emitter's surface with half-Maxwellian DFs and, further, move with no collisions. In both devices, the large amplitude oscillations of a current are observed with arise the different nonlinear structures.

In the diodes of the second type, a monoenergetic flow of electrons moves through a background of immobile ions uniformly distributed in the diode region, and the strong nonlinear oscillations arise, too. They are successfully applied as the high-power micro-wave generators [1]. For all devices mentioned above, it is inherent that, when the oscillations occur, involved are the potential distributions (PD) with a virtual cathode (VC), from which an amount of the electrons is reflected, returning to the emitter, the others overrun the potential barrier and reach the collector (Fig.1), the development of electron instability of a Pierce type being the origin of these oscillations. The results of this study of the non-linear oscillations are presented in the paper.

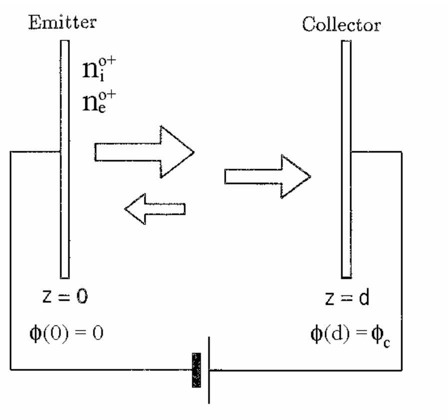

FIGURE 1. Schematic view of the electron dynamics in the diode.

When studying theoretically the time-dependent processes in the diodes with a rarefied plasma, a system of Vlasov's equations should be solved, i.e., the kinetic equations for the charged particles

$$\left[\frac{\partial}{\partial t}+v_z\frac{\partial}{\partial z}-\frac{e}{m_e}E(t,z)\frac{\partial}{\partial v_z}\right]f_e(t,z,v_z)=0,\qquad \left[\frac{\partial}{\partial t}+v_z\frac{\partial}{\partial z}+\frac{e}{m_i}E(t,z)\frac{\partial}{\partial v_z}\right]f_i(t,z,v_z)=0, \tag{1}$$

with a Poisson equation

$$\frac{\partial^2}{\partial z^2}\Phi(t,z) = 4\pi e\left[\int_{-\infty}^{+\infty} dv_z\, f_e(t,z,v_z) - \int_{-\infty}^{+\infty} dv_z\, f_i(t,z,v_z)\right], \tag{2}$$

where $E(t,z) = -\partial\Phi(t,z)/\partial z$, and the boundary conditions

$$f_{i,e}(t,z,v_z)\Big|_{\substack{v_z>0\\ z=0}} = f^0_{i,e}(t,v_z), \qquad f_{i,e}(t,z,v_z)\Big|_{\substack{v_z<0\\ z=d}} = 0 \tag{3a}$$

$$\Phi(0) = 0, \qquad \Phi(d) = \Phi_C \tag{3b}$$

In general case, a problem (1)–(3) can be solved only numerically.

NUMERICAL METHOD

To calculate the DF of the charged particles moving in the time-dependent electric field, we develop a new method [2]. In literature, it is called E,K-code. The code is of almost analytical accuracy. It involves the very fact that the DF is conserved along the trajectory of each particle in the collisionless case: $f(\tau,\zeta(\tau);u(\tau)) = f^0(\tau_0,u_0)$, where $\zeta(\tau) = \zeta(\tau;u_0,\tau_0)$, $u(\tau) = u(\tau;u_0,\tau_0)$, $\zeta(\tau_0) = 0$, $\zeta'(\tau_0) = u_0$. The electric field distribution is taken in the nodes of the spatio-temporal grid. To calculate the DF at the node, a number of trajectories of "representative" particles have to be computed. The main feature of the method is calculation of the trajectory of any particle backward in the time to the moment where it intersects the emitter surface. As a result, for a particle with the arrival velocity *u,* the injection velocity and injection time are determined, as well as the DF value for the velocity *u*. In order to guarantee the required accuracy of the DF, a step in *u* is chosen such that a difference between two DF-values for neighboring trajectories does not exceed a certain value: $| f^0(\tau^1_0,u^1_0) - f^0(\tau^2_0,u^2_0)| < \varepsilon_f$. As a result, step in *u* is taken automatically via DF gradient, and regions of its discontinuity are found with a high accuracy.

