Kinetic Theory in Vehicular Traffic Flow Modeling

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Abstract. This paper discusses (gas) kinetic models in vehicular traffic flow theory. In analogy to gas dynamics, models in literature are based on Boltzmann like equations with different mean acceleration features and interaction profiles of the drivers. After a discussion of their underlying assumptions together with their applicability in traffic flow an advanced model is introduced. It avoids their drawbacks by adding the acceleration as an additional variable to single car state and can be used also in dense traffic. The interaction profile of this model is based on the microscopic action-point approach using two velocity dependent distance thresholds. The equation is solved with an adopted DSMC like method and results for homogeneous and stationary traffic flow are presented. They are compared with measured data and agree qualitatively.

Keywords: Vehicular traffic flow, acceleration oriented kinetic modeling, DSMC like simulations
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INTRODUCTION

In vehicular traffic flow measurements the most observed quantities like traffic density, car density or mean velocity, together with their dependencies show typical behavior independent of place or time they are observed [1, 2]. This gives rise to the need of a theoretical understanding, resulting in a huge number of different traffic flow models (some surveys are [3, 4, 5]). Today these models are divided roughly into three categories depending on their focus. So microscopic models are based on the description of single car behavior like in molecular dynamics or cellular automata. They are solved via computer simulation methods. Macroscopic flow quantities are calculated from mean values of the single car quantities recreating the behavior of true measurement techniques. Where simulation of traffic dynamics is straight forward, starting from an initial condition following the cars in space and time, the calculation of traffic states needs a homogeneous road and the simulation is performed until stationarity occurs approximately. Then macroscopic mean values are independent of space and time coordinates. Problems can rise due to the explicit unknown distance correlation in the initial condition and that the reached traffic state is independent of the condition.

Macroscopic modeling introduces equations for the measured traffic flow quantities in time and space. For the time development of the car density on a road segment a partial differential equation can be constructed by the car number conservation law analog to mass conservation in continuum mechanics. Higher order equations for the mean velocity or scattering quantities can not be established in the same way, because in contrast to physical continuum mechanics there are no other conservation laws like momentum or energy conservation in a traffic flow. Therefore a wide spread of approaches are developed and discussed in literature. The idea of a given functional dependence between mean velocity and car density (often together with its gradients) results in a one equation kinematic model also used in water surface wave propagations. In contrast to this approach dynamic models use an additional partial differential equation for the mean velocity, mainly developed by heuristic arguments. The mean acceleration is constructed by a relaxation into stochastic equilibrium or traffic state, interactions are treated by space gradients of car density and mean velocity itself. Nearly all those model equations are hyperbolic of first order to ensure positive velocities of the cars. Some authors construct gas kinetic models mainly to calculate macroscopic equations using Grads method of moments or Chapman expansion method. Note that in analogy to continuum mechanics from macroscopic models traffic dynamic effects are calculated only, because they need traffic state informations as input.

In the third category the wide range of stochastic models is pooled, i.e. queuing and renewal approaches, aggregation models based on master equations and gas kinetic models. Especially the flexibility of the gas kinetic approach in modeling traffic states in analogy to classical thermodynamics and also traffic dynamics in a consistent manner makes them very interesting for traffic flow analysis.

The next section presents literature results of gas kinetic models with main focus on traffic state analysis. All those

models follow the ideas of the Boltzmann equation for the quantity $f(x,v,t)$, which in traffic flow is interpreted as the single car state probability density. As will be shown main differences lie in the interaction profile between cars.

One of the major drawbacks of gas kinetic models is their limited applicability in the high car density region. In the last section of this paper a solution of this problem extending the probability density by acceleration is outlined. An extended Boltzmann equation is presented, acceleration oriented interaction profiles are discussed and some results together with comparison to measured data in traffic state are shown.

GAS KINETIC MODELS IN TRAFFIC FLOW THEORY

The gas kinetic approach in traffic flow, first introduced in [6], orients itself on the Boltzmann equation, with cars treated like extended gas particles and a modified interaction profile. The quantity of interest $f(x,v,t)dxdv$ is the probability to find a car between place x and $x+dx$ with velocity between v and $v+dv$ at time t. From f all macroscopic quantities of interest are calculated via moments. Under the assumptions

- a car with velocity v at place x interacts only with its leading partner having velocity $\bar{v}$ at distance $h \geq h_{\min}$ (two car interaction assumption),
- the leading car is not active involved into the interaction (so its kinematic state is not changed),
- during the interaction, the leading car does not interact with another one, this means the durance of an interaction is small against the time between two interactions of a car,
- the interaction is behavior oriented and not place x or time t dependent

an equation can be constructed based on the same arguments found by heuristic derivation of the Boltzmann equation, i.e.

$$\begin{aligned} \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} \quad &= \left(\frac{\delta f}{\delta t}\right)_{\text{acc}} + \\ &N\int_{v',\bar{v},h\geq h_{\min}} \left(\varsigma(v|\bar{v},v',h)Q(\bar{v},v',h)f(v',x,t) - \varsigma(v'|\bar{v},v,h)Q(\bar{v},v,h)f(v,x,t)\right) f(\bar{v},x+h,t)\,dv\,d\bar{v}\,dh\,. \end{aligned} \tag{1}$$

The total change of f in time is given by an acceleration of the car without interaction (first term on r.s.) or by interaction with a leading car (second term on r.s.). The interaction integral is similar to the Boltzmann collision integral with the difference that the density of the leading car $f(x+h,\bar{v},t)$ is extracted from the brackets, because it is not changed due to interaction. N is the car number on road segment of length L and $h_{\min}$ the mean length of a car or the front bumper distance of the two cars at rest, resp.. ς is the transition probability density of a change to the new velocity v. The interaction rate Q has always the structure

$$Q(\bar{v},v',h) = (1-P_o(K))\chi(h)(v'-\bar{v})\Theta(v'-\bar{v})\,. \tag{2}$$

The relative velocity $(v'-\bar{v})$ comes from the same arguments as mentioned by explaining the Boltzmann equation, the Heaviside Θ function ensures that only following cars with velocities $v' > \bar{v}$ interact, the χ-function considers the non vanishing distance during the interaction analog to the Enskog function for dense gases and P_o is the overtaking probability reducing the interaction rate. P_o increases with increasing car density K reaching a maximum at moderate densities and decreases further on. In the simplest case it is modeled by a triangle-function.

Regarding the acceleration term, there are two approaches presented in literature. In the first approach an explicit term is defined, where in the second it is incorporated into the interaction integral. Typical examples for the explicit acceleration term most common are

$$\left(\frac{\delta f}{\delta t}\right)_{\text{acc}} = \begin{cases} -\dfrac{f-f_w}{\tau} & \text{(Prigogine 1960, [6])} \\ \dfrac{\partial}{\partial v}\left(\dfrac{W(v)-v}{\tau}\cdot f\right) & \text{(Paveri-Fontana 1975, [7])} \\ \dfrac{\partial}{\partial v}\left(\dfrac{W(v)-v}{\tau}\cdot f\right) + \dfrac{\partial^2}{\partial v^2}\left(\mathscr{D}\cdot f\right) & \text{(Helbing 1995, [8]).} \end{cases} \tag{3}$$

The relaxation approach first introduced by Prigogine following the ideas of BGK-equation, does not include acceleration on a single vehicle level, but moves the whole density towards the desired velocity density f_w, which is an input

quantity. For the most simple interaction profile Prigogine neglects the car length $h_{\min} = 0$ and allows only interactions at bumper distance[1] $\chi(h) = \delta(h)/L$, where the velocity of the following car is reduced to this of the leading one $\varsigma(v|\bar{v}) = \delta(v - \bar{v})$. The interaction integral in eq. 1 then can be rewritten using eq. 2 in

$$(1 - P_o(K))K(x,t)(V(x,t) - v)f(v,x,t) \tag{4}$$

with car density $K = N/L$ and mean velocity V. In traffic flow theory traffic state analysis is most important, because from state diagrams quantities can be extracted for planning reasons. A state is a homogeneous stationary flow independent on initial conditions, often called stochastic equilibrium. In this case the state probability density f_{eq} only depends on v and the l.s. of eq. 1 vanishes. In this case only for the Prigogine model the exact solution is known to be