To obtain a high accuracy of calculations of the trajectories, the electric field within each cell is approximated by a linear function of coordinate and time, and particle location and its velocity are found in a form of time series, for which coefficients, the simple recurrence formulas are obtained. Such approximation gives continuity of the field when passing through the cell boundaries and a high accuracy of the trajectory parameters, the latter being especially important for the trajectories with reflections of the particles. The code is tested with a series of the time-dependent fields giving a particle motion with reflection, where we found the analytical solutions for the trajectories and particle density. The validity of calculations is managed via the conservation laws.

NON-LINEAR OSCILLATIONS IN THE KDSI

Using E, K-code, we create a theory of nonlinear oscillations in the Knudsen Diode with Surface Ionization (KDSI) where both ions and electrons are supplied by the emitter surface with the half-Maxwellian DFs ($f^0_{i,e}(t,v_z) = n^0_{i,e}\sqrt{2m_{i,e}/\pi kT}\cdot\exp(m_{i,e}v_z^2/2kT)$, here $n^0_{i,e}$ is density of ions (electrons), left the emitter, *T* is the emitter temperature) and move with no collisions in the interelectrode spacing [2]–[3]. The threshold of oscillation is determined. It is shown that the oscillations are characterized by periodic sequences of two types of stages: the slow one, related with ion motion, and the fast one, determined by a time-of-flight of the electrons through the gap, two, at minimum, fast stages being presented for each period of oscillations. During one phase of slow stage, there are the PDs with the potential barrier, VC, which reflects certain amount of the electrons and thereby limits the passing current (see, Fig.2a). Other phase of the same stage is characterized by the PDs with huge potential hill. The ions roll downhill on the electrodes producing the accelerated ion beams, the beam energy being, eventually, essentially higher than the mean ion energy at the emitter (see, e.g., Fig.2b, presenting the ion DF at the collector). As a result, a spatial localization of the ion kinetic energy occurs (Fig.2c). All cited structures occur during the fast stage being resulted from the electron Pierce-type instability [4].

These investigations lead to two important conclusions. 1) If there is a directional collisionless electron motion in the diode, with a higher velocity than the thermal one, and the Pierce threshold is exceeded, then, independently of a manner in which ion background is forming, the Pierce instability should arise in plasma. As a result, strong potential distribution rebuilding-up occurs which can essentially limit the electron current involving its total cut-off. 2) It is necessary to study the Pierce instability in more details.