$$f_{\text{eq}}(v) = \frac{f_w(v)}{1 - \tau(1 - P_o)K(V - v)} + c\,\delta\left(1 - \tau(1 - P_o)K(V - v)\right) \tag{5}$$

with some constant c and therefore a lot of analysis was done based on this approach [9, 10]. The equilibrium state density eq. 5 is bimodal in principle. Compared to measurements it very often agrees insufficiently [11, 12]. So several improvements are discussed in literature. The most common way to enhance the model was introduced by Paveri-Fontana [7] changing the relaxation term into a mean acceleration term of relaxation type for each vehicle as shown in eq. 3. There $W(v)$ is the mean desired velocity of a car driving with velocity v and must be included as input. This approach contains correlations between actual and desired velocities of cars to some extend. The numerical analysis of eq. 1 together with the Paveri-Fontana acceleration and the interaction assumptions leading to eq. 4 for the traffic state shows always mono modal, Gaussian shaped densities [13]. Using the method of moments in this case and under the assumption of a Gaussian equilibrium velocity density an approximate expression for the important fundamental diagram of traffic flow $q(K) = K \cdot V(K)$ can be calculated depending on the mean and the variance of the desired velocity [3]. To include driver individuality and acceleration scattering effects Helbing extends the Paveri-Fontana approach to a velocity scattering using an empirical diffusivity $\mathscr{D}$ depending on car density K [8, 3]. He used this modified equation mainly to calculate macroscopic dynamic equations for mean velocity and velocity scattering with an applied moment method. To extend this model to higher car densities he also introduces a modified Enskog factor χ into eq. 2.

Up to now, there is no rigorous analysis on the influence of the transition density ς on traffic dynamic and state quantities done in literature. There are only a few publications, where the simple Prigogine approach is changed, allowing now transitions to lower velocities than this of the leading car. The common ones

$$\varsigma(v|\bar{v}) = \begin{cases} \dfrac{1}{(1-\beta)\bar{v}}\, e^{-\dfrac{\bar{v}-v}{(1-\beta)\bar{v}}} & \text{(Helbing 1996), [14]} \\ \dfrac{1}{(1-\beta)\bar{v}}\Theta(v - \beta\bar{v}))\Theta(\bar{v} - v) & \text{(Klar 1996, [15]; Wagner 1998, [16])} \end{cases} \tag{6}$$

are not analyzed with all their aspects in detail. The first one shows an exponential decay to smaller velocities, where the second is an equal density up to some value $\beta\bar{v} < \bar{v}$ with the parameter $0 \leq \beta < 1$. The limit case $\beta = 1$ results for both in the Prigogine model.

In literature the Paveri-Fontana model is enhanced to multiple lanes by introducing a lane index and transition rates between them [17]. Additionally significant different driver/vehicle behavior is added using classes and introducing an index to the state function f and the interaction profile Q, ς [18]. Both features extend the model to a system of gas kinetic equations.

Gas kinetic models of the second category have no explicit acceleration term. Instead Nelson includes it into the interaction profile by introducing a "ghost" car, which shows the desired behavior of the following one [19]. Such a car pair produce an interaction resulting in the desired behavior of the following car also. This model is the base of the distance threshold interaction model presented in [15, 20]. It uses the measured experience that drivers interact at given velocity dependent distances called action points or thresholds $H_i(v)$. These thresholds are incorporated into the interaction profile via $\chi(h) = \delta(h - H_1(v))/L$, restricting to one threshold for simplicity. For braking ($h < H_1$), the authors applied the second transition density in eq. 6. For acceleration ($h > H_1$) analog to braking an equal distributed

[1] $\delta(x)$ is the Dirac density or point measure.

new velocity larger than the actual one and lower than the desired velocity W is used, $\bar{v} \leq v \leq W$. This is the first gas kinetic model, where the interaction profile orients itself on microscopic car following measurements. The authors calculated velocity distributions in stochastic equilibrium (i.e. a state) for different car densities showing that they are also mono modal (but with some skewness) and developed macroscopic dynamic flow equations via the method of moments [21, 22, 23]. The next section presents an actual acceleration oriented model based on the same interaction principles.

AN ENHANCED ACCELERATION ORIENTED KINETIC MODEL

In gas kinetic models interactions lead to jumpy velocity changes of the following car. This approximation is only valid for thin to medium car densities as long as interactions of the leading car during a velocity change of the following one are negligible. If it is not the case, a decision change of the driver of the following car during the interaction occurs, which in principle can not be modeled by Boltzmann or Enskog like equations. A possible solution is looking for the kinematic process variable with the shortest timescale of change and approximating this by a jump process, creating again a modified Boltzmann or Enskog like equation by adding this variable to the state. Car following measurements, i.e. [24], show that a good candidate for this quantity is the acceleration[2]. As an additional gain of such an approach acceleration, as the flow control variable of the driver (by operating the gas or braking pedal), can be measured in behavior experiments and can directly be used for modeling interaction profiles in analogy to scattering experiments for the scattering cross section in physical transport theory.

In the following the acceleration oriented approach is outlined. Details can be found in [25, 26]. The assumptions presented at the beginning of the last section remain valid in an adapted form. In a car pair at time t the following car is in kinematic state (x,v,a) with acceleration a and the leading car is in state $(\bar{x},\bar{v},\bar{a})$ with acceleration $\bar{a}$, building up a car pair state vector $\mathbf{y} = (x,v,a,\bar{x},\bar{v},\bar{a})$. Places and velocities of both cars change continously, where the acceleration change is approximated by stochastic jumps. Assuming that the driving process only depends on the present and not on past states, $\mathbf{y}$ performs a random Markov process with state probability density $f_2(\mathbf{y},t)$. It can be calculated from Feller-Kolmogorov equation of jump processes. Inserting the assumptions, this equation reads

$$\begin{aligned} \frac{\partial}{\partial t} f_2(\mathbf{y},t) \quad = \quad & \underbrace{-v\frac{\partial}{\partial x} f_2(\mathbf{y},t) - a\frac{\partial}{\partial v} f_2(\mathbf{y},t) - \bar{v}\frac{\partial}{\partial \bar{x}} f_2(\mathbf{y},t) - \bar{a}\frac{\partial}{\partial \bar{v}} f_2(\mathbf{y},t)}_{\text{continuous part}} \\ & \underbrace{+ \int_{a'} \left(\varsigma(a|\mathbf{y}',t) Q(\mathbf{y}',t) f_2(\mathbf{y}',t) - \varsigma(a'|\mathbf{y},t) Q(\mathbf{y},t) f_2(\mathbf{y},t) \right) da'}_{\text{jumping part}}, \end{aligned} \tag{7}$$

where the first part on the r.s. describes the continuous change of f_2 due to space and velocity variables for both cars and the second part the change due to accelerations. Note that it is assumed that during an acceleration change of the following car there is no change of the leading one, which is acceptable due to the short endurance of an acceleration change even at high car densities. Q is the interaction rate, ς the interaction strength or transition probability density of an acceleration change and $\mathbf{y}' = (x,v,a',\bar{x},\bar{v},\bar{a})$. To get a single car state density $f(x,v,a,t)$ eq. 7 must be integrated over the state variables of the leading car. On the l.s. and in the continuous part on the r.s. f_2 changes to f, where the derivatives in $\bar{x}$ and $\bar{v}$ vanish. Analog to the Boltzmann equation the f_2 in the jumping part have to be transformed into some functional of f. This is done by the vehicular chaos theorem from [19]. f_2 is written as a product of conditioned probability densities

$$f_2(x,v,a,x+h,\bar{v},\bar{a},t) = l_1(\bar{v},\bar{a}|x,v,a,h,t) \cdot D(h|x,v,a,t) \cdot f(x,v,a,t)\,, \tag{8}$$

where $h = \bar{x} - x$, D is the conditioned distance density and l_1 some unknown function. Note that the transition from $\bar{x}$ to distance h only changes the integration limits in the $\bar{x}$-integration of the jumping part of eq. 7 to $h_{\min} \leq h < \infty$, when calculating f. The function l_1 is a density of the leading car variables, which do not depend on those of the following

[2] Note that such modified model can not be called "gas kinetic" any longer. Therefore the author calls it "kinetic" only.