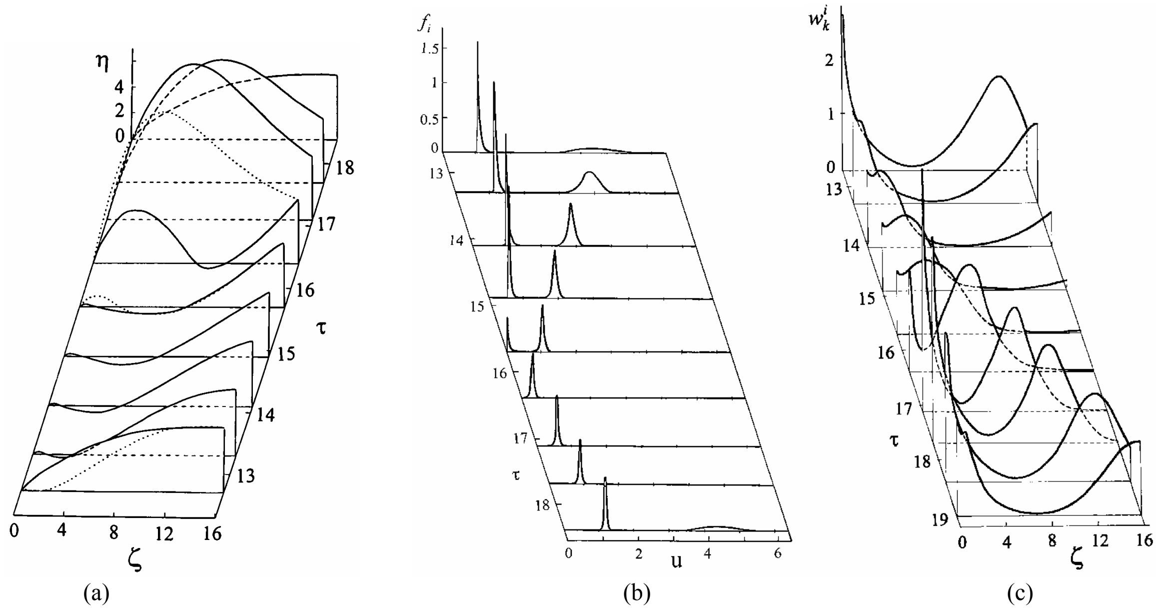

FIGURE 2. Time evolution of distributions of the potential (a), ion velocities at the collector (b), and ion kinetic energy (c) during a single oscillation in the KDSI with $\gamma = 2$, $V_C = 5$, $\delta = 16$. Here, $\gamma = n_i^0 / n_e^0$ is the emitter neutralization parameter, $V_C = e\Phi_C / kT$ and $\delta = d / \lambda_D^E$ are the dimensionless collector potential and electrode gap, respectively; $\zeta = z / \lambda_D^E$, $\tau = t\sqrt{kT} / \lambda_D^E$, and $u = v / \sqrt{kT}$ are the dimensionless coordinate, time and velocity, respectively; $\lambda_D^E = [kT / (4\pi e^2 n_e^0)]^{1/2}$ is the emitter Debye-Hukkel length.

TIME-DEPENDENT SOLUTIONS IN THE ELECTRON BEAM DIODES

In order to better appreciate the mechanism of electron instability in bounded plasma, we study the processes in a generalized Pierce diode, where its emitter supplies a monoenergetic electron flow, i.e., their DF onto emitter is $f_e^0(v_z) = n_e^0 \delta(v_z - v_0)$, and the ions are distributed uniformly through a gap and immovable. Rate of electron charge neutralization by the ions $\gamma = n_i / n_e^0$ varies from 0 to infinity. A special case $\gamma = 0$ refers to the vacuum diode, or the Bursian diode [5], and $\gamma = 1$ refers to the classic Pierce diode [4]. The Debye–Hukkel length of a beam $\lambda_D^b = (2\pi^2 e^2 m_e)^{-1/4} W_b^{3/4} j^{-1/2}$ is a length unit for the diodes with electron beam, which is determined via energy $W_b = m_e v_0^2 / 2$ and electron current density j at the emitter. The time-independent solutions are determined via three dimensionless parameters: γ, a gap width $\delta = d / \lambda_D^b$, and external voltage $V = e\Phi_C / (2W_b)$. In [6], a full classification is given and the regions of existence of the solutions of different types onto a plane (ε_0, δ), where ε_0 is the electric field strength at the emitter, are built for a number of γ values. It is shown that there are three types of such solutions: with no electron reflection, with a partial electron reflection from a VC and with a total reflection. If, additionally, an external voltage is fixed, the solutions are presented by the relevant curves on a $(\varepsilon_0, \delta)-$plane, i.e., solution branches. Solution stability in a regime with no electron reflection was studied using the dispersion equation [6]. And aperiodic stability of the solution with electron reflection was investigated by an $\eta, \varepsilon-$diagram technique [7].

More detailed time-dependent solutions are studied by us for a special case of the Bursian diode with no ions at all. Such diodes are used as the micro-wave generators of very large power, i.e., Vircators, Redutrons, and others (see, e.g., [1]). As it was shown by Bursian [5], for such a diode, a gap width threshold, higher of which there are no solutions with a total current transfer and only solutions with a VC partly reflecting the electrons towards the emitter, in this manner, limiting the transferred current, is inherent. In Figure 3 branch 1 refers to the stable solutions with no electron reflection, whereas branch 2 corresponds to aperiodic unstable solutions. The solutions with electron reflection are located on branch 3. Although they are stable relatively small aperiodic perturbations [6], a portion of them could be unstable relatively oscillatory perturbations. The nonlinear oscillations do arise on this branch, of which energy is converted in micro-wave radiation.