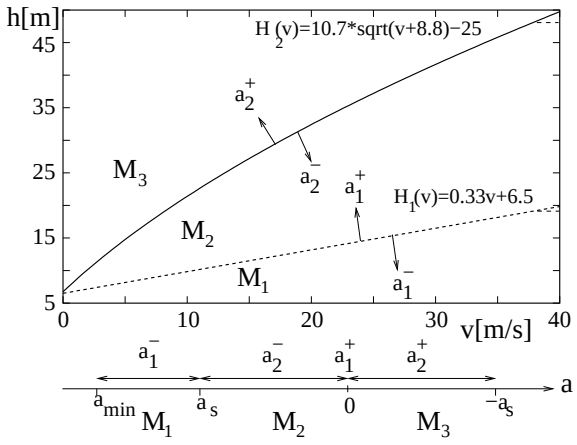

FIGURE 1. Sketch of the interaction profile. The plotted quantities are explained in the text.

one, i.e. $l_1 = l_1(\bar{v},\bar{a}|x,h,t)$. The simplest approach is to identify l_1 by the space conditioned state density of f

$$l_1(\bar{v},\bar{a}|x,h,t) = \frac{f(x+h,\bar{v},\bar{a},t)}{\mathscr{F}(x+h,t)} \quad \text{with} \quad \mathscr{F}(x+h,t) = \int_{\bar{v},\bar{a}} f(x+h,\bar{v},\bar{a},t)\,d\bar{v}d\bar{a}\,, \tag{9}$$

bearing in mind that the leading car is a following one in another car pair, therefore taking its state from f too. The resulting Boltzmann like equation for f then reads

$$\begin{aligned}\frac{\partial}{\partial t}f(x,v,a,t) + v\frac{\partial}{\partial x}f(x,v,a,t) + a\frac{\partial}{\partial v}f(x,v,a,t) = \\ \int_{\substack{a',\bar{v},\bar{a},\\ h\geq h_{\min}}} \{\varsigma(a|\mathbf{y}')Q(\mathbf{y}')D(h|x,v,a',t)f(x,v,a',t) - \varsigma(a'|\mathbf{y})Q(\mathbf{y})D(h|x,v,a,t)f(x,v,a,t)\} \\ \cdot\ \frac{f(x+h,\bar{v},\bar{a},t)}{\mathscr{F}(x+h,t)}da'\,d\bar{x}\,d\bar{v}\,d\bar{a}\,,\end{aligned} \tag{10}$$

where ς, Q and D must be specified by interaction profile and distance correleation function. The car density of N cars on a road segment of length L is $K(x,t) = N\cdot\int_{v,a} f(x,v,a,t)dvda$, where the mean distance h_{mean} of the cars used for D (see below) is in implicit form given by

$$\int_x^{x+h_{\text{mean}}(x,t)} K(x',t)\,dx' = 1\,. \tag{11}$$

Note that in a homogeneous stationary flow the car density is $K = N/L$ and from eq. 11, $h_{\text{mean}} = 1/K$.

The interaction profile is based on two velocity dependent distance thresholds

$$H_1(v) = \alpha_1 v + h_{\min}\,, \qquad H_2(v) = \alpha_2\sqrt{v+\beta_2} + \gamma_2 \tag{12}$$

taken by a fit procedure from measured data [27] (see fig. 1, details in [28]). Because interactions only occur on the thresholds, the interaction rate Q is[3]

$$Q(v,a',\bar{v},h) = \sum_{i=1}^{2} |v-\bar{v}+H_i'(v)a'|\delta\left(h-H_i(v)\right), \quad H_i'(v) = \frac{dH_i}{dv}\,, \tag{13}$$

following the ideas in [15, 26]. It includes the typical relative velocity dependence corrected by an acceleration term. Depending on the sign of $S_i = v-\bar{v}+H_i'(v)a'$ an acceleration a_i^+ or de-acceleration a_i^- takes place. So assuming a

[3] Here no overtaking is included. This can be done analog to gas kinetic models, eq. 2, by multiplying Q with $(1-P_o(K))$.

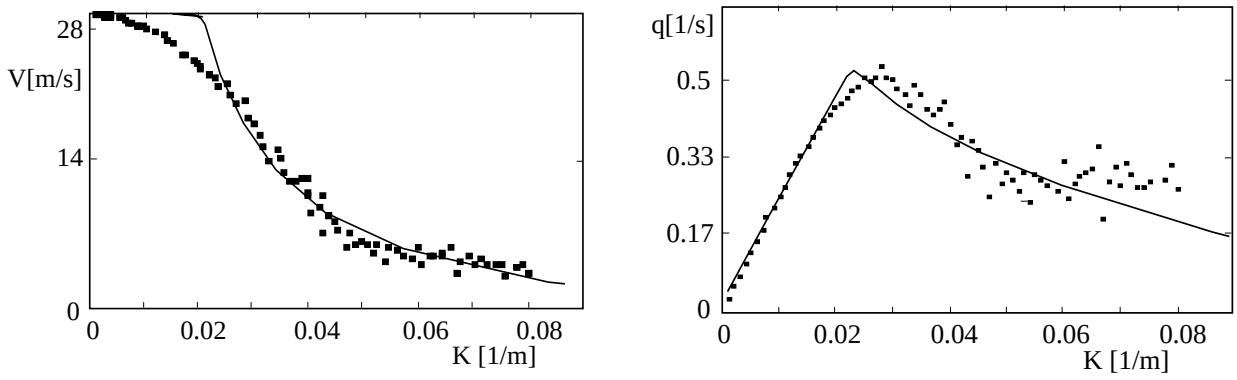

FIGURE 2. Mean velocity V and traffic density $q = V \cdot K$ as functions of car density K. Measured data are taken from [3].

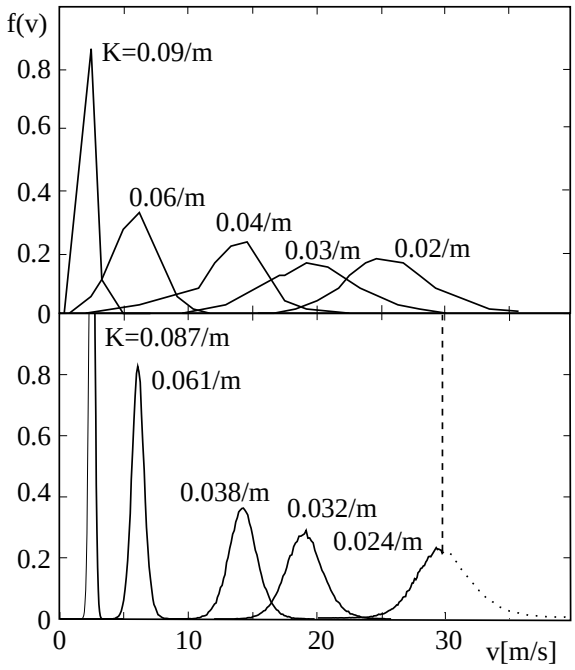

FIGURE 3. Velocity distributions for different values of car density K. Upper part measured data from [3], lower part model results.

simple acceleration approach without scattering, the interaction strength ς is

$$\varsigma(a|v,a',h=H_i(v),\bar{v}) = \begin{cases} \delta(a-a_i^+(v,\bar{v},h=H_i(v))), & S_i<0 \quad \text{(acceleration)} \\ \delta(a-a_i^-(v,\bar{v},h=H_i(v))), & S_i\geq 0 \quad \text{(de-acceleration)} \end{cases} \quad i=1,2. \tag{14}$$