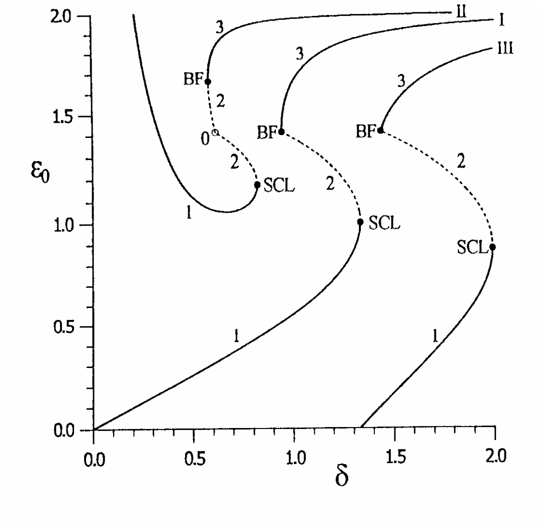

FIGURE 3. Typical branches of time-independent solutions 3. in the Bursian diode. $V = 0$ (I), $V < 0$ (II), $V > 0$ (III).

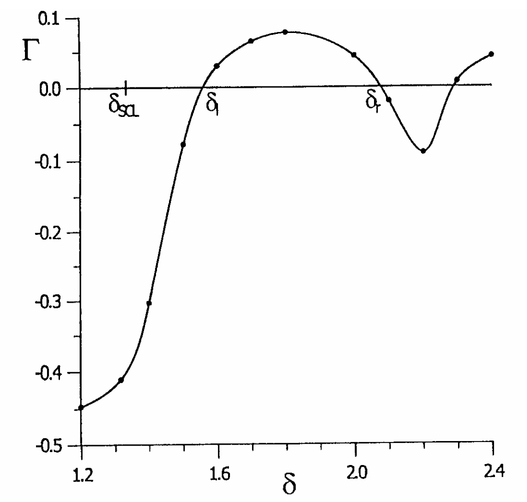

FIGURE 4. Growth rate as a function of δ for the branch V=0, $\Delta = 0.02$

In order to study the stability of solutions with electron reflection, a dispersion equation is not obtained as yet. This problem we solve numerically using E,K-code. The development of a small perturbation of time-independent solution has been under study. The following values were chosen for the computational grid parameters: N=200 spatial cells of a constant width $\Delta\zeta = \delta / N$ (the spatial step), and a time step $\Delta\tau = 0.05$. The DF of the emitter electrons has a small spread in velocity, i.e. $f_e^0(v_z) = (2\Delta)^{-1}\Theta[\Delta^2 - (1-v_z)^2]$ with $\Delta = 0.02$. High accuracy of the code gives an opportunity to determine the eigenvalues of leading mode, i.e., a growth rate Γ and a frequency Ω. Figure 4 shows that, at all gaps being smaller than a value δ_l, the solutions with electron reflection are stable relative to the small electron perturbations, simultaneously $\delta_l > \delta_{SCL}$. The obtained result do not support a contention occurred in literature that all time-independent solutions on a branch 3 are unstable.

From other side, above the threshold, namely in a region (δ_l, δ_r), the solutions in question are unstable. It is proved that the instability develops into the periodic nonlinear oscillations. Figure 5 shows an example of a limit cycle, i.e., a potential dependence at a point corresponding to the maximum electron density on the maximum location at different time moments. A filled point refers to the time-independent solution. An example of electron density distribution is shown in Fig. 6. One can see that the density varies over several orders of value, and its calculation is no so simple problem. Analyzing the initial conditions, we shown that every oscillatory solution in a region (δ_l, δ_r) is the unique solution. In the following region, which locates between $\delta = \delta_r$ and $\delta = 2.29$, oscillatory solutions exist along with time-independent ones, and there are more than one time-dependent solutions to the right of the point $\delta = 2.29$. Figure 7 shows, in what manner a VC oscillation amplitude $\Delta\eta_m = \eta_{\max}(\tau) - \eta_{\min}(\tau)$ change as a function of a gap size. Note, that, there is the classic Andronov-Hoph bifurcation at the threshold, i.e., near a point $\delta = \delta_l$, a curve $\Delta\eta_m$ is well approximated with $a(\delta - \delta_l)^{1/2}$–law at $a = 0.47$. The oscillatory solutions spread over whole region above the threshold, i.e. $\delta = \delta_l$. It is precisely these oscillations which are used in the generators of micro-wave radiation.

Under analysis are the physical phenomena relevant for the nonlinear oscillations. Particularly, on a spatial distribution of electron density, it is revealed that the pronounced jump occurs periodically to move towards collector and disappears there (Fig. 8). It results in development of a sharp front on the time dependence of the collector convection current with a wide much less than a time-of-flight of electrons through the gap (Fig. 9).

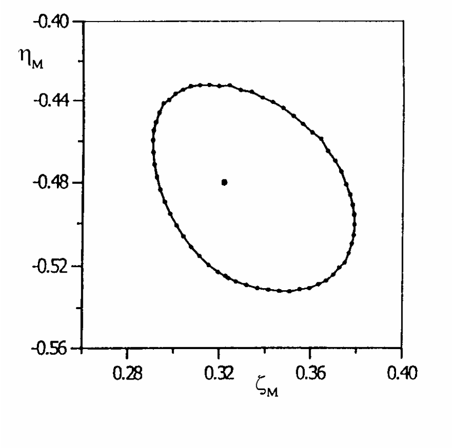

FIGURE 5. The limit cycle. $\delta = 1.7$, $V = 0$.

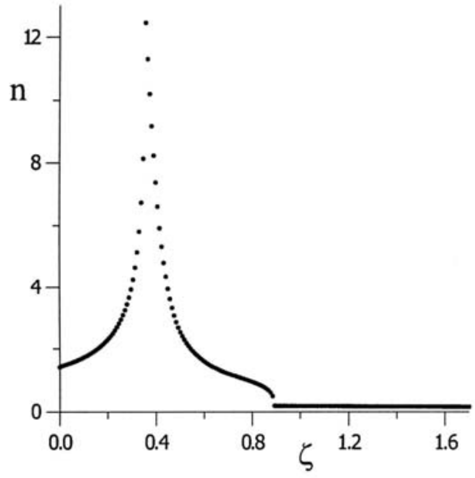

FIGURE 6. The spatial distribution of electron density.

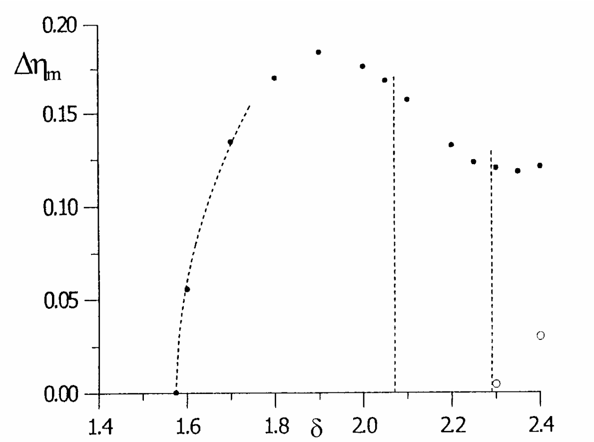

FIGURE 7. Oscillation amplitude as a function of a diode length δ . $V = 0$.

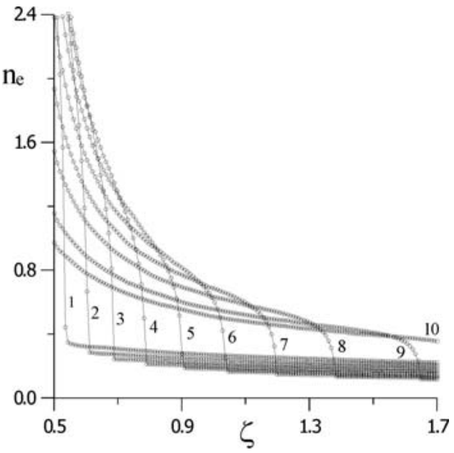

FIGURE 8. Time evolution of the electron density in the VC – collector region. $\delta = 1.7$, $V = 0$. Curves 1–10 refer to $\tau = 33.6$–35.4.