The functions $a_i^{\pm}$ are defined on the thresholds H_i, depending only on v and $\bar{v}$, with parameters $a_{\min}$, a_s and ε

$$\begin{aligned} a_1^-(v,h=H_1(v),\bar{v}) &= \min\left\{a_s;\max\left\{a_{\min};\frac{-v^2}{2(H_1(v)-h_{\min})}\right\}\right\} \\ a_1^+(v,h=H_1(v),\bar{v}) &= 0 \\ a_2^-(v,h=H_2(v),\bar{v}) &= \max\left\{\ a_s;\varepsilon\frac{\bar{v}-v}{H_2(v)}\right\}, \quad \bar{v}<v \end{aligned} \tag{15}$$

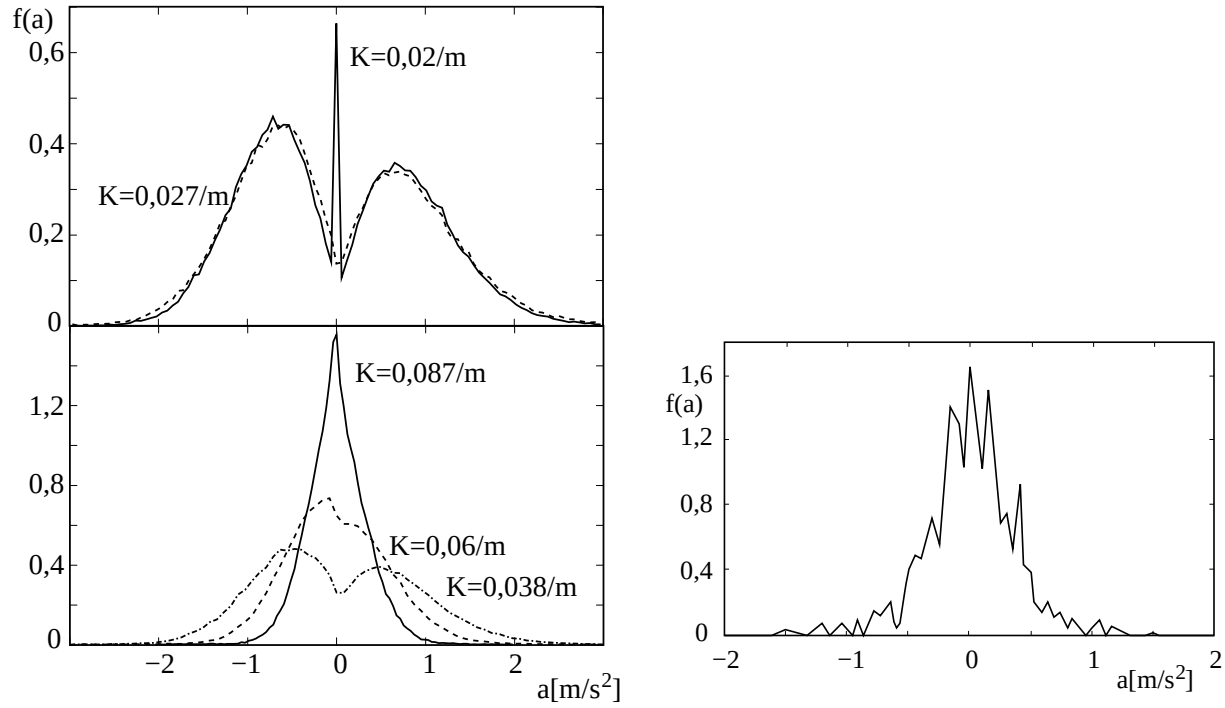

FIGURE 4. Acceleration distribution for different values of car density K (left) and measured data from [27] (right).

$$
\begin{aligned}
& = U_{[a_s,0)}, && \bar{v} \geq v \\
a_2^+(v, h = H_2(v), \bar{v}) & = \min\left\{-a_s; \varepsilon \frac{\bar{v} - v}{H_2(v)}\right\}, && \bar{v} > v \\
& = U_{(0,-a_s]}, && \bar{v} \leq v\,.
\end{aligned}
$$

On Threshold H_2 a standard car following law proportional to the relative velocity $\bar{v} - v$ and inverse proportional to the distance H_2 is applied [29]. Decision on acceleration or de-acceleration depends on the sign of $\bar{v} - v$. The case $\bar{v} \geq v$ together with $S_2 \geq 0$ – which means a crossing of H_2 to lower distances – only happens, if the following car has a too high acceleration value. It is then changed to a small equal distributed $U_{[a_s,0)}$ de-acceleration value. The same procedure is used, when the following car accelerates, even though $\bar{v} \leq v$ holds. On threshold H_1 the de-acceleration function avoids accidents. There is only zero acceleration, which means that the driver of the following car waits accelerating until the distance H_2 is reached. In the case that a car obtained zero or desired velocity acceleration is changed to zero too, independent on the state of the leading car. This behavior is included via a velocity boundary condition (note that eq. 7 is differential in v) analog to Bose gas condensation in physics [25].

Because of the threshold character of the interaction (eq. 13), distance densities D must only be known at $h = H_i$. The parameters in the eqs. 15 are chosen in such a way, that to each acceleration value the distance areas M_1, M_2 or M_3 can be identified one-to-one (fig. 1). So D reduces to the unconditioned density $\tilde{D}$

$$
\begin{aligned}
D(H_1|v, a' \in M_1, \mathbf{m}_f(x,t)) = \quad & D(h \to H_1 | h \leq H_1, \mathbf{m}_f(x,t)) && = \frac{\tilde{D}(H_1|\mathbf{m}_f(x,t))}{\int_{h_{\min}}^{H_1} \tilde{D}(h|\mathbf{m}_f) dh}\,, \\
D(H_2|v, a' \in M_3, \mathbf{m}_f(x,t)) = \quad & D(h \to H_2 | h \geq H_2, \mathbf{m}_f(x,t)) && = \frac{\tilde{D}(H_2|\mathbf{m}_f(x,t))}{\int_{H_2}^{\infty} \tilde{D}(h|\mathbf{m}_f) dh}\,, \\
D(H_i|v, a' \in M_2, \mathbf{m}_f(x,t)) = \quad & D(h \to H_i | H_1 \leq h \leq H_2, \mathbf{m}_f(x,t)) && = \frac{\tilde{D}(H_i|\mathbf{m}_f(x,t))}{\int_{H_1}^{H_2} \tilde{D}(h|\mathbf{m}_f) dh}\,, \quad i = 1,2\,.
\end{aligned}
\tag{16}
$$

$\tilde{D}$ only depends on macroscopic moments of f summarized in the vector quantity $\mathbf{m}_f(x,t)$. From measurements $\tilde{D}$ is often assumed to be a gamma density with mean h_{mean} and variance chosen here proportional to the mean velocity V [26]. The integrals in eqs. 16 then are done analytically resulting in explicit but lengthy formulas for D. Eq. 10, together with the specified interaction profile is too complex for analytical treatment. Therefore a DSMC-like computer simulation method of Nanbu/Babovsky type, together with the parameters specified in [28] is applied. Note that all parameters of the interaction profile are taken from microscopic car following measurements of different sources.

In the following traffic results for a homogeneous stationary flow (state) are presented. Fig 2 shows the car density K dependence of the mean velocity V and the traffic density q together with measured data from a Danish highway with speed limit. There is a surprisingly good agreement at higher K values. In the free flow at low K values the differences in the $V(K)$ diagram are due to the boundary conditions. Where in the model all cars have the same desired velocity, in reality a mixture with broad scattering occurs. This can also be seen in fig. 3, where measured free flow velocity distributions (small K) differ strongly from the simulated ones in their scattering. Velocity scattering is a superposition of the scattering due to different car classes not included in the model up to now and scattering due to interactions. The second part starts to be relevant at medium car densities, where the first one vanishes at high car densities. There the curves in both figures match better. Fig. 4 shows acceleration distributions for different K values. There appears a transition from a single peak over a double peak back to a single peak distribution with increasing car densities. A direct comparison to the distribution taken from a car following measurement (r.s. of the figure) is not possible, because it is not car density resolved. But the triple peak at the maximum and the small peak at each flank can be interpreted as a superposition of distributions of the simulated type for different K.

The results show that the model can be applied to traffic flow, esp. at higher car densities. Never the less a lot of features like a more realistic free flow concept, mixtures of car/driver classes with different behavior and multi lane extension must be included.

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