The interesting physical phenomenon, the long-living electrons [1], i.e., the particles arriving on a VC, being their potential barrier, and oscillating together during several periods of oscillations, is studied in detail. Basing on the self-consistent calculations, we advance a semi-analytical model of a potential distribution

$$\eta(\zeta,\tau) = -\frac{1}{2}\alpha^2[(\zeta_m^0)^2 - (\zeta - \zeta_m^0)^2] - \alpha^2\zeta_m^0\kappa\cos(\Omega\tau)\zeta \ , \tag{4}$$

here, $\alpha^2 = -2\eta_m^0/(\zeta_m^0)^2$, η_m^0 and ζ_m^0 are a height and location of VC under the time-independent field at $\kappa = 0$, and Ω is a field oscillation frequency, which provides the analytical expressions for the trajectories:

$$\zeta(t) = A_+ \exp\{\alpha(t-\tau_0)\} + A_- \exp\{-\alpha(t-\tau_0)\} + \zeta_m^0\left\{1 + \left[1 + (\Omega/\alpha)^2\right]^{-1}\kappa\cos(\Omega t)\right\} \tag{5a}$$

$$A_\pm = -\frac{1}{2}\left\{\zeta_m^0 \mp v_0/\alpha + \zeta_m^0\left[1 + (\Omega/\alpha)^2\right]^{-1/2}\kappa\cos[\Omega\tau_0 \pm \arctan(\Omega/\alpha)]\right\} \tag{5b}$$

It is seen from (5) that, at $\alpha(t-\tau_0)$>>1, the electrons reach a collector ($A_+ > 0$) or an emitter ($A_+ < 0$) leaving a gap. However, there is an alternative group of the particles, which can oscillate for long time near a VC. These electrons are just the long-living ones. Their velocities and time of their escape the emitter are arranged in the very narrow vicinity of the universal curve corresponding to $A_+ = 0$ in (5) (see, Fig. 10):

$$v_0 = \alpha\zeta_m^0 \left\{1+[1+(\Omega/\alpha)^2]^{-1/2}\kappa\cos[\Omega\tau_0 + \arctan(\Omega/\alpha)]\right\}, \tag{6}$$

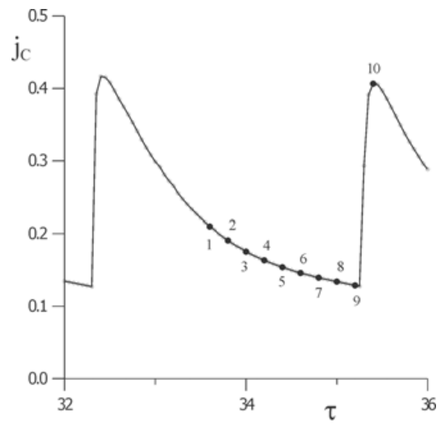

FIGURE 9. Collector current vs. time. $\delta = 1.7$, $V = 0$. Numeration of points is as in Fig. 8.

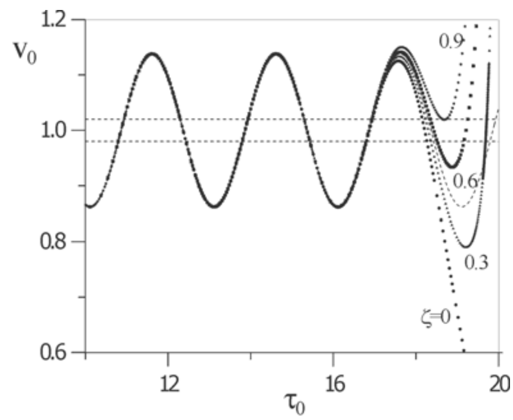

FIGURE 10. Dependence $v_0(\tau_0)$ for various points ζ in the gap. $\zeta = 0$ (emitter), $\zeta = 1$ (collector). Solid curve refers to (6).

Earlier, this property was revealed in a course of numerical calculations of the self-consistent processes [9]. Thus, the long-living electrons arise always during the VC oscillations. A number of new properties of the long-living electrons are revealed. Their DF is found, and its amount is estimated.

CONCLUSION

Thus, it is advanced an original numerical code, which gives an opportunity to build-up, with almost analytical accuracy, the velocity distribution function of the charge particles, moving in the time-dependent electric field. Use of this code provided to create a theory of the non-linear oscillations in the KDSI. For the first time, is proved the occurrence of the stable solutions in a regime with electron reflection, as well as is studied the time-dependent solutions and their features for the electron beam diodes.

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